Digital Communication
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Lecture – 20
Passband Digital Modulations – IV: MSK (contd..)
Passband waveforms for M'ary Signalling

Some of the important things that we try to bring out regarding the MSK waveform, regarding the MSK modulation scheme are the following, first it uses a co sinusoidal pulse shape right instead of actually half cosine pulse shape instead of a rectangular pulse shape second because the I and Q channels use the cosine, sine pulse shapes respectively half cosine and half sine pulse shapes respectively.

It turns out that the final modulated waveform which is the sum of I and Q waveforms has a constant envelop right, third the phase changes now do not occur abruptly in this waveform, they occur continuously over the entire bit interval right and the it is therefore similar to offset QPSK signal in that every T b seconds, every T sub b seconds you have a phase shift of pi by 2 but that phase shift is gradual rather than abrupt.

So there are no sudden phase transitions in fact because we have continuous phase transitions you can also think of an MSK signal as some kind of FSK signal because you find that in either in every bit interval you can interpret it instantaneous frequency to be either f c minus 1 by 4T b or f c plus 1 by 4T b right and therefore that brings us to the last property that we are trying to complete last time but could not do so, we will now take that up.

In FSK interpretation of MSK that is what you wanted to discuss right.

Student: The graph spectral density which we have got, that particular (())(3:06) why do we want to have smooth variation of the phase (())(3:14)

Professor: I do not have a spectral graph there with me but what is your difficulty.

Student: How do you get that comparison?

Professor: Oh how do I get the various graphs?

Student: (())(03:28) formula for SS for MSK also.

Professor: Yes I have not gone to the details of spectral calculation of MSK right because that is a bit involved and I think it is outside the scope of our discussion so.

Student: Mainly why do we want to have gradual phase change I mean (())(03:45) this before and also in OQPSK we have abrupt.

Professor: QPSK, OQPSK and MSK you can think of these three as some kind of (())(04:00) improvement of every waveform instead of a previous one, the improvement is in QPSK normal QPSK you have 4 phase constellation and every symbol interval or after every pair of bit intervals right you have to, you may have a phase transition of either 0 degree or 90 degree or 270 degree or 180 degree whatever right.

So these are the possible (())(04:35) you can have phase transitions of 0 degree plus minus 90 degree and plus minus 180 degrees there plus minus 180 degree phase shifts are rather large right similarly in the offset QPSK you will have phase transitions more often they will be now distributed every bit interval rather than every two bit intervals but the extent of the transitions will be only going upto plus minus 90 degrees.

So this that magnitude of this continuities will be smaller right and thirdly in MSK you have no phase discontinuity at all it is in fact what is called a continuous phase waveform right in fact the FSK that we have also said that MSK can be rather as a FSK waveform it is an FSK waveform that has special feature that it was a continuous phase FSK signal right, now why the advantage that is the question that Deepankar has asked.

You can appreciate that if a waveform contains a lot of discontinuities it will have larger extent in spectral domain that is reasonable intuitively clear but if you have a discontinuous waveform in time it will have a larger energy distributed at higher frequencies in the frequency domain right therefore if you try to band limit such a waveform you will expect more distortion in this waveform than if you try to band limit a waveform which does not have such discontinuities in time domain, this is intuitively clear?

Right because in a continuous waveform most of the energy will be considered in a narrow region where as in a discretely discontinuous waveform at several points along its locations in the time you will have significant energy at high frequency which will be cut out by the filter and because it has been cut off the waveform will be highly distorted the form of this distortion for phase discontinuous waveforms will be that your amplitude of waveform will no longer be constant it will go through amplitude and phase fluctuations.

Where amplitude fluctuations are a serious thing in systems where there are non-linearities right because if you are going to go through a large dynamic range of amplitude fluctuations that means all the components of your communication system it maybe receiver it may be transmitter must operate in a linear fashion over this entire dynamic range of operation.

Whether it is a low amplitude of times or amplitude at times now it is very difficult at very high frequencies like microwaves or even high frequencies to design such linear amplifiers or linear modulators, power modulators right typically they will have a nominally linear region of operation around a fixed amplitude point right and if your amplitude fluctuations occurs outside that region then we are bound to encounter further distortions due to non-linearities.

And those distortions are much more severe and much more difficult to eliminate than if you were to take care of it right in the first instance right because do you know what kind of distortions are introduced by non-linearities typically, harmonics, inter modulation terms that is for every frequency that you may have you will have set of frequency you may have, you will have at the output either frequencies generated, new frequencies will generated in the system which were not present in the input waveform at all, which will further spread the spectrum right.

So it is going to be very bad for the communication system as a whole because it may further introduce what you call spectrum decays into other channels because it will be generating frequency components which may lie in the frequency band of another receiver close by right which you will not like to do, so therefore it is preferable to use waveforms which have constant envelop or constant amplitude.

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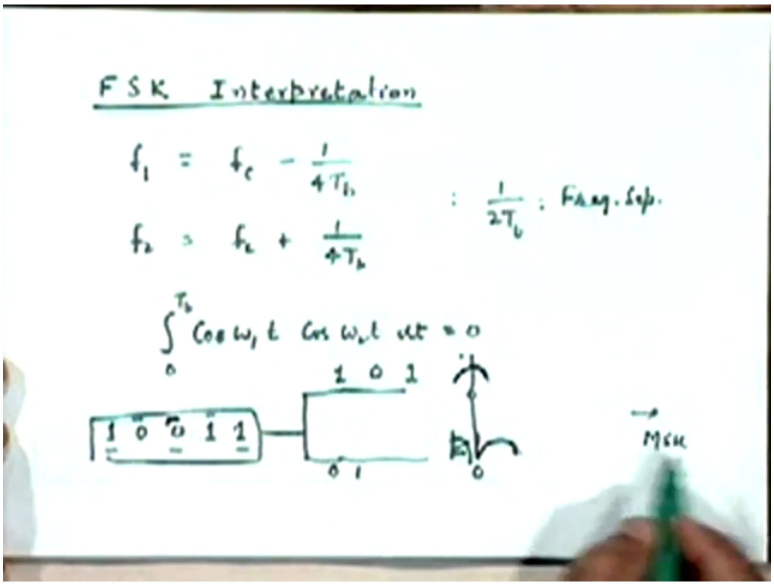

The MSK waveforms satisfies that requirement, is that answer your question Deepankar? Okay so we will proceed from this point onwards, what we have seen is that we can also interpret MSK waveform as an FSK waveform so look at MSK as FSK in fact the main MSK minimum shift keying comes from its relationship with frequency shift keying right we have seen that we can think of the two frequency, the waveform having two frequencies f1 and f2 corresponding to f sub c minus 1 by 4 T sub b.

