

Analog Circuits
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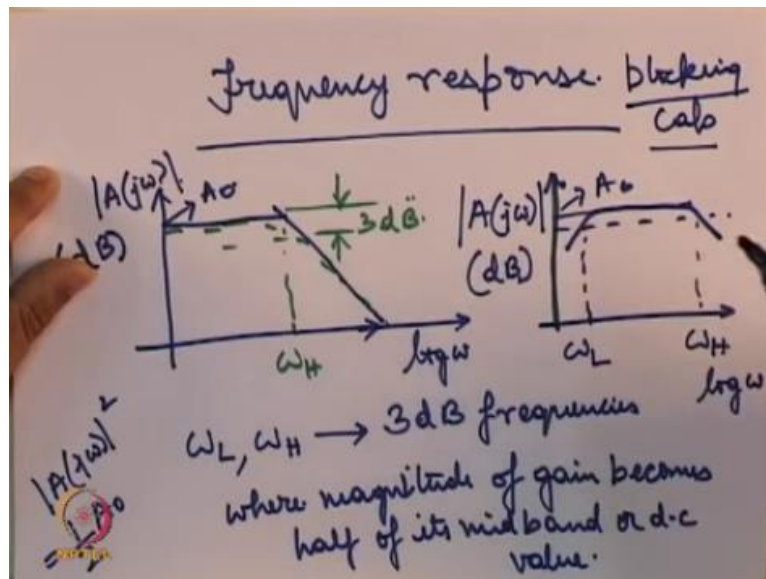
Week -02
Module -04
Frequency Response

Hello, welcome to another module of this course analog circuits, in the past module we had seen what are bode plots, bode plots as we have discussed they are the means to visualize the frequency response of any system not just an opamp or not just any analog circuits for any system in this module. What we are going to do is we are going to see the details of the frequency response of a typical opamp and then try to find a means to analyze what should be the frequency response of a typical circuit.

So, let us see what is the frequency response now, as you know frequency response is an important aspect of any analog circuit now we have studied the opamp and while studying it we had visualized it as a time domain device like as if the signals only change in time but unfortunately that is not the only way only analysis that can be done the reason being as you know any signal can be broken up.

First of all into its frequency components and then any analog circuit like that of an opamp they behave differently at different frequencies and that is why instead of a time domain perspective of the opamp it might be more useful to see the frequency domain perspective like how with variation of frequency the response changes.

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Usually the frequency response of an opamp or for that matter any analog device if we plot the bode plot of the magnitude then it will show, so this is in db it will show a variation like this, so these are the asymptotes and the overall curve will be something like this with of course one frequency Ω_H where the magnitude falls by 3 db sometimes depending on the circuit this response can be slightly modified like this

Here we have two 3 db frequencies whereas here we had only one 3 db frequency here actually we have two 3 db frequencies so we call that lower 3 db frequency as the ω_L and the upper one is ω_H now so this ω_L ω_H there are the 3 db frequencies what is the meaning of a 3 db frequency 3 db frequency means that frequency where magnitude of gain becomes half of its mid band or DC value.

So here this A of $J\omega$ when I say that A of $J\omega$ in db is falling by 3 db it means this A of $J\omega$ magnitude square is becoming half of its mid band value that is why when you take the 20 db of this or the 10 db of this you will find that there is a fall of 3 db from its DC value or its mid value, so this is the A_0 value this is the A_0 now these 2 cases arise depending on as I said at the beginning what kind of circuit you use and usually the frequency response is of this type.

But in some cases especially where at the input where the source is being inputted into the circuit we have something called a blocking capacitor, if a blocking capacitor is present then what it does it does not allow DC components to pass through it and therefore there is a dip

as the frequency decreases, so there is only a mid band gain there is no DC gain or there is neither for very high frequencies also the gain decreases.

We saw that either the response can be of this type or of this type and for this type usually this type of response usually a blocking capacitor is present which does not allow the DC to pass through it, so you know first let us try to find a formula for this omega L first so for that we will study what we call the low frequency response.

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Low Frequency Response.

$$H(s) = \frac{(s + \omega_{z1})(s + \omega_{z2}) \dots (s + \omega_{zn})}{(s + \omega_{p1})(s + \omega_{p2}) \dots (s + \omega_{pn})}$$

When, $s = j\omega \rightarrow \infty$

$$|H(j\omega)| \rightarrow 1$$

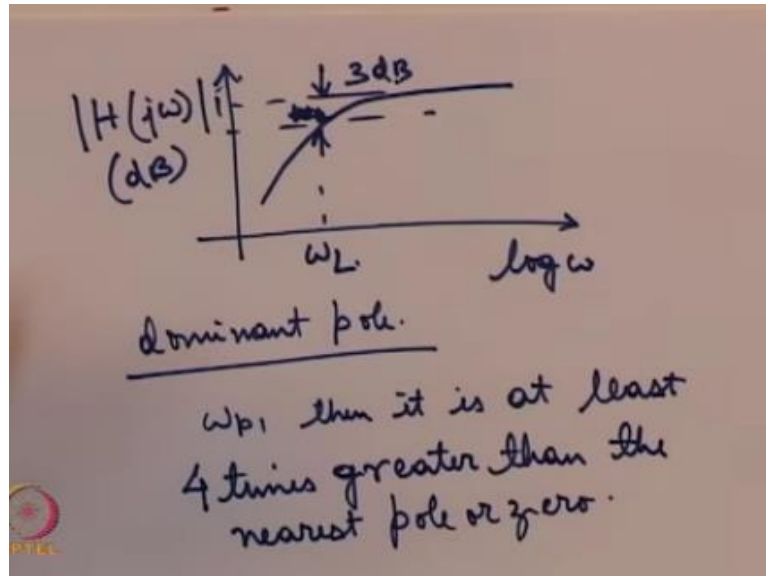
When, $s = 0$

$$|H(j\omega)| = \frac{\omega_{z1} \omega_{z2} \dots \omega_{zn}}{\omega_{p1} \omega_{p2} \dots \omega_{pn}}$$

