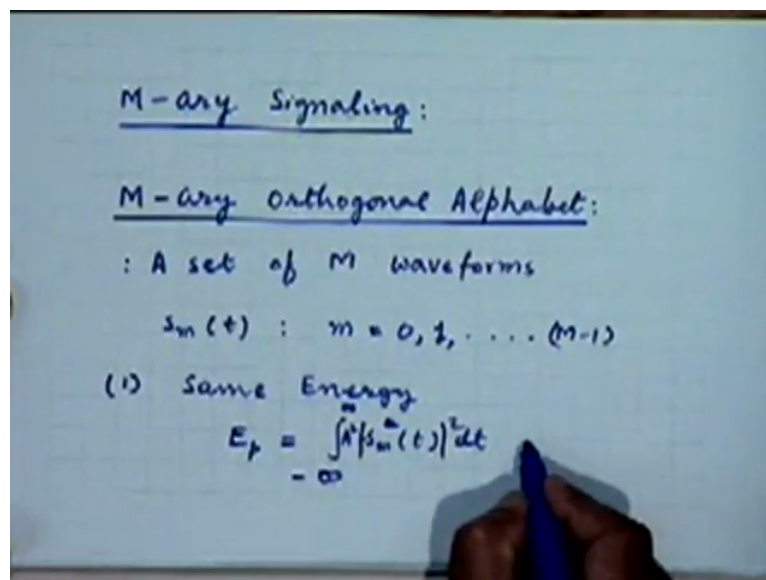


**Digital Communication**  
**Professor Surendra Prasad**  
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**Indian Institute of Technology, Delhi**  
**Lecture – 16**  
**M'ary Baseband Digital Modulation Techniques**

We talk about digital modulation schemes right and we have described the process of digital modulation as one of mapping a message sequence into a set of, a sequence of waveforms right this mapping essentially is carried out so that we can eventually communicate on a waveform channel right and we have made a review of what kind of constraints are present on a waveform channel namely power constraints and bandwidth constraints and how to design waveforms keeping these constraints in mind.

Basically last time we discussed the case of binary modulations and binary baseband modulations to be more specific and today we will extend our talk on baseband modulations to consider a case of M-ary situations, that is M-ary digital modulations in the baseband situation, now M-ary modulations, M-ary baseband modulations or baseband waveforms for M-ary modulations are what we are going to discuss today.

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And as before we can do this discussion we can carry out this discussion in the context of either bandwidth constraints channels or in the context of power constraints channels, at the moment we are going to look at waveforms which, in which there is no bandwidth constraint, there may be a power constraint but there is no bandwidth constraint and for this kind of

waveform channels primarily as I was telling you last time there are two basic kinds of M-ary alphabets that we can use right and these are orthogonal and simplex.

Let me first take the case of M-ary orthogonal signal, here we use a set of M waveforms so we use a set of this M is the same M symbol that we use for M-ary here, a set of M waveforms, let us call them  $s_m(t)$  where small m takes a value let us say between 0 to m minus 1 okay, having the following properties first they all have the same energy  $E_p$  right.

They all have the same energy, that is if you are going to compute the energy of each of these signals this  $E_p$  referring to the fact that this is analogy of the pulse p corresponding to the waveform  $s_m(t)$  which is  $\int s_m^2(t) dt$  more precisely you can write some amplitude A square and we can take the mod square if you are talking about complex waveforms right, so  $A^2 \int s_m^2(t) dt$  between 0 to infinity or minus infinity to infinity whatever you like to use.

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$$(2) \int_{-\infty}^{\infty} s_m(t) s_n^*(t) dt = 0, m \neq n$$

$$A = 1 \text{ or } \sqrt{E_p}$$

$M = 2^k$  : permits convenient mapping

$$(3) \int_{-\infty}^{\infty} s_m(t) s_n^*(t - \tau) dt = 0$$

$\forall m, n, \tau \neq 0$

So they have the same energy and secondly they are orthogonal to each other in the same way that we discussed for the binary case, so the second property that this satisfies that if you take any two of these waveforms let us say  $s_m(t)$  and  $s_n(t)$  conjugate, multiply them, integrate, the result is 0, yes that is important, for m not equal to n, because for m equal to n this will reduce to the energy integral right, the value of this constant A that you have got here in this expression, this can be chosen as per our convenience.

Sometimes we choose  $A$  equal to 1, sometimes we can choose  $A$  equal to square root of  $E_p$  so that this basic signal  $s_m(t)$  is having unit energy right so depending on the convenience so you can choose  $A$  to be either 1 or square root of  $E_p$ , when you choose  $A$  equal to 1 that implies that energy in  $s_m(t)$  is equal to  $E_p$ , when you choose  $A$  equal to square root of  $E_p$  that means you are considering a normalized version of  $s_m(t)$  whose energy itself is unity.

So that is a matter of convenience whatever you like to choose you can choose, what about the value of  $M$ , for convenience of mapping sequences into waveforms and typically we are going to work with binary sequences right which you are going to map into waveforms, it is preferred to choose  $M$  to be a power of 2, so  $M$  is equal to  $2^k$  which permits convenient mapping alright because we can take a sequence of  $k$  bits coming in and decide on depending on that sequence one of this  $M$  waveforms for mapping, for transmission right.

Now although I have talked about orthogonality in this sense, usually and I have taken this time limits to be infinite usually each of these signals will be time limited right so if for example you have a strictly time limited pulses then we can talk of, in fact let us ignore that fact for a time being but what is more important is we are transmitting these waveforms from one set of bits to another set of bits, one set of  $k$  bits will be mapped onto this waveform then the next set, then the next set and so on.

There is another condition of orthogonality which if satisfied with a waveform is really helpful at the receiver and that is  $s_m(t)$  is not only orthogonal to  $s_n(t)$  for  $m \neq n$  but also to  $s_n^*(t - T)$  and now it is for all values of  $m$  and  $n$  right including  $m = n$  that is we would prefer the signals set that you have selected to be orthogonal not only in this sense but also in this sense where we are looking at the correlation essentially you can think of this integral as some kind of a correlation between the two waveforms.

