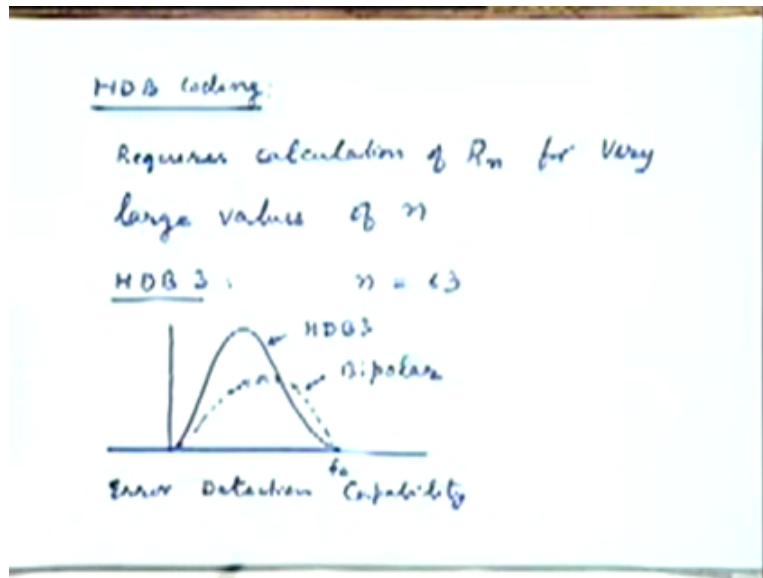


Digital Communication.
Professor Surendra Prasad.
Department of Electrical Engineering.
Indian Institute of Technology, Delhi.
Lecture-10.
Baseband Pulse Shaping: Nyquist First Criterion.

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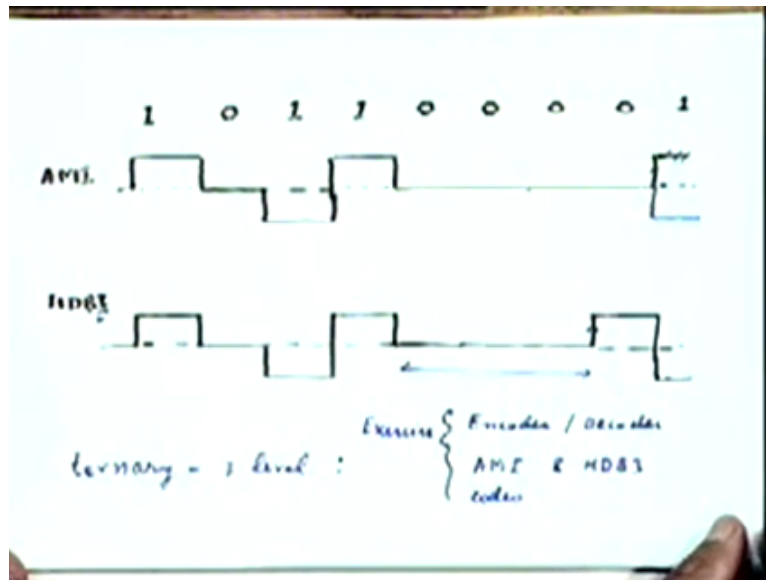
Professor: We said that one can do spectral analysis of hdb coding also in the same way that we have done for the other cases, however in this case it turns out that one has to evaluate a considerably large number of the coefficients r_m to be able to evaluate the power spectral density function. Because in all of the previous line coding schemes, it turns, it is so turned out that only a few of these coefficients were nonzero and they became 0 very fast after 1 or 2 or at most 3 coefficients, right.

But in this case because there is a considerable effect of 1 violation that you do in any point in time, to a large point later on, as to how data will be represented in time later on, will a considerable amount of correlation exists even for time instants which are largely separated from each other after coding, right. The waveform after coding has correlations existing for large values of n , large indices of, large values of the index n . Therefore it does not decay to 0 very quickly which means you have to evaluate a very large number of these coefficients to be able to get some kind of a good idea about the spectrum of hdb codes, right.

For example, for hdb 3 to be able to approximate well, you need at least n equal to 63, right. The other better methods of doing power spectrum calculation in such situations which we

have not considered but in many case we, that is also outside the scope of our present lecture. But there is a typical plot of the hdb 3 spectrum and with respect to the bipolar spectrum. So we find that the main lobes of both or within f_0 , that is 0 to f_0 , limited to this span, although the shape of the exact spectrum is slightly better in the hdb 3 case because it is limited, it is more, it is distributed in a more balanced in the available spectral band.

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The last point that we made was that the hdb 3 code also has error detection capability, just like the ami code has, which you may see by looking at the basic hdb 3 coding that we discussed some time ago. Basically we introduce a violation, right, of a string of 3 zeros, that is what hdb 3 coding does. If you notice that in spite of the fact that we are introducing violations, we still have error detection feature in this coding. For example let us say error occurs and this 1 or this negative voltage appears like a positive voltage here. So what you will see then is 2 consecutive 1s with the same polarity without 3 zeros in between them, right, which will be detected as an error, right.

And similarly can see any other kind of error that gets introduced will eventually get detected at the point of another violation, right. So like ami, hdb 3 also possesses error detection capability and for this reason because it has a slightly better spectral shape, in fact considerably better spectral shape than ami, hdb 3 is to be preferred over ami.

Student: Error detection is better than ami or it is equivalent?

Professor: It is like ami. Basically you can do single error detection both with ami as well as hdb 3, right, that feature is common to both. So with this i come to the end of our discussion

on line coding, right. Basically, let us review it quickly, what does line coding do for us? Line coding helps us to achieve a given power spectral density function, that is one function of line coding. It also helps to introduce certain desirable features in the encoded signal, right. Desirable from the point of view of dc content, from the point of view of timing extraction, from the point of view of transparency with respect to ones and zeros, right.