And fc plus 1 by 4 (())(09:55) I do not know how I define them last time this could be plus, this could be minus or vice versa, now what is the frequency separation between these two, 1 by 2T b is the frequency separation between the two frequencies right it can be shown that this is minimum separation between two co sinusoidal or sinusoidal waveforms which will be mutually orthogonal that is if you take two sinusoids or co sinusoids of period 2T b right, or period T b sorry bit interval.

Then this is a minimum separation that is required so that these two sinusoids or these two co sinusoids are mutually orthogonal over this interval not over minus infinity to plus infinity over 0 to T b right.

Student: Sir the statements are not clear.

Professor: Okay suppose you are interested in finding out cosine omega 1 t cosine omega you know there are two pulses with carrier omega 1 and carrier omega 2 and pulse durations are each T b you have to find out whether these two waveforms are orthogonal or not or whatever the condition for omega 1, omega 2 if they should satisfy such that this is 0 right suppose we ask ourselves this question, if we realize that f2 minus f1 should be a multiple of 1 by 2T b right and 1 by 2T b is the minimum possible separation that for which this will be true.

That is the reason we call it minimum shift keying that is generation of an orthogonal pair of waveforms in a minimum frequency shift between the two right that is where the name is derived from, it is derived from its similarity to FSK waveform and since the corresponding FSK waveform is orthogonal with the minimum possible shift in the two frequencies we call it minimum shift keying clear?

Okay if we have this interpretation of the waveform this also tells us that we could possibly have generated an MSK light waveform or MSK waveform strictly speaking by directly mapping the incoming zeroes and ones all to frequencies, so far how did we view the MSK waveform, we said we will take the incoming bit stream, you know it may be some other two bit stream right.

And you are splitting that into two bit streams I and Q bit streams of each having a rate of half of the incoming bit stream right so for example all even bits would go to, all odd bits will go here and all even bits will go here so and so on and then each of this was modulating was using the half cosine pulse here and a half sine pulse here right sorry other way around perhaps, yeah that is fine if you take this as 0 this as I am sorry it is a common reference point.

So this we are using a half cosine pulse and this we use as a half sine pulse right and then further modulating the carrier and then two (())(13:54) right that is how things were doing done now so this was a modulation process this was the way we are mapping our incoming bit stream on to finally on MSK waveform, I am not drawing the complete diagram because that is quite obvious, if you want we can do that, maybe we can quickly do that.

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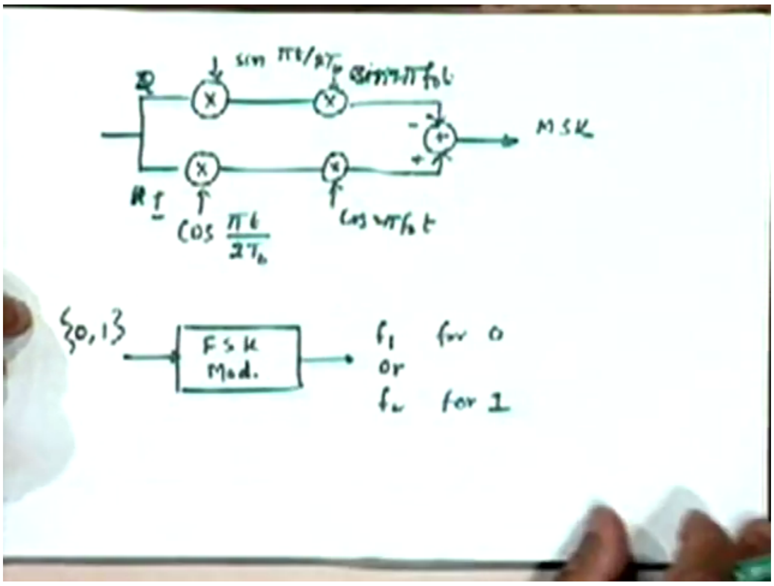

You have an incoming data stream, split it into two multiply with cosine what was it pi t upon 2T b and sine pi t upon 2T b right then we were multiplying with the carrier cosine 2 sorry this should be sine here, sine 2 pi f not t this would be cosine 2 pi f not t and then you are simply adding or subtracting or whatever alright, this is your MSK waveform, this is how we have been using the MSK waveform generation so far as offset QPSK waveform right.

These are the I and Q bits streams which are mutually offset with respect to each other by 1 bit interval this bit stream is modulating it should have been Q and I this way because I have taken the cosine over here, this is modulating a, this is being showed by a cosine pulse, this is being shaped, this is shaping a sine pulse right and then we are introducing a carrier right and adding them out.

This is how the offset QPSK waveform would be generated and this is how MSK waveform would be generated, the only difference between this and offset QPSK would be that offset would be using a rectangular pulse here and same rectangular pulse here with a shift of half bit interval, half symbol interval or 1 bit interval, right so this how we have viewed the MSK generation so far.

What I am now suggesting is that since we can view the frequency of MSK waveform to be either f1 or f2 in every bit interval, we could have directly generated an MSK waveform which could perhaps different from this waveform that we have generated here by directly mapping the incoming ones and zeroes all to frequencies keeping the phase continuous at every transition of bit, every bit transition.

