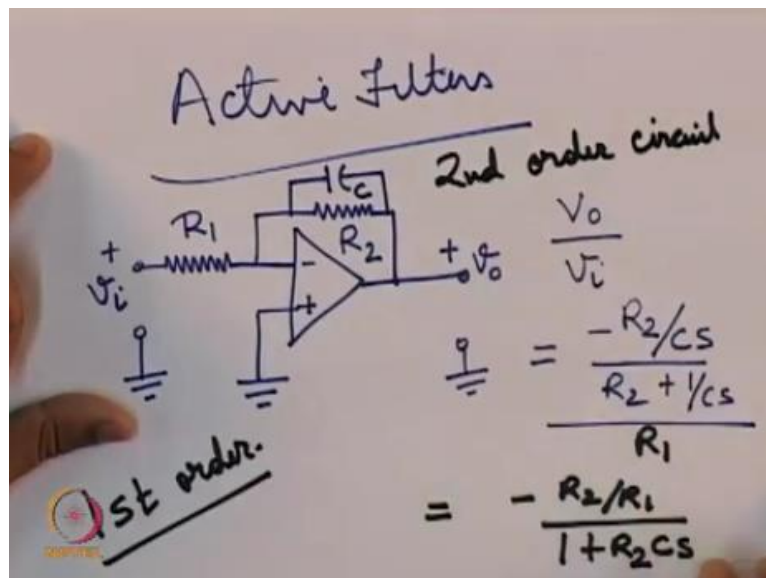


Analog Circuits
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Week -06
Module- 03
Active Filters

Hello, welcome to another module of this course analog circuits, in this module we are going to take up the topic of active implementation of filters, so in the previous module we saw how passive filters can be implemented. Let us now see how active filters so at the beginning let me mention that active filters are one where the gain of the filter can also be controlled which we cannot do in passive filters.

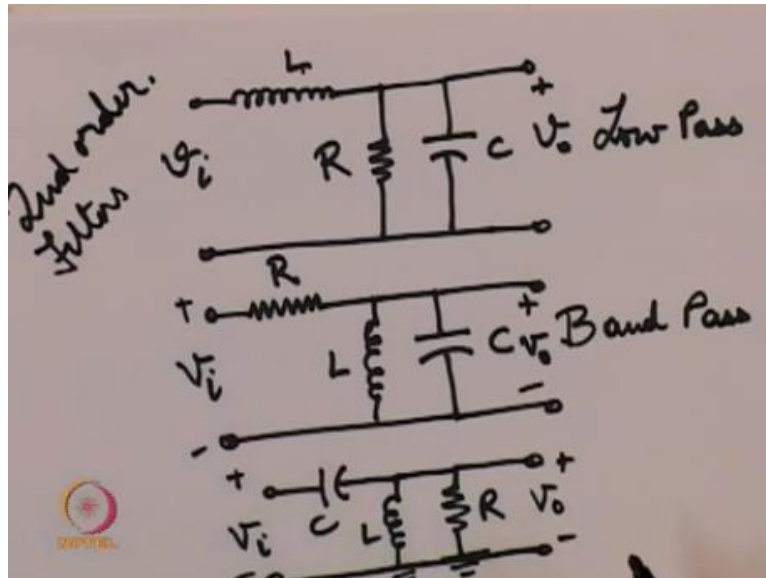
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Now one of the simplest implementation of an active filter is using a simple non inverting or inverting open loop, so we have so say you have opamp in inverting configuration so here what you see suppose we consider this circuit then you know that V_o upon V_i will be given by $-R_2$ upon Cs by $R_2 + 1$ upon Cs this whole upon R_1 , so this comes out to $-R_2$ upon R_1 upon $1 + R_2Cs$.

Now this kind of circuit as you can see using a simple open loop it is easy to implement this first order filters but this simple circuit is not enough to implement a second order circuit, so for a second order circuit what do we need, so for example you know a second order low pass filter will be something like this okay.