The correlation between  $s_m(t)$  and any translated signal from the set, the amount of translation is a multiple of the symbol duration, the waveform duration,  $l$  cannot be 0,  $l \neq 0$  right, because there is already here that condition is already here for  $l = 0$  we already have  $m \neq n$  now when you put  $l = 0$  then  $m$  and  $n$  have to be different that is why I have put that condition separately and this condition separately right.

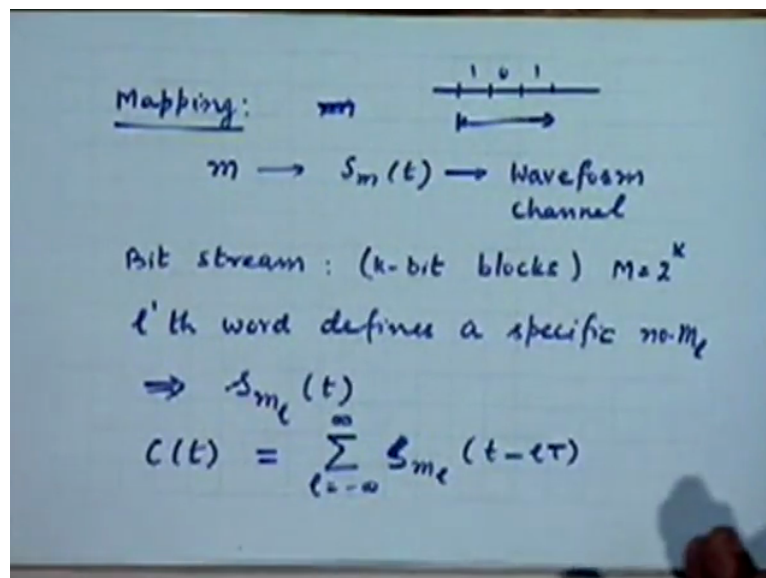
For  $l = 0$  we cannot allow  $m = n$  that is the only difference otherwise yes right they are the same, so this only says that a signal in the set is orthogonal to every other signal in the set with and without translation and the amount of translation you are talking about is

symbol duration or the waveform duration right of course it is better to call symbol duration as long as I am maintaining the limits to be infinity here but it is obvious that if each of this waveform is itself limited to T seconds then the second condition will be always satisfied without any problem right.

If not then we have to make sure that (( ))(10:16) through this integral so is this last point clear? The third condition that I have talked about here will be always satisfied for a certain waveforms in which every waveform has a duration of T seconds because the moment you translate it by a multiple of T seconds there will be no overlap between the original waveform and the translated waveform and therefore the product will be 0 and integral will be automatically 0.

Now this is a very useful and important property just like this is at the receiver, remember this condition is important so as to distinguish between different kinds of signals that you may like to transmit corresponding to different message signals, this condition is important again from the point of view of inter-symbol interference and things like that, that is signal transmitted in one interval does not interfere with that transmitted in the subsequent to right in some sense.

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And the sense that you are looking here is that of correlation that it has no correlation with signal transmitted in another symbol direction okay. So the mapping, let me although we have discussed the mapping sufficiently but just to complete the discussion a mapping that we are

going to do is such that we will take the  $m$ th symbol call it  $m$  we have  $M$  possible symbols, capital  $M$  symbols,  $0$  to  $M$  minus  $1$ ,  $M$  denotes the  $m$ th such symbol.

This will be mapped onto the waveform  $s_m(t)$  which in turn will be transmitted onto a waveform channel right and the way this will be done is that you have bit stream that is coming in, incoming bit stream because usually most sources will be binary in nature alright will be broken up into  $k$ -bit blocks right so broken up into  $k$  bit blocks or as you can call  $k$  bit bytes or  $k$  bit words or whatever you like to call them.

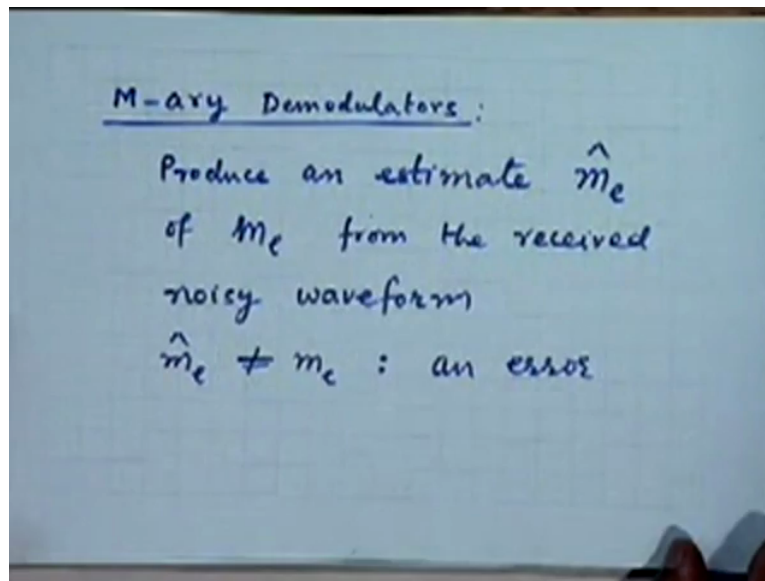
And the  $l$ th word that is during the  $l$ th time interval defines the specific number  $m_{sub\ l}$  right which implies that you will have to transmit the waveform corresponding to the index  $m_{sub\ l}$  right, the symbol  $m_{sub\ l}$  which will be  $s_{m_{sub\ l}}(t)$  and your overall transmitted waveform if we call that  $c(t)$  will be the sum of these waveforms from symbol to symbol for all  $l$  right the sum of all these pulses is what you finally transmit of course these pulses are all mutually displaced with respect to each other by symbol detection.