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$S_y(\omega) = S_x(\omega) |P(\omega)|^2$

PULSE-SHAPING

$p(t)$: Rectangular pulse
 : infinite bandwidth

0 to f_0 : A small fraction
 ↑ outside this band

: Distortion may be small for I.S.I

So these are the reasons why we do line coding and we have considered in detail so far how we can shape the spectrum of a line coded signal by mapping the data sequence to voltage levels in an appropriate manner, right. That is the basic tool that we have used so far. That is representing aks in a particular manner as far as their mapping into voltage levels is concerned. That is we have tried to control the power spectrum and introduce these desirable properties through control of $s_x(\omega)$, right. Because let me review, we said that $s_y(\omega)$ is $s_x(\omega)$ into $p(\omega)$ mod square where $p(\omega)$ refers to the power spectral of the pulse that is used, basic pulse shape.

And this depends on the way we introduced the, the way we carry out the coding of data into voltage levels right, as data comes along, right. This is essentially controlled by the coding aspect, whereas this is determined by the pulse shaping aspects. We also discussed the one example where the power spectrum was controlled by $p(\omega)$, namely that of the split phase or manchester coding, where you deliberately wanted to introduce a null at dc, right. And we said that we could do that by making the area under the pulse shape equal to 0, right. But

other than that largely we have been looking at the coding aspect and that is why the topic we call line coding, right.

It is equally important to look at the pulse shape, right. And now we will take up this issue and this is a very very important issue as we will soon see. We will now look at the issue of how to select this pulse shapes that we have been talking about, right. And that brings us to the question of pulse shaping. Of course pulse shaping will affect your power spectrum, so that becomes another tool with us to control the power spectrum. And now we will look at this in more detail, the issue of pulse shaping. What we have done so far, we have looked at pt, we have chosen pt to be essentially a rectangular pulse, right, in all our treatment so far, which in principle has infinite bandwidth.

So in principle it has not finite bandwidth but very large bandwidth, although we have argued that most of its energy will be concentrated in the main lobe of the power spectrum corresponding to the rectangular pulse. And therefore we can say that essentially it is band limited, right, in some sense. That is if we filter our signal before we transmit or if a channel is carrying out that filtering for us because the channel is band limited, then most of the energy of the pulse will pass through, but a fraction of its energy will not pass through, right, of the rectangular pulse.

That fraction we have seen is small, for example most of the energy lies between 0 to f_0 for most of the line coding schemes that way are likely, you are likely to use. Right. So most of the energy lies in this form, except for polar and some other allied line coding schemes in which case it lies between 0 to $2 f_0$, right. And for duo binary it lies between 0 to f_0 by 2. So depending on the line coding schemes the basic band differs but once you decided the basic band depending on the line coding schemes, we know that most of the energy lies in this interval, right. And a small fraction lies outside this band, okay.

Now let me transmit it over a band limited channel with a small fraction which lies outside the band can create problems for us. See, in practice we will not be allowed to use more bandwidth than is allocated to us for a, for transmission at a certain rate, right. If you are transmitting at f_0 and you are using a particular line coding scheme and therefore you are allocated a certain amount of bandwidth on that basis, you are not allowed to transmit anything outside the band, which means you have to do band limitation, right. And the moment you do band limitation and this small fraction therefore gets attenuated, it may not

cause much distortion but the little distortion it may cause, did can cause considerable problems for us, right.

So although the distortion may be small resulting in distortion, distortion in what, in the pulse shape, right. You are transmitting a rectangular pulse which has the bandwidth, you are not allowing all its bandwidth to be passed, that means it no longer remains a rectangular pulse, it changes into some other pulse shape, right, that is why it is distorted. This distortion, even though it may be small, it can introduce a very severe problem for us in the form of, this is the abbreviation i am using for intersymbol interference, okay. Let us talk about this a little more.

Student: Sir.

Professor: Yes please?

Student: (())(12:08).

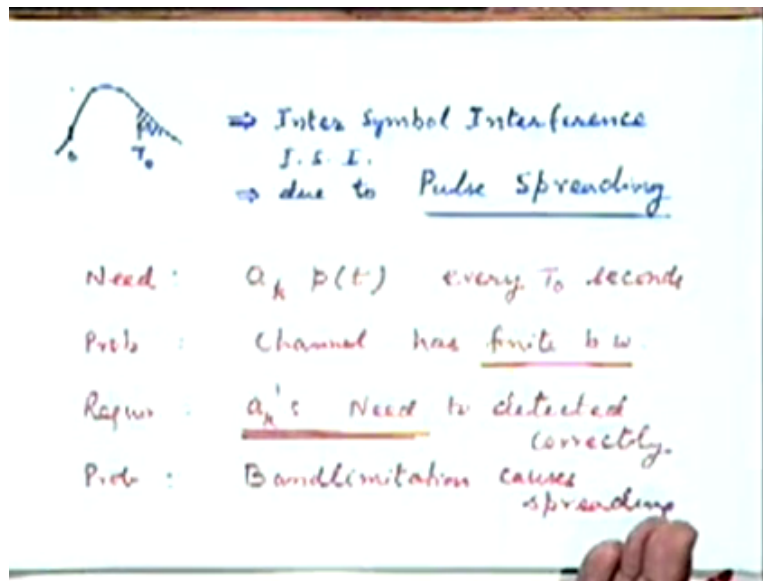
Professor: Of course. I will be just coming that, i will just introduce the topic to you but i would talk about it more. What happens when you band limit a pulse, what kind of distortion is like to occur? This is the main question to understand. Can you say something?

Student: Sir it will be (())(12:31). Infinite time.

Professor: It will try to be stretched in time, that is the essential thing. You know from the basic properties of fourier transform that something which we limits in frequency gets spread in the time domain and vice versa, right. This is the, in one of the talks also, somebody talked about the uncertainty principle of communication theory, right. That is if a function is limited in time is likely to have loss spreading in frequency domain and if it is limited in frequency domain, band limited in frequency domain, then it will try to, it will just spread out in time domain, right.