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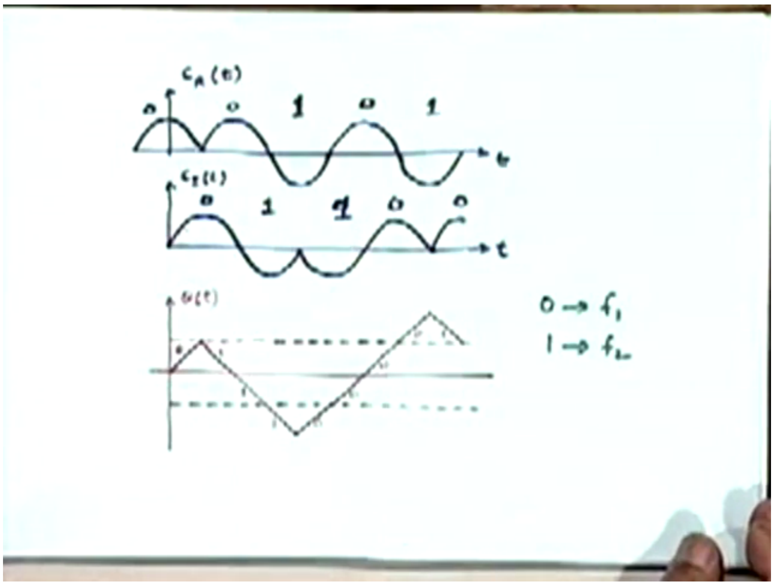

That is to say instead of thinking of shaping the I and Q pulses like deciding the I and Q pulse pattern in this fashion we could have directly looked at phase function in let us say corresponding to 0 we have an increasing higher frequency and corresponding to 1 you have a lower frequency just for the sake of argument that means for every incoming 0, I could increase the frequency to f1 so that you have a increasing phase in that interval or you would be decreasing the frequency to f2 right.

You changing the frequency either to f1 or f2 in every bit interval we have one of these two frequencies present in every bit interval and this could be done by a direct mapping of incoming zeroes and ones, say 0 going to f1 and a 1 going to f2 so I have the different modulator here the modulator now consists of essentially as a FSK modulator alright the incoming zeroes and ones directly map these bits onto frequencies onto waveform which estimates either f1 or f2 for 0 for 1 right.

And if f1 and f2 are selected to be same as we discussed earlier f c minus 1 by 4T b and f c plus 1 by 4T b qualitatively we will not be able to see any difference between this waveform and the one which you related using this method qualitatively that is if you just look at the waveform every bit interval you will find the frequency to be f c minus 1 by 4T b or f c plus 1 by 4T b qualitatively we may able to make any difference between the two waveforms is this point clear?

Student: Sir we said that (())(18:42) equivalent to the frequency shift of plus minus 1 by 4T b (())(18:46) but there we encountered 4 levels, the plus cosine and plus minus

Professor: Those 4 levels for every pair of bit intervals this is going to be happen

Student: There set appears as if it depends on the instantaneous frequency depends on just one bit instead of a pair of bits

Professor: So we can either regard it as a binary FSK waveform or as an offset QPSK waveform if you regard it as a offset QPSK waveform you look at pair of bits and do that mapping in this manner, if you use as a FSK waveform you can generate such a waveform by directly mapping every bit on to frequency in that case it would be a binary FSK waveform right so the two

Student: (())(19:32) MSK being generated by that scheme.

Professor: This is the point which I wanted to discuss in detail it would be equal but extend to the properties of the waveform of the same but the specific waveform that you will be generated here by the same sequence will not necessarily be the same waveform that is generated by this kind of a mapping right that is the specific sequence of f1 and f2 that you will get maybe different here and maybe different here right.

Corresponding to the same identical input bit pattern you will have two waveforms which are qualitatively similar but different in detail that is the, if you look at the corresponding bit interval and the frequencies of the corresponding bit intervals they may not correspond to each other.

Student: Only the mapping scheme

Professor: Only one can do the mapping either this way or that way that means if I modulate the waveform like this I cannot demodulate it like this or if I modulate it like this I cannot demodulate it corresponding to the demodulator of this kind of a mapping, this is the point to appreciate.

Student: Sir only thing will be that the bits will be a bit delayed

Professor: No they will be different I mean you will have to work it out, once we work it out we require a pre-coder if you want to do that right, if for example you want to map the waveform like this and demodulate as if it was an FSK waveform you will require a post decoder operation right whereas some kind of a transformation will be require on the bit sequence similarly if you modulate it choosing the FSK strategy and want to demodulate using the inverse of this strategy, you will require the pre-coder before you do the modulation here right.

Student: (())(21:15) demodulate BFSK, MSK as BFSK, this one OQPSK

Professor: Yes we can provided we have appropriate pre-coding or post-coding operations which will carry out the necessary equivalence operation in the bit sequences okay, let me complete this discussion then I will come back to your question, so what I am really saying is.

Student: Sir please show the diagram the last one.

Professor: This one? Alright, you have any question about it? Any question about this, this theta t comes from where you tell me, I showed you a phase trellis diagram last time for the MSK so basically this is one possible path on the phase trellis right depending on whether you have a 0 in a particular interval or a 1, my phase is either continuously increasing or continuously decreasing I could have mapped the other way around also I could have mapped a 1 to a continuous increase and a 0 to a continuous decrease right.

So this is interpretation of MSK waveform by direct mapping of bits on to frequencies right but you in fact this waveform very clearly illustrates if I do the mapping like this the bit sequence that you have is 0 1 1 1 0 0 0 0 1 whereas if you do the mapping like this and to generate the same MSK waveform the bit sequence will be 0 0 0 1 1 1 0 0 1 0 this what I have tried to illustrate in this picture and the two bits sequence is a different.

What I am trying to convey by this picture is that doing the IQ offset QPSK like modulation using this bit sequence or doing the FSK kind of modulation using this shown bit sequence we will generate precisely the same MSK waveform right now this is our obvious, this is not obvious, this I worked out for you alright, what I want to illustrate is that the two bit patterns are different, this is what I want to illustrate to you.

That is you have a different bit pattern here and a different bit pattern here so that if you modulate using this waveform and demodulate using this strategy you will get a wrong answer, wrong demodulator sequence, you have to reinterpret that modulation sequence or retransfer it so that it is the same as this similarly if you have to modulate using this strategy and demodulate using this you will have to again do the same kind of post decoding operation or you have to do a pre-coding the operation.

