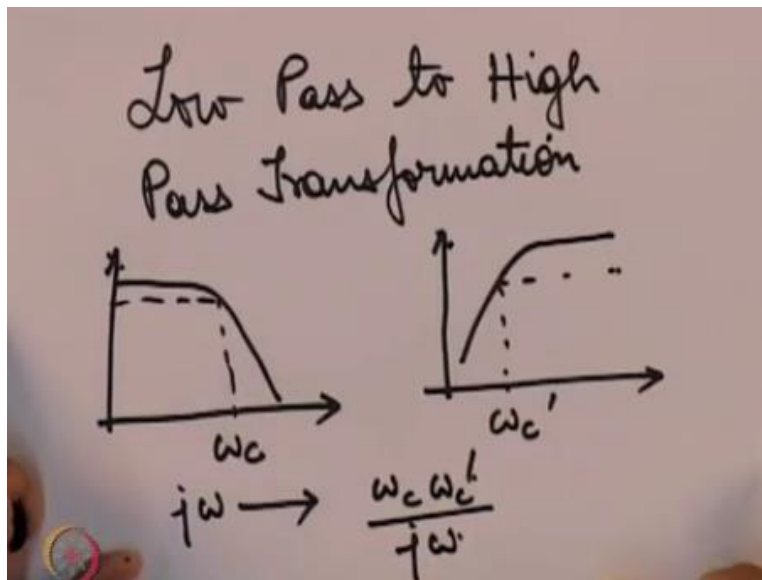


**Analog Circuits**  
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**Week -06**  
**Module- 02**  
**File Transformations (Contd....)**

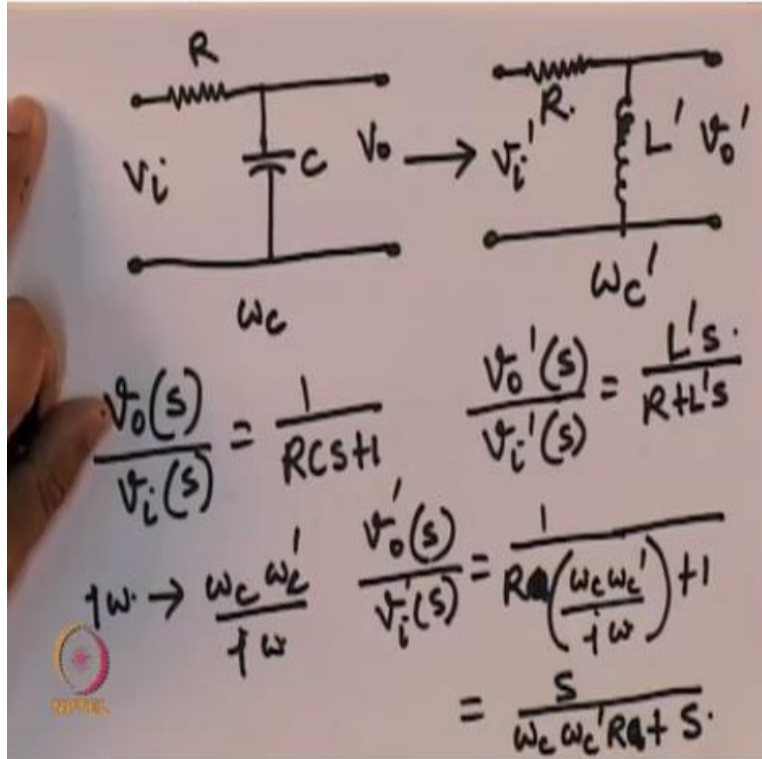
Hello welcome to another module of this course, in the previous module we had just started with filter transformations I showed you how to do a low pass to low pass transformation, in this module we shall be continuing our discussion and the first one that we will be covering is the low pass to high pass transformation.

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So low pass to high pass transformation involves the following, so the equation or the formula for or the substitution that we have to do for transforming from low pass to high pass is that  $j\omega$  should be transformed to  $\omega_c \omega_c' / j\omega'$  to again see how this is being done let us take the same RC circuit that we consider.

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So suppose our circuit is like this and so this is the low pass prototype and the high pass prototype is like this suppose the cutoff frequency here is  $\omega_c$  and this is  $\omega_c'$  then let us see so here this is  $V_o$  this  $V_i$  output voltage is  $V_o$  input voltage is  $V_i$  so  $V_o$  upon  $V_i$  is  $1$  upon  $RCs + 1$  and say this  $V_i'$  this  $V_o'$   $V_o'$  upon  $V_i'$  will be  $R + Ls$  upon  $Ls$ .