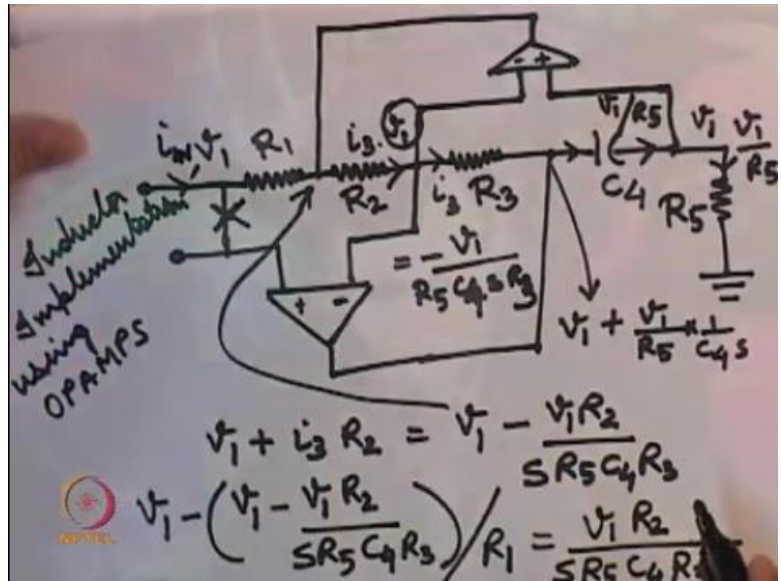
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So low pass filter will be something like this, a band pass filter will be something like this so what you see is that you know just by replacing or interchanging the positions of the LC and R we can get various topologies, so this is a band pass okay and then this one is a high pass okay, so you see elements are the same just how we place them if you want to you know it hits consonant this circuit consonant with the say others instead of putting the ground symbol we can just put it other terminal signal.

So here also this V_i this is V_i so the difference between these 3 second order filters, so these are all second order filters the difference is only that how these R's L and C are placed fine, so then let us see in an on chip and inductor is very difficult to implement because inductor has a significant height so it sticks up from the bode a simple implementation of an inductor using opamps can be done so we can replace an inductor with an opamp base circuit as follows.

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Now I am claiming that this is the inductor implementation using opamps, so first see that these opamps now in order to prove that this is really an inductor let us first assume that these opamps are ideal opamps if these are ideal opamps then there is no current flowing into each opamp, now in this first case what is the current flowing through this arm a current flowing through this arm is V_1 upon R_5 so that must be the current flowing through this arm again the current flowing the voltage.

So let us see since V_1 is the voltage at this terminal due to the virtual shot the voltage at this terminal also should be V_1 then what is the voltage at this point the voltage at this point is this voltage + the voltage drop across this capacitor, so it is $V_1 + V_1$ upon R_5 times 1 upon $C_4 s$ okay, then what is the current through this resistance the current is I can I say if I consider the current to be in this direction then this current will be the voltage difference between these 2 terminals divided by R_3 .

So this current I say I_3 is given by $-V_1$ upon $R_5 C_4 s R_3$ this is the voltage difference between these 2 terminals divided by R_3 then the voltage at this terminal would also be R_1 now this current is actually flowing in this direction that is the reason we get a - sign if we consider current to be in this direction then we get a negative sign now proceeding similarly the what is the voltage at this point it must be V_1 right now since no current flows in this terminal and this terminal therefore we can assume that the current flowing through R_2 is also I_3 and so the voltage at this point + the voltage drop across R_2 .

So the voltage at this point is $V_1 + I_3 \text{ times } R_2$ which is $= V_1 - V_1 R_2$ upon $S R_5 C_4 R_3 R_1$ this is R_3 this is C_4 okay, so that is the voltage at this point and finally what is the current flowing through this R_1 , now since again no current is flowing through this terminal to this line therefore the current flowing through this is to R_1 is $V_1 -$ the voltage at this point which is $V_1 -$ this whole upon R_1 which is $= V_1 R_2$ upon $S R_5 C_4 R_3 R_1$.

So this is the current that is going to R_1 and also the current flowing into the this network because no current is flowing through this terminal, now what do we see here the current it is the current I_{in} not that V_1 is the is the voltage at this point ok, so the V_1 is also the input voltage and i_{in} is the input current and that i_{in} is given by $V_1 R_2$ upon $S R_5 C_4 R_3 R_1$.