The rectangular pulse was time-limited to its interval, basic interval of 0 to t_0 seconds or 0 to t_0 by 2 seconds depending on whether it was a half width rectangle pulse or fullwidth rectangular pulse, right. But after band limitation to whatever band you are finally allowed it to use, it will get spread out. That is it will no longer be limited to this basic time interval of t_0 seconds, it will spill into a different time intervals, right, or just the same symbols, right. Therefore what you are transmitting, the energy that you are transmitting in the basic interval of 0 to t_0 seconds is no longer going to be confined to this interval, it is going to spill over.

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For example, the rectangular pulse perhaps may become like that, right. Some kind of spreading I am indicating, there can be number of ways of spreading the pulse. Energy of this pulse has spilled into this time interval, right. Therefore now when you do detection of impulse in a particular time interval, you no longer are sure that what you are looking at, whether it is energy you are looking at whether it is an attribute of the pulse you are looking at, is an attribute of the pulse that was transmitted in that interval. It also has contributions from what you transmitted in adjacent intervals, maybe adjacent and in fact a few adjacent intervals depending on how much is the spread, how many symbols it spills over into some right.

So therefore we get an effect which is called intersymbol interference. It is a very serious problem in digital communication in band limited channels, which we denote simply by ISI most of the time. We will talk considerably about this effect as time goes. This is essentially caused due to pulse spreading, right. Let us take a stock of what we have discussed so far. What is our need? Our need is to transmit pulses of the kind a_k into $p(t)$ every T_0 seconds, right, where a_k is the amplitude of the pulse that you want to transmit in this particular time interval.

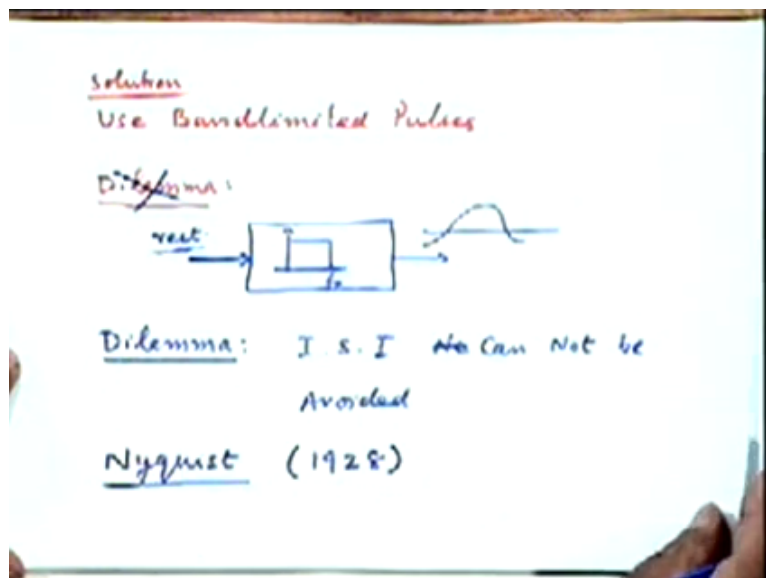
The problem is, channel has finite bandwidth, okay. Our requirement is that we should be able to determine a_k s correctly, a_k s needs to be determined correctly, right. Now the next problem is that this requirement is hampered by the fact that this band limitation causes spreading of the pulse, right. Band limitation causes spreading and that affects this

requirement, that asks needs to be detected correctly. Because when you are detecting, trying to detect ak by looking at the pulse interval corresponding to it, you may also get influenced by what was transmitted elsewhere, right.

And you may make a wrong decision. The probability of making a wrong decision is likely to increase because of disturbance from other pulses, right. In fact you can look upon this contribution from the pulses as also some kind of noise. Depending on whether adjacent pulses are positive or negative, the overall contribution will be dependent on how each of these pulses spills over into the record interval of time into which we are looking, right. It is kind of a random noise whose value is unpredictable because it depends on what data sequence is being transmitted, right, around that particular interval, right.

So therefore effectively, one way of looking at the effect of this intersymbol interference is as if your overall noise power has increased. We already have some noise in the channel, right, due to various other sources and here is another source of noise that has got introduced because of the pulse spread. So the rms value of the, total rms value of noise gets increased and once the total number of rms value increases, the error probability increases, right. So intersymbol interference indirectly affects your performance.

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That is your liability of determining asks currently comes down, reduces, okay. Is that understood? So what is the way out? Any suggestions on that? Obviously one suggestion would be why not use band limited pulses to start with because you are going to carry out band limitation, so you use band limited pulses, right. But it appears that you solve the

problem if you use my limited pulses. One possible solution that you can think of is use band limited pulses. But the moment they are band limited, once again they will be stretched in time and again appears, it appears they will cause intersymbol interference. So we have a kind of dilemma here.

Student: Why do you say that even this kind of a band limit pulse will actually stretch up on time?

Professor: Any kind of band limitation means, in time, it has to be stretched out.

Student: We are already using band limited, (())(19:55) limited to band.

Professor: One question at a time.

Student: Earlier the problem was the initial signal had an infinite spectrum and the channel had limited bandwidth so that the (())(20:14).

Professor: Yes, that means the same. Unless you choose the single very very carefully, because it is band limited, that itself is going to be spread in time and therefore either you will have to decrease your data rate depending on the spread, right or else you will have to again limit intersymbol interference. That is the point i am trying to make, right. It is just the same fact that we had been talking about earlier that if we, suppose you are saying that okay we use pulses whose bandwidth is strictly f_0 and does not have anything more than that, f_0 is the rate at which we are transmitting. Then the obvious thing is that it may not be, it may not be prickly time-limited to $1/f_0$ seconds or t_0 seconds, it will exist even outside t_0 seconds. So we have the same problem again, right.

Whether the channel limits the bandwidth or whether we limit the bandwidth, the ultimate result is the same, right. It is not, after all there is band limitation. It does not matter who limits the bandwidth, unless when i am limiting the bandwidth i do also something else which is what we are going to talk about. Not clear?

Student: No.

Professor: Why not?

Student: The problem is (())(21:30).

