**Digital Communication**
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**Lecture – 11**
**Baseband Pulse Shaping: Raised Cosine Family of Pulses**

Let us see recapitulate what we are talking about last time, we are looking at the design of band limited pulses that we could possibly use in a digital communication system and the motivation for that was we are simply not allowed to use pulses which are not band limited that is one motivation, we must restrict our bandwidth to within the allotted one that is given to us and secondly if we do not bandwidth ourselves the channel will do it for us and that will be worse.

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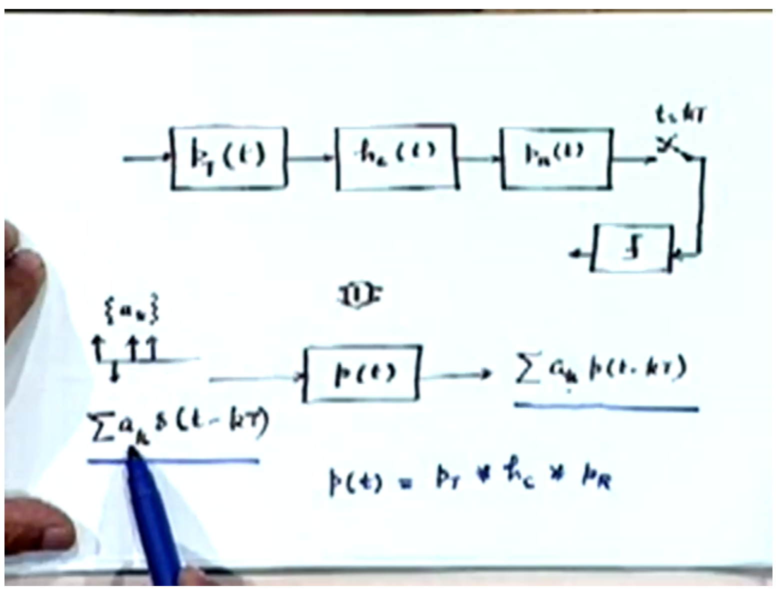


Because that will introduce, you remember what? Inter-symbol interference, which will be a kind of a digital noise into the system affecting the performance of the system. To this baseband pulse shift design so that we are restricted to bandwidth of interest in an appropriate way we develop this model, equivalent channel model for communication system which we said roughly could be thought of as consisting of three filters.

One at the transmitter to shape the pulses, one in the channel which is a characterization of a channel itself and one at the receiver which could be perhaps the match filter that you might be using at the receiver and the three together, the output of this is done is sampled every single interval and then it is sent to a decision making device to decide whether in a particular interval a 1 or a 0 was transmitted or whatever be the symbols that you are looking for.

This very crude model of the channel of the communication system can be represented by an equivalent filter which has an impulse response p t right, so what you are seeing is if you excite this equivalent filter with a sequence of impulses over here that is this impulse train, the strengths of this impulse is corresponding to the data sequence of interest which we are trying to send, the output will be another sequence of pulses, long pulses with pulse shape given by the impulse response p t alright.

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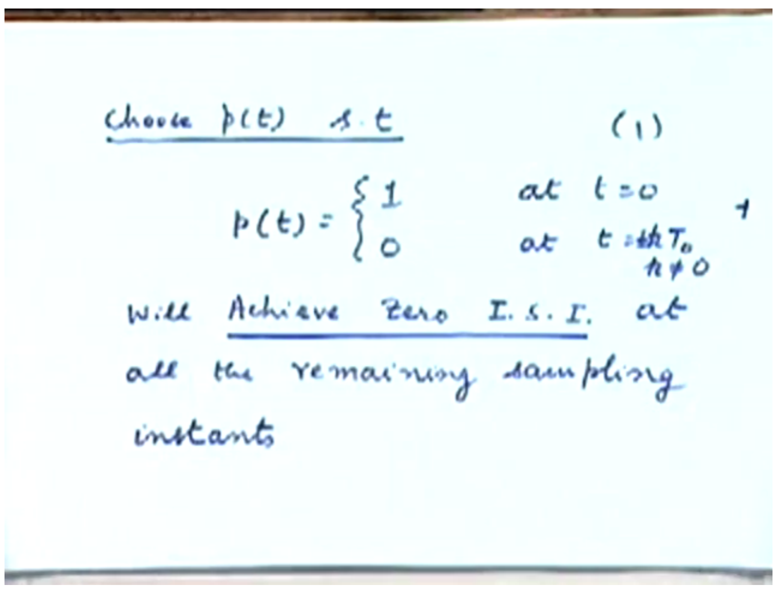


And Nyquist proposed that we must choose these different components of the system such that this equivalent impulse response p t does not exhibit inter-symbol interference and the way to do that would be to make, to choose this equivalent impulse response p t such that it is equal to 1 only at the sampling instant t equal to 0 and it is 0 at all other sampling instance, remember this discussion we had last time.

This if we achieved a pulse shape p t which has this nice property and which also satisfies our bandwidth constraint then we are through because that will avoid having inter-symbol interference which is of primary concern over here, the inter-symbol interference will actually be still present but what is important is that at the sampling instance we will not see it okay because at the sampling instance while one symbol is going to its peak value the other symbols will be exhibiting zeroes the response of this filter to other symbols would be 0 at each sampling instant.

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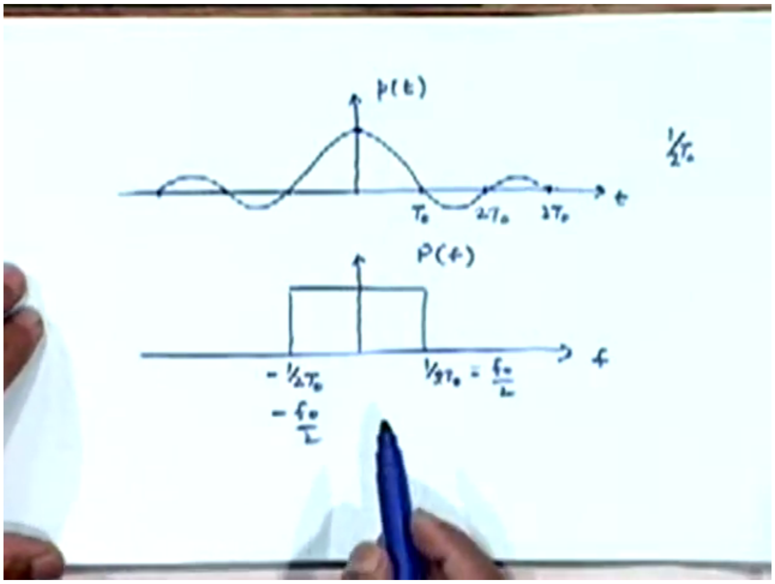

So that successful symbols do not exhibit any interference with each other, so this was the broad idea of Nyquist who suggested therefore that one could choose this method to design pulses and what we also appreciated was that the minimum bandwidth pulse which will have this nice property for us is the so called sinc pulse right, that is if you are transferring symbols having T0 seconds, we need to have a bandwidth only of f0 by 2 hertz right.

