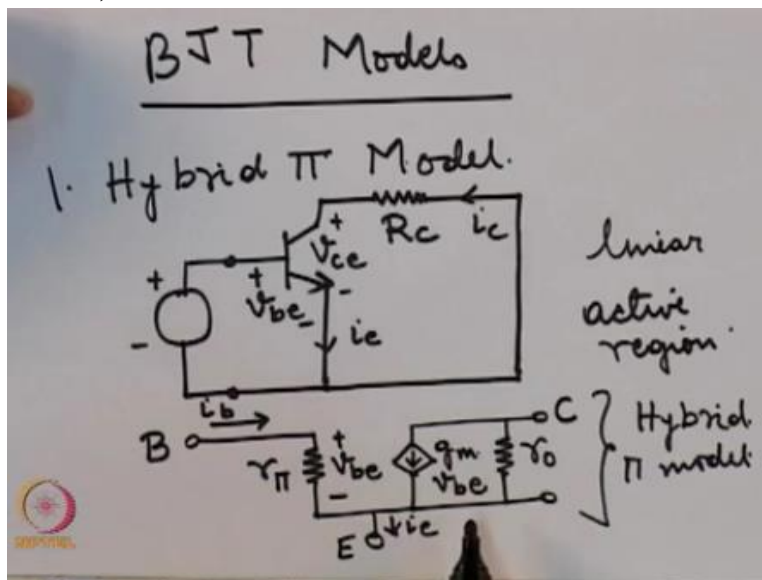


Analog Circuits
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Week - 08
Module - 05
Current Mirror

Hello, welcome to another module of this course analog circuits, so in this module we shall be introducing you to a class of circuits known as current mirrors which are also DC circuits based on the BJT devices but and they serve a very important purpose in providing the required bias currents various circuits, but before we go to the current mirror let me also introduce you to the various small signal BJT models that are used.

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So, there are a number of models for BJT's reported in literature of which 2 are very popular the first one is known as the Hybrid pi model ok, so in this module let us see first consider a BJT which is connected with proper bias voltages and currents, now note in this circuit I am not adding any DC sources I have not added any DC source sources like the v_{cc} or v_{bb} that I mentioned in the previous module instead all the sources here are AC or the actual signal that has to be processed by the BJT ok.

So, the equivalent circuit or equivalent hybrid pi circuit or hybrid pi model for this BJT is given like this, this is the hybrid pi model first thing to note about this model is that this is a linear model even though the BJT is a fundamentally nonlinear device that is the voltages and currents do not exactly have a linear relationship this particular model linearizes the circuit over its active region.

So, this particular model is applicable only in the active region not in the saturation or in the cutoff region, the active region by the way is also known as the linear region of the BJT, because in this region the voltage and currents are somewhat linearly related not strictly but somewhat so once we linearize our BJT then the model we get is something like this, now the important thing is what are these parameters, to understand what are these parameters let us consider the currents and voltages as represented by this model.

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$$\begin{aligned}
 i_e &= i_b + i_c \quad \checkmark \quad I \\
 &= \frac{v_{be} + g_m v_{be}}{r_{\pi}} \\
 &= \frac{v_{be}}{r_{\pi}} (1 + g_m r_{\pi}) \\
 i_e &= i_b (1 + \beta), \quad \boxed{\beta = g_m r_{\pi}} \\
 i_c &= I_s e^{v_{BE}/V_T} \\
 \frac{\partial i_c}{\partial v_{BE}} &= g_m = \frac{I_s e^{v_{BE}/V_T}}{V_T} \\
 &= I_c / V_T
 \end{aligned}$$

So, I have my emitter current I_E , so this is my hybrid pi model the curve current I note that this I_E is the AC current okay not the DC bias currents this is the actual signal current that is present and note also that this hybrid pi mode or any other AC model of a BJT will always exclude the DC sources, because since we have linearized our circuit we assume that the DC circuit the final response of this circuit will be a combination of the DC and AC outputs or the DC or AC currents and voltages at the various nodes, hence because of the linearity property of this BJT at its active region.

