Digital Communication
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Module 01
Lecture 21
Passband Modulations for Bandlimited Channels

In our discussion on digital modulation techniques if you may recollect we have been talking about passband digital modulations and so far we have talked about the digital modulations or passband digital modulations for channels where bandwidth is not a constraint right. Will now take up digital passband modulations for bandlimited channels ok.

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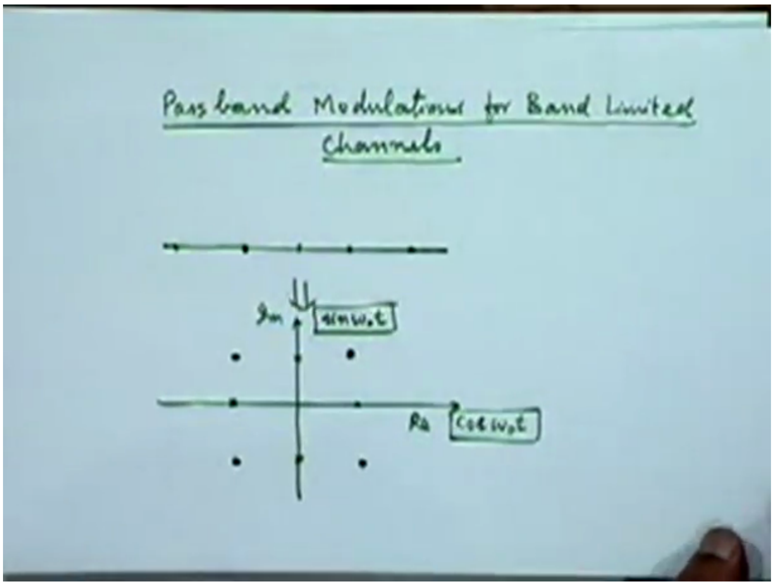

So passband modulations for bandlimited channels and the best point to start this discussion would be to recollect our (passband) and baseband modulations for bandlimited channels. What was the essential feature of base fare modulations for bandlimited channels.

If you may recollect the essential feature is that more or less we constrain to use either a single pulse shape which lies with the whose bandwidth lies within the channel bandwidth right or at best we can have multiple pulse shapes all satisfying the bandwidth constraint but if you may recollect we suggested for baseband bandlimited channels distant modulations essentially some kind of a P M kind of modulations right. We suggested constellation diagrams which were points on the real line right.

That is pulse a particular shape which satisfies the require bandwidth constraint a criteria maybe the Nyquist criteria is used with multiple amplitudes right depending on the number of levels you want depending on whether you want four levels or eight levels or two level, two level offcourse will become binary but we are now considering in general M-ary situation right. So this is what we did in baseband. We want to generalize this to fast fare now and if you recollect essentially we can have relationship in baseband and passband through the carrier. So the introduction of a carrier right and further generality we can introduce is that we can have two carriers in quadrature instead of one carrier right.

Infact we have discussed this point earlier. So I could have therefore this kind of a constellation independently available on two different carriers which are in quadrature right an I could combine them into a complex plane rather than on a real line. So essentially the difference between M-ary baseband modulations and M-ary passband modulations will be but I could define now such constellations on a complex plane with here an imaginary axis rather than essentially one real axis. So the generalization would be therefore to go from here to here ok. So example if I have a binary modulation on each of the two carriers two courage a carriers right.

I will get a four point constellation which now looks like this. If I have a binary modulation on cosine omega knot T and another binary modulation on sin omega knot T right. Maybe one this modulation will be governed by the first or even data bits and this may be governed by the odd data bits, the odd data bits maybe modulating this carrier and the even database maybe modulating this carrier right or we are taking a block of two bits and one of them is modulating this and other is modulating this I mean this is a very two we have looking at it but this is one we have looking at it alright.

So if one inner bit is modulating this and the other which is modulating this simultaneously and both of these modulations are taking place together then effectively when you sum of these two channels together will get a point which is a resultant of this point at lets say if both are one right the resultant of these two which will be this point. If we are getting a 0 and a 1 the resultant will be this point. If we have on this channel this point and on this channel this point then the resultant will be this point right and so on.

So depending on the specific combination of 1's and 0's that we have modulating the two quadrature carriers will get one of these four resultant vectors of points, we can represent it by one of these four points, is it understood? Very clear, so essentially we can therefore say that passband modulations bandlimited passband M-ary passband modulations can be represented by a constellation diagram on it complex plane by specifying if set of points on a complex (point) plane where the real and imaginary components of this can be thought of as quadrature carriers right.

Infact to start with to keep our discussion very general will not even being in the concept of this cosine omega knot T and sine omega knot T here all will say that these are four point in a complex plain with some real coordinates and some imaginary coordinate. Coordinates this is the real axis and this is the imaginary axis fine.