This waveform and this waveform will be same in fact this is a phase trellis corresponding to this MSK (())(24:32) okay in fact one can work out very precisely what this relationship should be and I want you to try to work that out yourself and see if you can do it precisely what, let me define the question.

Student: Sir MSK is sort of differential type of because it will depend, demodulation will depend on the phase previous phase.

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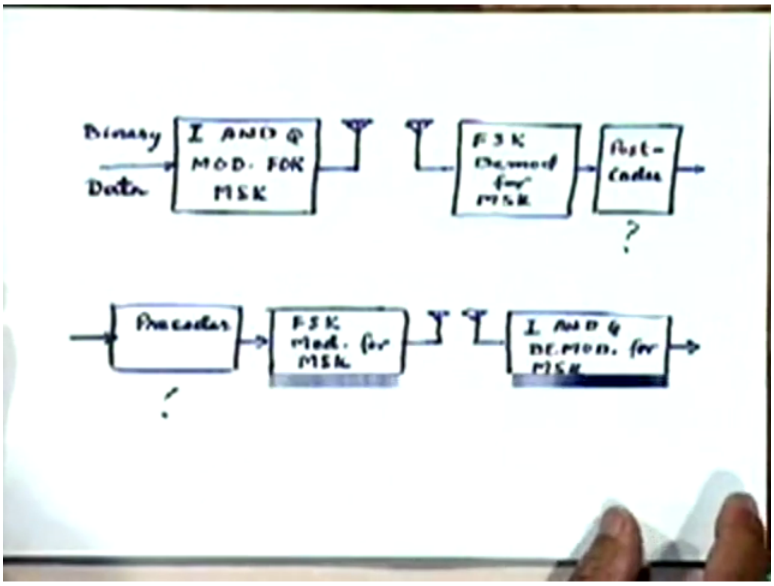

Professor: Yes there are phase relations, that is well is indicated by this phase trellis right okay you think about it I will define the question more precisely for you what I am really saying is that if you have some binary data coming in and you have done, you have generated the MSK waveform using the I and Q strategy right.

That is using the offset QPSK kind of strategy and you for some reason want to use an FSK demodulator for recovering a bit sequence, you can do so provided you fall it up with a post coder, a post transformer that is of post decoding transformation alternatively we can have a same binary data first processed by a pre-coder, convert it to a different bit sequence, use the

FSK strategy for modulation to generate the MSK waveform and now demodulate by the I and Q demodulator right.

Now what precisely should be this pre-coder or what precisely should be this post coder is what I am asking you to try to work out by yourself as to what should be the relationship between the bit pattern here and the bit pattern here so that you can do the post coding operation or the bit pattern here and a bit pattern here so that you can do the pre-coding operation right that is the exercise I am giving to you to work out, is it clear?

It is going to be fairly simple to work out in case you cannot work out let me know I will help you on that okay can we proceed further, (())(26:58) one question he wants to see the previous diagram for theta t yes here it is okay yes Varun what is your question.

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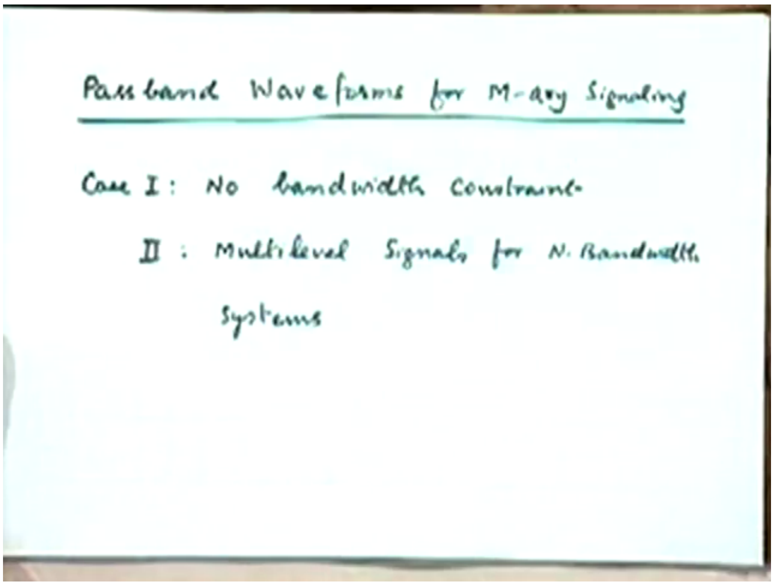

That is something you should work out, just think about it, do not think about it now, think about it later at home at leisure, let us proceed further and I will now like to discuss passband waveforms for M-ary signalling that is as far as we are going to discuss about MSK waveform we will return to MSK and all other modulation schemes that you discuss so far are going to discuss now when we talk about demodulation right because demodulation I want to discuss in a unified manner rather than individually for every stream right.

It is possible to do that to some extent, so to complete discussion on digital modulations remember how what is the hierarchy of approach we have followed we have talked about first baseband modulations both of the binary as well as the M-ary kind then we are discussing the passband modulations so far we have discussed mainly the binary modulations although QPSK offset QPSK have come in rather (())(28:43) over there.

And we will now like to do a formal discussion about M-ary waveforms, M-ary passband waveforms, again we can classify them into two types corresponding to the situations when there is no bandwidth constraint and case 2 where there is a bandwidth constraint in which case we typically use multilevel signalling for level bandwidth systems, so we will take each of these two separately. We are going too quickly for you alright let me slow down.

Now let us do the discussion more or less in the same frame work this is precisely the frame work we did use when we are discussing M-ary baseband modulations and we if you remember newest to discuss M-ary baseband modulations the basic strategy that we used was to first defy orthogonal set of waveforms and from that orthogonal set particularly for the case when there is no bandwidth constraint right.

And from that basic orthogonal set we could derive other kinds of modulation schemes like simplex, like bi-orthogonal and other takes right this is the approach we followed the so basically one has to talk about the orthogonal signalling set as the most important class of signalling set because from that basic orthogonal signalling set one can obtain the other classes mainly the simplex or any other kind.