So for low frequency response the transfer function can be written like of this form as you can see when $S = J \omega$ tends to infinity the magnitude of H of $J \omega$ tends to 1 $S = 0$, so when S is = 0 we have magnitude of H of $J \omega$ given by ok so now by suitably choosing this omega Z's and omega P we can make the value of this magnitude H of $J \omega$ very low so what we will get is a response like this okay.

So what this equation implies or what this graph implies is that for as the frequency increases there is a stable value of H of $J \omega$ magnitude let me draw it nicely so this equation the type of graph that we will get is like this.

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So this is again $H(j\omega)$ in dB this is \log of ω this is 1 and this ω_L sorry this is 3 dB and ω_L is here so at ω_L the value decreases by 3 dB from its stable high frequency value, so this equation we have so just I want you to want to convince yourself that this equation will give satisfies this kind of response okay of course we have to choose our ω_Z and ω_P is properly so that we do see this dip that we are seen but what is interesting is there is only a single ω_L here.

But here it appears that there are many poles and zeros to how so how do we reconcile with the presence of so many poles and zeros this particular graph, since you know while discussing bode plot we have seen that every 0 will increase the slope by 20 dB and every pole will decrease the pole by decrease the slope of this amplitude response by -20 degree, so if that is the case then how are we getting such a smooth curve

So to understand this first of all for a response like this to happen in any system we need to have what we call a dominant pole, so dominant pole is a pole for this low frequency response case this suppose this ω_{p1} is the dominant pole then it is at least 4 times greater than the nearest pole or zero that is what a dominant pole needs, so if such a pole exists then can we find out a formula for ω_L in terms of that dominant pole let us see.

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$$H(s) = \frac{(s + \omega_{z1})(s + \omega_{z2})}{(s + \omega_{p1})(s + \omega_{p2})}$$

$$|H(j\omega)|^2 = \frac{(\omega^2 + \omega_{z1}^2)(\omega^2 + \omega_{z2}^2)}{(\omega^2 + \omega_{p1}^2)(\omega^2 + \omega_{p2}^2)}$$

At $\omega = \omega_L$, $|H(j\omega)|^2 = \frac{1}{2}$.

$$\Rightarrow \frac{1}{2} = \frac{(\omega_L^2 + \omega_{z1}^2)(\omega_L^2 + \omega_{z2}^2)}{(\omega_L^2 + \omega_{p1}^2)(\omega_L^2 + \omega_{p2}^2)}$$

So suppose we have a system where there are only 2 poles and 2 zero's, so H of S is given like this okay so therefore modulus of H of J omega whole square is given by now at this omega = omega L we saw that the magnitude square of H of J omega will be half its stable high frequency value

Therefore we can write this modulus H of J omega square = half at omega = omega L, now from this we get half is = omega L whole square + omega z1 whole square omega L whole square + omega z2 whole square upon omega L whole square + omega P1 square omega L square + omega P2 square which in turn can be written as.

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$$\frac{1}{2} = \frac{1 + \left(\frac{1}{\omega_L^2}\right)(\omega_{z1}^2 + \omega_{z2}^2) + \frac{1}{\omega_L^4} \omega_{z1}^2 \omega_{z2}^2}{1 + \frac{1}{\omega_L^2}(\omega_{p1}^2 + \omega_{p2}^2) + \frac{1}{\omega_L^4} \omega_{p1}^2 \omega_{p2}^2}$$

$$\omega_L \approx \sqrt{\omega_{p1}^2 + \omega_{p2}^2 - 2\omega_{z1}^2 - 2\omega_{z2}^2}$$

Neglected $\frac{1}{\omega_L^4}$ $\Rightarrow \omega_L \approx \omega_{p1}$

$\omega_{p1} \gg \omega_{p2}, \omega_{z1}, \omega_{z2}$

Now then solving for omega L what we get is omega L is approximately = omega p1 square + omega p2 square - 2 omega z1 square - 2 omega z2 square, now here we have neglected 1

upon ω_L for term since this ω_L is much larger than the rest of the poles and zero's.

Now suppose this ω_{p1} is the dominant pole and it is as I said at least four times greater than the near is zero or pole, so ω_{p1} we can write it. It is much greater than ω_{p2} or ω_{z1} or ω_{z2} then what this implies is this ω_L is nearly = ω_{p1} or in other words the lower cut off frequency ω_L is = the dominant pole.

So in this module we covered the concept of the 3 db frequency and also the low frequency response of an open and what the lower cutoff or lower cutoff or lower 3 db frequency should be in terms of the dominant pole. In the next module we shall be covering the high frequency response where we will be deriving a similar expression for the upper 3 db frequency in terms of the dominant pole, thank you.