Student: Sir what is the  $k$ -bit block (13:38)

Professor: Remember I defined  $k$  a few minutes ago, this  $k$  is related by  $M$  equal to  $2^k$  to the power  $k$  right so you take an input incoming bit stream that is coming along let us say you select  $k$  equal to  $3$  so you select the symbol value here, here and here, look at this, this will define a specific symbol by which you are going to denote the sequence right call it  $m_{sub\ l}$  for example in this case you may use a symbol  $0$  to  $7$  right for a  $3$ -bit block and then depending on what this value is you will choose a specific waveform for transmission and the final transmitted waveform will be a sum of all this pulses coming one after another right.

And of course this capital  $T$  here will correspond to the direction over which these three symbols have been accumulated or this  $k$  symbols have been accumulated because from  $1$  set of  $k$  symbols you have to go to the next set of  $k$ -symbols right that is a way the mapping will be done, so instead of each waveform carrying binary information it is carrying information about a block of bits rather than a single bit.

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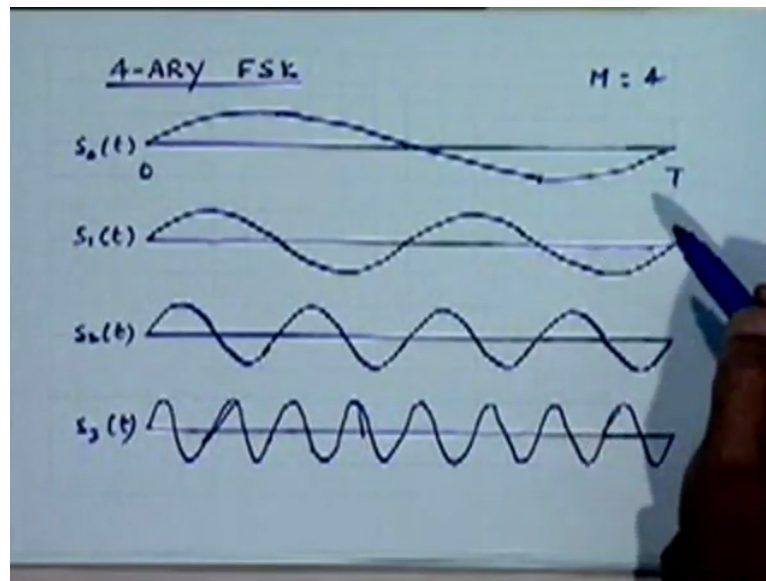
That is the essential difference between M-ary modulations, modulations and binary modulations right and there are advantages and disadvantages of doing things like that I am sure you can think of some of those yourself, at the demodulator which we will discuss in detail separately but broadly what the demodulator will now have to do because we have talked about binary demodulators at least very crudely earlier.

Let us very crudely talk about the M-ary demodulators, it will have to somehow produce an estimate  $\hat{m}_l$  of  $m_l$  from the received waveform which maybe noisy so from the received noisy waveform right, if somehow we produce an estimate which is different from what was actually transmitted you have committed an error this indicates that you have committed a mistake that is you are now going to decode your bit sequence wrongly, fine, is the basic concept of M-ary modulation clear and M-ary orthogonal modulation. Let me illustrate by means of a diagram.

Student: What is (( ))(16:40)

Professor: Okay that is something we will be talking about in detail when we come to demodulators right, just like in the binary case we have to produce a decision that whether a 1 or a 0 was transmitted here I want to indicate in both the demodulator functionally has to do, it has to produce functionally an estimate of  $m_{sub} l$  right, how it is to be done is something we will take up when we talk about demodulation in general.

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Now let me give you a few examples of orthogonal waveforms I have taken the value  $M$  equal to 4 for this examples, for illustration, there is a set of 4 waveforms which you may call  $s_0(t)$ ,  $s_1(t)$ ,  $s_2(t)$ ,  $s_3(t)$ , right which you may use for the situation that  $M$  equals 4, so essentially you see that over a symbol duration  $T$  between 0 to  $T$ , I have essentially a sine wave after different frequency going in and in the same way that we discussed the binary FSK you can think of this as a 4-ary FSK case.

All that is needed is that each of this sine waves has a frequency which is integer multiple of  $1/T$ ,  $2/T$  right, now I think at this point I will like to go back to the discussion of the  $T$  last time about orthogonal FSK kind of waveforms some of you has expressed the doubt that they will be in DC standing on the wave on the transmitted waveform right for the examples that we have selected it looked as if they will DC on the waveform, DC on the channel.

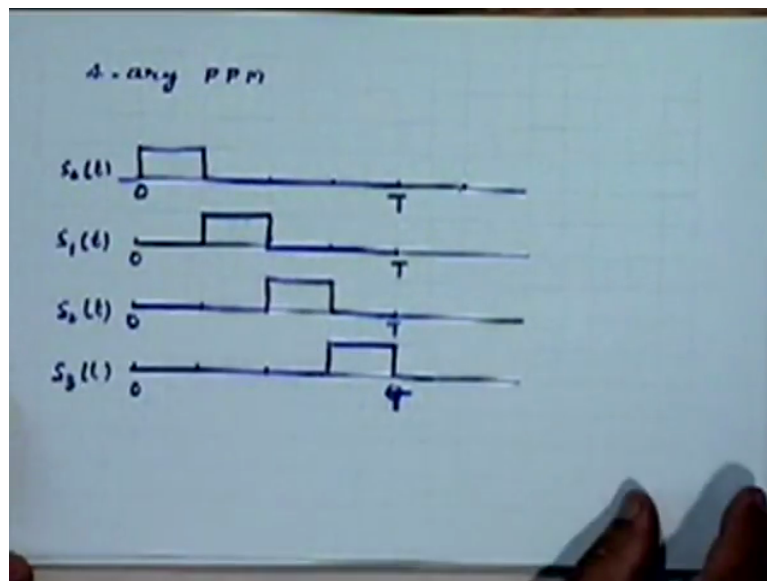
Well that was the only an example it was not to be taken that seriously that just showed an example of orthogonal set of binary waveforms right for example in this case you will see there is no DC because each waveform is balanced in terms of positive and negative cycles you could have selected even for those examples the waveforms set like that right so that was just an example to indicate that those are the kind of waveforms which can be orthogonal right.