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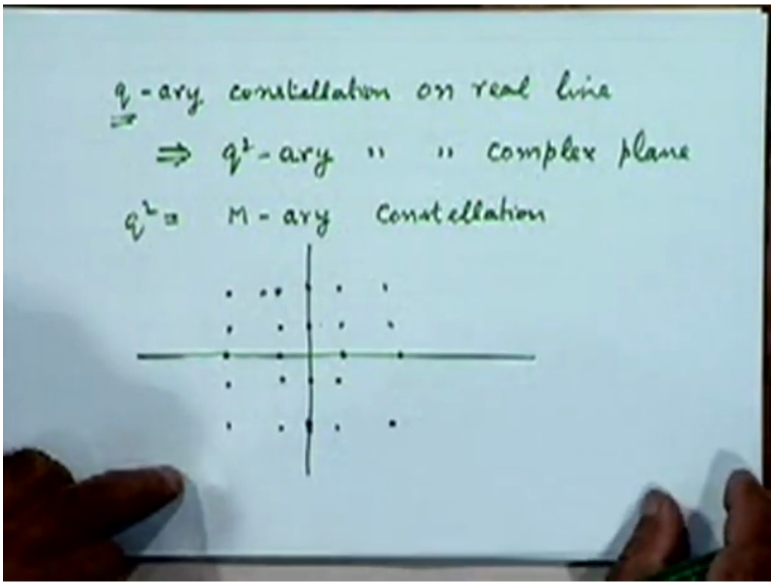

So in general for a Q-ary modulation that we had on the real line right. Suppose I had a Q-level modulation on the real line along I cannot define two Q-level modulations one along each of this quadrature axis and therefore I will get a overall constellation which will have how many points? Q square right, Q along one axis and Q along the other axis the product of these two will give you the total number of points in the constellation diagram.

For example if you just remember our discussion in couple minutes ago if I have a binary level modulation on each of these two carriers right I get a four point constellation in this complex plain. I had two points two possible points along this axis and two possible points two possible values along this axis and therefore the overall number of points that I have in the constellation is 2 into 2. If I had instead first Q possibilities along this axis and Q along this axis I will have an overall number of points which is Q square right.

That will give me some kind of a rectangular constellation that is I will show you some of these constellation in a few minutes. So a Q-ary constellation on real line by this procedure with the mapped into a Q square ary constellation on the complex plane right.

Student: (()))(09:47)

Professor: Suppose this value is M right suppose we define Q square equal to M so we have a M-ary modulation scale where M is Q square right. This is our new value of M now that is why I have defined this to be Q rather than M right. So if I have a Q-ary modulation scheme along the this is mind you just for example, this is not the only way of defining constellation in the complex plain, this is one possible way and this possible way of describing will produce for me a M-ary constellation in the complex plain which will be essentially a how will they be distributed in the complex plain? What kind of distribution you will see?

Some kind of a rectangular distribution or a square distribution right, just like this one is. This is a square distribution that will be a square with larger number of points essentially Q-square number of points. But I can many other kinds of distribution to.

Student: (())(10:57)

Professor: Because I have Q along this axis and Q along this axis and they are independently modulating the two, if I do things in this particular way, there are other ways of doing it right and if I independently can have Q equally spaced points along this axis, yes why don't we usually will not choose Q equal to 3, right, itself will be a power of 4 right. So lets take Q equal to 4, right, so it will have this then this right, similarly I could have four values here right.

So now depending on what combination of coordinates I have here, I have a resultant coordinate here then how is such resultant coordinate here? You will end up having (()) (11:57) that is the point that I have right and each of them will be obviously distributed in some kind of a fashion like that right, sorry it should be like that like that and so, infact I have drawn this constellation for you separately and I will show them to you, that is precisely what I said, it is square grid, right.

When I say distributed in a square shape essentially I meant that is points can be thought of as line on it square grid alright and this one is, this is on its square grid with 4 points. This will be in square grid with 16 points that is all.

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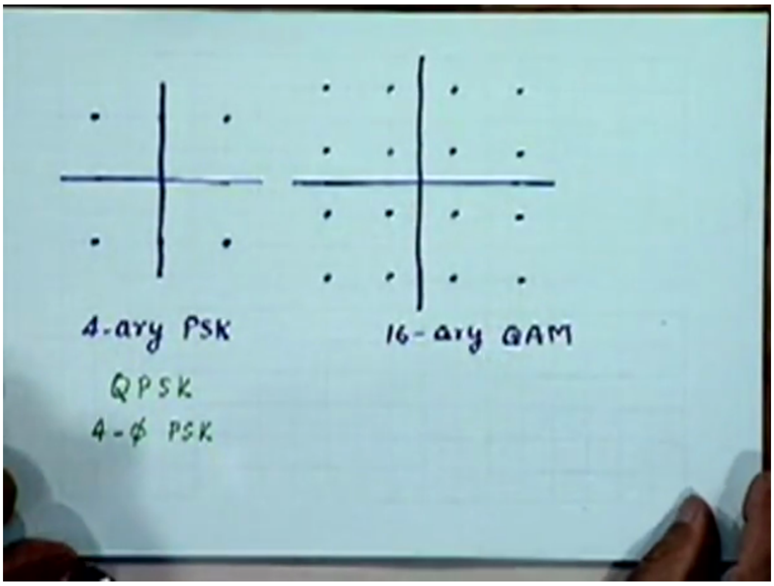

I have got them drawn more neatly here for you, this you can this is a same 4 constellation system that I have shown earlier and this is a same 16 level system that I have shown in few minutes ago.