And then this sinc pulse that is the Fourier transform of this spectrum exhibits those regularly space zeros which we are looking for the response is maximum at T equal to 0 and goes to zero crossings having T0 seconds and that is a point at which the next response will build up for the next symbol but it will not see any contribution from the previous transmitted pulse alright so this is what we discussed last time briefly.

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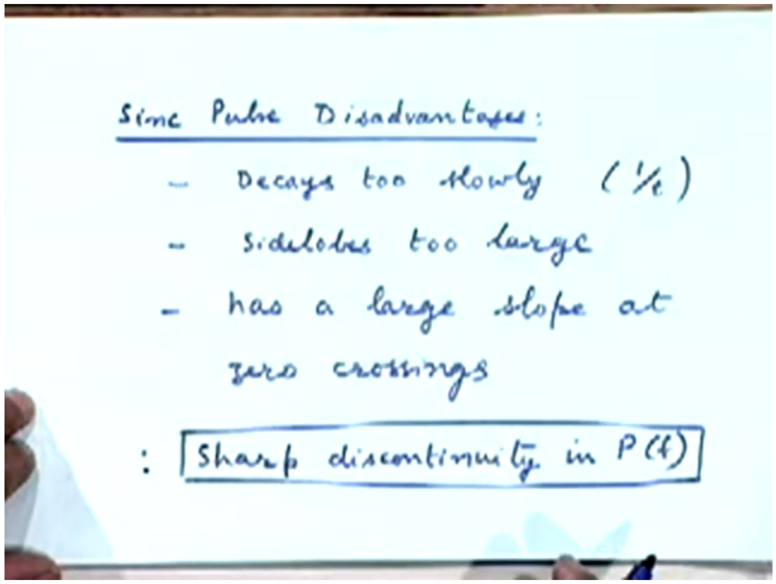


Now practical problems that we try to appreciate let me just summarize that that sinc pulse we found decays too slowly for us, its decay rate is only 1 by t, its side lobes are too large and that it has a large slope at the zero crossing right, now what is the significance of this three properties of a sinc pulse in reference to our requirement, all this three things make the sinc pulse very very sensitive to timing errors that might be present, if we are not sampling the pulses at the right instance if there is a timing error due to either a clock drift of the transmitter or a clock drift at the receiver or perhaps due to some timing deter that is come about somewhere then the sinc pulse will exhibit fairly large symbol interference.

A slight disorientation in time adjustment either at the transmitter or at the receiver will make the sinc pulse again give the same problems that you are trying to avoid in the first place okay and this is essentially coming about because of sharp discontinuities which are present in the spectrum of the sinc pulse there is a sudden fall from 1 to 0 in the spectrum which is not the desirable.

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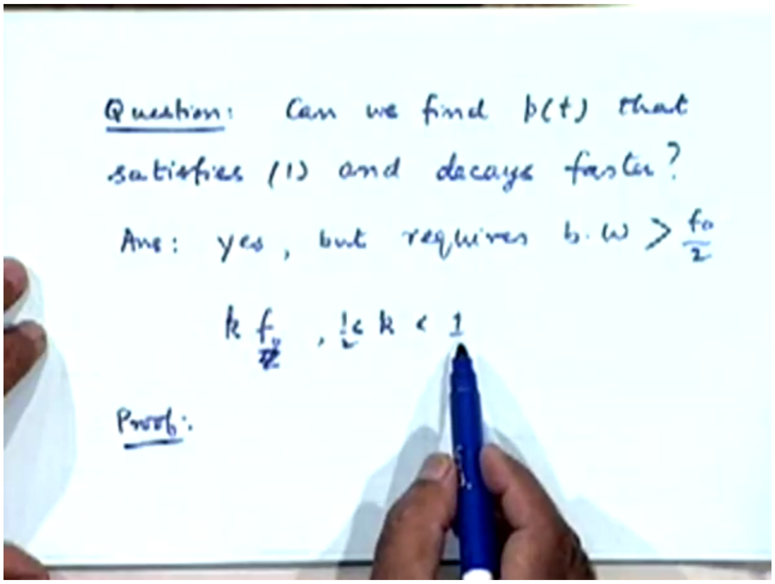


And the question that we posed at the end of the class was can we find a pulse p t that satisfies the nyquist criterion but decays much faster than that and what we said was perhaps it will be possible to do so and answer in fact is yes provided we are ready to use a bandwidth more than the nyquist bandwidth, more than the minimum required for transmitting at f0, the minimum required is f0 by 2 but if you agree to use a bandwidth which is greater than f0 by 2 of typically some k times f0 where k lies between half and 1 it is possible to obtain a pulse shape which is more perturbation tolerant then the sinc pulse is okay. So that is a quick revision of what we did last time, any questions on that. Yes, please.

Student: Sir, Nyquist Criterion which is stated earlier where it says that to transfer let us say bandwidth of W the minimum sampling rate we require is let us say 2f for W we need is 2W.

Professor: Say that again.

Student: The Nyquist Criterion which you have stated earlier.

Professor: The minimum bandwidth required is f0 by 2 to transmit at a rate of f0 symbols per second.

Student: Now obvious sir we have signals if even if the sampling rate is f0 then as we said the minimum bandwidth is W but the bandwidth can also be more than W.

Professor: Now what is W? W is f0 by 2 that is right.

Student: Yeah I am trying sir what we have done

Professor: Yes bandwidth can be more than W.