But if DC is a problem which you need to avoid you can choose this waveform more carefully right so that was just to

Student: (0)(19:12) giving us a smaller bandwidth requirement you could have (0)(19:15)

Professor: There will be other considerations typically of course here we are talking about baseband if you are talking about pass band you will have a cycle of mismatch in terms of DC will not make that much difference and actually speaking orthogonal waveforms really are useful when they are used with large values of  $m$ , not with small values of  $m$  and when you are using them with large values of  $m$  these considerations become very secondary okay.

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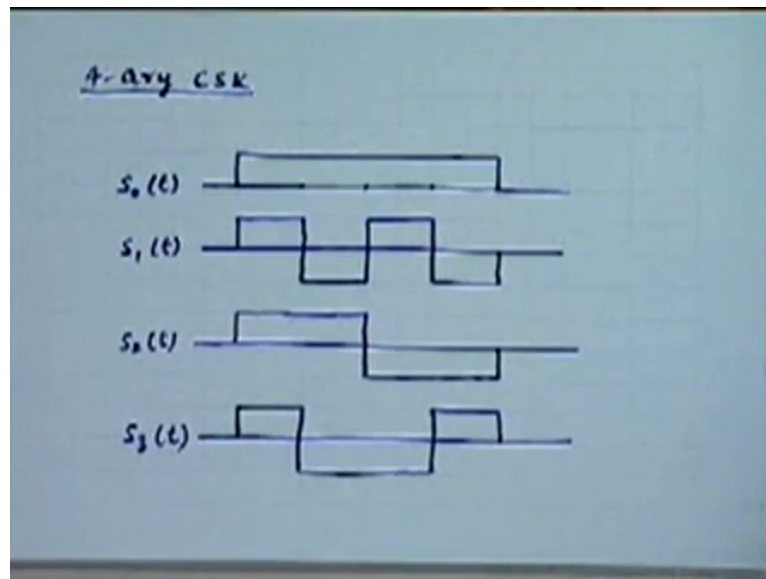


So anyway I thought since that point came up for discussion last time which I could not satisfactorily address I should have at least point this out to you at this point, so this is one example of orthogonal signalling, here is another example of orthogonal signalling, very simple trivial kind of example so here is your  $S_0(t)$  which is the waveform is to be regarded off duration  $T$  upto here it starts here and ends here right.

Again  $S_1(t)$  starts here and ends here right  $S_2(t)$  goes like that right and  $S_3(t)$  goes like that, sorry this is 0, this is  $T$ , so you can see that the waveforms are all continuing a pulse over a different portion of time long overlapping portions of time and for this reason you can possible call this waveform a 4-ary pulse position modulation right where the position of the pulse decides the nature of the waveform okay.



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Yet another example of orthogonal set of waveforms is a set of pulses like this right as you can see if you were to multiply them out and integrate the result they will be, the integral will turn out to be 0 any pair of them right and this is called 4-ary code shift keying C is standing for code okay so basically we have a different code for representing each waveform different binary code right.

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Larger families of M-ary orthogonal pulses  
:  $M = 32, M = 64$  : In Common Use  
: Loosely M-ary FSK : MFSK  
Energy Budget:  
 $E_p$  : distributed over  $k$  bits  
 $k = \log_2 M$

So these examples are sufficient to show that one can construct a fairly large variety of orthogonal set of signals right, now you want me to display that for a little longer alright, please say so if there is any problem of that kind and at any stage okay, one typically uses,

whenever one uses orthogonal set of signals one typically uses very large values of  $m$  right, in fact one can construct larger families of  $M$ -ary orthogonal pulses in the same way.

It will not be surprising if you come across this terms which use values as large as  $m$  equal to 32 or  $m$  equal to 64, okay they have in common use, another point to note is and this point we discussed also in the context of binary orthogonal signalling loosely speaking motivated by the fact that FSK is a very important member of this family, this class of signals are called collectively also loosely known by the name of  $M$ -ary FSK.

So generally we may also refer to them as  $M$ -ary FSK or  $M$ -ary FSK type signals right or sometimes simply refer to as MFSK right,  $M$ -ary FSK is briefly sometimes denoted by MFSK, let us talk about the energy budget here, we may have so many different possible ways of constructing  $M$ -ary signals we can choose different values of  $m$  and come up with the different modulation scheme right, for the same situation.

Now how do I compare all of them when I compare in terms of energy obviously when I choose a different value of  $m$ , the total energy that is being used to transmit that signal is representing different number of bits right because the moment I change the value of  $m$ , the number of bits corresponding to that also changes, so it is more useful therefore to talk about not just the total transmitter energy in  $M$ -ary case but the transmitted energy per bit, right.

So if remember each pulse here carries an energy  $E_{sub p}$ , right, this energy  $E_{sub p}$  is actually used for a symbol of length  $k$  in our binary to  $M$ -ary mapping right so it is where  $k$  is, so it is distributed over, this energy is distributed over  $k$ -bits where  $k$  is equal to  $\log_2 m$  so therefore energy per bit, I am sorry, energy per bit will be how much  $E_{sub p}$  upon  $k$  or  $E_{sub p}$  upon  $\log_2 m$ .

It is this energy which is important when you are comparing different modulation schemes or different  $M$ -ary modulation schemes for that matter right, so suppose you were to ask, you were interested in asking question what happens when I go for  $m$  is equal to 2 to the power  $k$  to 2 to the power  $k$  plus 1 well you look at the performance and look at the corresponding energy per bit and then you can make a meaningful comparison okay.

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$$E_b = \text{Energy / bit} = \frac{E_p}{k} = \frac{E_p}{\log_2 M}$$

Simplex Signals:

Any orthogonal family of  $M$  pulses ( $E_p$ ) can be used to construct a simplex family:

$$s_m(t) = \frac{1}{M} \sum_{n=0}^{M-1} s_n(t) \triangleq \underline{q_m(t)}$$

So  $E_b$ ,  $E$  sub  $b$  which is energy consumed per bit will be equal to  $E$  sub  $p$  upon  $k$  or  $E$  sub  $p$  upon  $\log_2 m$  to the base 2, so this serves as a common basis of reference let us say for performance comparison of different modulations, different  $M$ -ary modulation schemes right different values of  $m$ , so that is one important family of  $M$ -ary signals which one can use and are very commonly used.