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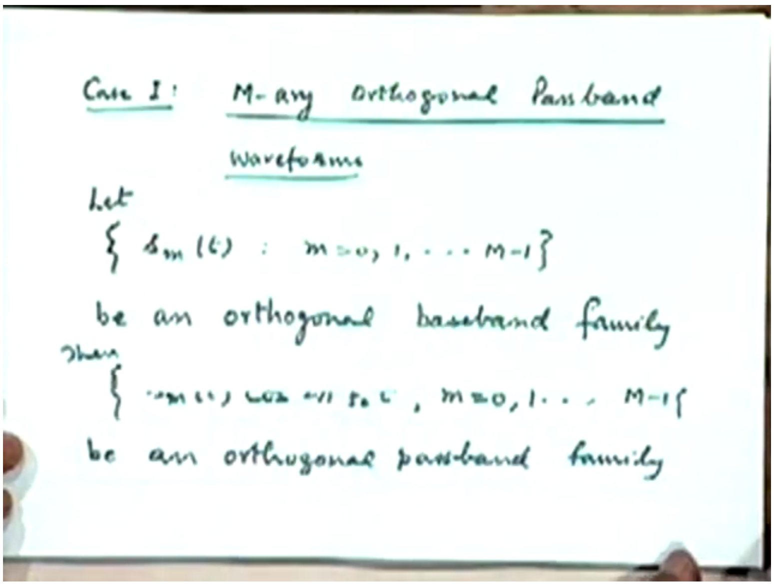

Now therefore considering case 1 first let us mainly emphasis on the orthogonal passband waveforms right, so M-ary orthogonal passband waveforms, similarly we could talk about M-ary simplex passband waveforms in fact, one can obtain M-ary passband waveforms by taking a corresponding baseband waveform as simply putting on to a carrier that is all one has to really do so basically suppose sm t, m equal to 0, 1 to m minus 1 is an orthogonal baseband family of waveforms right.

So let this be an orthogonal baseband family, you understand what it means that is you are going to use a dictionary of m waveforms and the way you are going to do the mapping we are made to return to this question again is we will take k bits at a time where 2 to the power k is equal to m right and assign one of these waveforms to every k bit sequence right and this m waveforms are selected in such a way that they are mutually orthogonal.

This is a set of baseband orthogonal waveforms, one can easily obtain a certain passband orthogonal waveforms by simply putting on, putting each of this waveform on to a carrier for example these set of waveforms will be, then this will be an orthogonal passband family okay this is very easy to check in fact we can generalize this, let me take the general case first, is it okay alright here it is again if you are not copied it, that is a problem in a monitor based instruction, on a blackboard the things stays there for you to keep on copying okay.

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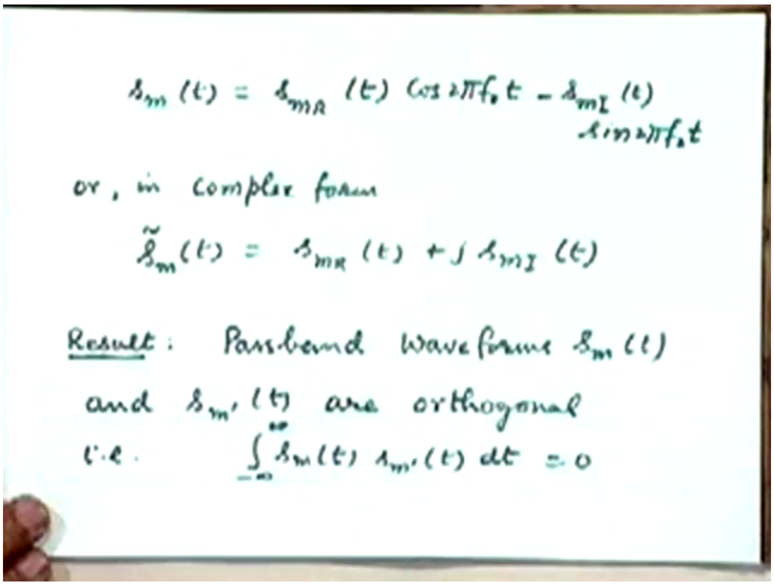

More generally I can take sm t then taking the situation where our baseband orthogonal waveform could be complex okay so it has a real part and an imaginary part this is the more general form of an orthogonal set of passband waveforms which we can write in complex form as follows, so the more general statement is that if I have a complex baseband waveform, a complex family of baseband waveforms which is an orthogonal family then from there I can construct a orthogonal passband family using this kind of approach right.

Of course as special cases we could have one or both of this component, one either of this components 0 we may not have both of them 0 either situation is rightly, the result which is very easy to verify and I like you to prove to yourself is that given that these waveforms are orthogonal even at the corresponding complex baseband representation are orthogonal, it has to be orthogonal for this to be orthogonal right.

In fact that is the result I would like you to prove given that these are orthogonal you can show that these will also be orthogonal if these are given to the (ortho), this is our starting point we start with a baseband family of orthogonal signals right for m going from 0 to m minus 1 alright see before this I considered the real case I started with a real family of baseband orthogonal waveforms as suggested that one could construct a corresponding passband formula using this operation upon generalizing this statement and saying that if you start with a complex family of orthogonal baseband waveforms.

We can obtain a corresponding passband family using this kind of a transformation right, this is what I am saying, so this is a more general statement corresponding to the previous one that just discussed therefore this is what I like you to prove that is the passband waveforms say sm t and s sub m prime t corresponding to 2 different indexes, two different indices m and m prime are orthogonal that is sm t these are real waveforms is not it? These are the passband waveforms, that is this integral is zero if of course one can write a very strict statement if and only if kind of thing.