Professor: No, that is not the point. It is not the question of reproducing, it is simply a question of. Okay, let me talk about that 1st. We have a rectangular pulse which the channel, which is being passed through the channel with limited bandwidth, right. That lets say have this kind of a frequency response, strictly between 0 to f_0 because they are not allowed to transmit outside it, right. It is this fact which spreads the rectangular channel into, it stretches it outside the time interval, right. Now whether i do this band limitation, or if the channel does this band limitation, this fact cannot be altered.

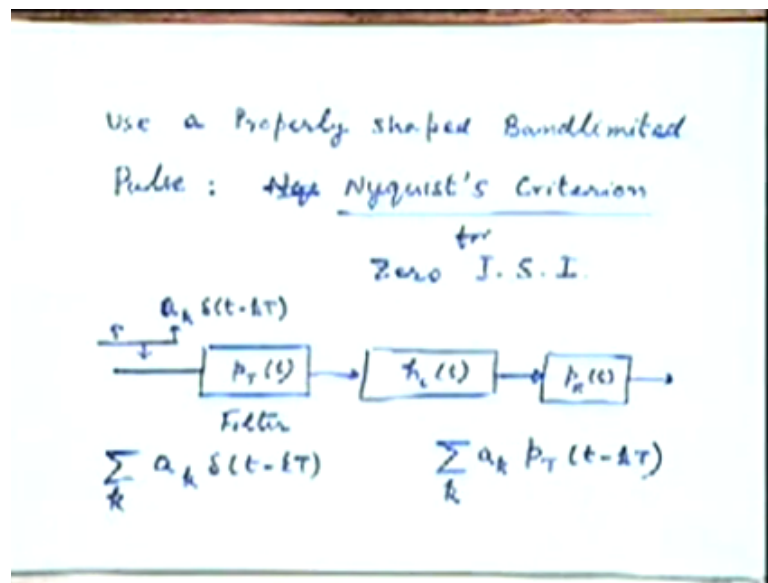
Student: (())(22:18) which is anyways band limited, then the output signal will also be the same signal.

Professor: How will you do that band limitation? The band limitation that you are going to do of a rectangular pulse, how will you do it, you will also do it in the same way, right. So therefore it does not really, it does not, that is why you are doing this kind of thing, that is the point of... i do not know, i think i have said all that i can say in this matter, it seems to be simple enough. Is there anybody else with this out? The point that i am trying to make is that band limitation seems like a solution but unless we do something special about it, it is not really a solution.

Because any kind of band limitation, any kind of band limited bus that you can think of is likely to be priced considerably in time, right. And if you are, if you are required to transmit at the same rate as before, we have a difficulty here, right. And that is the dilemma i was going to talk about, that it seems that no matter what we may do, whether we limit the bandwidth or whether the channel limits the bandwidth, there is going to be intersymbol interference which seems, it seems cannot be avoided. So either we have to live with it or we have to do something about it. And the answer to this dilemma was provided long ago by a gentleman whose name you are very well familiar by now, his name is nyquist, right.

And this result due to him which enables us to use band limited pulses and still eliminate intersymbol interference was provided as long ago as 1928, right, in a classic paper that is still referred to greatly. And the answer is, that is, answer lies in using a properly shaped band limited pulse, right.

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So of course we will try to understand what the proper shape is. Use a properly shaped band limited pulse. And what is the proper shape, this is what we need to understand. In order to do that, we suggest the criterion which is known by the name of Nyquist's criterion. There are number of them, at the moment we will discuss the 1st criterion but I will just call it the Nyquist's criterion, okay. For, this is the criterion for using band limited pulses with 0 intersymbol interference, okay. Now before we take up this criterion, let us see what we understand by this proper shape, okay and to do that I have to slightly digress.

Let us look at the basic, I am not going to the details of digital communication system but I want to look at functionally basic building blocks of a communication system, digital communication system from the point, point of view of pulse shaping which are relevant to pulse shape. We are not again going to review what the whole system looks like, we have already seen that. But from the point of view of pulse shaping, you can think of the following basic blocks. You have a pulse coming along, you have data coming along, let us put it that way, you have data coming along in the form of ones and zeros.

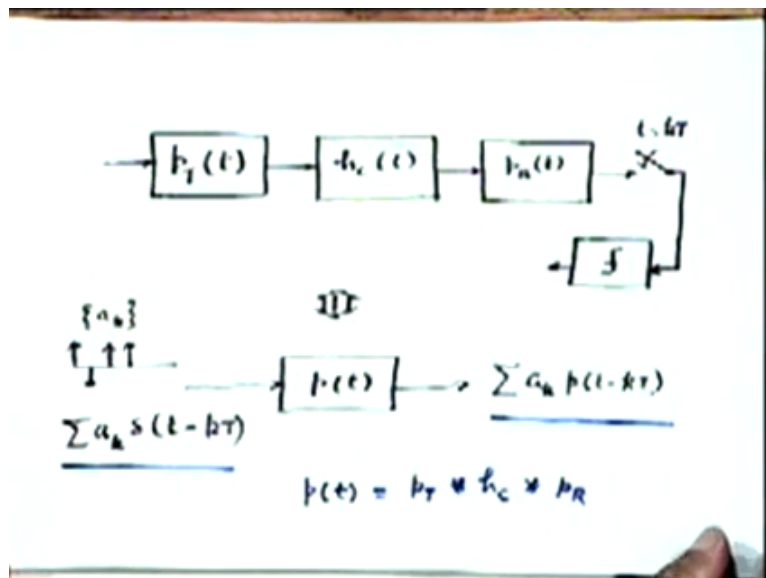
Which I can represent by, let us say impulses of $\delta(t - kT)$, right. Well let us say the kth impulse has a strength a_k and therefore data at time kt is being represented by an impulse, its strength $a_k \delta(t - kt)$, right. I can assume that this is being passed through a pulse shaping filter at the transmitter, and so this is a filter with the pulse response which I am denoting by $p_T(t)$, the subscript T is denoting the transmitter, okay. And obviously what I

will get here at the output of this is a superimposition of these pulses. The pulse shape here will depend on the transfer function of this filter or the impulse response of the filter, right.