Student: Last week Nyquist Criterion which we have done that is what I am not being able to do because there we said that if the sampling rate is f0 then the maximum bandwidth that will get transfer is f0 by 2.

Professor: No, the minimum bandwidth that we need to do that properly is f0 by 2 we can use more bandwidth than that, that is precisely what we are saying now, we are now going to look for solutions which use more bandwidth because as we try to appreciate last time we can find an infinite number of pulses which will satisfy the Nyquist Criterion, out of all this infinite number of pulses only one pulse will have the smallest possible bandwidth right.

And that is a Nyquist Bandwidth and that pulse shape is a sinc pulse now we are looking for other pulse shapes which will satisfy the same criterion but it will try to use more bandwidth so as to introduce certain desirable properties, other desirable properties in the pulse shape, not only the criterion itself but other properties like fast decay of the pulse with time as we go away from T equal to 0, does it answer your question? Is it clear now? Okay, anybody else have other problem okay, so then we proceed further from here.

Let us see how we can show that this indeed can be done, that is by using up more bandwidth by allowing the decay not to be, by allowing the fall in the spectrum not to be sudden but gradual we can introduce this desirable property in the pulse shape. (())(10:52)

Yes, we will discuss that separately there are many methods of generating or realizing a filter with a given impulse response right we can use any of those methods, we can use passive methods as well as active methods, you can use digital signal processors that methods so generation of those pulse shape is something perhaps I will show sometimes on separately right because we will see that a number of different kinds of pulse shapes may need to be generated and I will have a consolidate discussion of all of them together okay.

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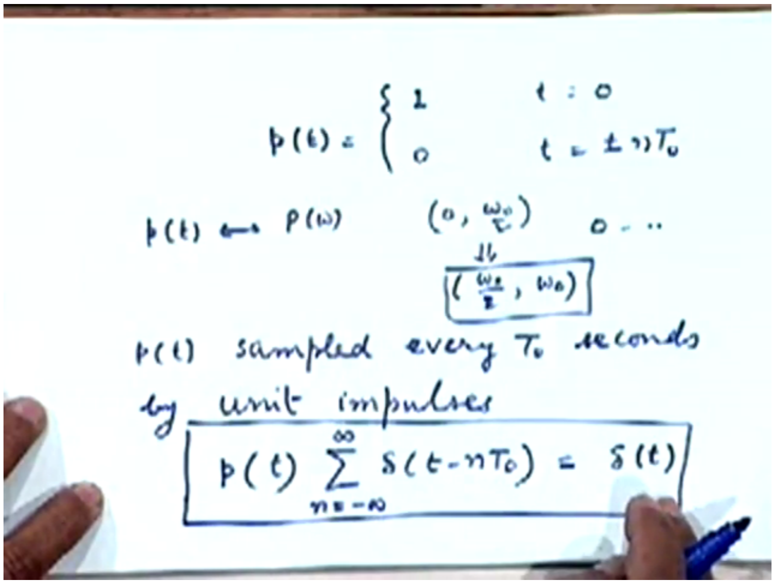


Now what we will do is you call that our Nyquist Criterion requires that our p t be equal to 1 for t equal to 0 and 0 for t equal to plus minus n T0 right, for n not equal to 0 of course let us say that this p t has a Fourier transform P omega and let us say that the spectrum of P omega is not constrain between 0 and omega 0 by 2 but is allowed to spill over beyond it let us say if somewhere the bandwidth extends to some point now between omega 0 by 2 and omega 0, of course the total band will be between 0 to whatever extra bandwidth we are using up.

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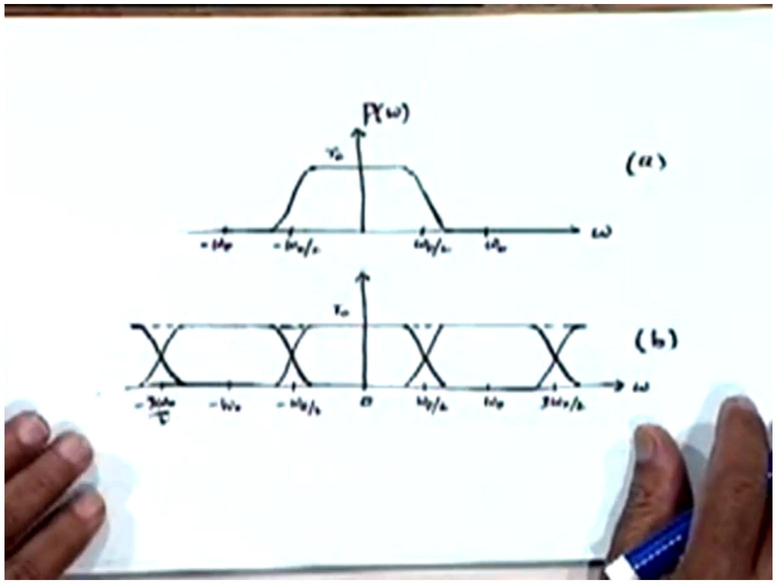

But our cutoff point now lies in this region rather than omega 0 by 2 right that is our (())(12:42) change you are now permitting P omega to have a cutoff beyond omega 0 by 2 alright I think I will show a picture of a possible situation, there is a possible P omega with this kind of a property that is your cutoff is no longer at omega 0 by 2, it is beyond, it is between omega 0 by 2 and omega 0 right and of course instead of a sharp sudden fall in the spectrum at omega 0 by 2 we now have a gradual roll off right, a smooth roll off.

Now first it is spectrum what we like to do is satisfy this property, what we are going to do is at the receiver we are going to sample p t every t0 seconds right that is how we are going to look at what the values are let us say p t is sampled by a sequence of pulse, impulses, sample every t0 seconds by unit impulses right, what that means is a sampling process will consists of multiplying p t with a impulse train of this kind alright. Is that okay?

And by requirement this sample impulse, this sample pulse after it has been sampled by this impulse train should be equal to an impulse itself because we would like it to be non-zero on their t equal to 0 and 0 at all other sampling instance that means at sample sequence that we have got is also an impulse sequence by itself so the Nyquist criterion requirement can also be put in the form of this simple equation and this should be equal to delta t. Why p 0?

