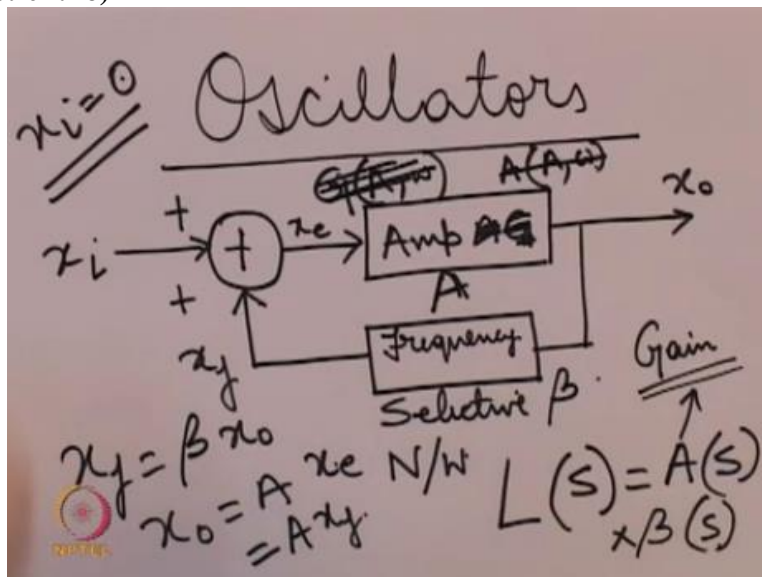


Analog Circuits
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Week 07
Module 01
Oscillators

Hello, welcome to another module of this course analog circuits, we are now in the week 7 of this course, So in the past week we had been discussing about some non linear circuits or non linear application of opamps, now in this module we will be talking about another set of non linear circuits which are known as oscillators.

Now oscillators we have already briefly discussed while discussing feedback we call in the past a few weeks ago that under certain circumstances a feedback loop can itself be oscillatory, so here we want a system to be oscillatory, so we already know the Barkhausen and criteria the criteria for making a system oscillatory will just go into a little bit more details about the phase and amplitude relationships of the loop gain and then after that we will be seeing some practical examples of oscillator circuits using opamp.

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So let us first go back to that feedback loop which we had discussed while discussing opamps, now note here I am choosing positive feedback as opposed to negative feedback okay, so this is my feedback loop this we had already discussed, now this is an amplifier whose gain is A this is the frequency selective network whose gain is B beta.

Now we had already discussed that beta is only dependent on the frequency whereas amplifier with a gain A is mostly a function of the amplitude and to some extent it is a function of the frequency also so maybe I should write the instead of using A I should use it on G where G represents the gain let me write the gain as G.

So this gain is dependent on both the amplitude and to a very small extent to the frequency also now the loop gain we call was the product of the forward path gain ok and the feedback gain now here I am reverting to this A here I said that the gain is G but here I am reverting to A as the terminology for gain, since that is more easier to understand while doing the derivations that we are going to do next.

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Characteristic Equation

$$1 - L(s) = 0$$

At $f = f_0$ $|L(s)| = 1, \angle L(s) = 0^\circ$

$$L(j\omega_0) = A(j\omega_0)\beta(j\omega_0)$$

→ Barkhausen criteria

So characteristic equation for this system is given by, So once again the characteristic equation is the denominator polynomial of the transfer function of the system and we had seen earlier that for positive feedback the characteristic equation is given like this, now at a particular frequency say F_0 this loop gain the magnitude of this loop gain is = unity so at $F = F_0$.

So then we can write it in terms of the loop gain as $L(j\omega_0) = A(j\omega_0)\beta(j\omega_0)$ here, even though I said that A is a function of the amplitude here we are concentrating more on the frequency part because we are trying to find out a solution at what frequency this system will oscillate.