Now the next family that I will consider, let us start with a different colour just to put some variety or simplex signals and the motivation one can derive is from the fact that perhaps you may feel that orthogonal signals may not be the best class signals from some point of view, intuitively you may feel like that at least from your binary experience you may feel like that because in binary experience that you conventionally have is that of On-Off keying verses polar keying right.

On-Off signalling is an example of orthogonal signalling that you are familiar with, polar signalling is an example of anti-podal signalling that you are familiar with right and you have a reasonable appreciation even though we have not gone into detail performance comparison so far because we have not looked at that, we have not looked at even optimum demodulation at the moment, so we cannot really talk about performance comparison but you have an intuitive appreciation of a fact that anti-podal signalling gives you better performance than polar signalling, than On-Off or orthogonal signalling at least for the binary case right.

So useful question to ask therefore in the  $M$ -ary context is can we construct generalization of anti-podal class of signals which constructed from the bin, is constructed in some way

which have similar properties as that of anti-podal, now what is the essential property that distinguishes anti-podal and orthogonal signals in the binary case or in the context of correlation, right now we are using correlation as a measure of similarity or dissimilarity of waveforms that we use right, is not it?

In orthogonal case the similarity or dissimilarity is measured by finding out whether or not the correlation various waveforms is 0 or not right, in the anti-podal case what is the similarity or dissimilarity measured, the correlation is negative in fact right, we like to go from to signal sets which are not only totally have 0 correlation in fact they have less than 0 correlation they have negative correlation right, that is if it is  $p t$  then other is  $\text{minus } p t$ .

And in general, in negative correlation between signals of signal set in M-ary schemes is a more desirable property than a zero correlation right that is something that will become clearer and clearer as we go along, particularly for demodulation because that basically means the various signals in the set have a larger distance in some sense with respect to each other than in the case of orthogonal signals.

So basically simplex signals are motivated from that kind of consideration okay so let us see, okay I will define, actually one can construct a family of so called simplex signals from any given family of orthogonal signals okay so any orthogonal family of M pulses, let us say each of energy  $E_{\text{sub } p}$  can be used to construct a simplex family right, the way it is done is as follows from each signal  $S_m t$  in the original orthogonal family, I subtract the average value of all other pulses, in fact of all the pulses including this, call this let us say  $q_m t$ , okay.

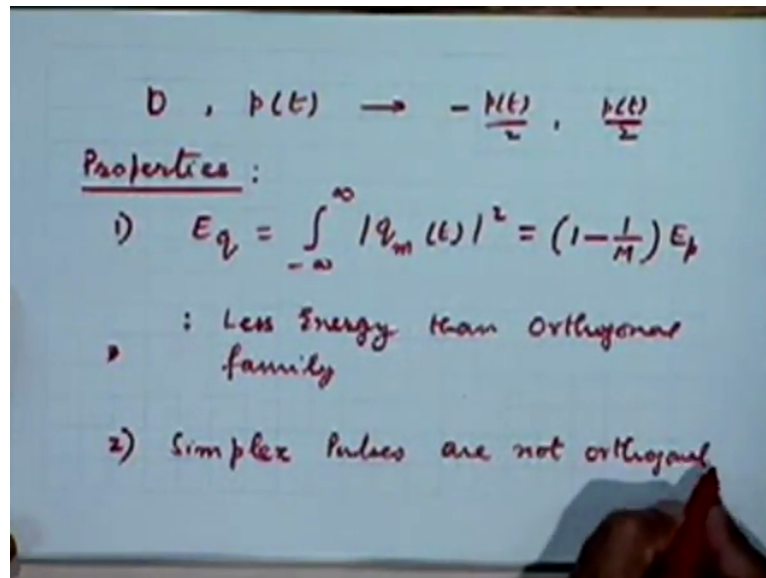
So from  $S_m t$  for any value of M between 0 to capital M minus 1, I am subtracting the average value of this signal right, average value of the signal set and generating a new form which I am calling  $q_{\text{sub } m} t$  so I get a new set of forms  $q_{\text{sub } m} t$  where m goes from 0 to capital M minus 1, right, this signals  $q_m t$  form the so called simplex family and I think the special case of binary becomes obvious that will lead to when you choose m equal to 2 it will lead to anti-podal signals right and it is quite obvious. (())(32:48)

For M equal to 2, for m equal to 2 it will lead to anti-podal signal, for binary case it will lead to M, is not that obvious

Student: So (())(33:00) I mean the basic definition of anti-podal like.

Professor: Basic definition of anti-podal is very simple, what was the definition we talked about for anti-podal signal, minus  $p(t)$  and plus  $p(t)$  that is precisely what you are going to get in the binary case of course we cannot talk about that concept in the M-ary case is not it? That is why we have not talked about the different concept which you calling simplex but binary case becomes a special case of this, alright, is not it.

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Suppose I start with the set 0 and  $p(t)$  which is On-Off set right that will lead to after this procedure minus  $p(t)$  by 2 and  $p(t)$  by 2 right so that will lead to anti-podal set of signals. So now let us talk about the properties of the simplex family, first of all if you are to calculate the energy of each pulse that results that is if you are to do the exercise of calculating the integral of mod  $q_m(t)$  square, right we expected to be the same? No, you are in for a slight shock, it will not be same.

I will like to do that, it is matter of very simple combination so please do that yourself I am not going to spend time on doing that it is very simple algebra and what you will find is this  $1 - \frac{1}{M}$  into  $E_p$ , so the energy carried by each set similar in the simplex family is smaller than the corresponding pulses as you peak orthogonal family from which the simplex family has been constructed okay.