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$$i_{IN} = \frac{V_1 R_2}{S R_5 C_4 R_3 R_1}$$

$$\boxed{\frac{V_1}{i_{IN}} = \frac{S R_5 C_4 R_3 R_1}{R_2} = Z_{IN}}$$

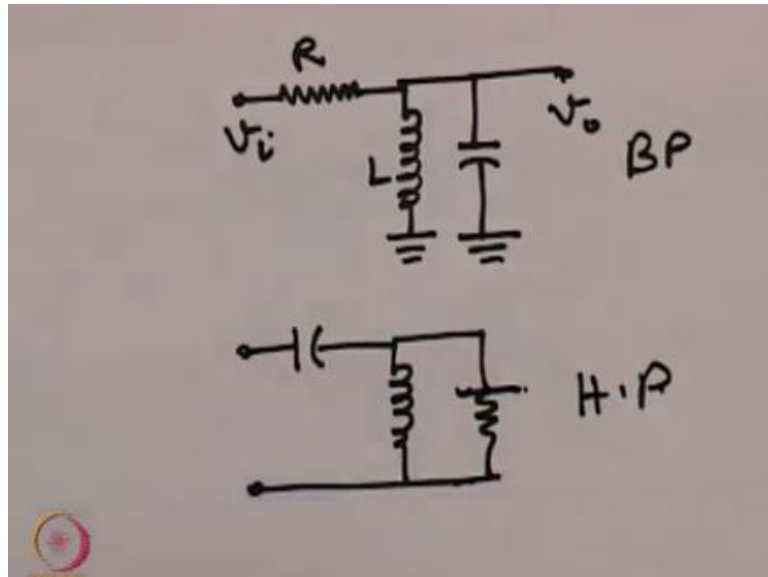
S L

So then V_1 upon i_{in} comes out to $S R_5 C_4 R_3 R_1$ upon R_2 okay this is my input impedance and since S is in the numerator so this has the same form as that of an inductor so by this circuit that I have mentioned we have actually implemented an inductor we have not used an ideal inductor but we have this circuit behaves exactly like an inductor, however please note that these opamps we assumed to be ideal that is no current is flowing inside these opamps and they have a virtual short between their 2 terminals which is not always present for practical opamps.

Now what is the utility of this circuit the first utility is that since we have implemented an active inductor so that band pass and high pass circuits, if we go back to them for a moment so this was our high pass (Refer Slide Time: 14:47), now these 2 circuits we see that 1 terminal of the inductor is grounded, grounded means like this so these 2 circuits can be

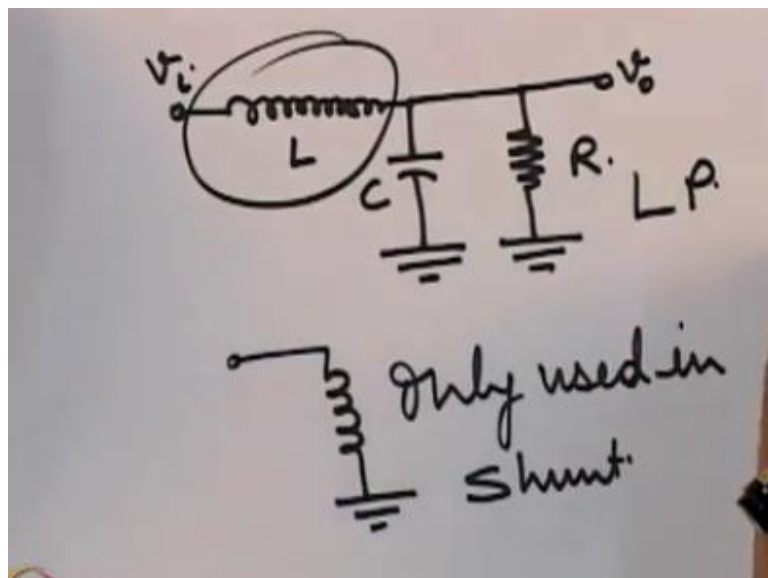
implemented using that active inductor circuit which I mentioned okay for a high pass and band pass filters.

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This is a band pass and this is a high pass prototype sorry this is a high pass prototype these 2 circuits can be implemented using this active inductor the low pass cannot be implemented because the low pass prototype has a circuit like this.

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So this low pass prototype has a circuit like this and that active inductor we saw the other terminal always has to be grounded so we have implemented an active inductor but 1 terminal of that inductor is always ground, so it can be used only in shunt it cannot be used in series like we have here and that is the reason we cannot implement a LP or low pass prototype using this inductor of course there are many other implementations of low pass

prototype. Let us see you and one of them you know let us once again go back to the pole position of our low pass and high pass and band pass prototypes.

