

Digital Communication
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Lecture – 19

Passband Digital Modulations – III: Minimum Shift Keying (MSK)

Kind of general behaviour as that of QPSK in as much as the kind of data rates you can have on this modulation scheme but has the advantage over QPSK that we, very abrupt phase discontinuities of 180 degree plus minus 180 degrees are avoided by trading of this factor with the factor that now you will have phase continuities every bit interval rather than every two bit intervals right.

Basically that is the trade off we have got within OQPSK, offset QPSK and normal QPSK both will give you the same kind of performance in terms of what we call the spectral efficiency that is data rate to bandwidth efficiency, data rate to bandwidth ratio right because for every two bits you are transmitting one of this 4 possible phases right so you will have the same kind of bandwidth efficiency for both.

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Minimum shift keying

Obtained by taking $s(t)$: half-cosine pulse

$$s(t) = \begin{cases} \cos \frac{\pi t}{T} & |t| \leq \frac{T}{2} \\ 0 & \text{Elsewhere} \end{cases}$$

$s(t)$

-T/2 T/2

With the advantage for QPSK that its spectrum will be perhaps slightly narrower as compared to the spectrum of QPSK signal which has very wide very fast fluctuations at every two T_b seconds, T_b seconds. Now we will now look at a variation of OQPSK called minimum shift keying, it is a very popular variation of QPSK, OQPSK in that whatever little problem

you still have with offset QPSK in terms of phase discontinuities or phase chumps that occur are completely removed in this version of the scheme.

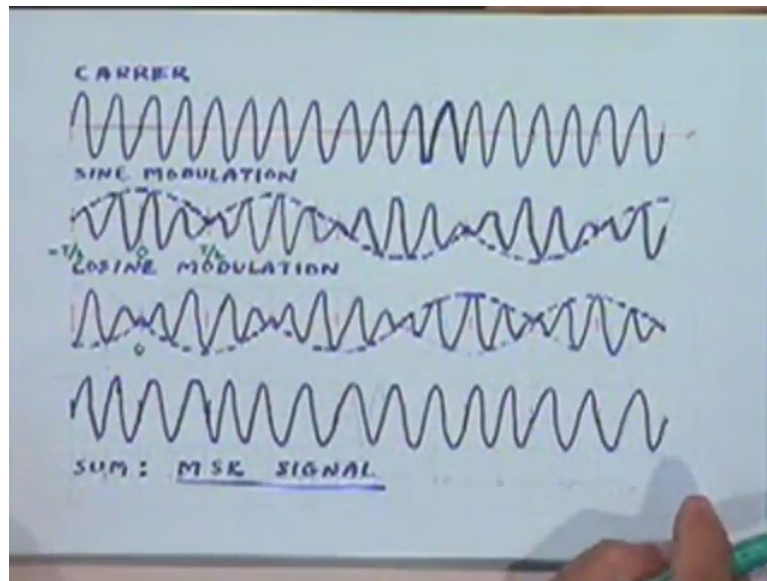
As to why it is called minimum shift keying there is a different reason for which we will discuss slightly later right now we will only discuss it as a variation of offset QPSK actually this name does not come from its similarity with offset QPSK, this name actually comes from its relationship with frequency shift keying right so we will talk about that name, the significance of this name slightly later right now let us see what it is all about.

Basically it is a same thing as offset QPSK in some sense except for a minor difference and the difference is that unlike the offset QPSK in which basically the baseband pulses we are using were rectangular pulses, here we use cosine shape pulses, half cosine shape pulses right, so we take this is obtained by choosing your baseband pulse waveform $s(t)$, $s(t)$ as the so called half cosine pulse which is defined as follows it is equal to $\cos(\pi t / T)$, now this capital T is the symbol interval right, which is twice the bit interval.

So T is equal to $2T_b$, this is equal to $\cos(\pi t / T)$ for $0 \leq t \leq T$, that is equal to 0 elsewhere, so what kind of pulse shape is that if you were to plot it, no that is all, it is a cosine pulse shape, one loop going from minus $T/2$ to plus $T/2$ right so this is the shape of your $s(t)$ instead of the rectangular shape that you use in the offset QPSK right.

In the offset QPSK the incoming bit stream is separated into the I and Q bit streams if you remember and these are represented by sequences of rectangular pulse trains which are offset with respect to each other by half the pulse width right now the only difference is we will replace those rectangular pulses with this kind of a pulse shape, everything else is same the incoming bits stream will be split into I and Q streams which will be in turn represented by pulse shapes of this kind which are mutually offset with respect to each other by half the pulse width right.