Student: p 0 is 1

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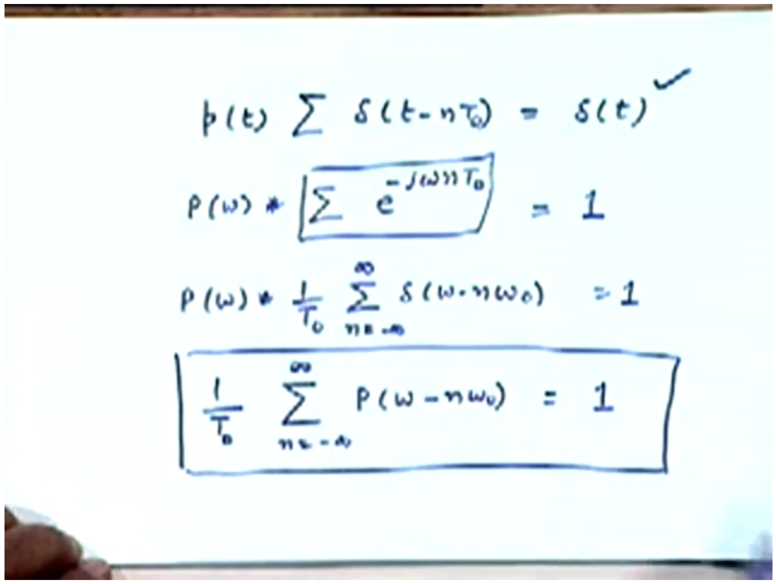


Professor: Oh okay I should make, p 0 is already 1 you are right, fine. So let us start with this equation again, let us take the Fourier transform of both sides of this equation, this is the product in time domain so we will get a convolution in the frequency domain and if Fourier transform of each impulse is e to the power minus, this should be n T0, j omega n T0 and this of course is equal to 1 they will also be, now again if you remember we had obtained or we have been using a formula which relates a summation of this kind.

With summation of impulses right which is equal to, that is a (())(16:26) formula for you, delta omega minus and omega 0 or if you convolve p omega with each of this individual impulse functions now in the frequency domain what we really get is this requirement, this criteria should be 1 by T0, what is the result of convolving p omega with each of this impulses, function p omega shifted to n omega 0 with its center at n omega 0 and all this shifting functions added up together from n equal to minus infinity to plus infinity dash equal to 1.

What does it mean? Any pulse shape p t whose Fourier transform p omega satisfies this would obviously satisfy this right, we are in fact derive this condition from our required condition, so we have obtained corresponding condition in the frequency domain now let us look at this condition by means of a picture.

Student: Sir p t is a actual signal transmitting or received?

Professor: Okay this what I have tried to recalculate for you at the beginning of today's class, p t is the impulse response of the model for the total communication system which consists of the transmitting filter, pulse shaping filter, the channel and the receiving filter, alright. So it is not either at the transmitter or at the receiver, it is really combination of anything, we have a separate job to do afterwards as to how to realize this overall p t in to the transmitter and receiver separately.

What power of it should be realized at the transmitter and how it should be realized at the receiver so that all three combined together gives you the required p t but that is a separate problem and we look at it separately when the time comes okay, at the moment we are just trying to design what p t should be like alright and what we are now saying is that our p t by the nyquist criterion requirement should satisfy this condition in a frequency domain, is that okay? Understood?

And can you picture this condition yourselves in your minds, basically you have your p omega which was restricted to 0 and some cut off frequency between omega 0 by 2 and omega 0, now you are replicating that spectrum around omega 0, around 2 omega 0, around 3 omega 0 similarly around minus omega 0, minus 2 omega 0 and so on by replicating or repeating this spectrum over the entire frequency axis and then summing all this functions you must get a constant value.

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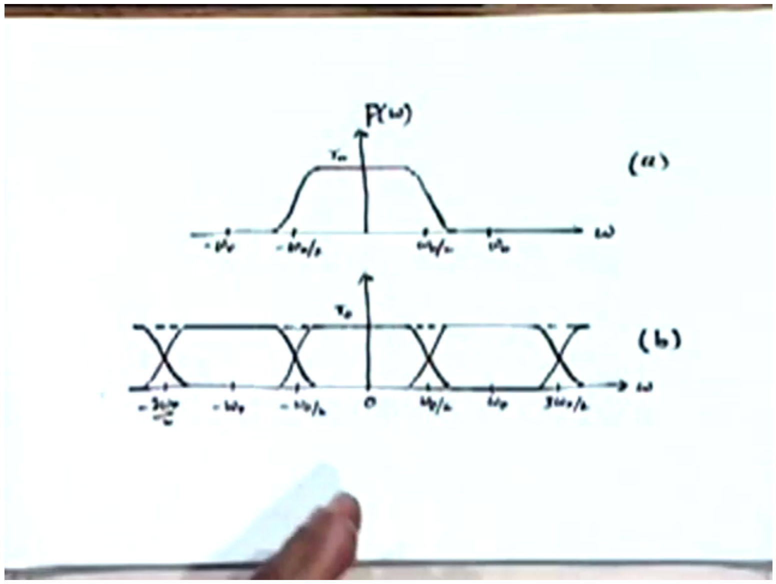


That is the picture that I would like you to have in your mind before I show the picture to you which is like this, right, this is your p omega in the base band which you are now replicating around omega 0 and minus omega 0 when 2 omega 0 and minus 2 omega 0 and so on, you repeating this function or replicating this function (())(20:07) and what we like to do the criterion tells us is that the sum of all this functions should be a constant equal to T0 right.

So if we design this base band spectrum p omega, no matter whether it is restricted to just the nyquist bandwidth or whether it spills out to the nyquist bandwidth as long as this criterion is satisfied we are guaranteed to have a pulse shape p t which satisfies our requirements, this is the important point to understand okay, any questions on that?

Obviously the sinc pulse function that we have used earlier also satisfies this criterion, why? Because in that case this was a brick wall characteristics and this replication would simply put those brick wall, functions side by side with respect to each other and you will anyway get a constant function right, so that was a special case of this more general results okay, so now we have a possibility of a whole lot of functions which can be the right kind of pulses to use.