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Low pass

$$T(s) = \frac{a_0}{s^2 + \frac{\omega_0 s}{Q} + \omega_0^2}$$

d.c gain = $\frac{a_0}{\omega_0}$

zero at 0

X. ω_0

X. $\frac{\omega_0}{2Q}$

So a low pass second order low pass filter will have a transfer function like this with a dc gain, so the poles are at this position 0 at infinity this radial distance of the pole from the origin is 0 and the horizontal distance from the Y axis for both is like this okay.

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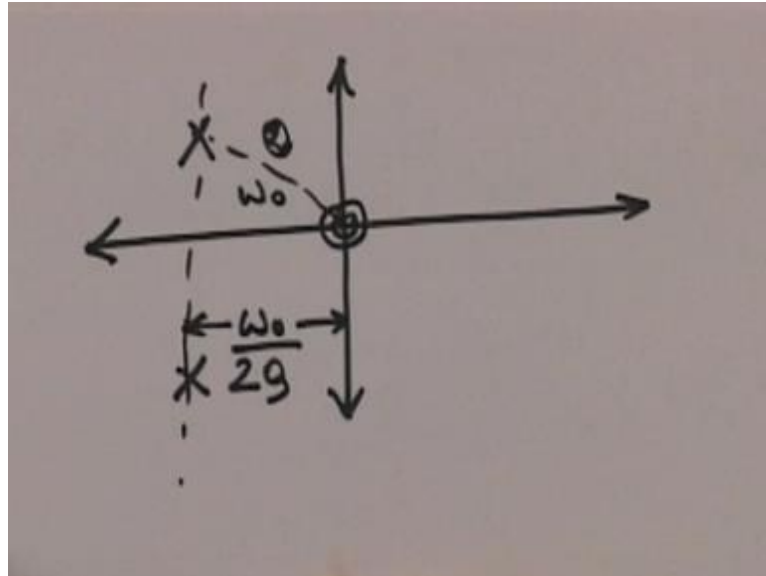
2nd order H.P
prototype

$$T(s) = \frac{a_2 s^2}{s^2 + \frac{s\omega_0}{Q} + \omega_0^2}$$

H.P gain = a_2

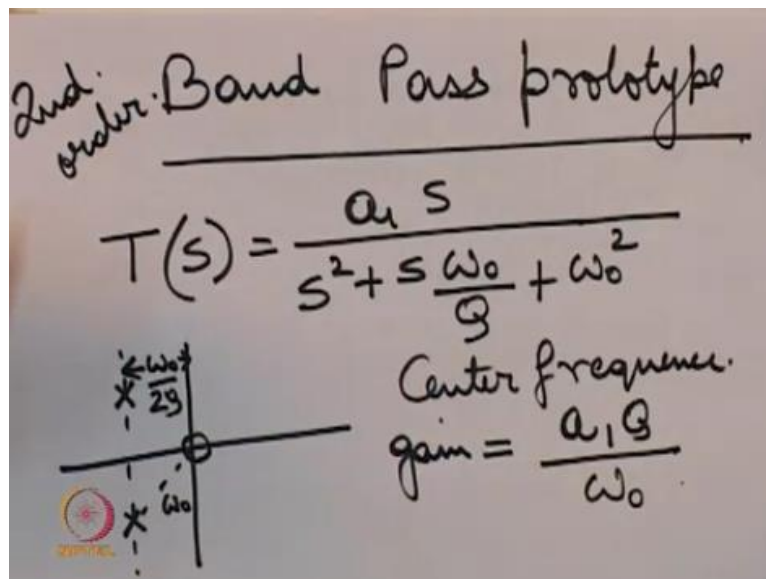
Then for the high pass prototype high frequency or gain at infinity is = A_2 .

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Now the pole positions again are like this two 0's at the origin and the pole locations are the same as for a low pass prototype ok and finally for the band pass prototype, now the reason I am elaborating on these transfer functions is that we will see certain similarities in their transfer functions eventually for the band pass prototype.

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So then for the band pass prototype the pole locations are the same zero at origin and center frequency gain, so for the low pass case we had the gain at zero at DC now for the HF case we had the gain at infinity or high frequency gain and for the band pass filter we have a center frequency gain of this one.

Now the first thing that I want you to appreciate is that so this is a second order is that that if you see the transfer functions, so this was for the low pass filter (Refer Slide Time: 22:05) the

characteristic equation for the low pass filter is same as the characteristic equation for the high pass filter which is also same as the characteristic impedance of the band pass filter.