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So if the incoming bit stream pulses are like that the I, sorry if the I bit streams are like that and the Q bit streams would be similar in shape except that they will be offset by $T/2$, $T/2$ sub b so if you the resulting this thing is called minimum shift keying, I think I best illustrate this by means of this picture here for minimum shift keying, here the treatment that I have given mathematically on the representation I talked about mathematically is a baseband representation, whereas this is a proper passband illustration of the same idea.

You start with a carrier right of whatever frequency because we are now talking of the passband picture and you have your incoming bit stream divided into two parallel bit streams one the I stream the other the Q stream right each of these streams is being represented by positive and negative pulses of this kind right depending on the particular bit sequence you have, this dotted envelop lines that I have shown here right, this one here and this one here.

The sign of this dotted lines that is this is a positive sign means perhaps the I bit here was corresponds to a 1, here also the I bit corresponds to a 1, here it corresponds to a 0, here it corresponds to a 0, and this interval is let us say this I will choose my reference as 0 here so this is minus $T/2$ and this is $T/2$ right and so on, in the quadrature bit stream same kind of pulses appear here but of course the bit sequence can be totally different from the corresponding bit pattern in the I bit stream.

Because the two bit streams are basically representing alternate bits in the, of the incoming bit stream got it, so in the Q bit stream we have a 0 followed by a 0 followed by a 1 followed

by a 1 right and obviously they will be represented by these kind of pulse, baseband pulses, and how do you do the actual modulation, actual passband modulation, what is the mathematical mapping which takes you from baseband to passband.

Student: Sir how do we get the envelop project this to be upper 1 or the lower 1?

Professor: No that is, that comes later, envelop actually in fact in the first instant if you just ignore what is there inside, in the first instance think of this dotted lines as the positive and negative pulses corresponding to the 1's and 0's that are coming in that particular bit stream, it is from starting from these pulses that we will go eventually to the modulated waveform right so ignore the modulation, the carrier in between to start with right.

To start with you only have this positive and negative pulses in the I stream similarly positive and negative pulses in the Q stream corresponding to some input data stream, alright, is that fine, can we proceed from there, this how do we convert this into a passband waveform, by multiplying this with the cosine carrier and multiplying this with the sine carrier right, I am multiplying the I bit stream or I pulses with the cosine carrier that is what it is.

Well one phase of the carrier this is actually the other way around this I have shown as sine modulation and this as cosine modulation, does not matter which way you do it and resulting waveforms will now be as shown over here after modulation by multiplying these pulses with the corresponding carrier of course the carriers in this and in this are mutually phase shifted by 90 degree because they have to be quadrature carriers, one has to be cosine the other has to be sine right.

Now if you sum them up that is your final passband modulation $I \cos(\omega_c t) + Q \sin(\omega_c t)$ right, you add this two up, you will get your final modulated waveform which now looks like this and what do you see, there is absolutely no phase transition anywhere, phase transition anywhere right you find that at each bit interval there is a absolute continuity of phase from one bit interval to the next bit interval right.

This is the physical picture now before I go into the mathematics and illustrate or demonstrate to you that such a thing should happen mathematically can you see what is happening physically here from the picture why there are no phase transitions although we are doing things in almost exactly the same way as we were doing in the case of offset QPSK, how do you avoid certain phase charms in this case by just using the pulse shape?

The basic thing that is happening if you notice is that instead of, see what was happening in OQPSK, at every bit interval a new data bit was coming along in either the I channel or the Q channel which was causing a phase jump of $\pi/2$ right either plus $\pi/2$ or minus $\pi/2$ right, now that phase jump is replaced by a continuous phase change over the bit interval instead of they are being a phase jump just at that instant that if equivalent amount of phase change is actually obtained through a gradual process of phase change over the bit interval rather than suddenly at the bit transitions, that is the essential difference.

And that is brought about by these envelop modulations by this right, in fact if you notice it carefully take this bit interval you will find the way I have plotted it exactly two cycles of a carrier take the next bit interval you will find a different number of cycles, in fact we will be able to see that one can listen in each bit interval the presence of one of two possible frequencies of the carrier right.