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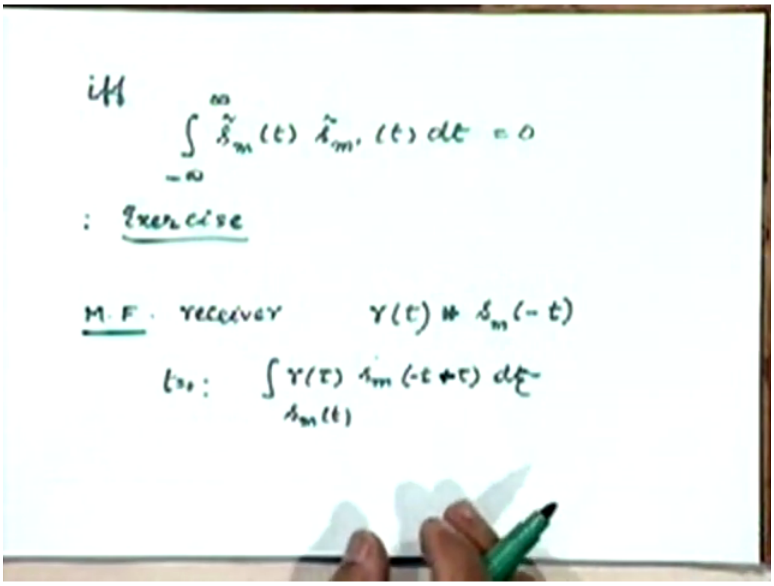

If and only of the corresponding baseband waveforms if you have not copied just show it you again okay these two are orthogonal if these two are orthogonal okay very simple to prove simple trigonometric identities are all that you need to use and I like you to, are going from you start from there substitute these in terms of the passband representation then you will get the (())(38:55) in actually one line almost in one line you can prove that thing so I would like to leave that also as an exercise for you to do, to show that this is all you need to.

Now let me try to make you appreciate this point I think I have tried to emphasize earlier the significance of orthogonality in the context of digital modulation schemes, the significance is related essentially to the fact that the way you carry out the demodulation so called optimum demodulation of a waveform requires you to basically correlate the waveform with its replica at its receiver right suppose you transmit a particular waveform in a particular bit interval or symbol interval and you are trying to demodulate it or detect what symbol was transmitted.

The optimum processor happens to be what is called either a match filter receiver or a correlated receiver right in match filter receiver I will tell you what it does I will although we will discuss it in detail later in match filter receiver essentially will take the incoming waveform r t and pass it through a filter whose impulse response is suppose you are trying to see whether this particular waveform always present or not sm t right.

Then you will pass it through a filter whose impulse response is sm minus t, well I have taken that t to be 0 f you already know about it but in general let me just say this much at the moment this is essentially what a match filter will be doing and then you will be looking at the peak of this output right so you looking at the, because typically these are pulses sm t is a pulse right I know because this concept is further down the time. I am just trying to motivate where this correlation operation comes from right.

Student: Sir you say it is match filter (())(41:17)

Professor: Yes what is this? This convolution? What is this convolution? This will be r tau t plus tau or something (())(41:39) it does not matter sm right d tau right and if you put tau equal to 0 there is something wrong this will be, that is why I wanted to avoid this at the moment we (())(42:03) how will you express this convolution interval? Minus t plus tau.

Student: Minus t minus tau sir.

Professor: Minus t minus tau alright, okay please it does not matter, it does not matter what is the correct expression, please it is very easy to interpret, it does not matter whether this is right or wrong we can still get to the point that I want to make, suppose I want to look at this waveform, the peak of this waveform it can be shown at the peak of this waveform for finite duration pulses will occur in this case for t equal to 0 okay this output will peak at t equal to, this is going to be function of time right.

Student: I tau is also of the form of the sinusoids carrier

Professor: It is whether they are same signal but it is in noise, it could be anywhere on this, quite true

Student: Sir then the (())(43:09) does not have to occur at, I mean.

Professor: Yes yes let me complete my statement, if I will complete my statement then you will not have this question, if r tau contains a signal sm t right maybe in white Gaussian noise when this signal peak at t equal to 0 right, if it contains a different signal at let us say it contains sm prime t rather than sm t then you produces 0 output provided the orthogonal signal okay.

Okay let us suppose there is no noise right then what is this waveform this will be sm t convolve with sm minus t right and at t equal to 0 what will be the output, essentially analogy of the signal right if you peak there, similarly if both are end points it will peak there but if one is m and other is m prime right provided your original signals to (())(44:16) orthogonal this would turn out to 0 at t equal to 0.

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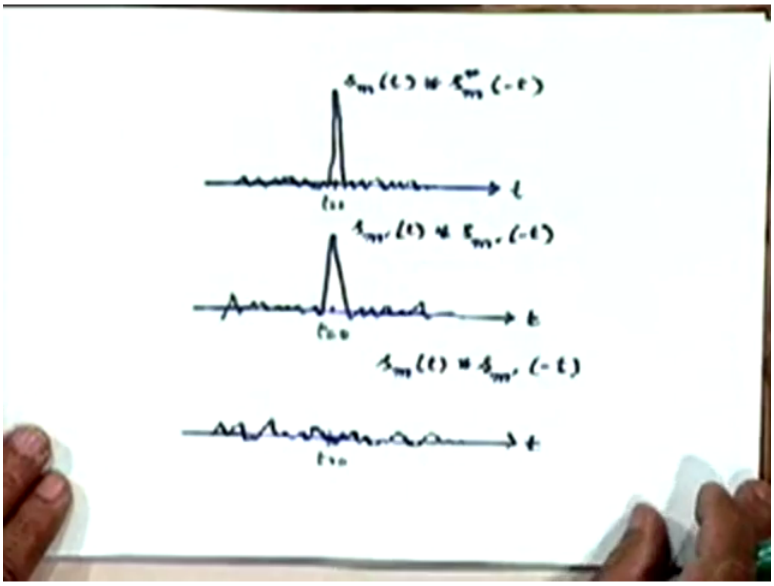

That is the significance of the orthogonal operation, I have illustrated this in this picture here for your convenience, this shows here the convolution of sm t with itself with sm star minus t with its match filter impulse response right the convolution of sm prime t with its corresponding match filter impulse response this peaks at t equal to 0 this is t equal to 0 this also peaks at t equal to 0 right. Whereas if you pass sm t through a filter which is match to sm prime right at t equal to 0 you are going to get a 0 output.