So if at the input to this filter the signal is a sub k delta $t - kt$ summation over k , right, that is the input. And the output will be a sub k pt, combination of this with an impulse will be simply pt , the pulse delayed by kt , right. So this is what we will get at the output, right. This is how typically we can do pulse shaping at the transmitter, by choosing a filter, pulse shaping filter of an appropriating pulse response. Now this is going to go over a channel, i am not going to other functions, other blocks of the system.

But broadly speaking, just before you trust it, let us say you have done this pulse shaping and this is going through a table which has a transfer function $h_c(\omega)$ and an impulse response $h_c(t)$, right. Finally at the receiver among other things that you do, as far as the section of a_k is concerned, we have agreed, although we have not discussed the theory of that so far but we will use some kind of a matched filter, right. So anyway there is another 3rd filter coming into the picture at the receiver, right. Which has perhaps an impulse response p sub rt , okay. All right.

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Now let us see what i am trying to say. How will you process the output of this p sub rt . I have got the same picture again drawn over here, please look at it bit carefully. Here is your transmitter pulse shape filter, here is your channel and here is your filter at the receiver which could be a matched filter or any other kind of detection filter which helps us to do reliable detection. Okay. Basically the idea of this filter is to average out the noise in each pulse shape

as much as possible and to get at the output some value, some signal which is proportional largely to the signal rather than to the noise.

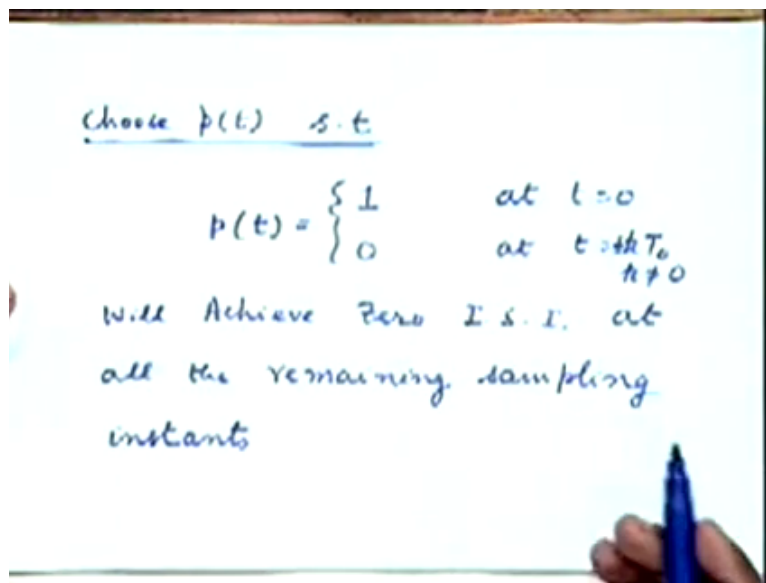
And typically what you will do is, the output of such a filter, matched filter will be sampled every kt seconds and depending on whether the sample value is let say above a threshold or below the threshold, you will declare is 0 or 1 to be present. This is a broad manner in which things will be done in a digital communication system from the point of view of a pulse that is transmitted, affected by the channel and processed at the receiver, okay. So from the moment that you have these impulses coming in which carry the data for you to the point at which final decisions are taken, you have 3 filters coming the picture. Right.

Which you can club together into a single filter which i call the pulse shaper p_t . The pulse shape i am going to talk about for the time being is not the pulse shape that is going to come out at the transmitter but which is what is going to, what you going to look at just before you do sampling here because it is that pulse shape which is really important as far as intersymbol interference is concerned. Because at the point of detection it is important that i do not have interference from other symbols, right. Where is it important that there should be no intersymbol interference? At the point at which i am going to decide whether a_k was 0 or 1.

Right, at that point i do not want any interference from what was a_{k-1} , how a_{k-1} was transmitted, how a_{k-2} was transmitted on how a_{k+1} is going to be transmitted and so on. So at that sampling instance of this pulse shape, if i do not have intersymbol interference i (()) (31:57). And therefore what i have really considered is that equivalent system whose impulse response is simply p_t which is the convolution of this p_t , $p_{sub t}$, the channel impulse response $s_{sub c}$ and the receiver filter $p_{sub r}$, right. So p_t is an impulse response of the overall system, overall baseband system and given by $p_t \star h_c \star p_{sub r}$.

And let us work with this equivalent system here. Okay. Input to this equivalent system is the data sequence, data sequence impulses, impulse train carrying the data sequence represented like this. Output is a pulse train sequence with pulse shapes $p_t - kt$, right. And we will now allow this p_t to be a band limited impulse response, strictly band limited impulse response, okay. Any questions so far? Now let us return, we again return to the nyquist's argument. But what nyquist proposed was that one can achieve 0 intersymbol interference if we choose this p_t such that it undergoes a 0 crossing every t seconds except for t equal to 0. Right.

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That means if we choose a pulse shape $p(t)$, just let this equal to some finite value 1 at t equal to 0 but is ensured and guaranteed to be 0 at all other sampling instances, t equal to kt , okay, $+ - kt$ let us say, k not equal to 0. So choose the overall pulse shape $p(t)$ such that this property is satisfied, we appreciate this, this is the important contribution of Nyquist. That is if we do that, we note that at the sampling instance we will not get any contribution from, because when I am sampling in the k th time interval, at t equal to kt , the only nonzero value from this summation will be coming from the k th pulse, all other pulses will be going through 0 crossing at that point.

So even though this summation is what you are really observing, right, the only term that contributes to the summation at the k th time instance is coming from a sub k and you will detect a sub k correctly. That is simple engineer's argument, right. And that is the contribution of (35:15), contribution of Nyquist way back in 1928 to eliminate intersymbol interference. So you let the pulse to spread out, it does not matter, make sure that it has regularly spaced 0 crossings at every t seconds, t_0 seconds, where t_0 is your time intervals of available, okay. This is a basic, very simple, ingenious argument given by Nyquist.