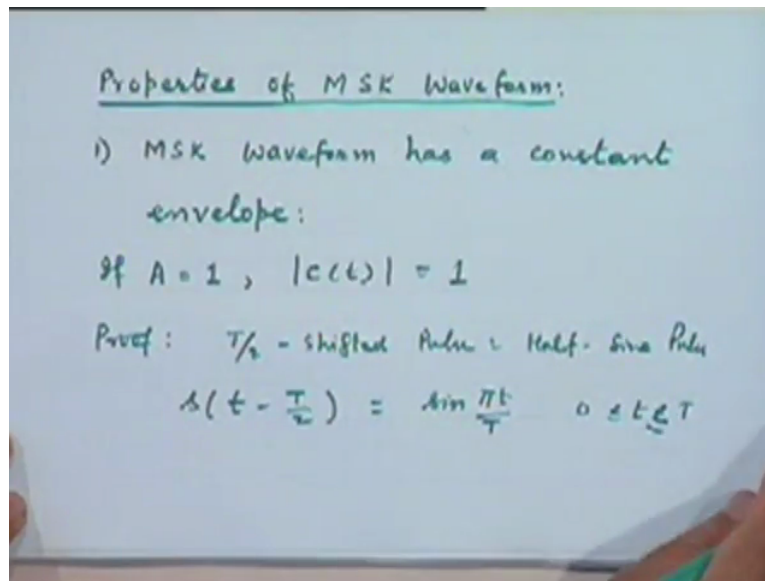
As if the carrier is not a fixed frequency carrier but you are shifting from one frequency to another frequency from one bit interval to the next bit interval because continuous phase change also implies a frequency modulation right, a continuous phase change implies a continuous frequency change in fact one can see that this final modulated waveform can also be thought of as an FSK waveform, as a frequency shift key waveform right because in every bit you will have one or the other of two possible frequencies which we will see right. So this is the physical picture if it is clear then we will go into some mathematics.

Student: Sir why is the frequency difference that you take when the

Professor: We will see that mathematically it is better to appreciate that mathematically but as you can see sometimes the intuitively I can tell you why it is happening let us get a intuitive picture right, remember in the, always go back to your offset QPSK picture, in the offset QPSK case there is a phase jump of 90 degree every bit interval but that phase jump could be a plus positive phase jump or a negative phase jump that means over the bit interval you like the phase either to increase constantly or to continuously decrease.

When it is increasing gradually you having a increase in frequency when it is decreasing gradually you have a decrease in frequency right, basically that is the idea, so is that okay? Is the intuitive picture somewhat clear? Then the maths will make it further clear if this is okay, can we proceed further? Any questions? So let us return to the maths now.

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By looking at some basic properties of this MSK waveform, this we will call by in short we will just call it MSK, the first property that I like to talk about is the fact that MSK waveform has a constant envelope right, okay that is intuitively clear here although the individual I and Q channel seem to be amplitude modulated DSBSC kind of waveforms the final overall modulated waveform MSK waveform is a constant amplitude or constant envelope signal right.

Let us see mathematically how we can prove that but all you have to do is look at the complex envelope of the corresponding baseband signal and look at the magnitude of that complex envelope right so if you write down the expression for MSK before doing that just what we will make, we will assume is that we are assuming A equal to 1 remember I said that this A is amplitude of this pulses okay.

We will take the amplitude of this pulses to be 1 and prove that the envelope which is the magnitude of the complex envelope $c(t)$ will also be equal to 1 so if we take the amplitude of this pulses be equal to 1 then the magnitude of the complex envelope will also be equal to 1.

Student: Sir one of them is cosine key and there is sine key.

Professor: That is right, so I think it is quite obvious proof is really a one line proof starts by the observation that we are basically in the I and Q channels having two basic pulse shapes the cosine pulse shape, half cosine pulse shape and its $T/2$ shifted version which is the sine

pulse shape right so $T/2$ shifted pulse is a half sine pulse right that is $s(t - T/2)$ we can write as $\sin(\pi t/T)$ for $0 \leq t \leq T$.

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$$c(t) = \sum_{l=-\infty}^{\infty} a_{2l} \delta(t - 2lT_b) + j \sum_{l=-\infty}^{\infty} a_{2l+1} \delta(t - (2l+1)T_b)$$

$$= \pm \cos \frac{\pi t}{T} \pm j \sin \frac{\pi t}{T}$$

$$|c(t)|^2 = 1$$

\Rightarrow No amplitude fluctuation in an MSK waveform

And using this fact going back to our expression for $c(t)$ if you recollect from our last times discussion we have this expression for offset QPSK, $a_{2l} s(t - 2lT_b) + j a_{2l+1} s(t - (2l+1)T_b)$ this corresponds to the I bit stream, take the Q bit stream, quadrature bit stream or the odd bits, this will be $a_{2l+1} s(t - (2l+1)T_b)$ can you see it there.

Student: (18:31) lot of reflection down, something is gone wrong, sir something is gone wrong here, sir even yesterday, sir contrast is bad sir, even yesterday same thing happened.

Professor: Okay can I move it over here, what is happened?

Student: The camera is focused here.

Professor: It is okay, so can I proceed ahead or there is still a problem, some light has to be switched off right.

Student: Sir the cassettes are available now?