So now if you know make that substitution that from  $\omega_c$   $\omega_c'$  upon  $\omega$  here so then  $V_o$  upon  $V_i$  will become and suppose the substitution is such that we have to make match this  $=$  this then is become  $=$  this so this becomes  $= \frac{R \omega_c \omega_c'}{s + \omega_c \omega_c'}$  upon  $\omega_c \omega_c' RC + 1$  ok and now this matches with the so if you know in now if we substitute in place of  $\omega_c \omega_c'$   $s$  so this becomes  $s$  upon  $\omega_c \omega_c' RC + 1$  so we already see the similarity with this sorry this one be  $C$  be  $C$  so  $\omega_c$  upon  $\omega_c'$   $\omega_c \omega_c'$   $R + s$ .

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$$\frac{V_o(s)}{V_i(s)} = \frac{1}{RCs+1}$$

$$\frac{V_o'(s)}{V_i'(s)} = \frac{L's}{R+L's}$$

$$\uparrow \omega \rightarrow \frac{\omega_c \omega_c'}{\uparrow \omega}$$

$$\frac{V_o'(s)}{V_i'(s)} = \frac{1}{\frac{R}{\omega_c \omega_c'} + \frac{1}{s}}$$

$$= \frac{s}{\omega_c \omega_c' R + s}$$

$$\frac{V_o'(s)}{V_i'(s)} = \frac{s / (\omega_c \omega_c' C)}{R + \frac{1}{\omega_c \omega_c' C} \times s}$$

We can slightly manipulate this equation and get something like this, so we have  $V_o$  dash upon  $V_i$  dash we write it like this  $S$  upon  $\omega_c \omega_c' R + 1$  upon  $\omega_c \omega_c'$  times  $S$  so see what we see is that this equation this equation is of the same form as this equation okay and so what have we got here I think I will just go back to this equation I think the I missed out here is to  $C$  there should be a  $C$  here sorry yeah I should so then this becomes  $= C, C$  like this is the correct form sorry I crossed out this  $C$  earlier so this is the  $V_o$  dash upon  $V_i$  dash formula okay.

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$$\frac{V_o'(s)}{V_i'(s)} = \frac{s / (\omega_c \omega_c' C)}{R + \frac{1}{\omega_c \omega_c' C} \times s}$$

$$= \frac{L's}{R + L's}$$

$$L' = \frac{1}{\omega_c \omega_c' C}$$

$$j\omega L = \frac{j\omega}{\omega_c \omega_c' C}$$

$$\omega L' = \omega \times \frac{1}{\omega_c \omega_c' C}$$

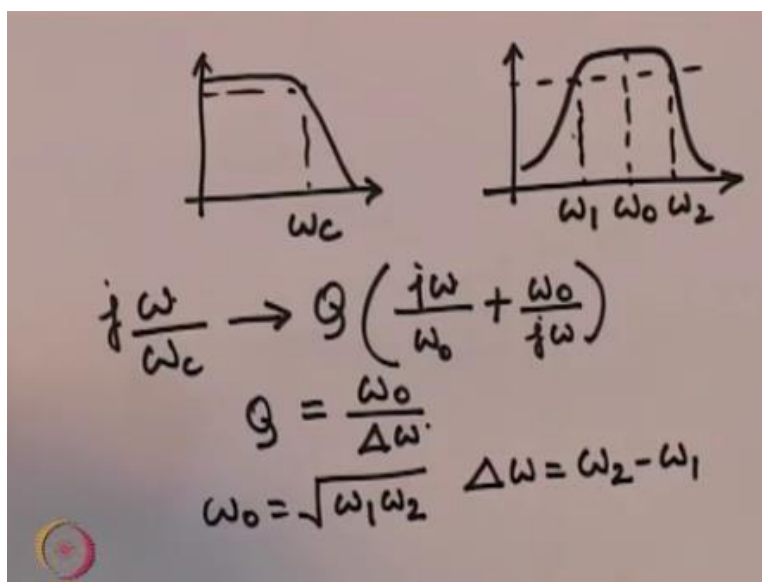
Now here whereas my original  $V_o$  dash upon  $V_i$  dash using the inductor circuit was  $= L$  dash  $S$  upon  $R + L$  dash  $S$  so what do we see here then the  $L$  dash is  $= 1$  upon  $\omega_c \omega_c'$  dash

times  $C$  and so my inductor needs to be modified the inductor or rather I should say the new inductor that I am introducing in the high pass prototype with the cutoff frequency  $\omega_c$  needs to have a value given by this and so this should be  $=$  so then  $J \omega L$  should be  $= J \omega_c$  upon  $\omega_c$   $\omega_c$  dash  $C$  right so that is how this transformation is coming actually.