It uses the pulse shape $p(t)$ such that it has this property. All of you understand this? Because if it is not understood, you can understand the Nyquist theory in a very easy way. It is a basic important point, rest is all maths which is very easy to follow. So such a pulse shape will achieve 0 intersymbol interference at all the remaining sampling instants.

(36:32).

Okay, I am talking about, this is the 0th pulse, it is going to spill into the remaining pulses that you are transmitting but at their sampling instants, sorry...

Student: ISI (36:44).

Professor: Yes, it will not interfere with the other pulses at the sampling, at their sampling instants. The pulse that we transmit at time 0 will not affect or will make no contribution to the pulse that is transmitted at t equal to t_0 , it is t_0 here at the sampling instant t_0 . Right. There will be intersymbol interference as such, right, they will keep on spilling into each other and adding into each other, at the sampling instance there will be no effect.

Student: But then synchronising will be very crucial.

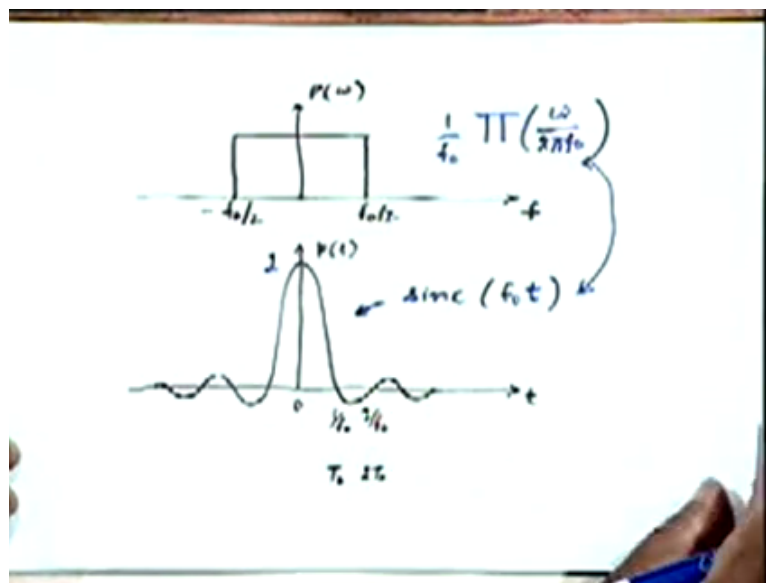
Professor: That is very important. That is crucial anyway, in digital communication that is crucial anyway but yes this is an important point which one has to worry about because a small timing error will destroy the 0 crossing property, right. It is a very important property you have made and we will return to this point soon enough, right. We will see that that point also matters to our choice of pulse shape, we will soon see that.

Student: Do you have internal policy of band limiting?

Professor: That is right, see that is the point, we are saying that the pulse may be band limited and therefore stretched time, but we do not care as long as spread out pulse has this property, right. Okay. Can we proceed further? Now let us look at some questions, maybe let me ask you this question. Suppose I ask you how many such pulse shapes exist, what will be the answer?

Student: Infinite.

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Professor: Infinite number of pulses because what we are saying that there is, you want to pass, find a band limited curve which passes through this point and which passes through all these points, right. And you can think of infinite number of functions which will do that for you, right. Suppose i ask you, the next question is supposed i ask you what is the smallest bandwidth pulse, such that... right, very good. Smallest bandwidth pulse with bandwidth 1 by t_0 , right, which will pass through this function, it is the sinc pulse, very good. The smallest bandwidth function which will pass through these points is the sinc function, right.

So now its extent is infinite, we are using an infinite, infinite pulse duration. But we have ensured that it is band limited strictly to, what is the band limitation, band interval? Not 1 by t_0 it is of the, the smallest bandwidth required is not 1 by 0, it is 1 by $2 t_0$, in fact that is the correct answer, sorry i made a mistake, 1 by $2 t_0$. Because the sinc function corresponding to that will have... we call this frequency 1 by $2 t_0$ as f_0 by 2, right, this is $- f_0$ by 2, okay. We have drawn that better here, you can look at it again.

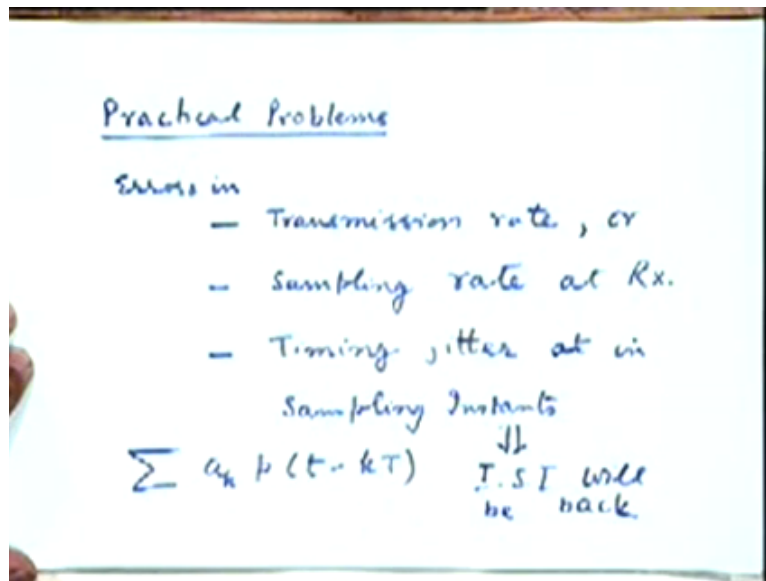
If you have the spectral shape like this, $p(\omega)$ shaped like this, then you have a regular 0 crossing every 1 by f_0 seconds or every t_0 seconds. So that is the kind of pulse shape we need to use, right. And we already have one answer as to band limited pulse which will have this property, namely the sinc pulse. Not only it will have this property, it has a very small required bandwidth. It is, in fact the since it is the smallest bandwidth pulse which will have this property, we say that the minimum bandwidth required to transmit data at the rate of f_0 is f_0 by 2.