Professor: I believe so, I was told that they were going to be available, I have not checked, I will check today. I can see from here, there is obviously some glare coming from somewhere. I think it is an angle at which you are sitting. I am asking these directors. Okay so let us look at the complex baseband representation of an offset QPSK from which the MSK is derived right

and all you have to do is substitute for this, this is a cosine waveform, cosine pulse and this is a sine pulse.

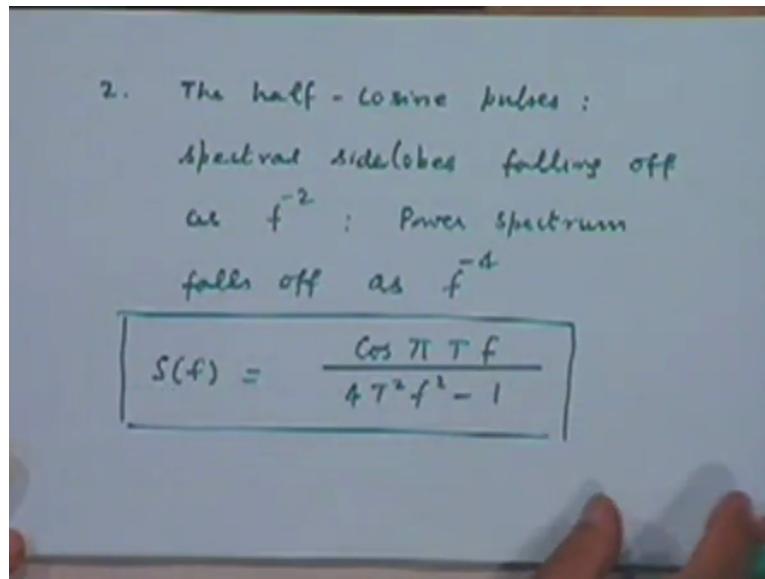
Basically this is what you have to appreciate right because this is a same pulse shape as this except that it is shifted by $T/2$ or $T/2$ right and since both a_{2l} and a_{2l+1} essentially are positive or negative amplitudes equal to plus minus 1 right so we will we can write this as equal to plus minus cosine $\pi t/T$ plus minus j times sine $\pi t/T$ right and it is quite obvious that no matter what this signs might be as far as the magnitude of the envelop is concerned, it is obviously going to be equal to 1 right.

So that shows that there is no amplitude fluctuation whatsoever in an MSK waveform, now that is a very very desirable property in any digital modulation waveform right for reasons for which we have discussed a number of times in the last two classes, for example this will make it relatively insensitive to non-linearities which may be present in your system right and that is a measure reason for the popularity of MSK waveform in such application.

Particularly let us say satellite channels where non-linearities are a fairly important class of problems to worry about, now the second reason for its popularity comes in the second property that we are going to discuss which I will discuss very briefly here that has to do with the power spectrum of the MSK waveform, you see bandwidth efficiency is a very important factor for us, the QPSK and MSK waveform modulation are very similar in many respects.

But because of the fact we are using this pulse shapes to be cosine which have a gradual decay, not really a gradual decay at least it do not end up as abruptly as rectangular pulses do, they have some kind of a roll-off between from one bit interval to the next, the power spectrum corresponding to a MSK waveform has a sharper decay compared to a corresponding decay of the QPSK waveform or OQPSK waveform.

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So the half cosine pulses that you use here they are associated with a better spectrum, their spectral side lobes, the rectangular pulses have a sinc function as a spectrum or if you look at the power spectrum it is sinc square right the side lobes of that we know do not decay very fast whereas the cosine pulses that we using here has spectral side lobes which fall off as f to the power minus 2 rather f to the power minus 1 actually power spectrum will off more rapidly is not it? f to the power minus 4 right.

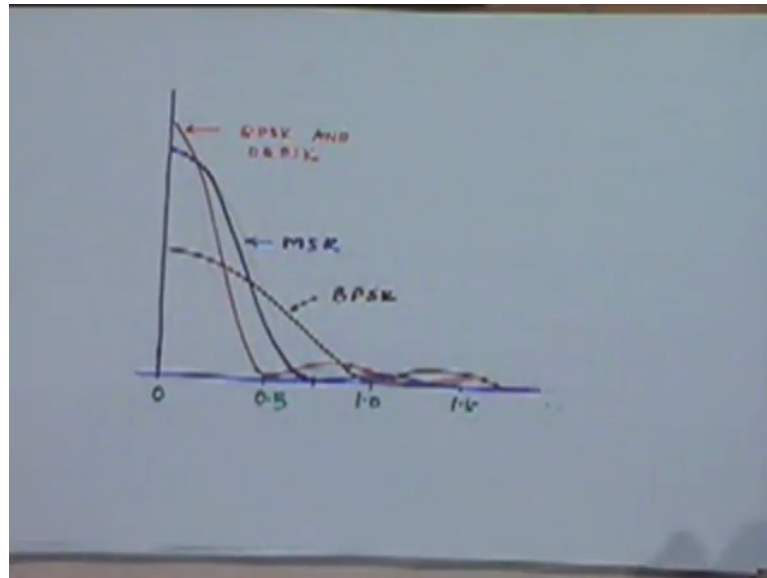
If the side lobes themselves fall off at the rate of f to the power minus 2 and this is a spectrum of the pulse as far as the power spectrum is concerned they will off even more rapidly as f to the power minus 4, in fact one can show the precise spectral expression for an MSK waveform when you assume the consecutive bit streams to be independent of each other under that assumption can be seen to be given by this expression.