Now we know that at $\omega = 0$ this is the value of the loop gain where $\omega = 0$ is the frequency of oscillation and we know that at the $\omega = 0$ the magnitude of the loop gain should be $= 1$ and the phase of this L of S should be $= 0$ degree ok so these are the 2 criteria for oscillation which we had already discussed and we call that this is also known as Barkhausen criteria.

Now going back to my block diagram for a moment (Refer Slide Time: 05:46) what is my X of F , X of F is $= \beta$ times the output okay and what is X of 0 , X of 0 is $= A$ times X of e if I call the signal as X of e then X of 0 is A times X of e now for an oscillatory system the input is absent so input is absent means X_0 is produced without any input X_i may be 0 so $X_i = 0$ if $X_i = 0$ then I can write X_e as simply $= A$ of X_F .

So we have X of $F = \beta X$ of 0 and the X of 0 itself is $= A$ times X of F okay so I will just revert back to the terminology A the gain of this amplifier is A , so X of F is $= \beta$ times X of 0 and X of 0 itself is A times X of F which means so from this what do we get from these 2 equations.

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$$A\beta = 1$$

$$X_y = \beta X_o$$

$$X_o = A X_y = A\beta X_o$$

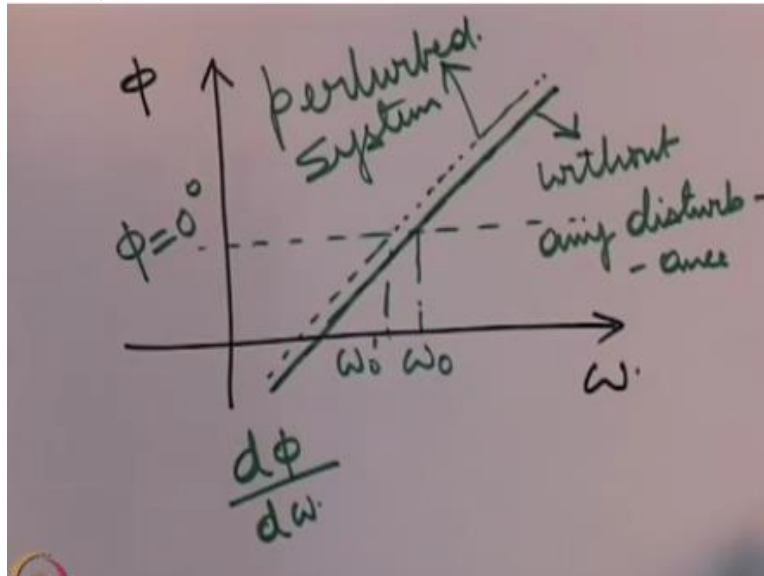
$$|A\beta| = 1 \quad \phi = \angle A\beta$$

$$\angle A\beta = 1$$

What we conclude is that this $A\beta$ should be $= 1$, why let us once again write it down X of F is $= \beta$ times X of 0 and X of 0 is $= A$ times X of F , so we can write this as A times βX_o equating the this with this we get this condition from which of course we get the other way of expressing the Barkhausen criteria like this okay now suppose if Φ represents the argument of

A beta or the loop gain then how should this Phi vary with frequency so as to give a stable oscillation.