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$$\frac{V_{hp}}{V_i} = \frac{a_2 s^2}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

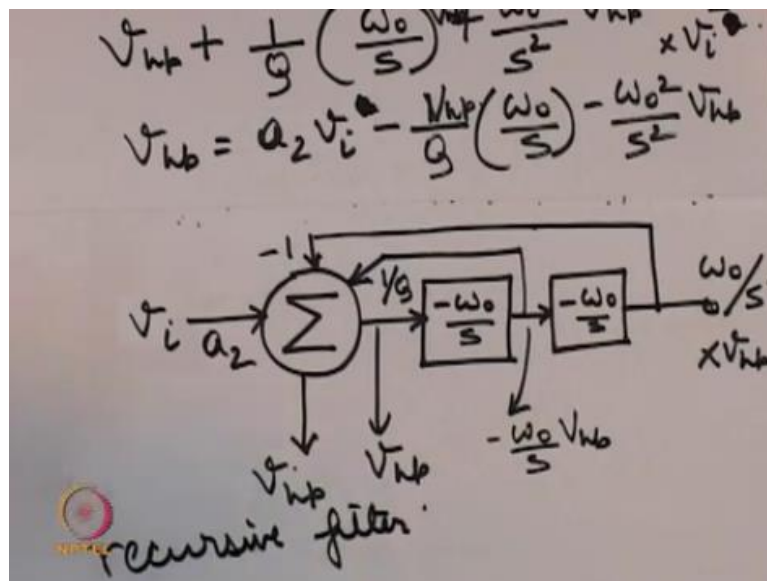
$$V_{hp}(\omega_0^2) + V_{hp}\left(s\frac{\omega_0}{Q}\right) + V_{hp}(s^2)$$

$$= V_i a_2 s^2$$

$$V_{hp} + \frac{1}{Q}\left(\frac{\omega_0}{s}\right) + \frac{\omega_0^2}{s^2} V_{hp} = \frac{a_2}{V_i} s^2$$

If they are same, then let us rewrite suppose I write the high pass transfer function like this, so we have ok this I can also write it in a different way as $V_{hp} \omega_0^2 + V_{hp} s \omega_0 Q + V_{hp} s^2 = a_2 V_i s^2$ if I divide both sides by s^2 then what I will have is $V_{hp} + \frac{\omega_0^2}{s^2} + \frac{\omega_0 Q}{s} = \frac{a_2}{V_i} s$, now suppose we write a signal flow graph diagram signal flow diagram is a diagram which shows the various voltages as they are processed, so instead of trying to explain what a signal flow graph diagram is let me draw it.

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So after you know this block diagram give me the total picture of this equation look here see what I have is this V_{hp} okay can I right V_{hp} like this as you know the summation of $a_2 V_i$

square - 1 upon $Q \omega_0 S - \omega_0^2 S^2$ Vhp suppose I write that same equation like this then you see that Vhp is the summation of 3 quantities $A^2 V_i^2 - 1$ upon $Q \omega_0 S$ and $-\omega_0^2$ upon S^2 Vhp I should have a Vhp sorry I forgot that so there should be Vhp here sorry so you see that Vhp itself is the sum of another 2 terms which are dependent on Vhp.

So it is like a recursive to get the current value of Vhp I have to use the past values of Vhp to arrive at and that is what I am implementing here actually see here this is a summation okay of V_i multiplied by A^2 okay there is no square I beg your pardon is not square in this term sorry this is note this correction I had mistakenly put a square at of this V_i^2 making it V_i there is no square in the term V_i so this Vhp is = the summation of these 2 terms.