In general well lets first talk about this one, you can think of this as Q PSK right because essentially you will see that the 4 signals which will now have will produce a resulting carrier with same amplitude right the only difference which will be there in the from one point to the other point is to the phase right and that phase is the angle that you make lets say with respect to the real axis right. So you can think of this as a Q PSK or 4-ary PSK system modulation system right and similarly we can think of this as a 16 level system or 16-ary now we cannot say that there is no amplitude variation and there is only phase variation right.

You can see as go from one point to another point there is possibility of both amplitude as well as phase variation of the carrier. The resultant carrier if you get after combining the 2-quadrature carriers the in phase in the quadrature carriers will have a resultant amplitude and phase which will be different from point to point. So we simply then call this kinds of modulation schemes as quadrature amplitude modulation schemes, which one? Yeah we can call it Q PSK sure, I said Q PSK to start with, 4-ary PSK or Q PSK is the same thing, Q PSK stands for quaternary PSL which is again 4-ary PSK right, this are ways of describing the same constellation.

No this means quaternary that is 4 phase PSK, basically I mean these are the different ways by which you describe these things

Student: ())(14:53)

Professor: There is nothing like quadrature PSK in general we talk about quadrature amplitude modulation right, quadrature PSK quaternary PSK is also an example of QAM it is a special case of QAM in which there is no amplitude fluctuation right, but in general such calculations for which the resultant carriers corresponding to different point, have different amplitudes and phases essentially known by the name of quadrature and amplitude modulations right yes what is your question.

Student: (())(15:28)

Professor: In the Q-PSK I have always said that this you can regard this as a special case of QM in which there is no amplitude fluctuation you didn't listen me completely, this is what I said alright fine this is a possibility.

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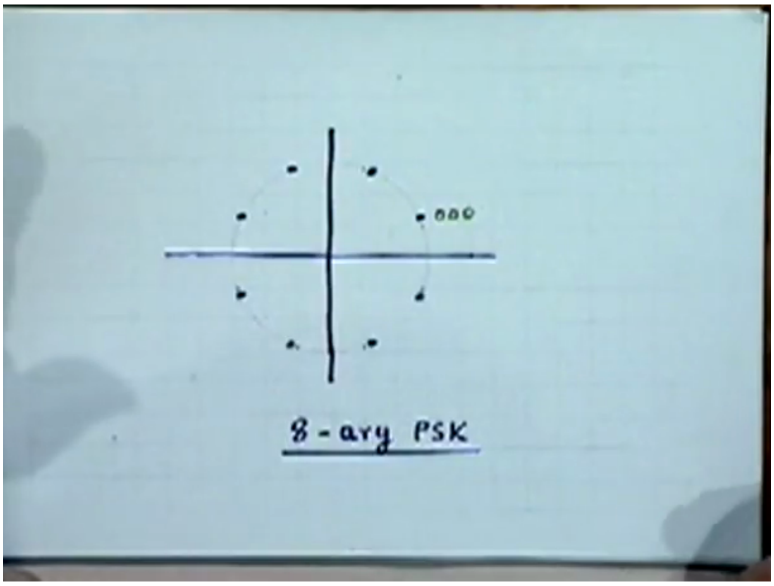

Now here are some more possible kind of constellation you can have. Here is another constellation for you, I have again 8 points here now this time 4 right. Now this is not produced in the way I described few minutes ago. The manner in which I just did when I was talking about these constellations I said ok we can think of this as two grid data mixed independently modulating that two quadrature carriers right.

That is why we were then forced to lie on a square grid right because of the manner in which we were talking about things there, however there is no way , there is no necessity to think of a general quadrature amplitude modulation in this particular way right. You can basically choose any set of points on a complex plane and associate certain number of bits representing that point right, to be present in this point. For example, I could choose 8 points which lie on a circle right equally spaced and equally spaced in angle all this cycle and I could represent a sequence of 3 bits by these 8 points right, depending on the different values on these 3 bits.

In which case I will have an 8 phase PSK or 8-ary PSK because again in this constellation the amplitude in is constant so resolve here phase fluctuation right and the phases are equally spaced you can have one of this eighth possible phases in the representation depending on the input seek for example, I could associate zero-zero-zero here with this phase and this some other binary sequence at this phase.

Now ofcourse that allocation is done usually by following the Grey code philosophy with which we have quite familiar already, these are other examples just to show that we have an infinite choice here in designing constellation right, there are lot of variations possible I can have in eighth level QAM in quite a different way this is yet another way of designing a QAM system in which I have amplitude and phase variations possible.