As long as that function has a p omega which satisfies this criterion we are through okay, now let us restrict our attention to those functions of which the cutoff lies between omega 0 by 2 and omega 0 right and let us restrict our attention in that case only to the frequency band between 0 to omega 0 because if our function is like that, has a cutoff upto here then if you are reasonably sure that it is going to be constant in this interval because only three functions will come to the picture in this interval then.

Which one? p omega then p omega minus omega 0 and p omega plus omega 0 right because only these three functions will contribute to the sum in this interval, in the interval between minus omega 0 to plus omega 0 if these three function when edit together give rise to a constant value in this region will be assured that the say will also hold for rest of the frequency domain okay, so therefore let us restrict ourselves now to a P omega of this kind rather than general p omega.

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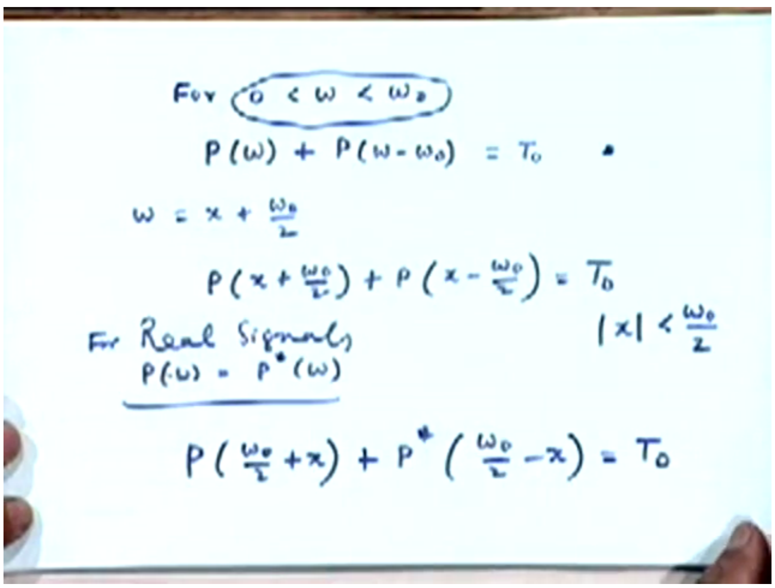


And then we can write for omega between 0 and omega 0, P omega, in fact between this range only two functions come to picture P omega and P omega minus omega 0, this should be equal to T0 okay so the condition becomes even simpler here for these kind of functions instead of having to work within infinite summation, I only have to work with two terms in this summation.

Let me now define a variable x which is related to variable omega through this, the motivation be x will essentially denote, suppose I choose any point omega on the axis, x simply denotes the distance of it from omega 0 by 2, x can be both positive as well as negative so depending on if omega is less than omega 0 by 2, x will be negative, if it is more it will be positive, so I am defining a new variable in the frequency domain which is related to omega through this selection.

And then we can put this condition in this form x plus omega 0 by 2 and here it will become this minus omega 0 which will give you x minus omega 0 by 2 is equal T0 and this limit I will now put in terms of x, magnitude of x less than omega 0 by 2 okay please stop me if there is a problem, as you mean that our pulse shape is real which is the case over here, you know for real signals the Fourier transform satisfies this condition of P minus omega is equal to P star omega right for real signals okay.

Then we can write the same condition as P omega 0 by 2 plus x plus P conjugate omega 0 by 2 minus x, so first I have taken the negative of this so as to write P conjugate and then you get this for x in this range, any questions with the maths so far? Alright, now let us make our life still simpler by saying that the spectrum is also real it is not really very big restriction provided you are ready to work with zero phase and linear phase functions in frequency domain.

We can later if you want permit P omega to have a linear phase function which will correspond to a constant delay but to start with let us say even P omega is real, the frequency, the spectrum of the pulse is also real which only means that the, for example the sinc function that we discussed earlier will have a real spectrum right in fact any symmetric pulse of that kind will have a real spectrum.

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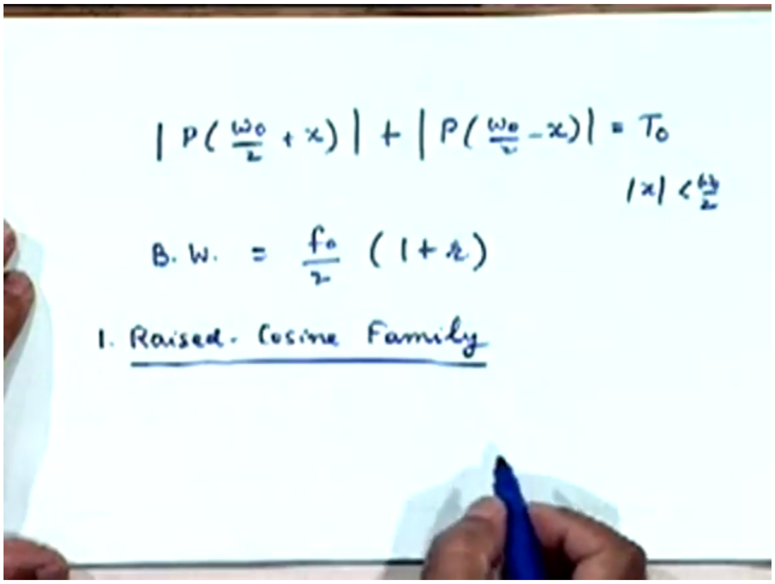


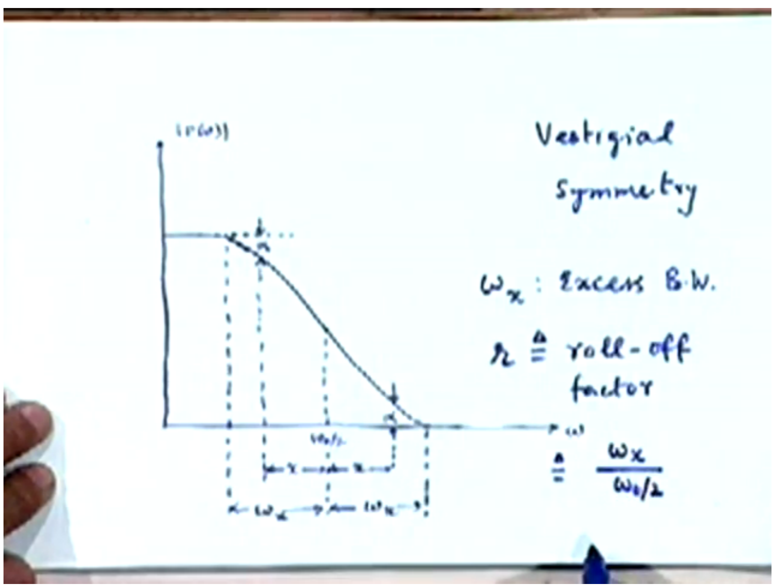


So let us say we are having pulses of that kind so in that case we do not even have to worry about this complex convolution and we can say that our magnitude function should satisfy this required property. Have you come across this property ever before in some other context, vestigial, very good so you remember that from your previous course, okay and pictorially what that means is let me have what is called vestigial symmetry around omega 0 by 2 okay that is our function P omega, magnitude of P omega should have vestigial symmetry around omega 0 by 2.