We can separate the AC circuit out and just analyze the AC solution of this circuit later on, we can combine this solution with the DC solution and the final output will be the sum of the 2 or the super position of the 2 so ie my emitter current is equal to as I can see off the collector current and the base current, so this is equal to $i_b + i_c$ also note that I am using this small letters the small letters that is small ie's the or small v they represent the AC signal whereas capital V or capital I they represent the DC signals.

So, what is my i_b I can see that i_b is nothing but the v_{be} upon r_{π} okay and i_c is equal to g_m times v_{be} okay here we are for some time ignoring this resistance R_0 okay if we ignore this resistance R_0 assume that the r_0 is infinite and no current is flowing through this then this equation will be true.

So, I can take v_{be} upon r_{π} common ok and that will give me $1 + g_m r_{\pi}$, here this π should have been here like this and this I can say is equal to i_b because v_{be} upon r_{π} is equal to i_b and suppose I write it like this then I get back my original relationship between i_e and i_b , i_e and i_b are related by this equation therefore the beta of this small signal model is given by $g_m r_{\pi}$, so this is the equation relating the traditional beta this beta is irrespective of whether you linearize the circuit or you have an equivalent model beta is always present but this g_m and r_{π} are specific to this model and this is the equation that relate the 2.

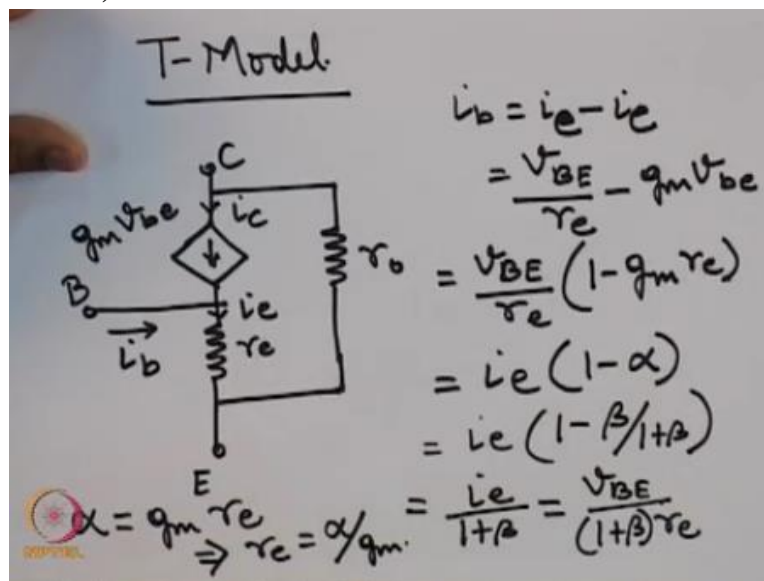
I can further proceed along the same similar lines by considering the equation for I_c , I_c should be equal to I_s times e raised to v_{be} upon V_T here my v is the AC voltage and i_c is the AC voltage if I differentiate this with respect to v_{be} then that will give us the value of g_m because after all g_m is the dynamic trans-conductance that is g_m is equal to the change in i_c due to a change in v_{be} g_m is not equal to simply i_c upon v_{be} this is the DC trans-conductance but the AC trans-conductance is that or the dynamic trans-conductance is the change in i_c due to a change in v_{be} .

So, once I get this value of g_m what is if I now evaluate this derivative then that comes out to be equal to $I_s e$ raised to v_{be} upon V_T this whole upon V_T ok and this is again equal to I_C upon V_T

ok so then g_m is equal to I_C upon V_T this is the formula so these were my 2-unknown's odd for this model one was this r_{π} and the other was g_m .

I found out g_m was given by I_C upon V_T and r_{π} was given by β upon g_m from this equation so first I can find out g_m from this equation substitute it here and I can find out r_{π} so this is how this model is defined now another module that is very commonly used for the active region of a BJT is the T model, so let us see what is the T model.