I will just showing this for the sake of completeness, a result which you can easily appreciate is not it? Because you have seen what kind of pulse shape you will get if you have a raised cosine kind of power spectrum when you did the nyquist criterion all that right there we working with this kind of function in time, we now have a dual function in frequency right because the corresponding pulse shape is the cosine pulse shape in time domain.

So the results are easy to appreciate from what we have discussed earlier so this is the power spectrum, this is not the power spectrum this is the spectrum of the cosine pulse, half cosine pulse right, the power spectrum would be magnitude square of this, so this is the second

advantage which I have plotted here in terms of a comparison with other spectra, spectra for the pulses.

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I have compared here in this picture the spectral efficiency of binary phase shift keying, quadrature phase shift keying as well as offset quadrature shift keying and MSK and you can see the relative standing of these, no it is actually better and not between the two in some sense yeah we will have to see that, that is very easy to appreciate. As far as BPSK is concerned its spectral efficiency is poor as you can see.

Because here your bandwidth is nearly governed by the bit interval right and depending on the data rate you have, your first main lobe, first zero crossing occurs at $1/2T_b$ of that right whereas QPSK since you are looking at two bits at a time your effectively your main lobe bit comes down by a fraction of 2 right for the same data rate, your spectral efficiency increases by a fraction of 2.

However there is a considerable amount of energy in the side lobes and this is what is of great concern, when you do the final band limitation although roughly say that the spectral efficiency is good in practice the spectral efficiency is going to be very poor, reason is we are not really going to restrict the bandwidth through just the main lobe, we are going to have to restrict the bandwidth upto a point where there is significant energy in the spectrum right because if you do not do that we are going to have the other problem.

Which is other problem? Suppose I band limited very strictly according to this main lobe width the non-constant envelop problems right, so one has to choose between the two.

Student: Sir, how to correlate this non-constant like I do not understand the problem.

Professor: You see basically what does it is a band limitation which causes a problem right, the more sever the band limitation, the more severely your sudden transitions in time domain will be flatten out, will be smeared out and they will be therefore the amplitude variations will become larger after filtering, if you filter signals which have sharp discontinuities and you filter them very heavily they are going to be much larger magnitude variations in the envelop.

Then if you were to allow most of the spectrum to pass through, if you allow most of the spectrum to pass through in the filtering process that means you are not really carrying out much filtering right and (())(28:56) the waveform from too much, it will not introduce unnecessary amplitude fluctuations right so you have a trade off to worry about because there is a significant amount of energy in the side lobes in the case of QPSK, no QPSK.

On the other hand as far as MSK is concerned there is very little energy in the side lobes in fact even what I have drawn here comparatively speaking should be much lower than what I have shown right as far as side lobe power is concerned of course you know the total energy is constant so if you remove the energy from the side lobes it has to come somewhere else it has to come in the main lobe and therefore you will see that the main lobe width increases as compared to QPSK is not it?

Because the total energy is same of the signal so if you are taking away the energy from the side lobes it has to come in the within the main lobe therefore you will find that the main lobe width increases but I still now have the advantage that just increasing the bandwidth slightly I have most of the energy of the signal accounted for and therefore we can expect very little envelop fluctuations or amplitude fluctuations after filtration of such a signal before transmission right which is a reason why MSK is really popular in those kinds of channels.

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3) MSK Viewed as FSK:

$$c(t) = \pm \cos \frac{\pi t}{2T_b} \pm j \sin \frac{\pi t}{2T_b} \checkmark$$

$$= \pm e^{\pm j 2\pi \left(\frac{t}{4T_b}\right)} \quad \left\{ \begin{array}{l} T_b \text{ at } t=0 \\ T_b \text{ at } t=T_b \end{array} \right.$$

: Within each interval: freq. of $\left(\pm \frac{1}{4T_b}\right)$

$$c(t) = e^{j\theta(t)}$$

Is it alright? Appreciate this point now better by looking at this spectrum, relative spectra of the various related schemes modulation schemes, any questions? Okay, the third property we will discuss is what we discussed intuitively a few minutes ago that is MSK viewed as an FSK signal and that is also extremely easy to appreciate again let us start from the complex envelop and a few minutes ago I showed to you that we can write this complex envelop of the MSK waveform as this kind of an expression.