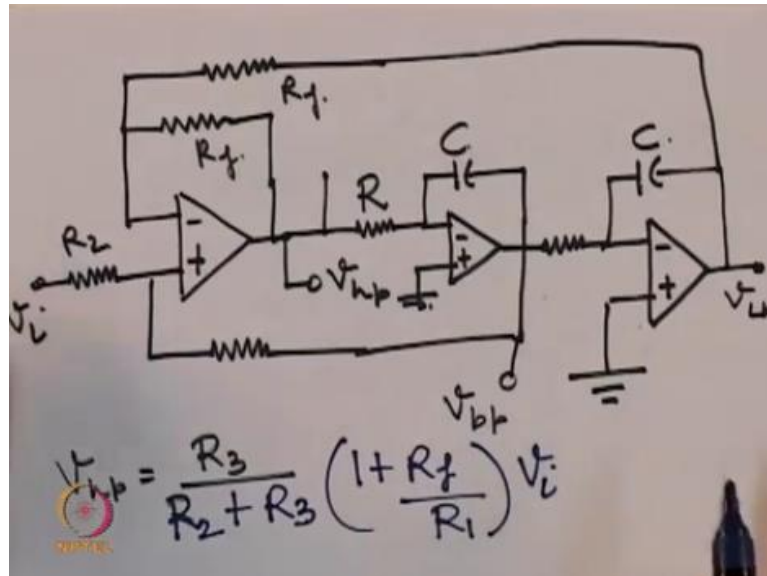
So what I did was suppose I have the signal V_i here I multiply with A^2 and then add it to this summer other inputs to this summer are this $\omega_0 S^2$ upon Vhp now the output of this summer is the signal Vhp okay so this Vhp this is also Vhp this is being first multiplied by $-\omega_0 S$ if I multiply it by $-\omega_0 S$ then scale it by 1 upon Q then this is this signal this quantity so this is $-\omega_0$ upon S Vhp.

If I pass this signal through this block it will be again multiplied by $-\omega_0$ upon S so at the output here we will get ω_0 upon S^2 times Vhp this I then scale by -1 and the feed it to the summer okay so then I get back so this summer itself now generates Vhp, so this is what is known as a recursive filter an implementation of this recursive filter would be something like this.

So this was the signal flow graph diagram just I want (Refer Slide Time: 28:54) repeat a signal flow graph diagram is a frequency domain is shows the frequency domain flow of signal and the relation between the quantities and with the input signal, so here this is when I say $-\omega_0$ upon H Vhp is actually proportional to the band pass signal and then this one which is ω_0 upon S^2 Vhp this is actually the low pass signal proportional not exact but proportional, because the exact signal here the exact signal has to be scaled with the appropriate gain values.

A possible implementation of this circuit is using opamps could be like this so we just follow the signal trail how the signals are added and by following them properly we will get our (()) (29:46) circuit.

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So this circuit is implement this signal flow graph is implemented by the following circuit, so this is my circuit these are the various points at which I obtained by high pass band pass and low pass signals and the output equation that we get if we follow the opamp analysis will be something like this.

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$$V_{hp} = \frac{R_3}{R_2 + R_3} \left(1 + \frac{R_f}{R_1} \right) V_i$$

$$+ \frac{R_2}{R_2 + R_3} \left(1 + \frac{R_f}{R_1} \right) \left(\frac{-\omega_0 V_{hp}}{s} \right)$$

$$- \frac{R_f}{R_1} \left(\frac{\omega_0^2 V_{hp}}{s^2} \right)$$

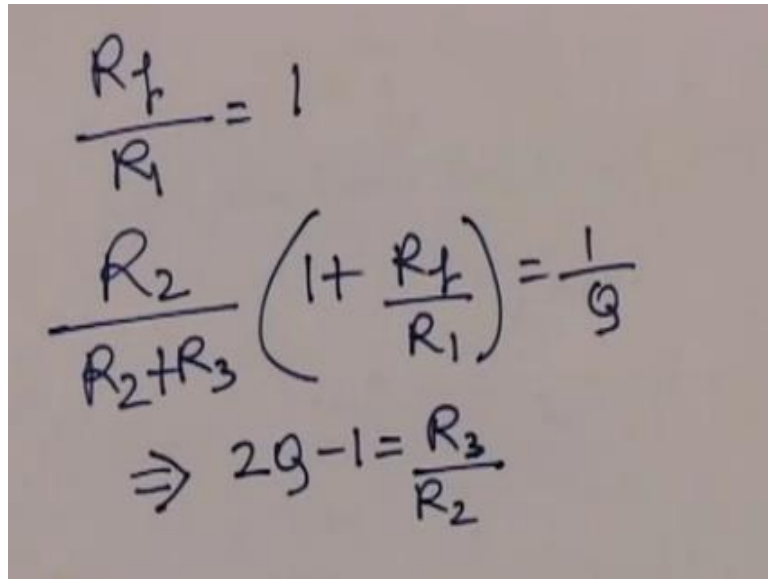
$$\Rightarrow K V_i - \frac{1}{Q} \frac{\omega_0}{s} V_{hp} - \frac{\omega_0^2}{s^2} V_{hp}$$

So this V high pass will be = R3 upon R2 + R3 1 + Rf upon R1 Vi + R2 upon 2 + R3 okay, so this is of the same form as Vhp - ok here this gain from the these capacitor based opamps,

opamps where the capacitors are connected in the feedback loop is - 1 upon CSr and this is = omega 0 upon S, so then omega 0 is = 1 upon Rc, so the value of this omega 0's is 1 upon Rc.