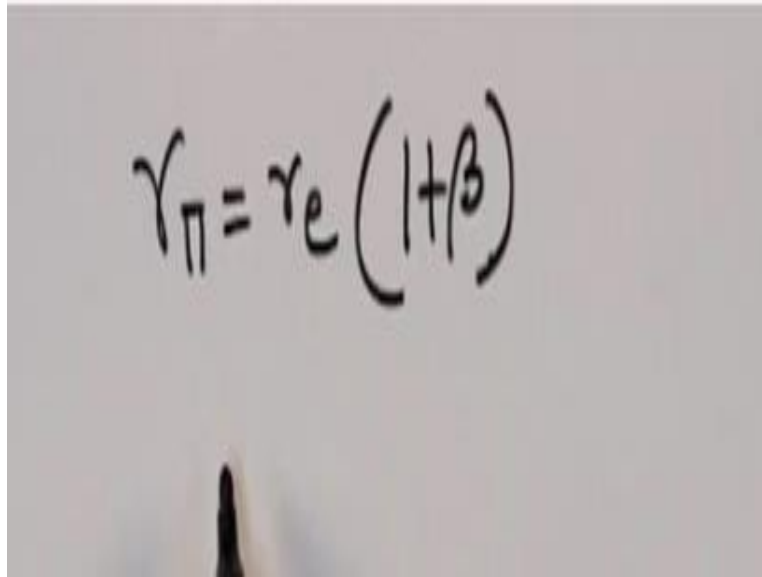
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Now the r_0 that I had mentioned in the previous hybrid pi model is will be defined by this, when we discussed this particular model so here we see that this i_b base current is equal to the difference between the collector and the emitter current or I should say this is equal to $i_e - i_c$ and i_e is given by v_{be} this is my base upon r_e and i_e is given by g_m times v_{be} so this whole comes out to be equal to v_{be} upon r_e multiplied by $1 - g_m r_e$ and this is equal to of the form i_e multiplied $1 - \alpha$ which can be simplified to i_e upon $1 - \beta$ upon $1 + \beta$ which is equal to i_e upon $1 + \beta$ and which is equal to v_{be} upon $1 + \beta$ r_e .

So, here this α my traditional BJT α is equal to $g_m r_e$ which implies r_e is equal to α upon g_m so this how this r_e resistance is defined. Now comparing the value of r_e and r_{π} that I had defined in the for the previous model we can obtain a relation which I leave for you as an assignment to find out.

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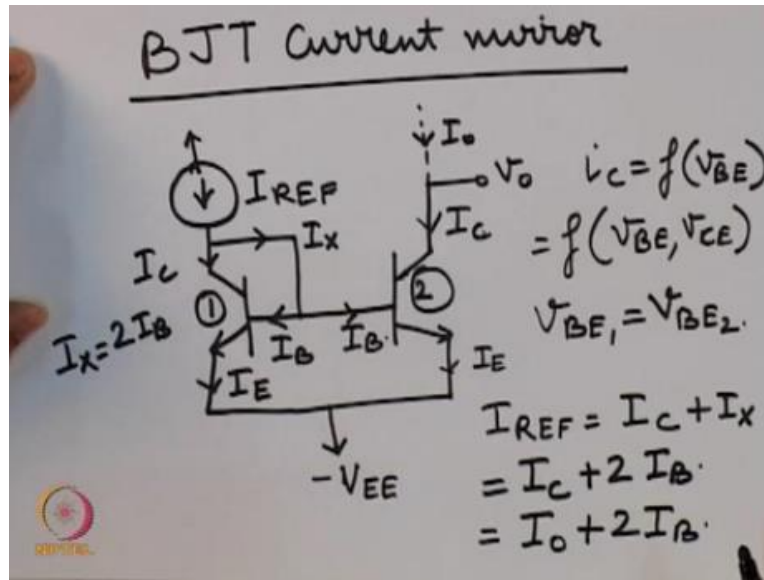
A photograph of a whiteboard with the handwritten equation $r_{\pi} = r_e (1 + \beta)$ written in black marker. The equation is centered on the board, and a black pen is visible at the bottom center, pointing towards the equation.

That r_{π} will be equal to r_e times $1 + \beta$ now once these models are known we can use these models for various AC finding out the AC response of various circuits, for example the common emitter circuit that we had discussed earlier we can find its equivalent model either in terms of this T model or the hybrid pi model here connect the AC source that is been applied to the base of the original circuit and find out the voltage and current at the base and collector junction, so that is how we solve for the AC response of a BJT.