So my  $\omega_c$  the new  $\omega$  so what I had was initially  $\omega L$  dash and that should be  $=$   $\omega_c$  dash and to the new value which is  $1$  upon  $\omega_c$  dash  $\omega_c$  dash  $C$  ok so this  $\omega L$  dash will be  $= \omega$  times  $1$  upon  $\omega_c$   $\omega_c$  dash  $C$  ok so this is the basic formula know from which we get that transformation this transformation that I mentioned earlier that my  $J \omega$  gets modified to this value that is coming from this equation that the new the new  $L$  dash for realizing the high pass prototype will be given in terms of the  $C$  of the low pass prototype using this equation.

So that is how we can achieve a low pass to high pass transformation the next 2 transformation are from the low pass to band pass transformations now low pass to high pass transformation is understandable because in both cases there is just a single cutoff frequency how do we achieve a low pass to band pass transformation because in a band pass filter there are 2 cutoff frequencies or 3 DB frequencies.

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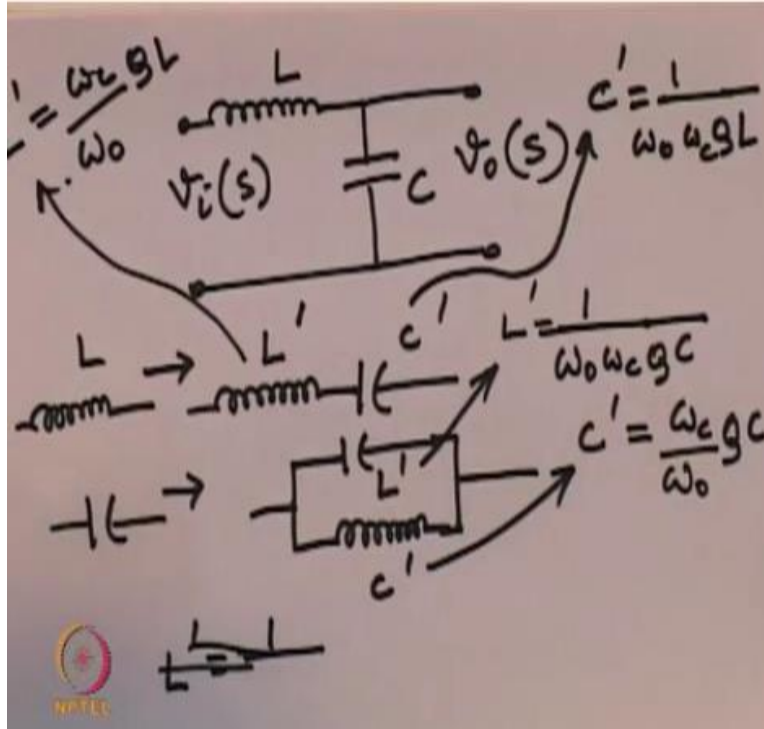
Now that is possible if we consider so we have our low pass prototype like this okay and our band pass prototype is like this okay now without going into the circuit details I will just give you the transformation ok so if suppose  $\omega$  is the frequency variable in this domain the low pass domain then that will simply get transformed to this variable.

Now see here in the band pass case even though there are 2 cutoff frequencies, they are basically mirror images of each other what I mean is that the cutoff see the properties are same along both  $\omega_1$  and  $\omega_2$  they are mirror images.

So if we just considered half of this and this looks like a high pass prototype and that is why so the properties of the low pass prototype will be reflected in both the cutoffs both the edges that is how we are able to do this had this mean that in the lower cutoff (freq) cutoff range the properties of the or the fall of this characteristic is different from the upper half then we could not have done this and this  $Q$  is  $= \omega_0$  is the geometric mean of  $\omega_1$  and  $\omega_2$  okay and  $\Delta\omega$  is  $= \omega_2 - \omega_1$ .