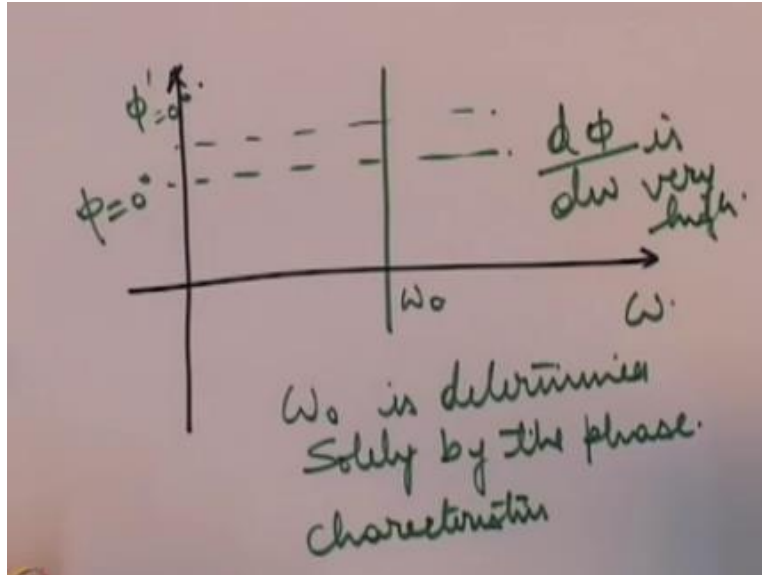
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So if I draw a graph with frequency on the X-axis and I argument Phi on the Y axis ok, now so suppose this curve let me make it a solid curve this solid curve represents the variation of Phi with omega now if suppose I try to find out this dPhi the variation of the phase with omega and say there is a small perturbation to the oscillation because of which the new curve is like this okay so this is the curve without any disturbance or perturbation and suppose I disturb the circuit of it as a result of which the new curve for ϕ vs. omega becomes like this I call this perturbed system, So this dotted line dashed line is for the perturbed system.

Now what do we observe here for this particular new for this perturbed system, the new frequency of oscillation is here suppose this is my $\phi = 0$ degree point okay, so we see that if the slope of slope dPhi upon d omega is finite then upon the system getting a little disturbed or perturb the frequency of oscillation will shift a bit and if it shifts a bit then that is a problem isn't it on the other hand suppose my curve was one where this slope was between the Phi and the omega that is dPhi upon D omega was very high.

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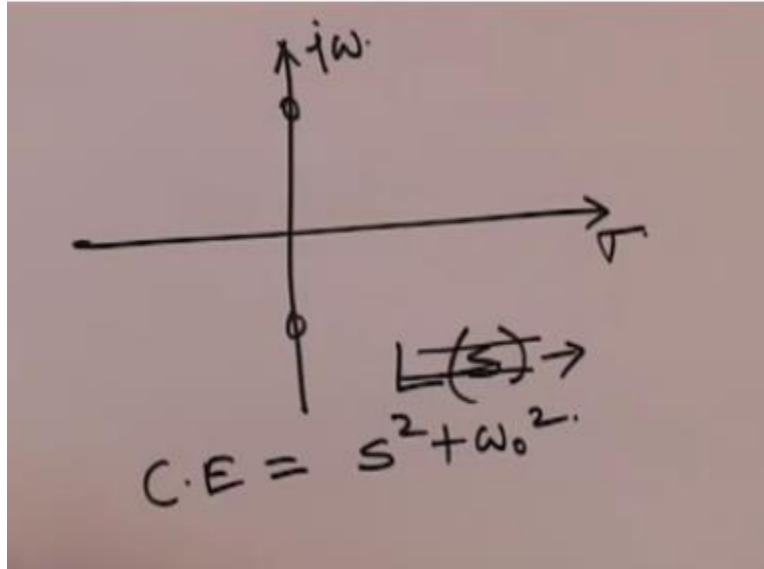


So suppose my curve was like this were okay, it was like this so this is omega this is Phi now even if there is small perturbation the you know the frequency of oscillation will continue the same the reason being that even if suppose since my $d\phi$ upon D omega is very high the shift in this curve due to any perturbation will be minimum also one more thing that we note is that this omega 0 is determined solely by the by the phase characteristics.

So in this case suppose this was my initial $\phi = 0$ degree okay now this is for the disturbed system this is my $\phi = 0$ degree still so even though there is shift in the $\phi = 0$ degree point but the frequency of oscillation because the slope $d\phi$ upon D omega is very high the frequency of oscillation remains the same so in other words for good stability of oscillation $d\phi$ upon D omega should be very high okay.

There is one other aspect that when a system is oscillation what should be the position of the poles from the position of poles and the root locus theory that we studied earlier we saw that for a stable system the poles are always on the left half of the S plane for an unstable system the roots poles are always on the right half of the S plane and for an oscillatory system the poles will be on the J omega axis okay.