Since time is short and we don't have many modules left so we will I will not be covering any further topics on the AC response of BJT's, but as promised at the beginning of this module I shall be showing you a circuit which is known as a current mirror. Now current mirror is a circuit that replicates a current, so it is a mirror if the word mirror is used because the same current that is applied at one terminal appears across another terminal and it is very commonly used in various circuits.

For example, if we want identical operation of 2 amplifiers then we cannot just simply design the circuit to ensure that the equal currents flow through them. We need to actually ensure it using current mirrors so we connect 2 branches of the current mirror to the 2 amplifiers and since they are in mirror there is a mirroring effect so the currents will definitely be the same in both the amplifiers so let us see how this current mirror is constructed.

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The circuit for this mirror is given like this now at the beginning of this at the beginning when we this first started discussing BJT's I had mentioned that the current i_c is a function of only v_{be} this is true to some extent but actually in reality i_c is also a function of V_C , however this dependence is very tenuous not V_C doesn't influence or variation in V_C doesn't change i_c to that extent to which i_c is changed due to changes in V_{BE} and we shall be assuming that i_c or any of the other currents of a BJT are only functions of V_{BE} .

If that is the case then from just from the way these 2 NPN transistors these 2 NPN BJT's have been connected we can see that the V_{BE} voltage will be same for both so V_{BE1} , suppose I call this transistor 1 this as transistor 2. V_{BE1} will be equal to V_{BE2} , hence we can say that all the currents flowing through all the transistors the respective currents in the various terminals of both the transistors will be the same, so that's why we have the same current I_B flowing through the base of both the transistors of course these 2 transistors are themselves identical okay and so will be the emitter currents and also the collector currents.

So, we have we can then write this current I_{REF} as equal to $I_C + I_X$ and I_X as we can see is equal to twice of I_B where I_B is the base current going through the base of both the transistors so I can write this as equal to $I_C + 2I_B$ ok now I_C should be equal to the collector current of this transistor which in turn is equal to I_O , so I can replace I_C with I_O like this I_B in turn is a function of I_C and given by.

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$$I_B = \frac{I_0}{\beta}$$
$$\Rightarrow I_{REF} = I_0 + 2 \frac{I_0}{\beta}$$
$$= I_0 \left(1 + \frac{2}{\beta} \right)$$
$$\Rightarrow \frac{I_0}{I_{REF}} = \frac{1}{\left(1 + \frac{2}{\beta} \right)}$$

I_B is equal to I_0 upon β now this implies I_{REF} is equal to $I_0 +$ twice of I_0 upon β which is equal to I_0 in to $1 + 2$ upon β so this implies that I_0 upon I_{REF} is equal to 1 upon $1 + 2$ upon β as we can see I_0 and I_{REF} are not totally same if β is quite high of course then I_0 will be equal to I_{REF} but if β is still the time β is not very high there will be a small difference between I_0 and I_{REF} and that is one problem with this kind of current mirror true that I_{REF} and I_0 match each other very closely, since β is usually quite high be we have some other designs where this closeness of I_{REF} and I_0 is even more.

So, in this module we covered some topics on transistor modeling we discussed about the 2 common AC models used in the active region of the BJT that is the hybrid pi model and the T model and we also discussed a simple current mirror using a BJT, now this is the last module of this course that I shall be taking, after this module the TS for this course will be taking some tutorial classes.

So, those will be the subsequent modules of this course it was really an enjoyable experience discussing the various topics of analog circuit design with you throughout the past few weeks and I hope you it you found it and enriching experience.

I hope you got some value out of the discussions with we had and please send me an email if you have any doubts, my email ID is please contact me if you have any questions, I shall be very

happy to provide answers to your questions also please let me know if you felt that some more topics needed to be covered or something which we had to cover in more detail, of course we have a time constraint of 20 hours for this course and that's why I try to squeeze as many topics I could within this time that we had but I would definitely if I receive some emails from you regarding some particular topic that you are interested in we can discuss on that, thank you.