Now when we do this transformation just like we saw for the high pass case where this in the high pass prototype the  $L$  dash would be obtained from the low pass prototype  $C$  using this formula in the case of a band pass prototype what happens is that if we have any inductors in the low pass.

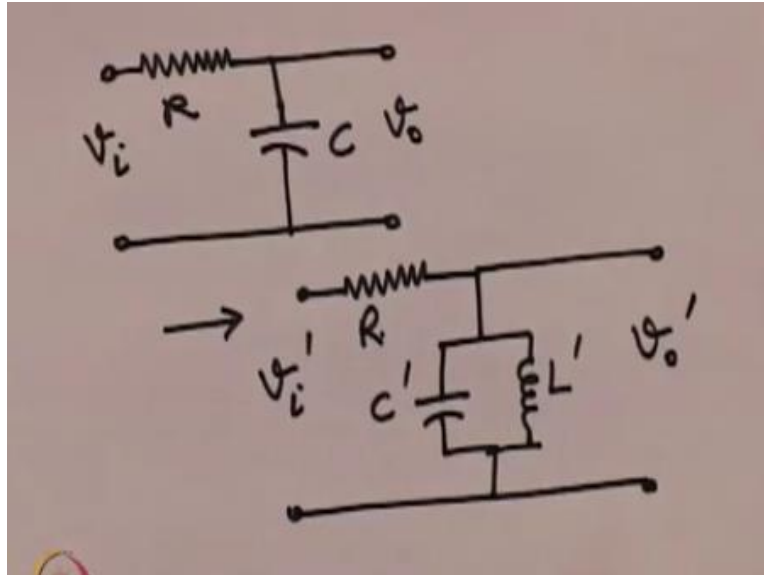
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So for example, suppose our low pass prototype was like this  $L$  would get transformed to a series combination of another  $L$  and another  $C$  like this okay and the capacitor would get transformed to a shunt combination of  $C$  and  $L$  where this  $L$  dash was given as for this series combination the formula for the  $L$  dash is and  $C$  dash is so this  $L$  dash refers to this  $L$  dash and this  $C$  dash refers to this  $C$  dash and this  $L$  dash is given by this formula and this  $C$  dash is given by this formula ok.

So that is it now inductors get converted to a series combination of another inductor and capacitor and the capacitor gets converted to a shunt combination of another capacitor and another inductor whose values are given like this and so then suppose if we have a RC low pass prototype that gets transformed like this you ok.

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Now I leave the task of you know of converting from low pass to band stop as an exercise that you can do in your free time, let us move on to something else now the realizations as I was speaking you know like if you want to realize the filter using a actual hardware how do you do that now we already saw some sample circuits but these were simple RC passive circuits using opamps or using active circuits how do you do that so let us study that.

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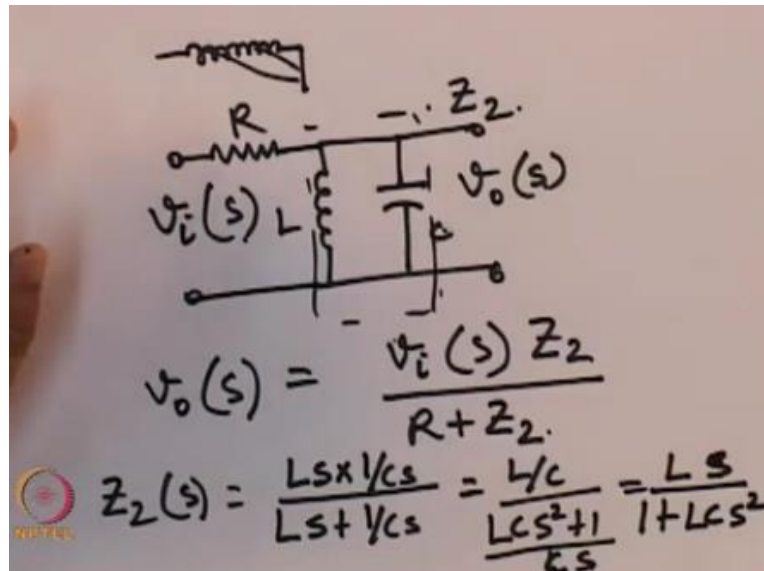
Filter Realization

$$\frac{V_o(s)}{V_i(s)} = \frac{1/Cs}{R + 1/Cs} = \frac{1}{RCs + 1} = \frac{Y_{RC}}{s + Y_{RC}} = \frac{k}{s + \omega_0}$$