So that is the first thing that they have less energy than the orthogonal family right, second important point to notice is that simplex pulses are not orthogonal right so simplex pulses are not orthogonal in fact they have the more desirable property of negative correlation right, can I remove this?

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: Stronger Property of -ve correlation  
For  $m \neq m'$   
$$\int_{-\infty}^{\infty} q_m(t) q_{m'}^*(t) dt = -\frac{E_q}{M} (?)$$

That is a stronger, we can say that they contain a or they have a stronger property, stronger in the sense that they are more desirable in this kind of applications of negative correlation, for example if you choose two different values of  $m$ ,  $m$  and  $m$  prime let us say and they are not equal the correlation is given by  $q_m(t)$ ,  $q_{m'}(t)$  of course prime should come here and conjugate should come here, integral between minus infinity to infinity will be equal to minus  $E$  sub  $q$  upon  $M$  or minus, can I write it in terms of  $E$  sub  $p$  what will it be, okay I think just leave it like that.

Just check whether this is correct or not I am having a small doubt about it.

Student: We will check it for the binary case.

Professor: For a binary case it is okay because that will give rise to 2 right,  $E_q$  by 2, that is correct, okay in any case please verify this, now why we call it a stronger property is I have already mentioned, let me put it in the writing over here we later study later this, we will study this point later but I will just like to mention this property here that we will find that the simplex set gives the same error probability as the orthogonal set from which it has been derived right. So what is the advantage? Advantage is it is giving the same performance with a smaller energy right.

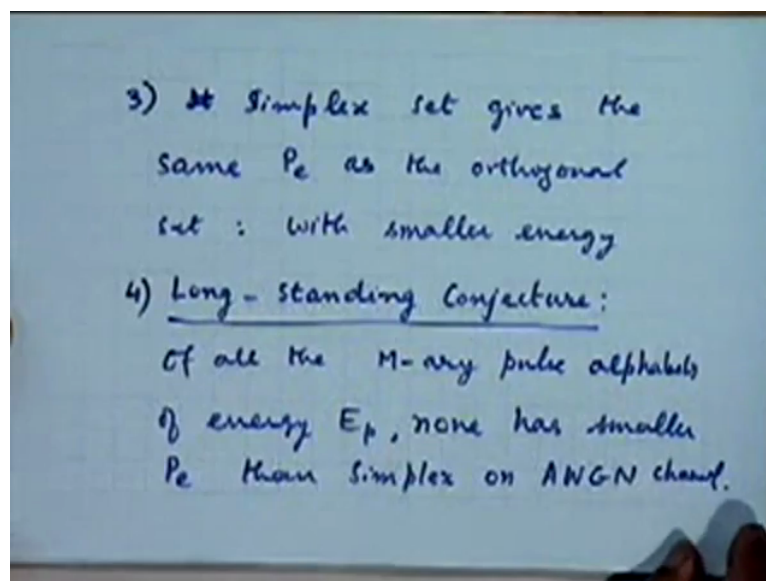
Student: Sir point here is that the energy transmitted might be less but (( ))(38:30) that we are constructing it from the orthogonal signal that much energy is being used up.

Professor: What is important is how the energy is used on the channel, how much energy is actually how much power is  $(P_e)$  (38:45) to the channel right the construction is a very trivial process at the transmitter.

Student: What I am trying to say is some  $(P_e)$  (38:55)  $S_m$  minus summation of all this and by that encoding we are wearing down the energy level but the energy that we are giving to the system, channel included is still the same  $(P_e)$  (39:12)

Professor: Pulse generation mechanism is a local mechanism of your circuit of the transmitter right even do the whole exercise that will be very low power level but it is again important is how much energy level finally counted on to the channel right so in fact this construction mechanism is only artificial in that sense that does not define how much energy that we need put on the channel, that will be finally decided where the power amplifying is have right okay.

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Okay the point that I want to mention which I have mentioned you simplex set gives the same error probability as the orthogonal set with smaller energy so it is more energy efficient right and something that is more energy efficient is more useful at least in a power constraint channel right that is the meaning of power constraint that when we are short of energy and we like to make very efficient use of energy that we might have at our disposal like in satellite communication.

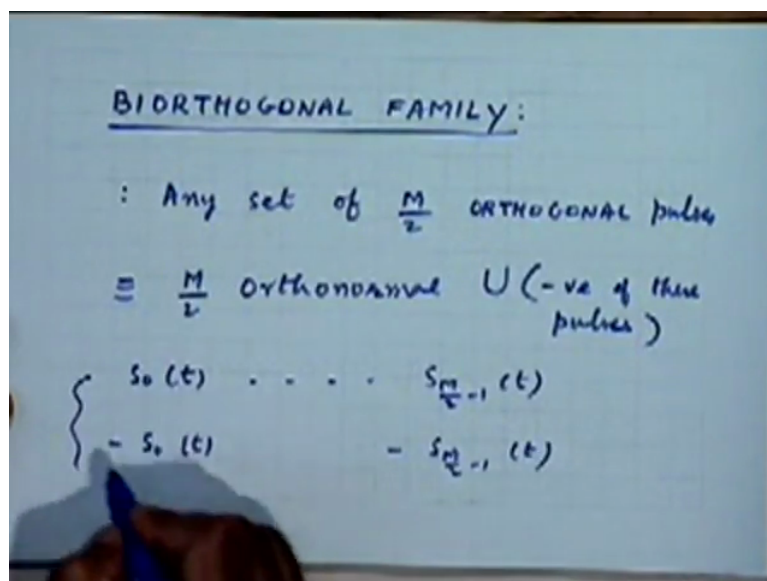
And now this is something that I will just mention to you as an interesting thought and maybe some of you can take that off for your personal research there is a very long standing

conjecture although nobody have been so far been able to prove this or disapprove it for that matter which is a conjecture because it is neither has been proved nor a counter example found so far.