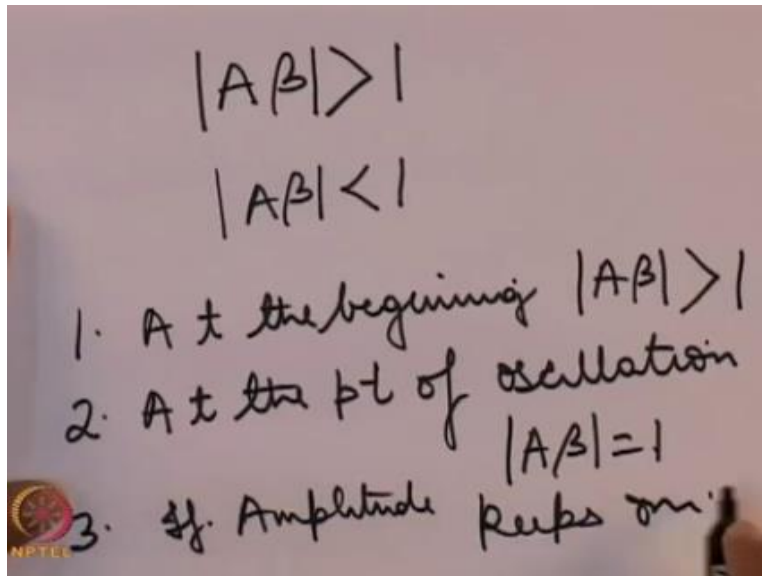
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So then we can say that since LS determines the characteristic equations of the system, so this LS itself should be of the form or the characteristic equation of the system of an oscillatory system should be of the form this one at the frequency of oscillation, however one thing that we have to note here is that even though at the point of oscillation or at the frequency of oscillation the poles of the oscillator are on the $j\omega$ axis in order to start the oscillation where should the poles of the system be you know as you can understand in order to start the oscillation means to make the system unstable at the beginning.

So if you have to make a system unstable the beginning and the initially the poles should be on the right half of the s plane, once the system reaches the point of oscillation the poles should shift to the imaginary axis okay so one way to do that to ensure that an oscillation is started successfully or that initially the poles are indeed on the right half of the s plane.

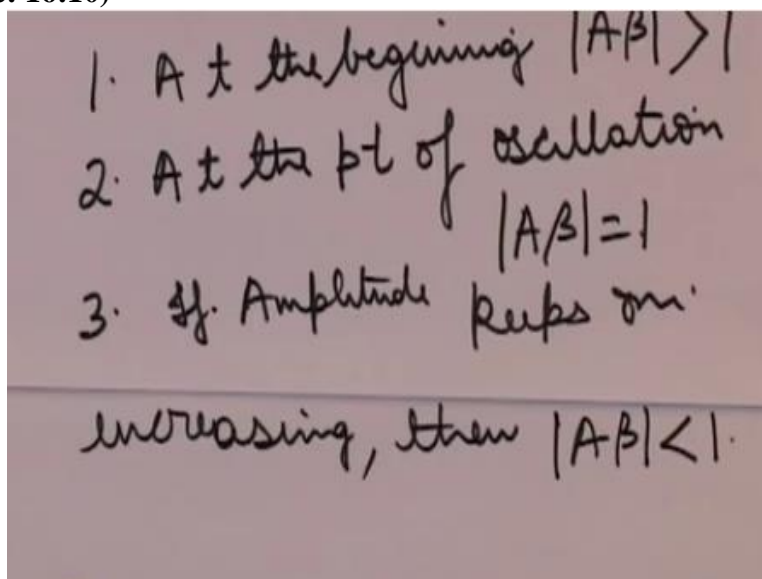
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You see that we have also studied while discussing the feedback that if the loop gain $A\beta$ has a magnitude greater than 1 then the amplitude of oscillation increases on the other hand if the loop gain $A\beta$ has a magnitude lesser than 1 then the amplitude of oscillation keeps on decreasing so this loop gain is $A\beta$ is a useful way to control the amplitude of oscillation and thereby ensure that the oscillator is indeed at the point of oscillation.