This is where the minimum comes from which I talked about a number of times in the previous lecture, right. The smallest bandwidth required to transmit data at the rate of f_0 per seconds, f_0 bits per seconds or f_0 symbols per seconds is $f_0/2$ hertz, right. And the way to do this is to use a sinc pulse for transmitting the data. Of course we must remember that the pulse shape you are referring to here does not refer necessarily to the transmitted pulse shape. In fact that is not important, what you really transmit, what is important is the combined effect of transmitting pulse shaping filter, the channel filter and the receiver filter should look like this, this is what is really important, right. The actual pulse shape that goes onto the channel is not that important, right. All right.

So is that all or there are problems here? Let us just before coming to that, note that this function is $\text{sinc}(f_0 t)$, right, I am assuming this amplitude here is one. And this function here, which is a Fourier transform of this is 1 by f_0 or t_0 rectangular function, well in f domain it is f_0 , in ω the domain it is 2 by f_0 , right and these are Fourier transform pairs. And such a pulse shape can be generated by passing an impulse through a filter of this kind, right, all that is very clear. That is talk about some practical problems with this pulse shape.

One problem we have already been pointed out by our friend and that is for small timing error which will be due to maybe the transmission rate is slightly changed due to the instabilities in the clock, maybe the sampling rate at the receiver is slightly changed due to again instabilities of the clock. Or maybe the timing extraction circuit has some timing jitter associated with it, after all no timing extraction circuit will be perfect, right. Noise will affect the performance of even this timing extraction circuits, right. So if that is the case, the points of sampling will change with respect to the 0 crossings, right.

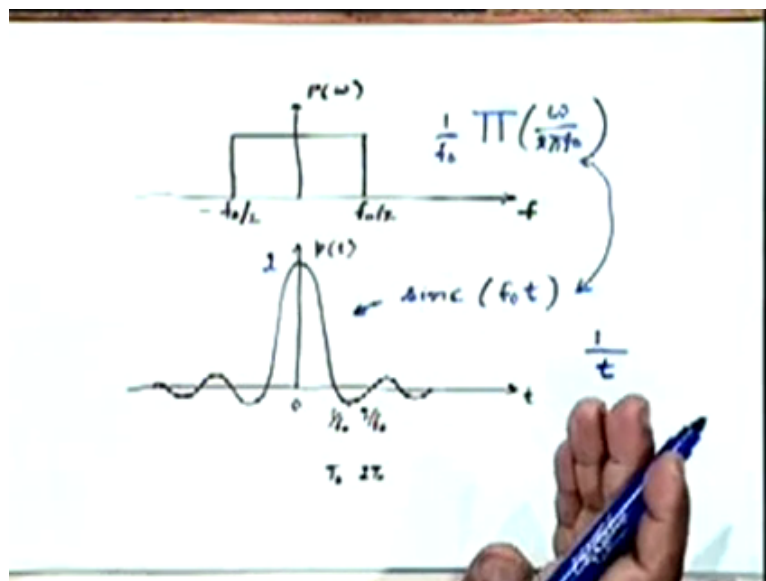
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Now that is going to create trouble. We will talk about that. So the practical problems that you must worry about are errors in maybe the transmission rate or maybe errors in your sampling rate at the receiver, right. Or there may be a timing jitter, a small timing jitter, that is your recovered clock at the receiver does not have fixed timing intervals from one to the next but there is a small jitter in it at the sampling instance, in sampling instance. Now in any of these situations we can again back to the same problem which he wanted to avoid, namely intersymbol interference, is that clear.

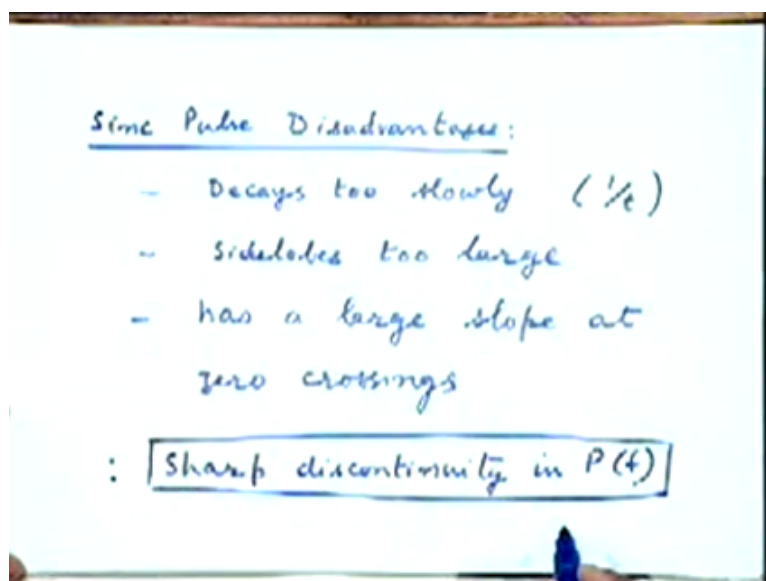
Because this summation $a_k p(t - kT)$, again at the sampling point is altogether pulses are not being looked at as being 0, they will contribute to your sample values here, right. And they will again introduce intersymbol interference. So the net effect is that any of these situations, ISI will be back, right, intersymbol interference will reappear because of these problems. Now particularly the sinc pulse is bad for this, from this point of view for various reasons. One is this function, it decays to slowly for us, right. So if the function decays very fast, a slight timing error will not cause that much of timing, will not cause that much of intersymbol interference if the decay is very fast.

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What is the rate at which it decays asymptotically with respect to time? Is it important? It is $\sin x$ by x . So asymptotically it decays as $1/t$, right, so that is a bit too slow. In fact you may know that the main lobe of this is only 13 dB below, sorry, side lobe, the 1st side lobe is only 13 dB below the main lobe value. If you take the ratio of this to the peak value, it is only 13 dB in magnitude, which is not much. That means a small timing error can have the potential for large ISI associated with it. Number one, number two the slope of the 0 crossings is quite large, right.

