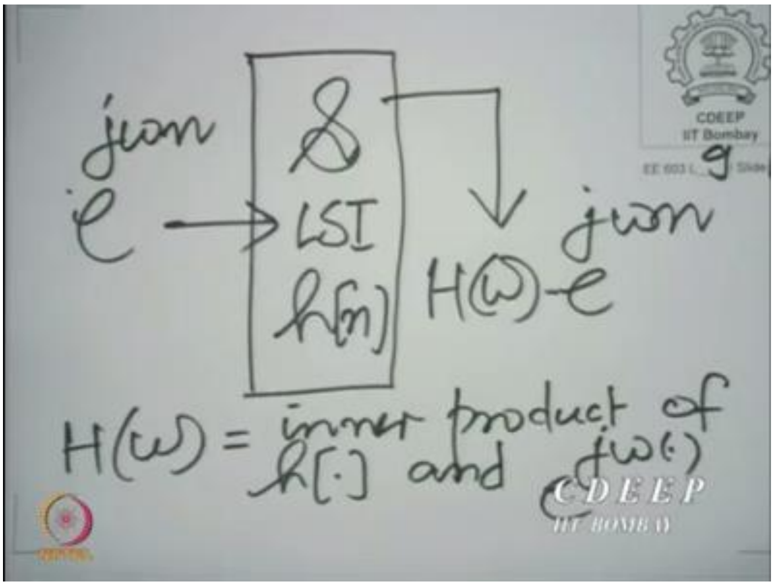
Digital Signal Processing and its Applications
Professor Vikram M. Gadre
Department of Electrical Engineering
Indian Institute of Technology, Bombay
Lecture No. 09 D
Interpretation of Frequency Response as Eigenvalues

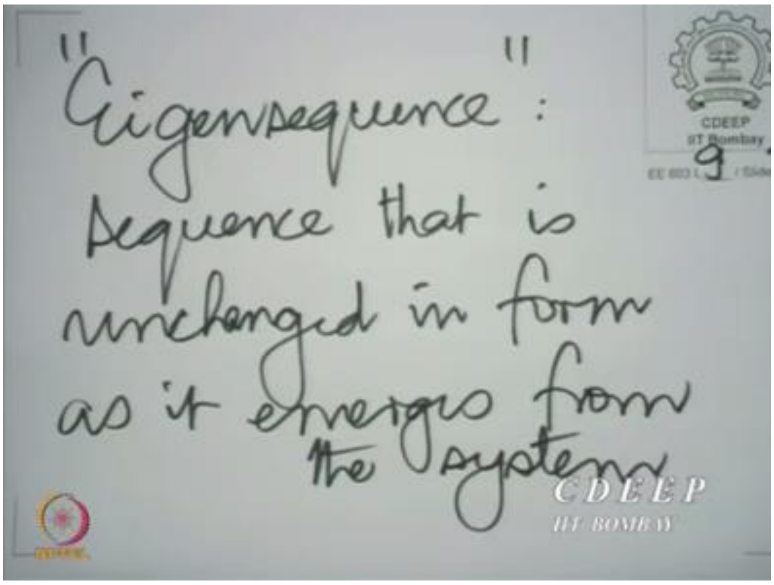
(Refer Slide Time: 00:19)



So, in fact here another way of understanding this is through the notion of eigenvalues and eigen vectors. You see what are we saying in effect? We are saying here that when we put the sequence into the system what comes out is the same sequence multiplied by a constant. Now, this is not going to happen for all kinds of sequences. If you have a sequence which is square wave like in nature it is not going to happen for LSI system in general.

But for complex exponential this happens when you put in a rotating phasor into the LSI system with impulse response h[n] out comes the same rotating phasor, but multiplied by a constant.