That is if I were to fold this portion of the spectrum back into this region between 0 to omega 0 by 2 by just carrying out a folding operation at this point then this function and this function should adapt to a constant value, that is vestigial symmetry right that is precisely what this states over here, right this condition, you can please if you want copy this picture, in the meantime if you have point to discuss.

So this is a vestigial symmetry condition which you must have discussed when you discussed vestigial sideband modulation in analogue modulation systems, right, this is the requirement for vestigial sideband filters right, now here is the notation omega 0 by 2 is the nyquist frequency, x is this new variable we have introduced and omega x denotes the region over which we are allowing this function to have roll off right.

And obviously the function has a bandwidth exhibits a bandwidth which is greater than omega 0 by 2 by an amount equal to omega x so omega x is also called the excess bandwidth that we are using up now right, excess over and above the minimum nyquist bandwidth that we could have used alright so omega x is excess bandwidth and this is a meaning of vestigial symmetry for example if at the point x this value is alpha then this value is alpha on this side that is the meaning of vestigial symmetry.

Let me also define a factor r here which is known by the name of roll-off factor and it is defined as equal to the ratio of the axis bandwidth over the nyquist frequency which is omega 0 by 2 so in terms of this roll-off factor, can I remove this now? Is there anything else that you want to look at here? Okay.

Then you have bandwidth of this pulse will now be given by f0 by 2 into 1 plus r, f0 by 2 is a nyquist bandwidth and this is additional bandwidth that you are using up r times f0 by 2 so that was a result with the P wanting to show in the first place that if you use up a bandwidth equal to k f0 where k lies between half and 1 right, it is precisely the situation here 1 plus r by 2 will lie between half and 1 then we can have other pulse shapes which will perhaps be better suited.

We still have not seen that aspect whether these pulses will be better than the sinc pulse or not is something you still have not try to appreciate but at least we now have pulse shapes other than just a sinc pulse available to us, a whole lot of them as long as you have a P omega it satisfy which has this kind of a shape we are guaranteed to have the nyquist criterion satisfied and we can obviously construct many such functions okay.

Student: Sir.

Professor: Yes, please.

Student: (())(32:21) that you see that a signal in any pulse is also real in time and real in the frequency domain.

Professor: That is not a very big assumption because any symmetric pulse, pulse shape around zero will have that kind of a property, symmetry in time will give you reality of the spectrum right?

Student: Sir the moment you say symmetry in time it is not a real time domain like it is a not in the (())(32:47)

Professor: No no, when I said real pulse what I really meant was real valued pulse that is the function p t is real value, it is not a complex valued function okay of course it can have extend both in positive time as well as negative time, I did not mean real in the sense of not using the negative time okay because basically use in a mathematical sense real valued function p t is a real valued function, okay so I hope that confusion is not there, similarly when I see t omega is real what I really mean is real valued function.

Student: Sir what (())(33:25) the pulse that we have seen in the negative time.

Professor: Well strictly speaking such a pulse shape is not truly realizable but any such pulse can be always realized by introducing a sufficient amount of delay right, I will discuss this issues separately right and using any communication channel will have certain amount of delay associated with it, so at any point in time you will not only see what the response to the channel due to the past but in some sense also response to the future with respect to that time instance not the true future of course but future at the transmitter.

Because receiver and transmitter are separated by some delay right, so you will see some feature corresponding to the transmitter at time t also at some time t minus t d at the received right, so we will discuss both issues separately because that will mix up the issues anything else, okay, now let us see some typical, excuse me, yeah now what we have to really decide is how to choose this pulse spectrum shape right so as to obtain some desirable property, to some extent the fact that we are shaping it like this also is (())(35:16) a certain nice desirable properties as such.

Because we have come out of that sharp fall and we now have a gradual roll off, the time frequency properties of signals of such that if one function let us the function is very sharp in one domain it will have a very slow decay in the other domain right, we have avoided that situation here by not having a sharp fall so to some extent we are guaranteed that we will have nice properties, the only other thing is whether we can come up with a nice mathematical function which will have this vestigial symmetry and also be very easy to realize.

A comparatively easy to realize as it stands truly speaking this functions are not strictly realizable, I am not really going to go into that theory now but if you see in the bandwidth is 0 to omega 0 which is of perhaps of interest to us here there is a region over which the function is absolutely 0 such frequency domain functions are strictly speaking not completely realizable in time domain they are, we do not satisfy the causality condition required for real functions in the sense that one of our friend mentioned.

But in fact the condition that they do not satisfy which is a well-known condition I do not know whether you know about it or not is known as (())(36:42) condition alright, so this functions do not satisfy the (())(36:46) condition, however even you do not know about it let us not worry too much about it however the important thing is that because they have a nice shape we can be approximated to realize to fill a good degree of approximation, fairly closely realized, right, by various methods of approximation which will again go into separately some other points which I.

So I will not go into the realization issue here but to say that if we have a reasonably smooth function they will be easy to realize and one family of functions which have gain popularity in the digital communications literature for this purpose is a very well-known family it is called the Raised Cosine Family of spectrum and they satisfy this vestigial symmetry condition since I cannot write it here I will write it separate sheet here.