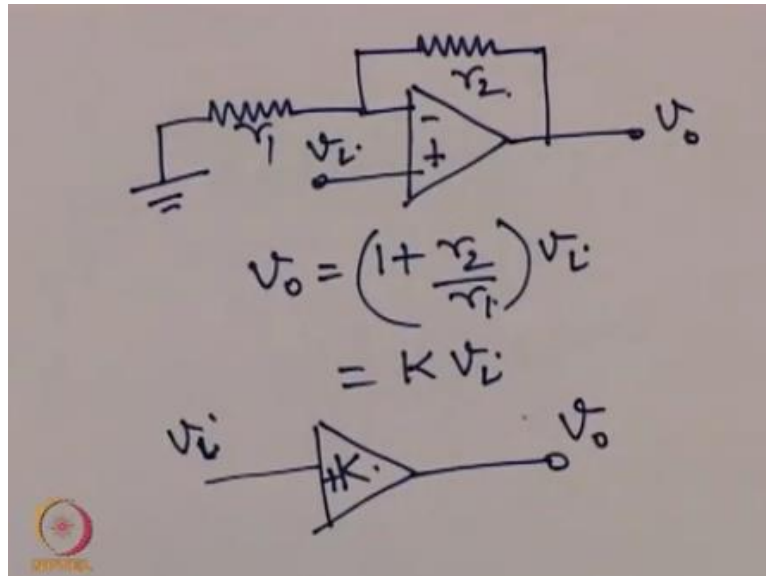
So this is of the exactly the same form that we discussed a few minutes ago, here this was the equation this one represented the equation we obtained from the signal flow graph diagram and this is the one we get from the actual circuit.

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$$\frac{R_f}{R_1} = 1$$
$$\frac{R_2}{R_2 + R_3} \left(1 + \frac{R_f}{R_1} \right) = \frac{1}{Q}$$
$$\Rightarrow 2Q - 1 = \frac{R_3}{R_2}$$

Now of course to equate these 2 equations the values of these R1, R2 and R3 should be such that Rf upon R1 is = 1 R2 upon R2 + R3 1 + Rf upon R1 is = 1 upon Q okay so from here what we get is that the Q value to Q - 1 should be = R3 upon R2 so if we choose our Rf, R1, R2 and R3 according to this equation then we can implement this now once circuit that I did not mention which is frequently used for gain control in an active filter is a what is called a buffer amplifier.

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So we have already studied about buffer amplifiers while studying non inverting opamps, a simple buffer amplifier is like this okay so here V_o is $= 1 + R_2$ upon R_1 V_i which is of KV_i so for example say when we take the output of these filters that I was mentioning suppose let us consider the this filter (Refer Slide Time: 36:43), if we want to take the outputs V_{hp} V_{bp} and V_{lp} and when one to scale it to the appropriate gain values we simply can connect it to a buffer amplifier whose gain is controlled using this equation so this is nothing but this $V_i K + K$.

So in summary in the past lectures we have covered a number of topics on a active filter design how to implement a filter using opamps for the first order case and then for the seconds order case how to replace a passive inductor by an active implementation of an inductor, however we saw that that active inductor so implemented can one be used for the band pass and high pass cases and then finally we saw a full implementation of low pass band pass and high pass filters using what we called a recursive technique and the same topology can give all the 3 types of filters.

There are some more implementation of second order active filters, such as the Sallen key filters and which we have not covered and one thing I would like to emphasize here you saw that the difference between an implementation of a filter of an active filter between the first order and the second order is totally different.

A second order implementation of a filter is much more involved than a first order implementation and the third order implementation would be even more difficult for passive

case, it not so difficult all we have to do is add an extra ladder to increase the order of the filter but for active filters it becomes very difficult that is why whenever we want to implement an active filter the order of the filter should be as low as possible and therefore the prototype functions like the Butterworth the Chebyshev prototypes that we use we should judiciously use them so as to keep the order of the filter to a minimum for the given specification, thank you.