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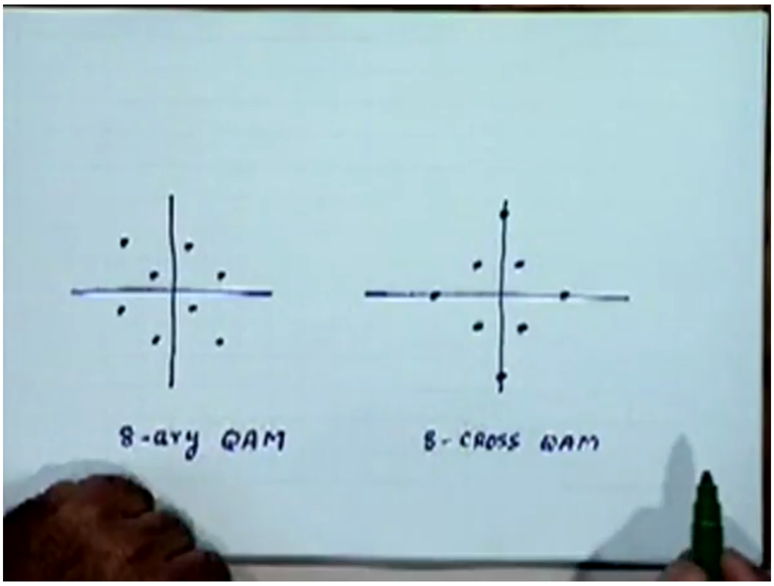

Some kind of a regular grid not necessary the square grid that I had to start, right here what I have got, in this what I called it cross QAM you will find that I have a four level QAM of this kind and another four level QAM with a different amplitude which is some kind of crossing this some kind of a crossing relationship between these two perpendicular 45 degree shifted versions of each other right, 45 degrees rotated versions of each other. This constellation the outer constellation and the inner constellation you can think of a crossing each other right.

So we call it (())(18:56) if I have same amplitude I get back to PSK

Student: Even if the we assume that the phase is not the uniformly distributed

Professor: Yeah it may not be in this case it is but in general QAM modulation scheme the phase will not be uniformly distributed even the amplitude may not be uniformly distributed. For example just see here right, you can have you have infinite choice here it is upto us to choose a constellation to represent our given number of bits that we want to encode or use for modulation at a time ok. How do these things all fall within the class of bandlimited modulations?

Because it is understood or it is implied that each of this points will be using either a common pulse shape with a same bandwidth or maybe at just you can use different pulse shapes all satisfying the Nyquist criteria or required bandwidth constrain. Offcourse usually one uses a common pulse shape there are situations when more than one shape pulse shape maybe employed but most of the time we use a common pulse shape to represent any of these points ok. The only attributes that before our the resultant amplitude and phase of the carrier that goes in that pulse or offcourse (in the).

So the overall modulated weight from that comes out depends on not only tis amplitude phase combination but also in the pulse shape which you use to describe it which is actually the same. Any questions so far? About this constellations and the concept of in general quadrature amplitude modulation or sometimes these kind of things are also known by the name of amplitude phase shift key, in general it is QAM systems APK right stands for Amplitude Phase Shift Key and they are

Student: (())(21:19)

Professor: Confusing? What is confusing? They all imply the same thing right, you can see each of these names makes sense because in these schemes those amplitudes help from one representation to another representation both amplitude and phase are being varied. Whereas in pure PSK only phase is being varied just to distinguish these from the pure PSK kind of modulations right but the most common name today is QAM. So if you remember that it is fine right.

Now when I have this choice of modulations available you can see that I can design a constellation in any particular manner that might fascinate me right. But is there some pattern behind these designs or should we consider some parameter when you are designing this constellations right, if so whatever this parameters we like to look at those. So I will just first give a few definitions and then will come to this point.

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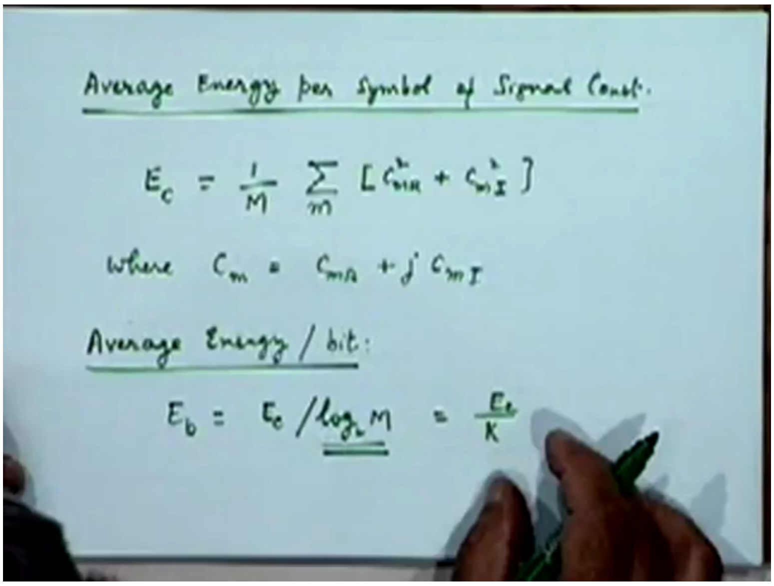

Let me first define average energy per symbol of the signal constellation, you have a given signal constellation right lets say this constellation, you have 8 symbols here corresponding to possibly 3 bit sequences right, 8 symbols here. What is the average energy per symbol? Associated with this constellation, what is the average energy per symbol associated with this (constellation)?