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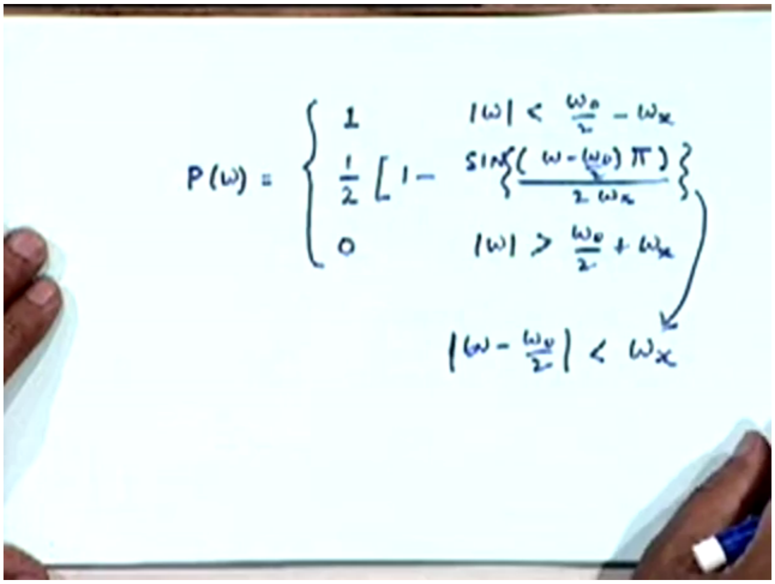

It is given by the following equation P omega for the Raised Cosine Family it is a good family from a practical realization point of view is given as follows if you might notice you could realize this function into essentially three parts there is a constant part here, a zero part here and a roll-off part here right, so I will specify each of this three parts separately, as far as a constant part is concerned you can say it is equal to 1 for omega between omega 0 by 2 minus omega x right.

For values of omega whose magnitude is less than this that corresponds to this interval here right omega zero by 2 minus omega x is this much from here to here right so in this interval of omega it is constant, it is equal to zero for omega greater than omega 0 by 2 plus omega x and it has this roll off shape in the remaining interval sin omega minus omega 0 by 2 into pi sorry this is getting a bit cluttered I hope you can make it out.

This sine of omega minus omega 0 by 2 into pi upon 2 omega x and the interval corresponding to this is omega minus omega zero by 2 less than omega x that is on either side of omega 0 by 2 which is less than omega x okay and if you want to plot this function basically that is what it looks like when part of a sine function or you can think of some kind of a cosine function really either this or this.

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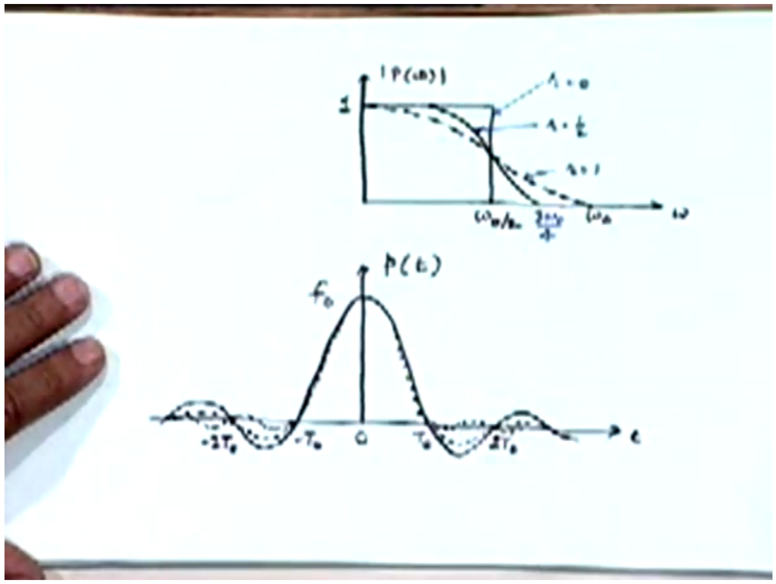

You can either hold lot of functions depending on what value of omega x you choose right, omega x is a parameter with us remember you could choose different values of omega x depending on how much excess bandwidth we are ready to use right, the maximum we can go to in this scheme of things is omega x equal to omega 0 by 2 right but we could use less than that if we wished so we could therefore have a whole family of functions that satisfy that this condition.

This is a function corresponding to r equal to 0 that is same good old brick wall function, that is a function corresponding to r equal to half which will use an excess bandwidth equal to omega 0 by 4 that will go upto 3 omega 0 by 4 this a 100 percent roll-off situation where you go upto omega 0 right, now important question is what kind of pulse shapes do these various spectra have right okay before I show the physical picture I will just do a little bit of maths for that, it is a bit more involved to do it for the general case to get the pulse shape for the general case what I will do is take it for the special case when r is equal to 1. (())(42:08) In the this equation here, (())(42:15), yes.

Student: We get at 1 minus sin of pi by 2.

Professor: No omega minus omega not by 2 this will be 0, so this is 0,

Student: Okay okay

Professor: So this disappears altogether you simply have half right which is what we want if this is 1, we will like this to be equal to half right, try it out you just have to plot this function to verify whether the picture conforms to this or not alright, so coming back to the maths, what I was saying was that we will consider the case when r is equal to 1 to get the corresponding pulse shape and for the rest I will just tell you what they are like, yes please. (())(43:12)

T0 is very easy I can multiply everything by T0, okay yes that is the point I could have multiplied everything by T0, T0 here, T0 here. What that will do is in fact it has shown here instead of p t having amplitude equal to 1 you will have a amplitude corresponding to 1 by T0 right which is f0, so it is a method of putting a constant either here or here it does not qualitatively effect our argument.