(Refer Slide Time: 01:27)



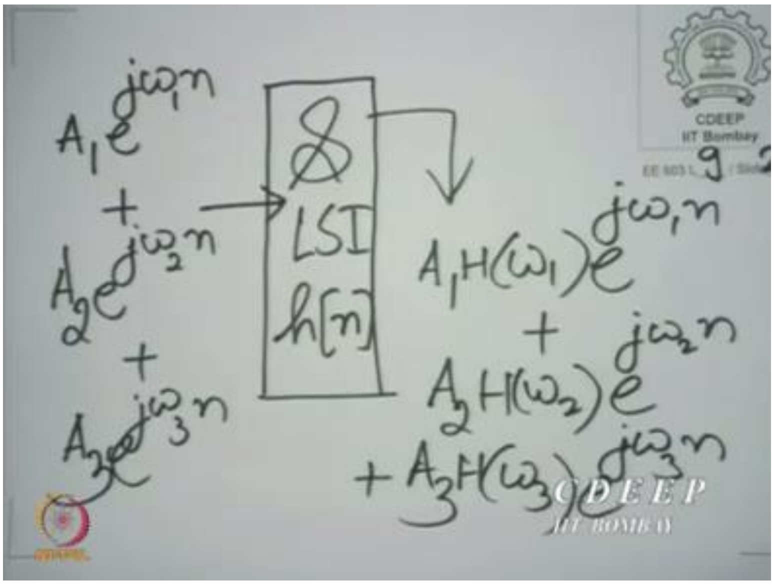
Now, such a sequence which goes into a system and emerges as the same sequence, but multiplied by a constant is called as "eigen sequence" of the system. Sequence that goes in that is unchanged in form as it emerges. It is like saying that the action of the LSI system is decoupled for each of these sequences. We understand this better if we can do one more thing.

Suppose by whatever means now you could express any sequence x[n] in terms of these impulse sequences $\delta[n-k]$. So, you wrote any sequence x[n] as $\sum_k \quad x(k)\delta[n-k]$. So, you were able to express any sequence in terms of the unit impulses. Now, suppose by whatever mechanism we are able to express any sequence as a linear combination of such rotating phasors.

Rotating with different angular velocities then what we are saying here is, if I take any one of those phasors that comprise the input, the response of the system to that component of the input is only the same phasor multiplied by constant. See if I decompose the input along each of these phasors so to speak for different ω, then what I am saying is the response to the phasor at a particular angle of frequency ωhas nothing to do with the phasor at some other ω.

They are all decoupled, they can be treated separately; slightly deeper issue but we will understand this better as we go along. So, in other words what we are saying is, let me try and bring this little more, it takes a little time to absorb this.

(Refer Slide Time: 03:50)

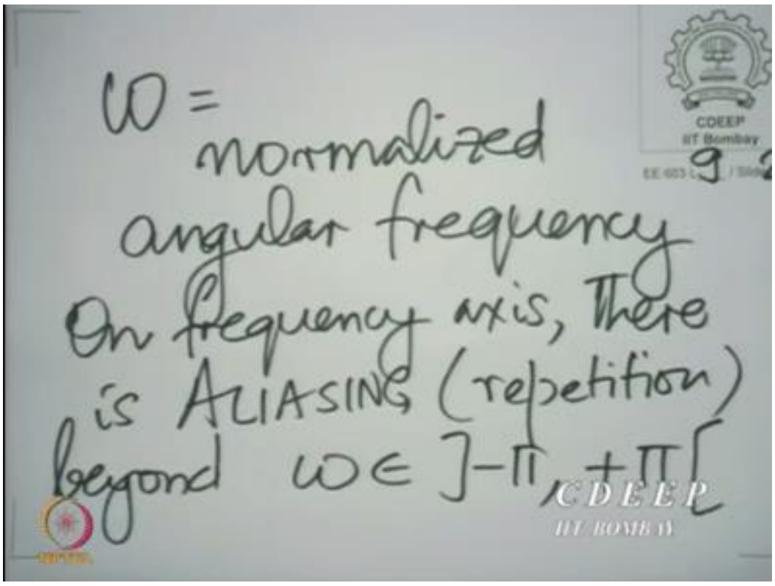


What we are saying is take the same system S LSI h[n]. Suppose instead of giving $e^{j\omega n}$ you gave $A_1 e^{j\omega_1 n} + A_2 e^{j\omega_2 n} + A_3\, e^{j\omega_3 n}$ output comes to be$A_1 H_1(\omega_1) e^{j\omega_1 n} + A_2 H_2(\omega_2) e^{j\omega_2 n} + A_3 H_3(\omega_3) e^{j\omega_3 n}$.