I wrote t there, I am replacing that with $2T$ sub b which you can appreciate can also be written as plus minus e to the power plus minus $j 2 \pi t$ upon, I have just deliberately introduced this factor of 2π instead of π and t by $4T$ sub b for let us say t in the l th interval going from lT sub b to $l + 1 T$ sub b right in any between, in the $l + 1$ th bit interval we can write the complex envelop either in this form or in this complex exponential form right.

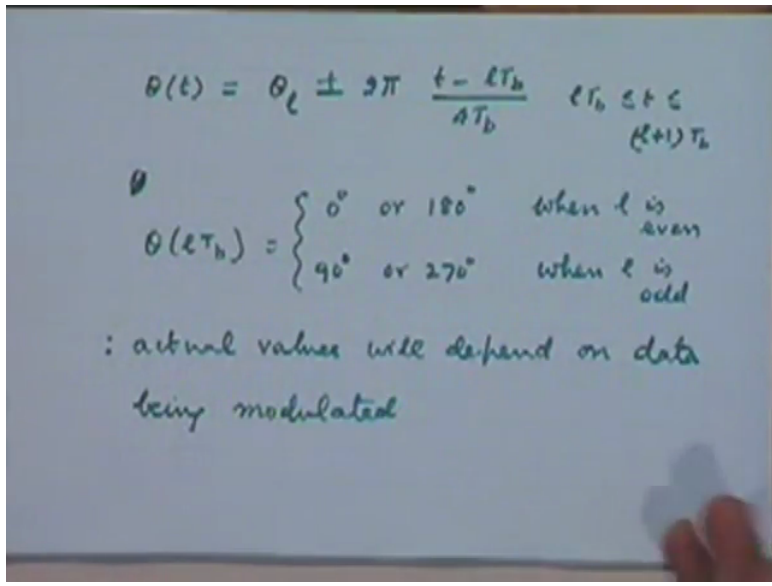
And of course this signs will be dependent on what specific data stream is coming along this signs are really the function of the data stream that is modulating the carrier and this signs will change abruptly from one bit interval to the next because as data stream keeps coming in, now if you can look at this, if you look at this complex envelop waveform particularly looking at this exponent, what does the exponent tell you?

This exponent $j 2 \pi t$ by $4T$ sub b , what kind of phase function is this, it is a linear phase function or a constant frequency function within the bit interval of you were to look at the frequency which is the instantaneous rate of change of phase right or derivative of the phase function, it

is going to be a constant function right so within each interval, we see a frequency of either plus how much plus 1 by 4T b or minus 1 by 4T b right.

A constant frequency of either plus 1 by 4T b or minus 1 by 4T b right, of course if you were to look at this waveform in association of the carrier that will be f not plus 1 by 4T b or f not minus 1 by 4T b so that is the frequency shift keying that we were talking about right so this you can read the frequency is of the complex envelop as frequency changes of the actual modulated signal finally right.

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So that is the first thing that we appreciate let us rewrite this as this c t is equal to e to the power j theta t right, if you were to rewrite like this I can write an expression for theta t as follows, theta as an function of time now within each bit interval we know what is the behaviour is, you either increase linearly, the phase either increases linearly or it decreases linearly right that means you start from somewhere theta sub l which is a phase starting phase of the previous, which was the okay ending phase of the previous bit interval right.

And you are now getting to the next bit interval and from there onwards your phase is either increasing or decreasing linearly eventually going upto, so this is the mechanism by which your phase is in, is not it? What is the expression between, what is the difference between this expression that I wrote here and this, I have taken cognisance of the fact that we have a starting initial phase which will dictate what is whether this is plus or minus here right.

And then we have right so this plus minus sign here I have put in θ itself as a function right so depending on what your initial phase was or what your ending phase of the previous bit interval was let us call that θ the final signs will be decided right which I am not going in details which is easy to appreciate that the sign of θ will have a bearing on whether you will have a plus here or minus right because after all you can split this up into $e^{j\theta}$ to the power $j\theta$ into $e^{j\theta}$ to the power plus minus the rest of the function right.

So depending on whether $e^{j\theta}$ gives you plus, I mean it can give you 4 possible phases again.

Student: Sir, θ is a previous phase.

Professor: Yes θ is a previous phase alright so I have subtracted this minus $t - T_b$ because we are now looking at the phase behaviour in the after $t - T_b$ after the time instance $t - T_b$ right and from there onwards it is increasing either linearly with time or decreasing linearly with time and to what extent will the phase change, let us say if you put t equal to $1 + T_b$ it will go to 2π upon $4T_b$ right, π by 2.