Of all the M-ary pulse alphabets of given analogy  $E_{sub p}$ , there is no other alphabet which can be smaller probability of  $(\cdot)$ (41:29) than a simplex set okay, I will mention it, of all the M-ary pulse alphabet of energy  $E_{sub p}$  or  $E_{sub Q}$  whatever none has smaller  $P_{sub e}$  than simplex of course you had to specify the conditions on a normal kind of channels we do this analysis for which is the additive white Gaussian noise channel.

That is when you encounter White Gaussian Noise on a channel then simplex signal set, M-ary simplex signals that is the best, and you already know that for the binary case right we have discussed that for a binary case this is a corresponding result for the M-ary case but it is an interesting point that I made here for that maybe some you can try to prove or disprove it.

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Okay so what we have done so far is the M-ary orthogonal family and the M-ary simplex family which can be constructed or expressed in terms of the M-ary orthogonal family, now there is one more family of M-ary signals one can derive from the orthogonal family and simply call it Bi-orthogonal family and it is very simple you take any set of start with any set of M by 2 orthogonal pulses.

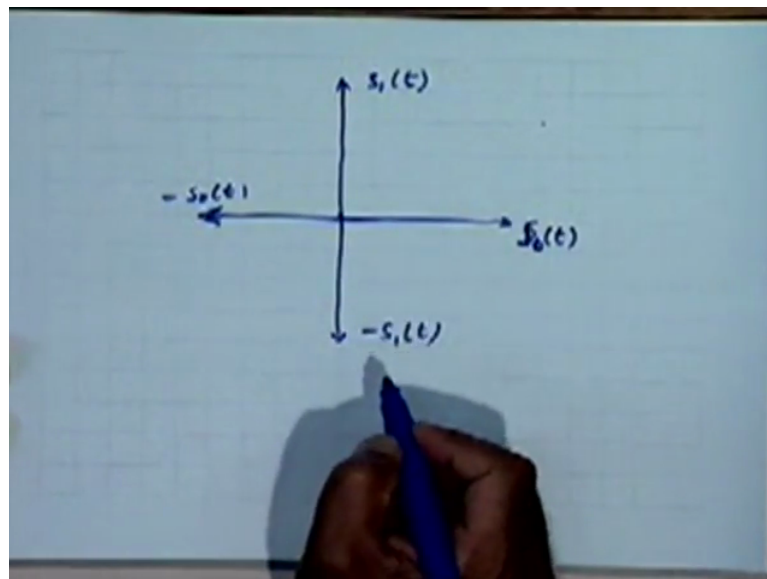
And we can now construct a set of M so called bi-orthogonal pulses which is simply the M by 2 orthogonal pulses that we started with include those in the set along with this is I am just saying union with the negative of each of this pulses, okay so for example if you start with S



sub 0 t to S M by 2 minus 1 t right then just add the negative of each of these to the set you have a family of m signals which are called bi-orthogonal okay.

You can think of a random  $(\cdot)$ (44:38) space right in which each orthogonal access of the space, orthogonal basis function of the space is used to represent one and different signal and then you also taking the signals corresponding to negatives of each of this basis functions right that is the bi-orthogonal, for example in a signal space representation which will be taking up separately later.

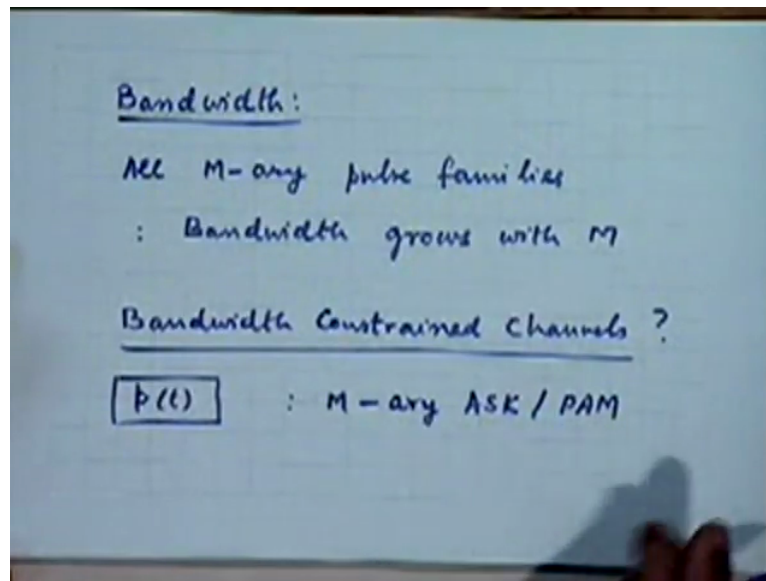
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Suppose these are, these represent two orthogonal set, orthogonal signals in a binary orthogonal set for example this  $(\cdot)$ (45:15) let us say S 0 t and this represents S 1 t right they just being used as basis function for abstract space of this kind, then a bi-orthogonal family would be simply obtained by using this along with this and using this along with this right so what you will notice is the dimensionality of the space will not increase we will be working in the same space, M by 2 dimensional space.

But there are number of signals that we are going to use is larger right, now some other points of interest in the context of M-ary orthogonal and other M-ary schemes that we have discussed so far, we have talked about energy calculations and how we will like to compare energies whenever required, we initially mentioned that these are signals that can be used when bandwidth is not a limitation right.

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I will like you to appreciate that fact that is here, let us talk about what happens to bandwidth as M increases in this class of signals if you look at typical examples that I have given you, you will get a feel for that right for example look at the 4-ary FSK example that I gave you and so on, as I increase the value of M you will have to put more and more such signals and it may appear intuitively that necessarily the bandwidth has to go up right.

Increasing the value of M that is putting, using larger and larger chunks of K-bits to map into waveforms is necessarily associated with increase in bandwidth so all the M-ary schemes we have discussed so far have this problem that is bandwidth literally grows with M and there is no attempt on our part to constraint the bandwidth right, all we are interested in is that the orthogonality condition must be satisfied or a simplex condition must be satisfied and so on.

We are not even explicitly or consciously try to do anything about the bandwidth right, yes we will have to see the bandwidth efficiency but the overall bandwidth is going to go up right for as the value of M is going to increase, we have to still see how bandwidth closes with respect to increase in number of bits that we are simultaneously carrying out, no it will depend on the kind of signals that we use, we cannot make a very general statement about that, all you can say is it will grow in some sense.