This is not all difficult to show from the property of linearity or additivity and the property that when put in $e^{j\omega n}$ out comes $e^{j\omega n} H(\omega)$. So what we are saying is the action of the LSI system on each of these components is decoupled. I do not have to worry about what $A_2$and A_3 are when I am calculating the output to $A_1 e^{j\omega n}$.

When I am calculating the output to $A_2 e^{j\omega n}$, I do not need to worry what A_1 and A_3 are. They are decoupling. Now, carrying this argument to its limit. Here I am taking a very specific input which has only three such $e^{j\omega n}$ only three such frequencies. Suppose, I use a continuum I use all the frequencies $\omega$from 0 to 2π or all the frequencies from -π to π. Incidentally, now we make a remark about what range of values ω can reasonably take?

(Refer Slide Time: 05:52)

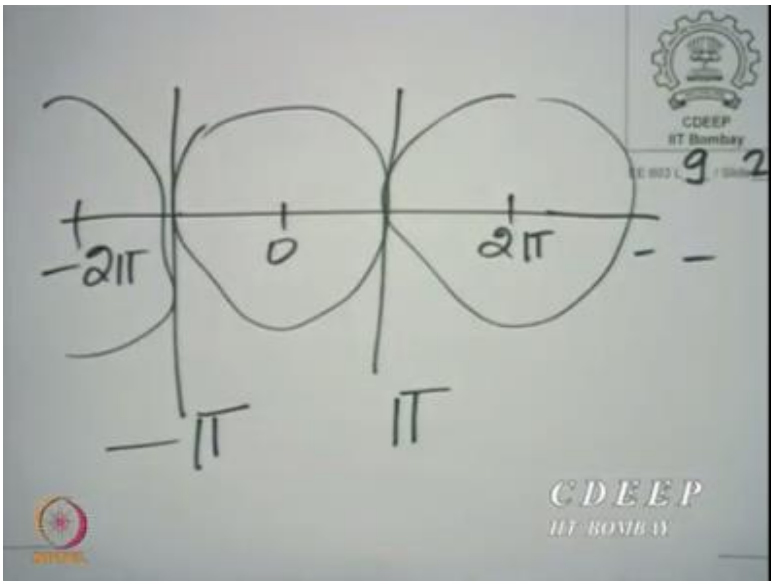


$\omega$is the normalized angular frequency. Now, on the frequency axis there is aliasing or there is repetition beyond ω between -$\pi$and +π. You see the reasoning for that is very simple. What does ω equal to 2π corresponds to? ωequal to 2π corresponds to sampling frequency and we have agreed that every sine wave is comprised of two oppositely rotating phases.

If a sine wave has frequency ω it comprises of two phasors, one rotating clockwise with frequency ω and one rotating anticlockwise with frequency ω. In other words, if you take both to be in the clockwise or anti clockwise direction one with frequency ω and one with frequency $-\omega$. Two phasors come together to form a sine wave. So, on the phase of angular frequency axis each pair of frequencies ω and -ω come together to form a sine wave.

And obviously their magnitudes the two phases have to have the same magnitude and opposite starting angle of opposite phase. Now, we've seen that when you sample you take the original set of frequencies or original frequency axis whatever it is, shifted by every multiple of the sampling frequency and these shifts are added. On this normalized scale the sampling frequency is 2π. So, when you take the original spectrum shift for every multiple of $2\pi$and add up the shifted versions.

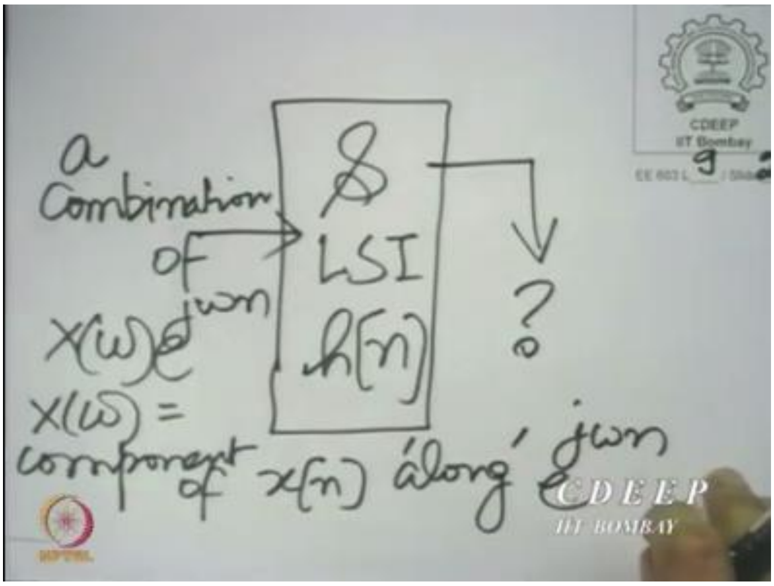
(Refer Slide Time: 08:21)