So plus minus π by 2 which is precisely what we expected in normal offset QPSK the only difference is instead of that happening suddenly at that point you have a gradual phase change eventually culminating in a total phase change of either plus π by 2 or minus π by 2 so therefore we can conclude so of course I should write here $t - T_b$ less than T less than or equal to $1 + T_b$, this is our expression for the next interval like that.

So we can now say θ we can broadly conclude will be either 0 degree or 180 degree right either it will be of this form, now this is from our appreciation of the QPSK offset QPSK signal that we have discussed a number of times before this will be the case when l is let us say even right and it will be 90 degrees or 270 degrees when l is odd, illustrate? Illustration will be bit difficult but it is very easy to argue it out.

Let us say you with some initial phase in let us say you are starting with the odd bit case or let us say you are start with 0 phase right after 1 bit interval you will either go to plus 90 degree or minus 90 degree right so it will be plus 90 degree or 270 degree after the next bit interval you will again go to either 180 or 0 degree right and so on and so forth so it is very easy to appreciate this fact.

Of course the actual values will depend on the data stream that you have okay and what theta l plus 1 has to be will be dictated by the data stream right only thing is accordingly this plus minus sign will change if theta l plus 1 has to be less than theta l there will be a negative sign so theta l is actually more than theta l by 90 degrees then there will be positive sign. Let us proceed further with this discussion and rewrite.

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$$\begin{aligned}
 c(t) &= \cos \left[2\pi \left(f_c t \pm \frac{1}{4T_b} \mp \frac{s}{4} \right) + \theta_c \right] \\
 &= \cos \left[2\pi \left(f_c t \mp \frac{s}{4} \right) + \theta_c \right] \\
 f_c &= f_1 \text{ or } f_2 \quad (1) \text{ or } (2) \\
 f_1 &= f_c + \frac{1}{4T_b} \\
 f_2 &= f_c - \frac{1}{4T_b}
 \end{aligned}$$

Now let me look at the modulated waveform with carrier since it does not really cause any confusion I will continue to use the symbol $c(t)$ so I am using here sometimes $c(t)$ for complex baseband waveform sometimes for the real modulated waveform without changing the notation because it does not cause any confusion if it causes any confusion let me know.

So the real waveform carrier with carrier can be written as I think I will put a bracket like this here all I am doing is appreciating the fact that the carrier is going to be associated with this phase, extra phase right you will have the term f not t of course and this is extra phase you have to keep that in mind which you can write as plus minus t upon $4T_b$ right, that is an increase or decrease minus plus, I am just substituting this expression.

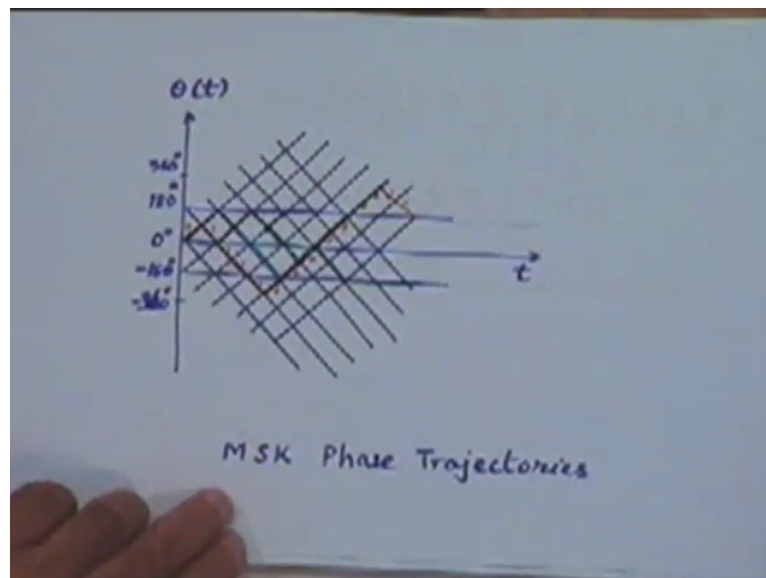
I think I let me keep it here right minus plus 1 by 4 plus theta 1 all I done is substituted this over here along with $2\pi f t$ right, so you can right the modulated signal with carrier in this form which you can write if I define I already appreciated the fact that f not plus 1 by $4T_b$ can be taken as one of the two possible frequencies and f not minus 1 by $4T_b$ can be taken as the other frequency.

So I will write this as $2\pi f_i t$, the f_i is the instantaneous or frequency in that particular bit interval, f_i into t , f_i into t minus plus $\frac{1}{4}$ plus θ_i .