Because different symbols have different energies. So first thing that I wanted to define is a concept of an average energy. So will define this to be E sub C which is obviously the average of the sum of the energies of all these symbols ok. Now as you can appreciate this energy for individual symbols will essentially define on the amplitude of individual symbols right or the distance of the symbol from the origin, right, which you can think of this as the sum of the real component and the sum of the square of real and imaginary components.

So if I define the real components of the Mth symbol or the Mth point on the constellation as C sub M R and C sub M R square plus C sub M I square which is which represents the imaginary component, this defines the average energy per symbol for a given constellation right. If you compute this earlier where C sub M is C sub M R the real component plus J times C sub M I right.

Student: (())(24:40)

Professor: Ok yes you are right this definition of average implies that that is we are assuming every symbol to be occurring with same probability which again would be true if every bit occurs with same probability independently of every other bit right which will be true. So yes this assumption implied in this definition, you are quite right thanks. Similarly we can define average energy per bit as this is a very important parameter finally the parameter which is most important is average energy per bit. It is, will denote it usually by E sub b, it is a fairly standard notation and this will be E sub c divided by Log M to the base 2 which you can also call 1 by K or E C upon K where K is this number right.

Repeat something. What is that? What do you want to repeat? This definition of average energy per bit? This basically this is average energy per symbol, each symbol represents how many bits? Log M bits log M to the base 2 right or K bits right, therefore average energy per bit is E C upon K.

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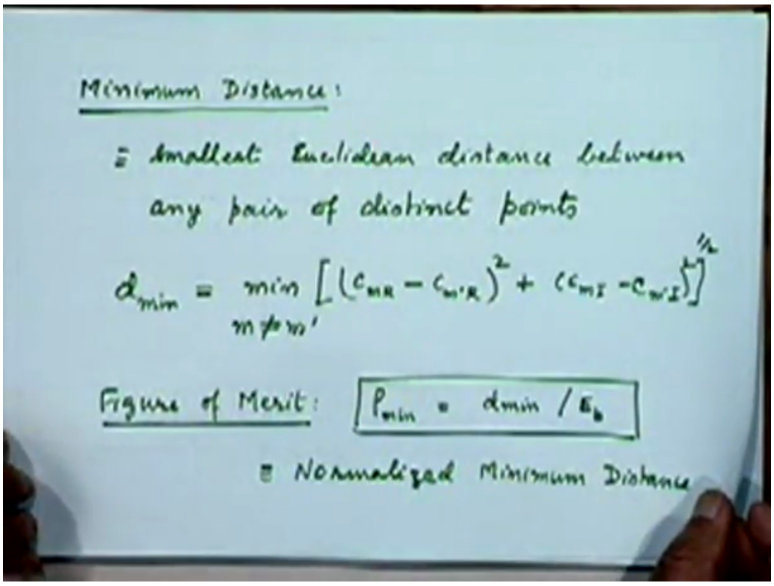

This is as far as energy definition are concerned the next parameter that is of interest is very important performance parameter of a constellation, it is called the minimum distance on a single constellation and I think by the very term you know what it means it denotes the smallest distance between any two points on the constellation right.

Smallest Euclidean distance between any pair of distinct points right on a single constellation we can write a mathematical expression for it D sub min is the minimum value of suppose I have two points represented by C sub M and C sub M prime alright and their corresponding the distance will be you take the component along the real axis square it up take the difference square it up plus similarly for the imaginary part right, this square root of that. Minimum of this value for all possible combination of the indices M and M prime right.

So minimum over all M and M prime which are not the same ok so that is a formal definition of the minimum distance on the constellation, makes sense. Now this is a very important parameter of a signal constellation but unfortunately this parameter alone will not survives because I can have two signal constellations which are perhaps identical in shape but have different domain by simply spacing out one constellation more than the other that is one constellation you have more power than the other.

But that is the power advantage, that is not a constellation advantage right.

Student: (())(29:04)

Professor: I can increase that D sub min of two constellation which are identical in shape but are different from each other in the sense that one has points which are spaced out more than the other, the otherwise identical in shape right. So I can increase the minimum distance by simply pumping in more power in the system by spacing them out, what am I doing? How am I spacing them out? By putting in more power in the system right.

So this alone therefore is not a good measure of how good a constellation is right, because you must normalize it with respect to the something which will otherwise make them similar right. So to obtain a proper figure merit for a signal constellation (we do) we define what is called Romain. Now you come to the figure merit which will tell us how good or how not so good is signal constellation is in a particular sense I will talk about that sense in a few minutes and will define it as D min yes you are quite right, you have to normalize it with something and the most reasonable thing to normalize it is E sub C average energy per symbol E sub D for that matter average energy per bit right.