Student: Sir why do we take sm of minus t

Professor: That has to be in a match filter derivation (())(45:12)

Student: Sir this mn m prime represents members of the orthogonal family.

Professor: Family right, so therefore now you can think of the receiver as consisting of a number of match filters right, one corresponding to each different member of the orthogonal family each producing a response and the correct filter will produce the peak at right point right that is the motivation for defining this orthogonal families right, you can easily distinguish different signals, different members of an orthogonal family through the use of corresponding match filters okay.

This has just to motivate as to where all this comes from otherwise you will be taking up these things in detail when we discuss the demodulation of this waveforms alright so do not worry too much about where this minus t comes from and whether this should be minus t plus tau or plus t plus tau we will discuss all that in detail correctly, is it okay you have any questions about it?

What I am basically trying to convey to you that orthogonal family of signals is a very basic family of signals both from the point of view of the fact that it is easily distinguishable that is one member of the family is using a distinguishable family other members through match filter kind of receivers, also because from starting from these basic orthogonal family one can design a whole lot of other families.

Particularly important class of simplex and bi-orthogonal signals right which are also easily distinguishable right in fact they some of them they give better performance than the orthogonal family for a finite alphabet although it can shown that for very large alphabet sizes where m is very large the orthogonal family is in fact the optimum family to use in some sense but those are things which we cannot discuss here alright.

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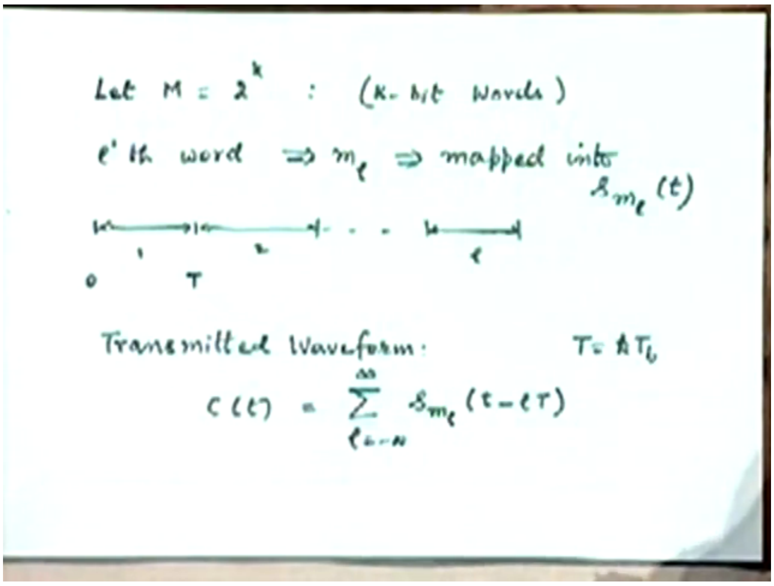

Let us return to further discussion of this, how much more time we have okay 5 minutes okay that is okay, so how we are going to map bit sequences to this waveforms we are going to choose an f bit value of M which is a power of 2 let us say 2 to the power of k so basically we are going to organise our incoming bit stream into segments of your k-bits or words of length k-bits.

Student: Sir you still talking about the receiver, match filter receiver.

Professor: No I am now talking about coming back to the orthogonal family of signals right that discussion is over, so data is segmented into k-bit words right and l th such word, let me denote that by m sub l, m sub l also can take the value either 0 or 1 or upto m minus 1 right but l refers to the (())(48:31) this is you know this is a first block of k-words, this is the next block of k-words right so we are now talking about the l th block of k-words.

Each of these you know could take any of these M value, this m sub l can take the value from 0 to l minus 1 so this is mapped into a waveform, a passband waveform sm sub l t alright, therefore your overall transmitted waveform is c t which is going to be the sum of all these waveforms, this waveform is transmitted every, suppose this interval is t seconds right from 1 block of l symbols, l bits to the next block of l bits right so every t seconds you will be transmitting a new such waveform right so sml t minus lT this corresponds to M.

Student: This will be l K t e.

Professor: Yes you could write it T equal to kT, no capital T would be k Tb right, l goes from whatever fine, no I have said it m sub l is mapped into this waveform whatever that waveform is, I have only discussed in passband pairs, sm lT represents the Mn th passband waveform corresponding to this particular word that we have right fine let me take one example just to finish this discussion, so we will take an example of an orthogonal family.

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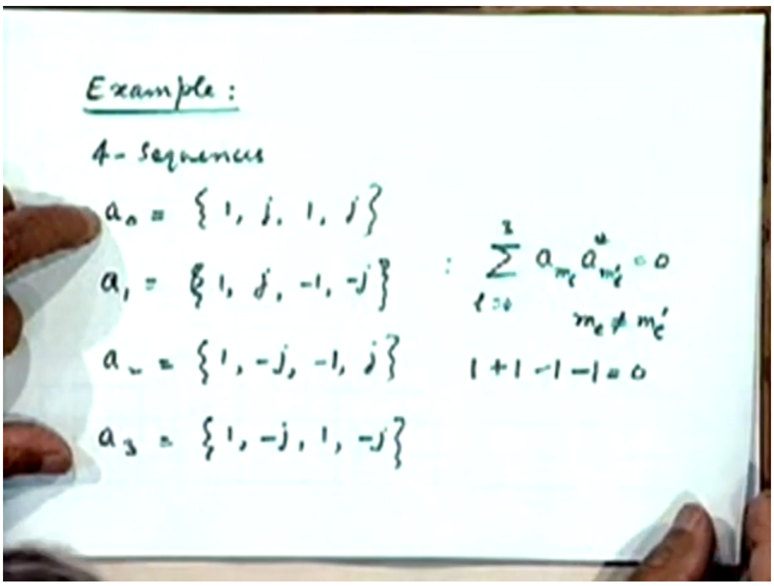

Let me start with foreign 4 sequences in the baseband a0 is the sequence, complex sequence 1, j, 1, j. a1 is similarly 1, j, minus 1, minus j. a2 is 1, minus j, minus 1, j and a3 is, okay you can suggest for yourself that these 4 complex sequences that I have written down here are indeed orthogonal sequences that is if you take any pair of sequences and compute these sum, you will find that a0 your ml could be for example, sorry this should be m prime l where ml prime or l prime sub l is different from m sub l.