Let us show that graphically. We are going to shift this by every multiple of 2π and you are going to add them. Now, obviously if you do not want these shifts to overlap this must remain between -π and $\pi$and of course that is also obvious from what we see in the sampling theorem. The maximum component that you have in the original signal should not be more than half the sampling frequency.

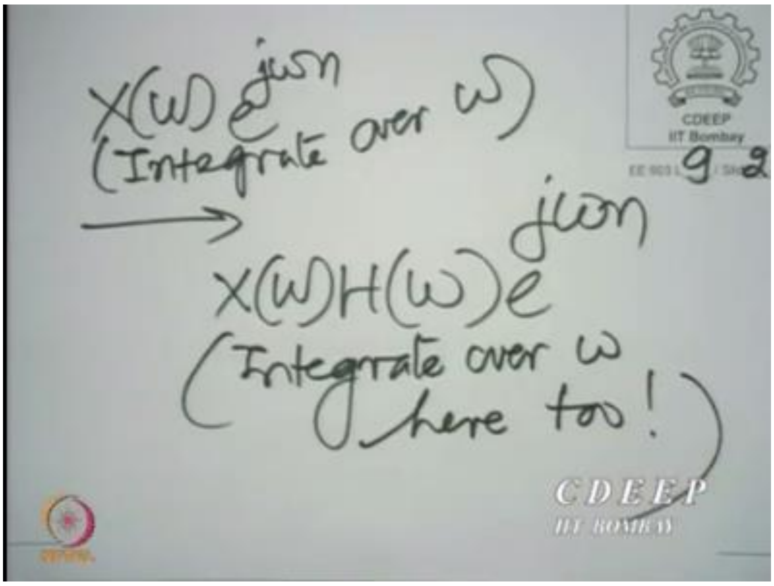
That means the so called unique ω's that we can deal with are only between -π and π. So when we take any input x[n] and ask whether it can be expressed as a linear combination of $e^{j\omega n}$ we need only to worry about the ω's going from -π to π. Of course the frequency axis is continuous so I am not going to be able to use a summation now. I need to integrate the limit of summation as the variable of summation becomes continuous even integral.

(Refer Slide Time: 09:47)



So, what we are saying is suppose, I mean I am trying to motivate the whole idea, suppose I take the same LSI system once again, give to it a combination of $X(\omega)e^{j\omega n}$. You know what I mean by X(ω)? X(ω) is the components of x[n] along $e^{j\omega n}$. So, what I am saying is suppose we are able to decompose; just like you decompose the impulse response, suppose you are also able to decompose the input along $e^{j\omega n}$ and you did this for all the ω's going from -π to +π then it means that since -π to +π is exhaustive putting those components back should give you back x[n]. So, what we are saying is see what we are trying to ask is what comes out of the LSI system and we are seeing in particular here when you take $X(\omega)e^{j\omega n}$ what will come out.

(Refer Slide Time: 11:23)



So, $X(\omega)e^{j\omega n}$ is going to give us $X(\omega)H(\omega)e^{j\omega n}$ and therefore if you integrate over ω on the LHS you can also integrate over ω on the RHS and that is because of the property of linearity. If for each ω I can do this, I can do it for the combination of the ω's. Now, this will leave us with a very interesting property of this inner product.

This inner product of a sequence with a rotating phasor gives us a new domain or the Discrete Time Fourier domain or Discrete Time Fourier Transform Domain. We shall see more of this in the lecture to come and build up in greater depth the whole idea of Discrete Time Fourier Transform in the coming lecture. Thank you.