Student: That is also going to find that you know moreover I mean the bitrate is increasing with bandwidth.

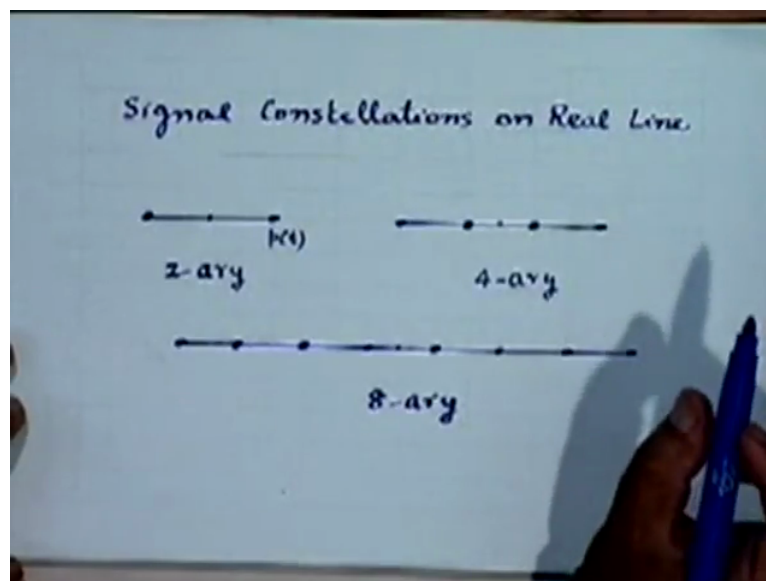
Professor: But the bitrate is only increasing  $\log_2(M)$  times whereas the bandwidth is increasing much more than, much faster than that right because you are having instead of M

signals persistently your bandwidth is going to go perhaps at least linearly not more right, so therefore that increase in number of bits that you are representing with these instead of M waveforms is not necessarily an off-setting factor as far as bandwidth is concerned okay.

So there is no attempt to constraint bandwidth and increasing M is associated with more bandwidth therefore the question arises what should we do in bandwidth constraint channels when we want to go for M-ary schemes? What approach should take? Here we cannot allow any increase in bandwidth at all no matter what is the value of M okay, now what kind of approach comes to your mind, we briefly talked about it last time.

We are going to necessarily have to work with band limited pulses right and typically we will decide on a band limited pulse shape or perhaps a set of band limited pulse shapes if you so desire, usually it is convenient to zero onto a single pulse shape  $p(t)$  right and then construct M-ary waveform around that pulse shape and then the options that you have are very few in number okay.

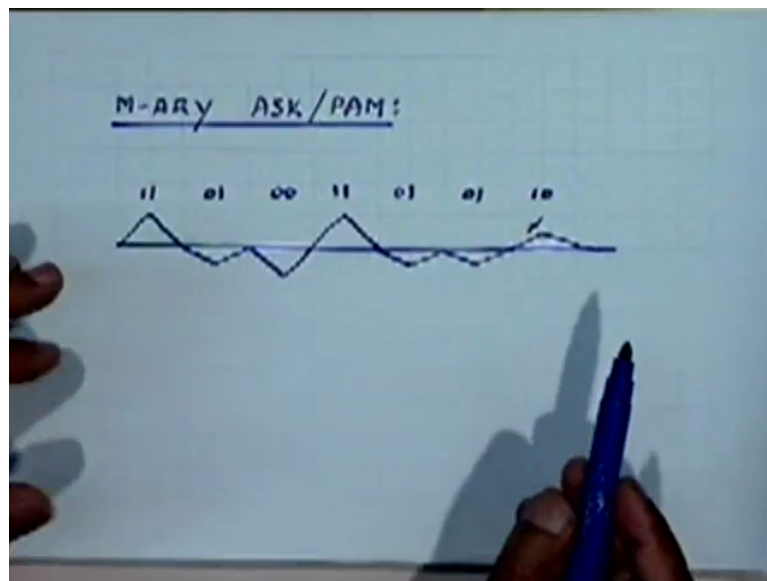
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You, it is going to be now very difficult to construct orthogonal or simplex other kinds of families, once you put this bandwidth constraint alright the most commonly used option is in fact what is called M-ary ASK or M-ary PAM okay, so what a signal space representation of the same would be a chosen a pulse shape, I think I have figure for that somewhere, you might have chosen a pulse shape and let us say this line represents that particular pulse shape this is the basis function and this in one dimensional space which is represented by  $p(t)$ .

So this point here represents the signal  $p(t)$  then in the binary case we know what the corresponding thing is you can use  $-p(t)$ , you can construct a 4-level signalling by using four different amplitudes right or an 8-level signalling by using eight different amplitudes but the basic pulse shape remains  $p(t)$  only you are using that pulse shape with different amplitude okay so these are the kind of constellations or signal space diagrams that you work with when you are working with bandwidth constraint M-ary signalling.

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An example here of a waveform, a 4-ary waveform corresponding to M-ary ASK where just for the sake of illustration chosen a triangular pulse a four possible different amplitudes right, this is one amplitude, this is another, a third and a fourth, these are the 4 amplitudes that are coming up and we have associated with each of these amplitudes a binary sequence of length 2 right so when the two successive bits are 1 1 maybe you transmit this amplitude 0 1 this amplitude so on okay.

So that is an example of M-ary ASK of course this is not a good example in the sense that this triangular pulse is not going to get strictly band limited also right typically you are going to use pulses which satisfy the nyquist criterion alright, namely the sinc pulses and that family, I think we will stop here and we will next time do consider passband modulation for both binary as well as M-ary case. They are quite different in philosophy and style okay the most of the material again will be available in any other books though not in this form.

The book that I am following at the moment will not be easily available to you so it is called Blahut, Digital Communications by Blahut.

Student: Could you spell it?

Professor: Yes, I could, dekhiye.