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So and what will that mean? That means even small timing errors will cause considerable shift from the 0 value and therefore considerable amount of ISI, considerable

amount of intersymbol interference. Alright. So let us notice these things. Sinc pulse disadvantages. One is that decays too slowly, right, only as $1/t$ asymptotically. Side lobes are too large, particularly the 1st side lobe is very large, right. And 3rd is, it has a large slope at 0 crossings, right. All of which are bad things for us, as far as robustness to timing errors is concerned.

So this is the basic pulse shape which satisfies the nyquist requirement that it wanted, but it is not robust to timing errors, right, this pulse shape is not robust to timing errors.

Student: () (48:37) a large slope matters.

Professor: All right, very easy. If you have a large slope at the 0 crossings, what it means is if instead of sampling here you end up sampling here, right. Because there is a large slope, the difference from 0 will be large, right. Instead of sampling here you end up sampling here, right. Because there is a large slope at that point, that means the buildup is large, buildup of a nonzero value is very fast, okay. So we prefer to have a small slope at 0 crossings as possible. Okay. What is the reason for all these things, in the sinc pulse? Why does a sinc pulse have these bad properties? The reason is if you look at the corresponding fourier transform of the spectrum, it has a sudden, in fact a very sharp or abrupt change from a nonzero value to a 0 value.

Any function which is, which is very very sharp in one domain will have, will exhibit such oscillatory behavior and such bad properties in the other domain, right. So the answer therefore lies in making the fall gradual rather than too sudden.

Student: You said the decay is too small.

Professor: That is right, decay of the pulse shape, pulse value.

Student: () (50:18).

Professor: I did not get your question.

Student: () (50:24).

Professor: So what is your question?

Student: () (50:49).

Professor: I thought you are talking about decay, what are you talking about?

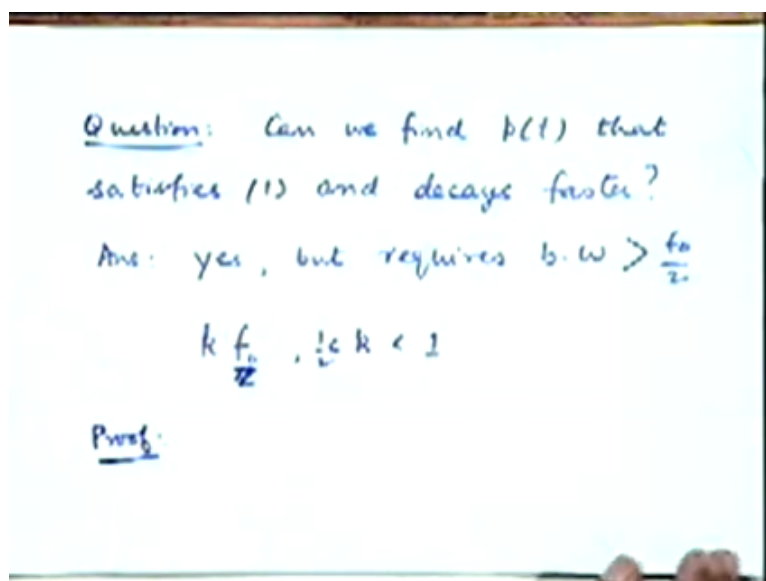
Student: (0)(50:52).

Professor: Decay, asymptotic decay. If the asymptotic decay is fast, that means its ability to interfere with pulses which are removed from it will become small, very fast, if on the other hand if the asymptotic rate of decay is slow, it will have considerable amount of side lobes, even with pulses, at location of pulses maybe a few symbols away from it, right. So its potential to cause intersymbol interference over a large transmission, over large intervals increases if the data rate is slow, that is the significance of slow decay. Alright.

So therefore we will like to have a large decay rates to that its potential, if it all it has the potential to cause intersymbol interference, it is limited to only a few symbols around it and not to many symbols removed far away from it, okay. That is a significant of that point. Now anyway, now turning back to the disadvantages of the pulse shape, we note that all these disadvantages are essentially contributed by the fact that it has a sharp discontinuity in the frequency domain, in $p(\omega)$ or $p(r)$, right.

So if we can get rid of the sharp discontinuity and also ensure that whatever we do, we do in such a manner that 0 crossings are maintained, then they are perhaps in business, right. We may of course have to increase the bandwidth, is not it. Because this was the smallest bandwidth pulse which could satisfies that requirement. Therefore any other shape which has a gradual cut-off rather than an sharp cut-off is the system which is going to have a larger bandwidth. That may introduce a better reliability in terms of timing errors, right, robustness to timing errors.

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So the question therefore you need to look at is, can you find a pulse shape $p(t)$ that satisfies the Nyquist criterion, it satisfies the equation which says $p(t)$ should be equal to 1. We call that equation as equation number-one, this particular equation. Right. We call this equation as equation number-one, okay. Which satisfies this equation and decays faster, right, as all the other nice properties that we want. The answer is yes but at the cost of some bandwidth, right. It requires bandwidth which is greater than the minimum $f_0/2$, right.

Typically what will work is the bandwidth value $k f_0$, $k f_0/2$ where k lies between, I am sorry, that means $k f_0$ and k lies between half and 1. Let us prove that we can do such a thing. Can we direct our discussion this way, it will be better. Yes, speak out your question.

Student: (())(55:00).

Professor: Exponential pulse, let us have your suggestion, whatever it is.

Student: You could multiply the sinc with something to make it decay exponentially.

Professor: That said, well, exponential decay will be bit too much but at least polynomial decay should be possible, right and that is what we will see is possible. That is $1/t^n$ rather than $1/t$, right. Which is what you want? We will try to do that, okay, just wait. So let us look at the proof of this statement that we have, okay, I think we should take that up next line, we are just finishing in time. So we will see a non-sinc pulse, that is one which is not a sinc function satisfying this requirement and having slightly larger bandwidth than this. Thank you very much.