Student: Sir what is this $\frac{1}{4}$ for?

Professor: This $\frac{1}{4}$ comes from this $\frac{1}{T_b}$ upon $4T_b$ right and that basically takes care of the fact that every bit interval you either increase your total phase by $\pi/2$ or decrease it by $\pi/2$ right because 2π into $\frac{1}{4}$ is a final phase right and it is this final phase change is occurring gradually through this frequency change process right, basically that is what the whole idea is.

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Now that corresponds to f_i equal to f_1 or f_2 that is i is equal to 1 or 2 and f_1 is f_0 plus $\frac{1}{4T_b}$ and f_2 is f_0 minus $\frac{1}{4T_b}$ alright, one can also whatever we have discussed so far one can you know this equation for $\theta(t)$, $\theta(t)$ behaves like this with time can be expressed in terms of a what is called a phase trajectory diagram right in which a basically in going from you see there is a linear either you are increasing the phase linearly or decreasing the phase linearly.

So in going from 1 bit interval to the next bit interval you could be anywhere along this grid depending on the input data sequence that you have right because depending on the input data sequence that you have in any specific bit interval let me take arbitrarily you might be initially here your θ_i value will be initial here and then you will be either increasing

linearly or decreasing linearly from there onwards right where all successive phase differences are π by 2 values.

And all this if I am exceeding 360 degree here you must interpret the phases to be modulo 360 right so I have plotted this grid here where a typical phase trajectory for example could be like that or another one could be sorry yes it could be like that right, why not they will depending on the input bit sequence that you have they may not be of phase change from one bit to the next.

Student: There has to be phase change (0)(45:32)

Professor: No it does not have to be, suppose the two bit stream, two successive bit stream both the (0)(45:38) are exactly the same as before right bit changes are required so in that case, yeah I think perhaps in this scheme there has to be phase change to the extent yeah I think that is a important point I will have to think about it, you also think about it, I think there may be a truth in what he says.

Yeah right there may be, there has to be a phase change I think but I will have to verify it and be sure so I think the trajectory we are talking about will avoid this horizontal, in fact you will notice that actually I have this are only reference lines so I got confused with that so you should really either go up or go down that is how these are the possible phase trajectories and so on.

So these are typical MSK phase trajectories and represented in this manner so we have seen the FSK interpretation of MSK now there is an interesting thought here we shall just like to share with you, you could therefore obtain the same modulated waveform not through the modulation process that we talked about, what is the modulation process that we have talked about.

Our modulation process consist of take your incoming data stream, split it into two parallel streams which are quadrature delayed with respect to each other right and represent the data in each of the I and Q streams by half cosine pulses of corresponding sine pulses right and then these half cosine pulses multiply the carrier and these two carriers are simply the quadrature carrier and they are simply added after modulation, that is the modulation, that is a mapping process from data bits to the MSK waveform that we have discussed so far.

But we could as well have carried out that modulation process through a different mechanism for example suppose the incoming bit was 0 right I could have linearly, I could have associated with the frequency f_1 the next bit is 1, I could have associated with the frequency f_2 right that is decrease in phase and so on and that would have resulted in a similar kind of phase trajectory, as far as phase trajectory is concerned of the signal it would have resembled an MSK waveform in terms of properties.

So I am not now splitting the incoming data into I and Q streams just looking at successive bits every bit interval and associating a frequency f_1 or f_2 with each bit right.

Student: Sir in the bandwidth would have been larger.

Professor: No because you have precisely the same waveform as far as basic waveform properties are concerned they remain the same is not it, because you are following the same phase trajectory.

Student: What do you mean by associating frequency () (48:44)

Professor: Mapping, mapping of the data to the waveform see after all what does a modulator do? Basic function of modulator do not forget about it, its basic function is to map data bits onto a waveform now we have discussed one mechanism in a context of MSK for this mapping, I am discussing a possible different mechanism right in which successive input bits are associated corresponding to each successive bits let us say 0, I generate a cosine, a carrier with a frequency f_1 right.

The next bit which may be a 1, I generate the same carrier with a frequency f_2 maintaining the phase continuity right so effectively I will generate a waveform whose properties will be the same as that of the MSK waveform you have discussed because MSK waveform also in successive bit intervals goes through frequency changes of f_1 or f_2 right, it may be f_1 it may be f_2 let me physically generate those f_1 f_2 by associating those with actual bit values directly I can do that, that is how it is an FSK waveform right you can think of this as an frequency shift keying waveform in that manner, not clear?

I do not think you are all very happy about that, is there a doubt about this? Aaj ke liye bahot ho gaya, yeah that is a question I was going to come to but he says aaj ke liye bahot ho gaya. We will start from here next time.