Just to give you an idea of so will call this (())(30:42) as normalized minimum distance alright. Since basically it is minimum distance which has been normalized by the average bit energy will call it normalized minimum distance. Just to give you a feel for what kind of values will have for this figure of merit.

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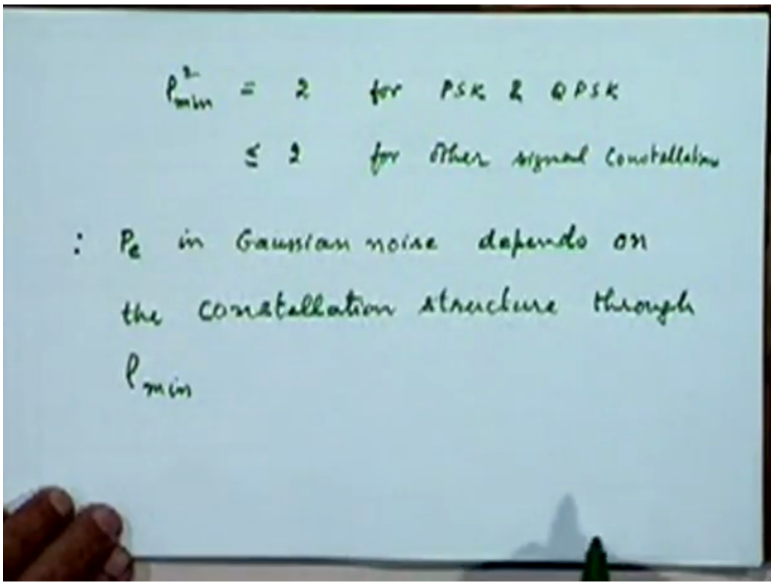

Let me consider the value of R min square for some typical modulation schemes. Can you make a guess for its maximum value just a blind guess.

Student: (())(31:37)

Professor: Maximum possible value for this normalized minimum distance by any constellation.

Student: 1.2

Professor: Somebody is saying 1, somebody is saying 2

Student: 2 sir

Professor: 2, good a lot of you are very close to it infact it is the right answer, it is infact equal to 2 and can you give me the constellations for which it will be equal to 2, PSK and QPSK are examples binary PSK is also a I am including this in a general discussion all though strictly speaking there is binary and (())(32:17). For PSK and QPSK and it can be seen you can convince yourself that this value is going to be less than 2 for all other constellation. Now why is this important? Why do we call it figure of merit for the constellation?

Yes that is true, ultimately in a digital communication system the final performance measure is our probability of error for a given system right and here we are talking about the contribution of the constellation design in determining the probability of error right and it turns out that if you look at the P sub E the error probability in Gaussian noise situation then it depends on a single constellation design specifically it depend on a single constellation structure this probability this other property essentially through this parameter right that the tilled constellation structure itself is not that significant as this parameter is right.

So the error probability is largely determined by given single constellation structure by this property of that single constellation structure right, that is if this value is large you will have a smaller error probability if this value is small you will have higher probability. So essentially the property of the signal constellation for the purposes of error probability calculation is largely through this parameter (())(34:16).

So depends this error probability depends on the signal constellation structure essentially through this parameter (())(34:40). So this is a useful measure by which you can compare different signal constellation that you may like to look at ok. Essentially what you are saying is if constellation are well designed they will be such that any two points which are closed together will have a large minimum distance right, with reference to Varun this disturbs me a lot your talking, if you have something to say I will really prefer that you say it to me and through me we can all discuss it together.

But if you constantly keep talking to each other it upsets my presentation please.

Student: (())(35:29)

Professor: Yes if you any questions so you are most welcome to asking yeah actually we look at this later when we talk about de-modulation error probability talking about error probability at this point in detail will not be of much significance unless we know how we are de-modulating this in, because ultimately the final error probability depends on both how you are modulating and how you are de-modulating right. So when we take up that issue in detail will discuss and comeback on this point.

Other word just keep this as a point of information in mind right that ultimately the most important parameter of a signal constellation which determines its overall performance in Gaussian noise mind you in other situations this conclusion may not be correct but in just Gaussian noise alone this is the most important parameter the normalized minimum distance ok but will come back possibly this question again when we discuss the error probability calculation of the we will discuss how we have to de-modulate this.

Student: Why Gaussian noise?

Professor: Why Gaussian noise is a very easy answer which conform you have dealt within your first course because most of the time the kind of noise that we encounter in communication systems can be modelled as Gaussian noise.

Student: why it is not applicable to other kind of things ?

Professor: To other kind of things means?

Oh yes, it will be very difficult from it and this there is a nice answer for that but will be difficult from you to take it out here ok for Gaussian noise you have very many very nice properties and this is one of them right. The fact that it is Gaussian makes life very simple for us infact that is also one of the reasons why we use Gaussian as a model. Sometimes when it is not even valid which is offcourse wrong ok.