So when I say realization you know a simple realization of a filter could be like using as I have already pointed out an RC network this acts like a low pass filter and this gives you what we call the first order filter isn't it so the  $V_o$  upon  $V_i$  will be given by okay so this is like you know of the so this becomes  $1$  upon  $RCs + 1$  and this is  $= 1$  upon  $RCs + 1$  upon  $RC$  and this of the form

of K upon S + omega 0 as this is the first order realization again if we want to say realize a band pass filter how do we do that?

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Consider this circuit suppose I consider the whole you know this to be Z2 this whole LC combination so then Vos is = Vis times Z2 upon R + Z2 and Z2 of S is = a less into 1 by CS upon LS + 1 upon CS on C on + 1 on so this So LCS square upon Cs from which this becomes LS upon 1 + LCS square okay, so if I substitute this value of Z2 in this equation and what do I get?

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$$V_o(s) = \frac{V_i(s)}{R + \frac{Ls}{1 + Lcs^2}} \times \frac{Ls}{1 + Lcs^2}$$

$$= \frac{V_i(s) \times Ls}{RLcs^2 + Ls + R}$$

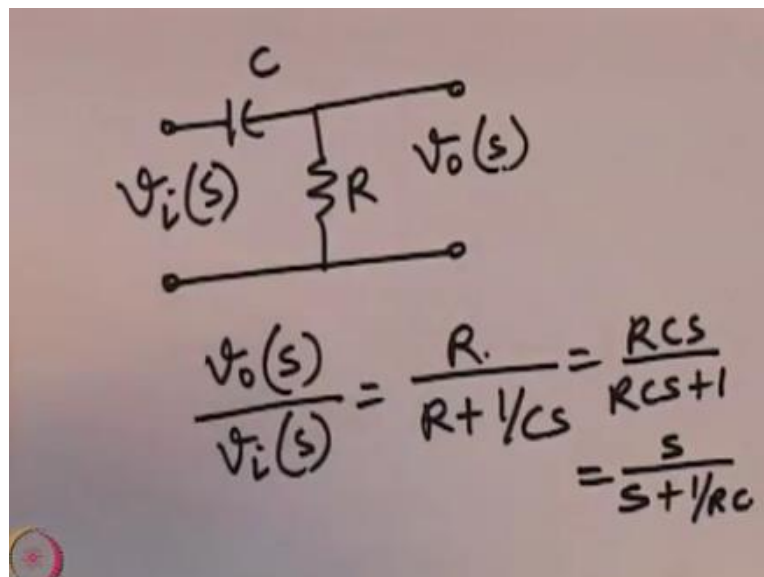
$$\frac{V_o(s)}{V_i(s)} = \frac{Ls}{RLcs^2 + Ls + R}$$



So  $V_o$  of  $S$  becomes = so that becomes =  $V_i$  of  $S$  upon  $R + LS$  upon  $1 + LCs$  Square times  $LS$  1  $LCs$  Square which is =  $V_i$  of  $S$  multiplied by  $LS$  upon  $LCs$  square +  $RLC$  square +  $LS + R$  okay, so then the transfer function overall becomes =  $LS$  upon  $RLCs$  Square +  $LS + R$  so this is a second order band pass filter because the order of the denominator polynomial is 2 is the second order filter.