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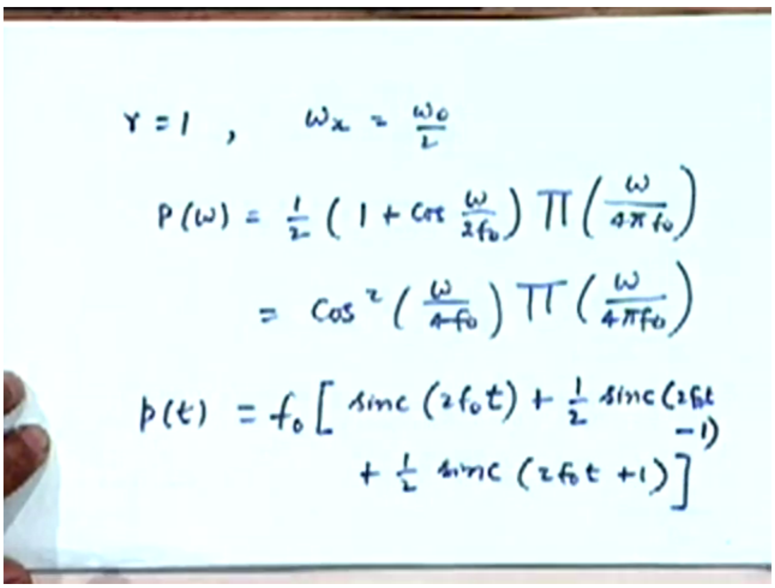


So consider a case for r equal to 1, that is omega x becomes equal to omega 0 by 2 in this case it is very easy to verify this statement that the same, in fact I have to specify only one region now there is only roll-off region right there is no constant region, no zero value region and in that region if you need the substitution there expression can be simplified and that sine function becomes cosine function of the simplification.

And this is to indicate that the function exists only between 0 to omega 0 that is a rectangular function spread between minus omega 0 to plus omega 0 right, which you can write as cosine square omega by 4 f0 okay, this is our maths corresponding to the special case when r is equal to 1 and if you want to appreciate what p t is like for this case, make use of this expression here, this is a rectangular function in the frequency domain with bandwidth omega 0 right.

And here we have additional terms, you can write this function, cosine function as sum of two complex exponentials right so we really have three terms, one corresponding to the brick wall function alone and two terms corresponding to brick wall functions multiplied by a complex exponential right, which will be like a delay operation in the time domain so basically you can appreciate physically I will leave the details for you to check yourself.

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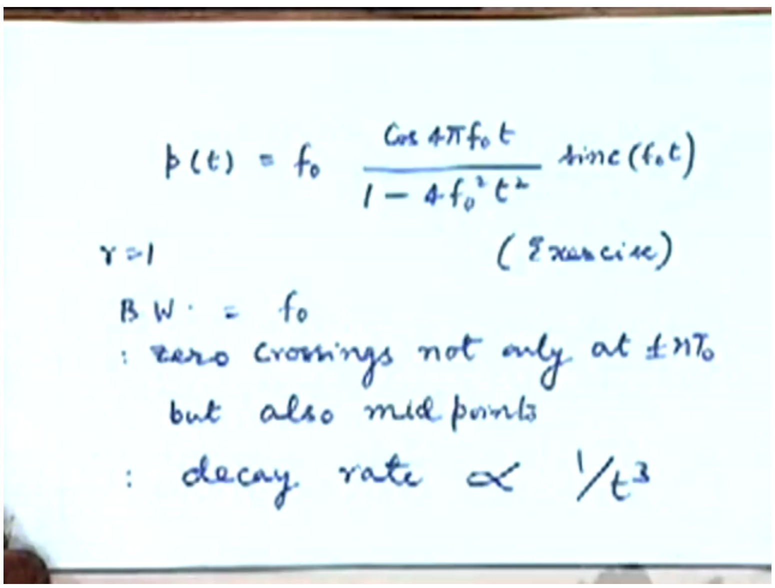


You can appreciate that p t will be essential sum of three sinc functions, three sinc pulses right like this, the basic sinc function corresponding to bandwidth f0 not f0 by 2 so it is 2 f0 t right, plus half of the same function but shifted by 1, plus half sinc to f0 t plus 1 right which will, if you were to simplify a little bit you will get p t equals I will just give you the final result, skip a few steps, this will be the final result after simplifying that function.

I will like you to check that yourself and if you have any problem get back to me so that is an exercise for you and the plot of this functions looks like this, excuse me, yeah here it is, they all look like the same good old sinc pulse in fact this solid line I have drawn here is precisely the good old function sinc pulse right that is it has zero crossings at t0 2t0 and so on multiples of T0 but what the effect of this roll off is you can see here in this dotted lines right.

This one here corresponds to r equal to half of course what we have just discussed is the case of r equal to 1 so what you will notice is as you keep on increasing r, the value of r from 0 onwards towards 1, that is as you keep on using more and more excess bandwidth your side lobes decay very fast, in fact for a case of r equal to 1, you can even mathematically see what is the rate at which it will decay.

What is the rate? Asymptotically let us say when t is very large 1 upon t cube because 1 factor of t comes from here and t square comes from here in the denominator so the decay rate increases to proportional to 1 by t cube rather than 1 by t which was the case of the sinc pulse for a special case of r equal to 1 we even have zero crossings midway between the desired zero crossings, you will see that here, may not have come out very well but that is what this function is like here. (()) (49:41)

For r equal to 1 specially this property holds for other values of r, they may or may not be roughly these additional zero crossings right but many case there is lot of less interest to us what is really of interest to us is that we have a zero crossings at these points which will be available okay so the net result is that, for continuing our discussion for that case of r equal to 1 our bandwidth will be equal to now f0 twice the minimum nyquist bandwidth which will be 0, the function will go through zero crossings not only at plus minus n T0 but also midpoints of these intervals.

Which only means that they are decaying much faster, the side lobes are decaying very fast, in fact the asymptotic decay rate is now 1 by t cube so as a result of these facts the effect is that our sensitivity to finding errors will be considerably reduced as compared with the sinc pulse, in fact you can see that the slope at zero crossing also cannot be very large because side lobes are smaller in fact for I equal to 1 from one zero crossing it has to come to another zero crossing, so it cannot possibly have a very large slope here right.

Therefore sensitivity to timing errors comes down and as a result this is also more closely realizable because you have a gradual roll-off, now it also has another advantage which we will discuss next time, this function, the raised cosine family of functions they also satisfy another criterion that was put forward by Nyquist which is known by the name of Nyquist's second criterion.

For zero inter-symbol interference, now we look at that second criterion more closely next time and in fact we will see that it is related to something that we have already discussed in the context of line coding and that was in the case of duo binary signal we discussed here we will see that there is a relationship between spectral shaping of pulses and line coding that we have discussed earlier okay I think this is a good stopping point for us today, if you have any questions.

Student: Sir second criterion is regarding

Professor: Again another criterion for inter-symbol interference removal but enabling us to double the bandwidth, double the data rate of the same bandwidth okay.