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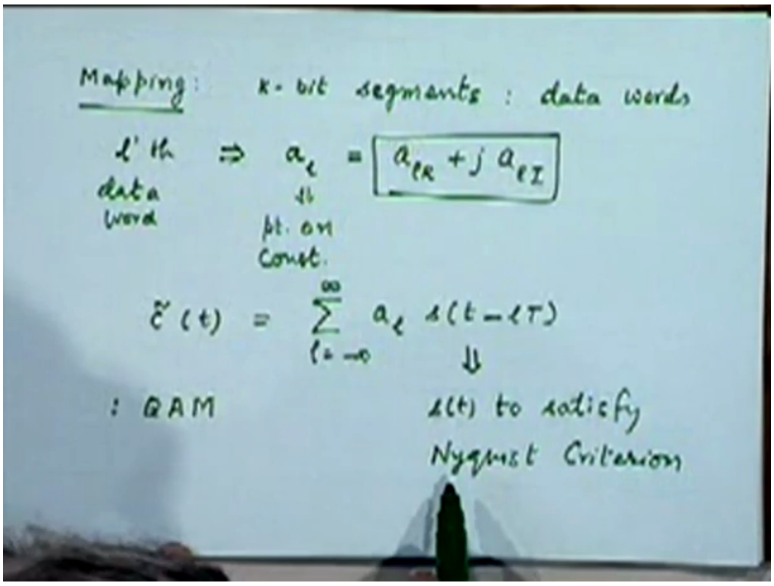

Let me just stay for to complete the discussion by first saying something reasonably trivial and obvious. The mapping of bits in sequences to this point in the constellation we all understand by now right, essentially a group of K bits will be taken so input data stream will be segmented into K bit segments K bit words. So first you will divide the incoming bit sequence into K bit segments and will call this data words tis segments right where say this a number of times the Lth data word lets call it A sub L right oh sorry it will be Lth data word will be sign to a point one of the points on the constellation.

Lets call that point (())(38:35) right, so Lth data word assign to a this is a point on the constellation which will have real component plus an imaginary component right. So basically this represents what? A complex amplitude or the pulse that you are going to transmit right. This (ampl) this Lth point or A sub L value dependency complex amplitude of the pulse that you are finally going to transmit offcourse with a carrier right. So infact this represents the amplitude and phase complex amplitude means it represents the amplitude of the carrier and the phase of the carrier, when we talk of complex amplitude.

So in general the overall complex signal that you are dealing with is going to be transmitted signal is going to be sequence of these with the pulse shaped S T, L going from lets say minus to plus infinity right, this is your transmitted waveform where tis A L's essentially govern the complex amplitudes of the transmitted pulse right. That is what we call QAM mathematically right.

Student: Sir like in (())(40:08) divided into even base and odd base same data stream, can we do that type of in here.

One can have I did say that when we are discussing orthogonal single stream last time I did mention to you that one can have M-ary MSK kind of signals but I didn't want to discuss them here right because I will become to detailed in discussion for which we don't have time you can read about it yourself. But other moment we are just doing a general QAM discussion alright. So this so another way of looking at QAM mathematically and in this typically this pulse S T that you will use will satisfy your, what kind of pulse shape you are going to use? Strike this criteria.

So S T to satisfy Nyquist criteria right, which offfcourse in kind of implies that this pulse shape that we are talking about is not necessary to be the pulse shape at the transmitter alone right. It is really going to be the pulse shape overall pulse shape that will result from staring from the transmission pulse shape to the channel impulse response to the receiver filter match (or whatever). It is a combined pulse shape that we talking about. That is what we require to satisfy the Nyquist criterion. Not just the transmitted pulse a lot ok, because what we finally want to make sure is that at the decision instance , there is no inter symbol interference right.

In a bandlimited channel that is how we go about designing of pulse shapes, this is just to recomplete for you when we what we discussed on Nyquist criteria.

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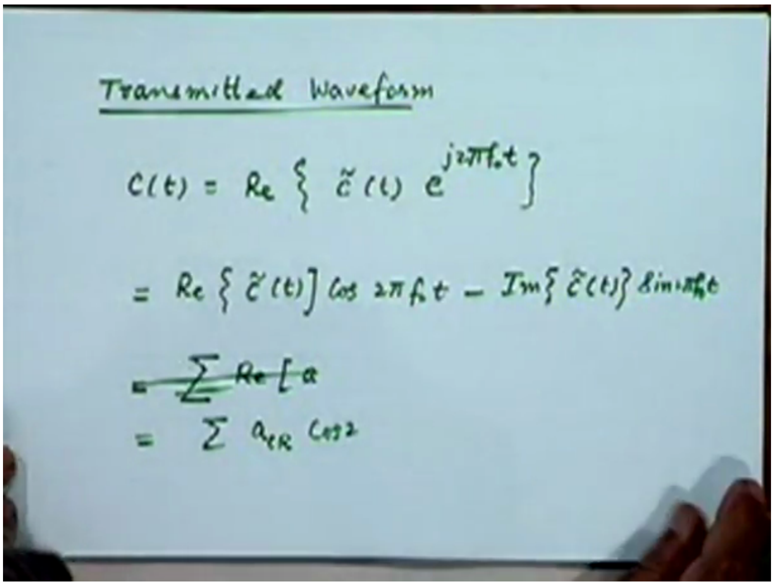

The final real transmitted waveform what will be nature of that? That is just a complex representation, what we have got here is just a complex presentation right, a final real transmitted waveform would be lets call it C T , that is right, so far I have not introduced the carrier right this tells some basic pulse shape therefore this is really baseband representation of a passband signalling right.