So if I choose the same m and m prime to be same of course then this will not be 0 then this will be obviously equal to 4 or may not 4 how much will it be, 1 square minus 1, it will be 0, it will mean it is 0, it is 0 for all situations yes, yes whenever you have complex waveforms the orthogonality is defined in terms of a conjugate, conjugation operation, remember I

Student: Yes sir that is true how do you do it in the case of 4 arguments, 4 parameters, you just illustrated

Professor: Alright let us take one of them, let us take this pair right, you basically multiply on a bit by bit basis right 1, this will be minus 1, this will be minus 1, oh I am sorry this will be minus 1, this will become 1, plus 1, minus 1, then minus 1 I think so the result is here okay, 1 plus 1, minus 1, this will become plus j so it will become, I think it should have been 4, I am in fact a bit confused, yes because you had conjugate that also so it will become 4 this is true for ml not equal to ml prime right.

So suppose you have this sequence of length 4, 4 sequences then I can now define 4 waveforms f 4T

Student: Sir what I mean is 4 sequences you have basis.

Professor: I will come to that, for example see you have decided to use, I want to define 4, I have (())(54:38) define 4 waveforms right basically 4 orthogonal waveforms is what I want to define therefore I should take two bits right this two bits are mapping 1 to one of these 4 sequences I can do that.

Student: (())(54:56) Sir 1 j 1 minus j

Professor: You could think of this as if you have rectangular pulses with

Student: Sir but why is the size is 4 of this sequence, it could have been 2 because you know, then 1 j, 1 minus j, minus j 1 and yeah, these could be complex numbers as well that is beside the going out what does it do to the waveform like

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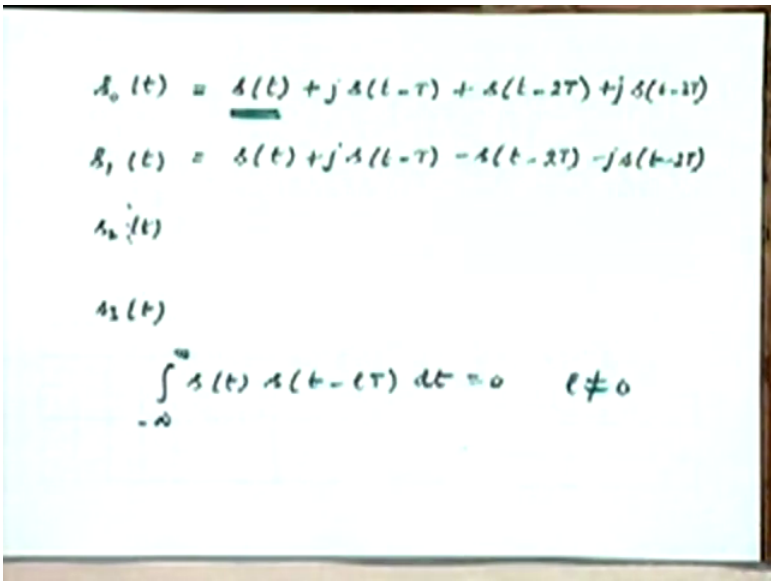

Professor: I have not fully defined the mapping yet if you let me define the mapping I think things will be somewhat clearer, let us go through that complete operation, the 4 waveforms that I am going to define from this sequence is on a forward s t, this is just an example they do not have to be done this way.

Student: There s t is cos cosine something

Professor: Where s t is any element requires which satisfies the properties right we will talk about that in detail and so on right you can similarly find s2 and s3 I think the mapping is obvious how we are doing it, so basically what we have done is we have taken a pulse shape s t right and from that basic pulse shape I have generated 4 waveforms which are mutually orthogonal right, now these are of course baseband waveforms right. To go from here to passband waveforms you would you know translate into real and imaginary parts (())(56:59) you want to introduce the carriers.

Student: Sir this s in itself could be a passband waveform because you have not defined s.

Professor: I am going through the steps that will correspond to the way I have defined orthogonal family right, the orthogonal family I said could be a orthogonal passband family, could be obtained by deriving it from a corresponding orthogonal baseband family right, so first I have constructed an orthogonal baseband family for you right and then from there you go to an orthogonal passband family.

Now how you allocate bits to waveforms is upto us now you have 4 waveforms and we were allocate upto 2 bits in any way you like, obviously this is not a very nice thing to do with the way I have done it right because it will occupy a total interval of upto 4T right whereas we are only transmitting a pair of bits right so its I mean ideally we would like to do it in 2 bit interval rather than 4 bit interval, so this is just an example of construction of an orthogonal family not necessarily to be used literally in a digital modulation scheme right.

Student: Sir only to have interference, because we have seen the 1

Professor: Okay the requirement will be you have to choose s t, no before, you have to choose s t such that st convolve with its on match filter or something it will think 0 not equal to

Student: Corresponding to 1 bit pair last still 4 bit intervals then at after 2 bit interval another waveform is saying

Professor: Yeah let me, you will require to choose st such that these are orthogonal right, the chip s t should be such that, these are orthogonal, if you want to use this method of generating baseband orthogonal waveforms, they are things we have already discussed actually right yes now coming back to your question Tanay

Student: Sir we see that for 1 bit pair we transmit a waveform which (())(59:20) for 4 bits period so one (())(52:22) interference between corresponding

Professor: I am not suggesting that you use such a waveform, this just an example of construction of an orthogonal family okay we will in fact never really use this kind of waveforms, one would generate a more efficient orthogonal representation of the bit pattern than the one which I have given in this example, in fact one can generate MSK like waveforms in the M-ary context but I will not go into those here which are more efficient representation.

I know you may not be very satisfied with this example but do not take this as an example which is a practical example for using a digital modulation scheme, just illustration of the ideas that I have try to give in this class, thank you.