So these are the rules of starting oscillation at the beginning $A\beta$ that is when oscillation has not started $A\beta$ magnitude should be greater than 1 at the point of oscillation at the point of oscillation $A\beta$ should be = 1.

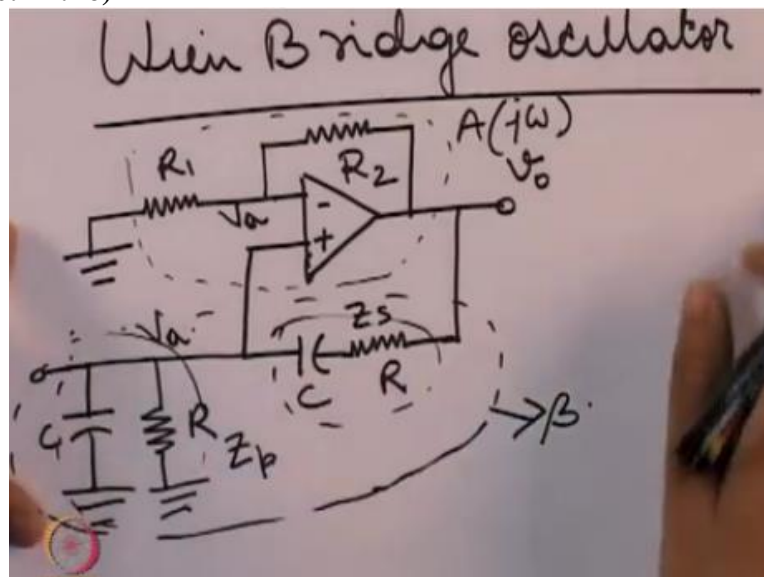
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if amplitude keeps on let me use another sheet, so these are the rules if amplitude keeps on increasing at the beginning increasing then make $A\beta$ magnitude lesser than 1 okay, so this is the way to design an oscillator at the beginning the magnitude of $A\beta$ the loop gain should be greater than 1 at the point of oscillation the magnitude of the loop gain $A\beta$ should be = 1 and if for some reason at the during stable oscillation the amplitude keeps on increasing then make the loop gain magnitude lesser than 1 that will ensure that the amplitude comes back to its stable operating point.

If for some reason on the other hand the amplitude keeps on decreasing during stable oscillation increase the magnitude of the loop gain $A\beta$ may and make it greater than 1, so that the amplitude goes increases once again and reaches the stable oscillating point now let us see some practical circuits we shall be discussing some number of circuits but let us see 1 which is known as the Wien bridge oscillator.

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So first let us see the circuit of this oscillator, now in this circuit this part this non inverting opamp comprises the A of $j\omega$ part and this part comprises the β now let us consider suppose the voltage at this point is V_a then due to the virtual short affect the voltage at this point will also be V_a now if I try to express V_a in terms of V_0 how will that expression look like, so see V_a is obtained from V_0 by a simple voltage division because no current flowing inside the opamp.

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$$V_a = \frac{V_0 Z_p}{Z_p + Z_s}$$

$$Z_p = \frac{R \times 1/c_s}{R + 1/c_s}$$

$$= \frac{R}{1 + RCS}$$

$$Z_s = R + 1/c_s = \frac{1 + RCS}{CS}$$

So I can write if I call this part of the circuit say as Z_p and this has Z_s it is this is Z_s and this Z_p then V_a is = $V_0 Z_p$ upon $Z_p + Z_s$ now Z_p is = R into 1 upon CS upon $R + 1$ upon CS okay so this is = R upon $1 + RCS$ Z_s is = $R + 1$ upon CS okay, so this = $1 + RCS$ upon CS fine so then what is V_a , V_a is given by this expression.