Your final C T would be you use that signal to modulate the carrier and take the real part of the take it so real part of this multiplying E to the power J2 Pie F O T or F C T whatever, which, now because we have taken our let me go through some little bit of so this will be real part of C power T into cosine 2 pie F O T right, minus imaginary part of C power T into sine 2 Pie F T, this is the quadrature that we have finally transmit right, which we can represent in terms of data by just going to it further simplification.

This in turn what is real part of C power T? where C power T is this, right, this is a real part of this S T is a real pulse shape right in general, I should say in general most of the time S T is going to be a real pulse shape in general I could have a complex pulse shape also whether real or imaginary component but for my current discussion lets assume that the pulse shape itself is real in nature. When do we have a complex pulse shape? When can you possibly have a complex pulse shape? Ok we go to that issue separately.

At the moment lets just stick to ourselves if just think about it we will just stick to ourselves to the situation where S T is real so that we can take this to be essentially to face it by whether you know the real part of A L and the imaginary part of here right. So it will be a real part of I think I can directly writes that right, sigma A L R cosine sorry I should have used something else I better do it fresh.

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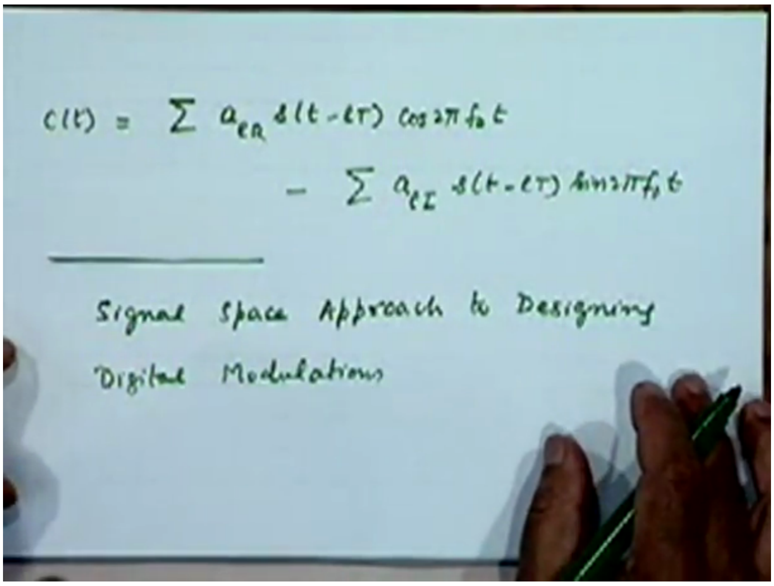

A sub L R S T minus L T cosine 2 Pie Fo T minus A sub L I S T minus L T sin 2 Pie Fo T, so that is a final transmitted waveform on the channel for real pulse S T.

Now finally I like to finish this discussion by this more or less completes our discussion by just mentioning to you the fact that this signal design in a two dimensional space that we have discussed so far right, the signal constellation for M-ary (baseband) passband modulations is actually a fairly important topic in itself and one can even think of higher dimensional spaces signal spaces this is a two dimensional signal space. Basically two dimensional because we are using the quadrature carriers as our basic carriers right, basic signals. But in general one can think of high dimensional abstract signal spaces in which to carry put digital modulation signal design right.

So there is a whole subject of signals space approach to signal design digital modulations, to designing digital modulations which will help us in the two dimensional case it will help us to design a good constellation and this kind of a theory will also helps us to possibly give us digital modulations in higher dimensional spaces where which I am not going to at the moment. But basically the approach consists of first defining a space of functions which you will permit which you will use in your modulations in your communication system.

Space a function means space of signals which will have some properties basically the set of waveforms and the you will be having a concept of in each waveform will be represented by a point in this signal space and that will be the concept of distance and essentially the design will be design requirement will be such design like a point instead of constellation in the space so that this normalize minimum distance or equivalent of normalized minimum distance is as large as possible right.

So offcourse that subject is outside our scope and will not really go into it but one has to do that kind of an exercise to achieve or design good modulations schemes. The job of the modulator is very simple, once you design a signal constellation the modulator is essentially a rule to map incoming data bits onto one of these points right that is all a modulator has to do. But a modulator designer has to design what set of points to choose and how to choose that ok, I think this is where we will stop this discussion.