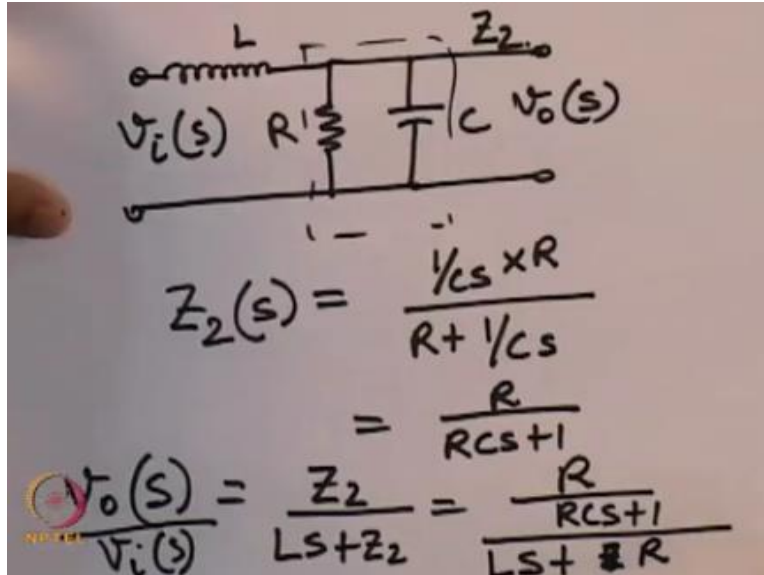
So this is so this circuit that we had this  $R$  in series  $LC$  in shunt this is a second order band pass filter this is a so this one was a first order low pass filter similarly a first order high pass filter will be like this so that we have already seen while discussing the filter transformation but just for completeness.

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This will be a first order high pass filter, suppose I asked you to make say first order or a second order low pass filter, then how will we go about the design so then the way to do it is like this.

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Consider this circuit just let me write it down so this was a first order high pass filter again let us consider this to be a single impedance then this combination R and C in shunt then  $Z_2$  of S is given by  $\frac{1}{CS} \times R$  upon  $R + \frac{1}{CS}$  which is  $= \frac{R}{RCS + 1}$  okay so  $V_0$   $V_o$  of S is  $= \frac{Z_2}{LS + Z_2}$ , so this which is  $= \frac{R}{RCS + 1}$  upon  $LS + R$ .

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$$\frac{V_o(s)}{V_i(s)} = \frac{Z_2}{LS + Z_2}$$

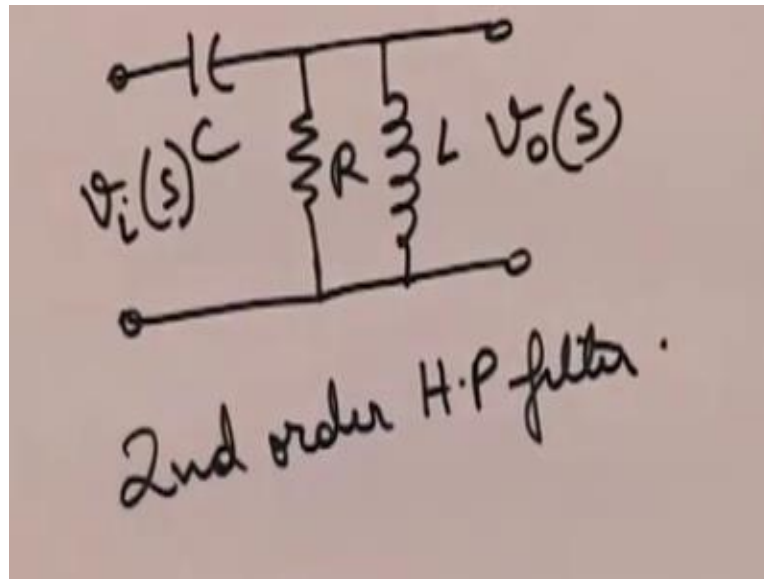
$$= \frac{R / (RCS + 1)}{LS + \frac{R}{RCS + 1}}$$

$$= \frac{R}{RLCS^2 + LS + R}$$

2nd order low pass filter

Let me us a fresh paper  $V_0$  upon  $V_i$  of S is  $= \frac{Z_2}{LS + Z_2}$  which is  $= \frac{R}{RCS + 1}$  upon  $LS + R$  upon  $RCS + 1$  upon  $RLCS^2 + LS + R$  okay so this is a second order low pass prototype low pass filter not prototype filter, so this is the one so this one this is our second order low pass filter suppose we want to realize the second order high pass filter then the circuit I am not going to the detailed derivation but the circuit would have been something like this.

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This would have this is a second order high pass filter, so overall in summary in this module we have studied some of the remaining transformations from that is low pass to high pass, low pass to band pass and I have left the low pass to band stop as an exercise for you and also we saw some passive realizations now all the realizations that we saw in this module were passive because you cannot control the gain however if you could have controlled the gain of the output then you would realize something what we call as active filters.

So in the next module we shall see the realization of active filters that is the same first order and second order filters but using gain control and we shall see how some of these circuits are realized, thank you.