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$$V_a = V_0 \times \frac{R}{1 + RCS}$$

$$\frac{R}{1 + RCS} + \frac{1 + RCS}{CS}$$

$$\beta = \frac{V_a}{V_0} = \frac{RCS}{RCS + (1 + RCS)^2}$$

$$A = 1 + \frac{R_2}{R_1}$$

$$L = A\beta$$

So I can write V_a is = V_0 into R upon $1 + RCS$ upon R upon $1 + RCS + 1 + RCS$ upon CS okay so this is my beta so beta is = V_a upon V_0 and that comes out to if I simplify this then that will come out to RCS upon $RCS + 1 + RCS$ whole square okay and what is this A part the gain A is = V_0 upon V_a $1 + R_2$ over R_1 so now if I try to find out so I can write down and just I rewrite this A , A is = $1 + R_2$ upon R_1 .

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$$L(s) = A(s) \beta(s)$$

$$= \frac{1 + R_2/R_1}{3 + RCs + \frac{1}{RCs}}$$

$$\angle L(j\omega) = 0$$

$$L(j\omega) = \frac{1 + R_2/R_1}{3 + j(\omega CR - \frac{1}{\omega CR})}$$

So now if I try to find out the loop gain L is $= A \beta$ and that will come out to be L of S is $= A$ of S times β of S and this is $= 1 + R_2/R_1$ upon $3 + RCs + 1$ upon RCs okay so that is the way it will be if you want to see how the you can work this out you know you can take simplify this expression, so I will just try to simplify it so this I if I simplify then this comes out to RCs upon $1 + RCs$ whole square $+ 3 RCs$ and from this I get 1 upon 1 upon $RCs + RCs + 3$ which is this expression this expression (Refer Slide Time: 23:42) the denominator.

So now in order to ensure the condition of oscillation I have to have L of $J \omega = 0$, so now in order to make this $= 0$ okay first of all let me write down what is L of $J \omega$ so let me move $\omega = 0$ this signifies that $\omega = 0$ is the frequency of oscillation L of $J \omega$ comes out to be $1 + R_2/R_1$ upon $3 + J \omega C R - 1$ upon $\omega C R$.

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$$\begin{aligned} \angle L(j\omega_0) &= 0 \\ \Rightarrow \omega_0 CR - \frac{1}{\omega_0 CR} &= 0 \\ \Rightarrow \omega_0 &= \frac{1}{RC} \\ |L(j\omega_0)| &= 1 \\ \Rightarrow \frac{1 + \frac{R_2}{R_1}}{3} &= 1 \Rightarrow \frac{R_2}{R_1} = 2 \end{aligned}$$

Now see in order to make the phase of this expression = 0 this quantity within parentheses should be = 0 magnitude L argument of L of J omega 0 = 0 this implies that omega 0 CR - 1 upon omega 0 CR should be = 0 and then from here we can get that the frequency of oscillation should be = 1 upon also we have to ensure the magnitude condition of the loop gain which is that have the frequency of oscillation the magnitude of loop gain should be = 1.

So we already have this quantity within brackets as 0 so then this simply translates to 1 + R2 over R1 upon 3 is = 1 from which we get R2 upon R1 is = 2, however this is not enough in order to ensure that you know this just says what are the conditions in the circuit at the stable for stable oscillation, however to ensure that the amplitude does not exceed stable oscillation value we have to provide some mechanism for amplitude control and we have already studied in the previous modules about the mechanism for amplitude control which was using the limiter circuits.

So for this circuit to successfully work as a oscillator see here it is still not it will not work even if I have this circuit it will still not work as an oscillator because we have no amplitude control it is still the linear circuit an oscillator has to have some non linearity for the amplitude control, so therefore that we will need some non linear control amplitude control circuits and we will be using the same circuits limiter circuits that we discussed in the previous modules, so that is something we will cover in the next module, thank you.