Analog Circuits
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Week- 06
Module- 01
Chebyshev Prototype, Filter transformation

Hello, welcome to another module of this course analog circuits, we are now in week 6 and in the past week we had studied about the Butterworth prototype filter realization, in this module we are going to study about other prototype function which is Chebyshev prototype and we will see some advantages associated with it and the beginning, let us note that the Butterworth prototype was very flattish isn’t it, both in the pass band as well as in the stop band the response was quite smooth.

Now the problem with having such a very smooth response is that the order of the filter increases I should say, so the more flat response you get the higher is the order of the filter that will be obtained the problem with realizing a higher order filter is that the circuit while the mathematics is quite simple it just the value of capital N changes when the order of the filter changes.

However to actually realize it on a circuit there is a huge difference between say are realizing a filter whose order is 1 and a filter whose order is 2, the Chebyshev prototype function on the other hand has a response which is not so flattish and the advantage of that this given us is that by allowing some ripples as we say in the in both the pass band as well as in the stop band.

We can reduce the order of the filter so for the same filter specifications that is A min in the A min in the stop band and A max in the pass band you can get a filter which of lesser order using the Chebyshev prototype as compared to the Butterworth prototype, so let us see what that what the how the transfer function looks like.

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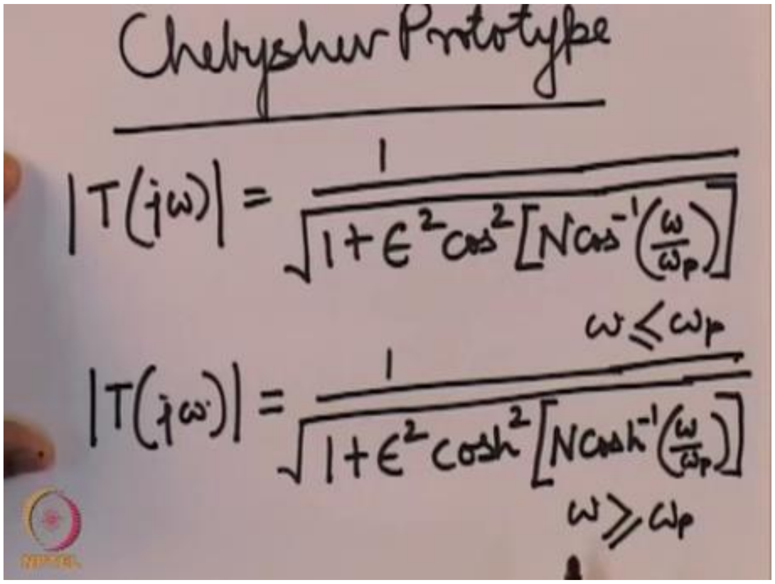

So the Chebyshev prototype has a magnitude response which is of the following form so below the cutoff frequency omega p the response will be of this form and above the cutoff frequency the response value of this form and now graphically this looks like this.

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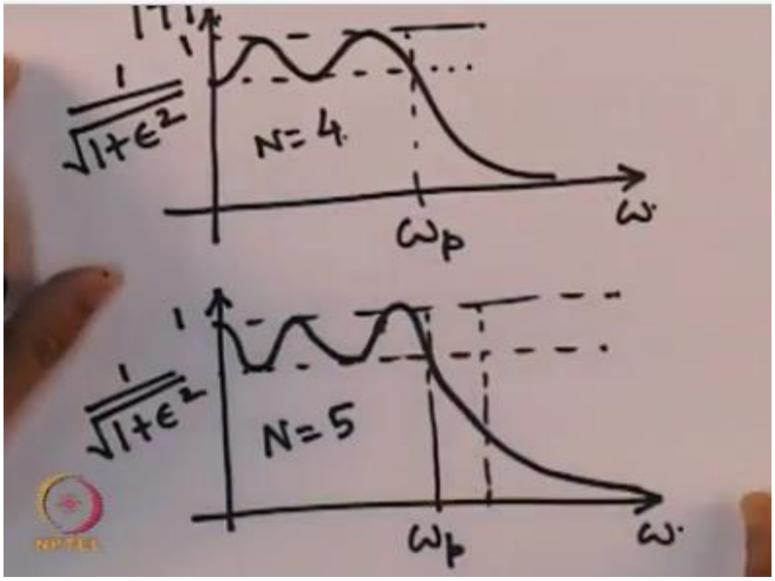

So this is the response of N = 4, we note that the ripples are present for frequencies which are below omega P for both case and this is the response for N = 5, now here what has to be known is that the number of up and down points 1, 2, 3, 4, 5 so number of the order number corresponds to the number of traps and crests of the ripples similarly here for N = 4 we have 1, 2, 3, 4, 4 traps and crest, now how do we go about the design to go about the design we again proceed similarly as we had done for the Butterworth case that is we have the specification of Amax in the pass band and Amin in the stop band.

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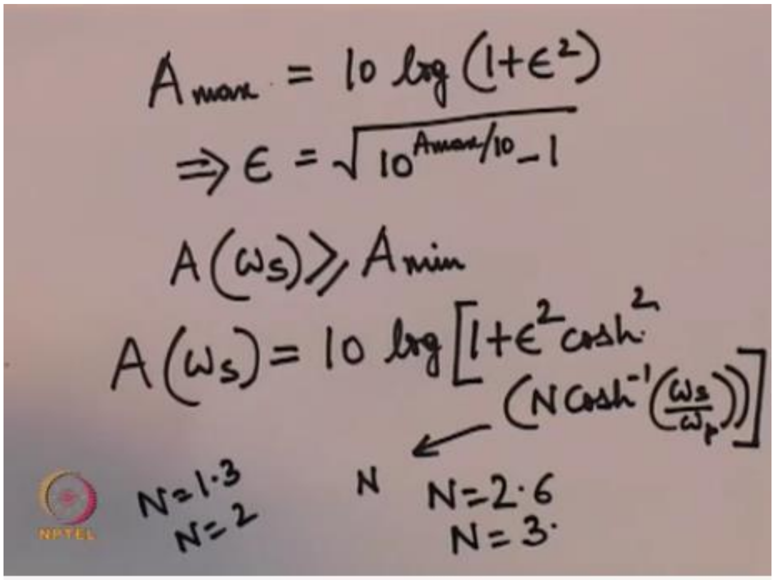

So A max is given by 10 log of 1 + epsilon square from which we get epsilon ok so this follows from the fact that you know from this formula that when omega is equal to omega p this function reduces to 1 upon 1 + epsilon square whole under square root, so now coming to the other specification which is the A min specification in the stop band and just in the case of the Butterworth prototype we have the requirement that in the stop band the attenuation should be minimum.

A min it is at any frequency omega S the attenuation should not be lesser than A min and as for frequencies which are greater than omega S the attenuation is usually larger so now A of omega S for this Chebyshev prototype it comes out to this ok so from here we can get the value of N and just as in the case of the Butterworth prototype suppose N comes out to be equal to 1.3 then we should take N = 2 the next higher integer if N is = 2.6 then N should be taken as 3 ok.

So from this we get the value of N and the value of epsilon and if we plug it in this equation the values of N and epsilon then we get the magnitude response and then from the magnitude response we can get the poles of the equation of the transfer function of the system and the poles will be given by this formula, so you can derive it if you want but I will just state the values of it is a very complex derivation the to derive the formula for the poles.

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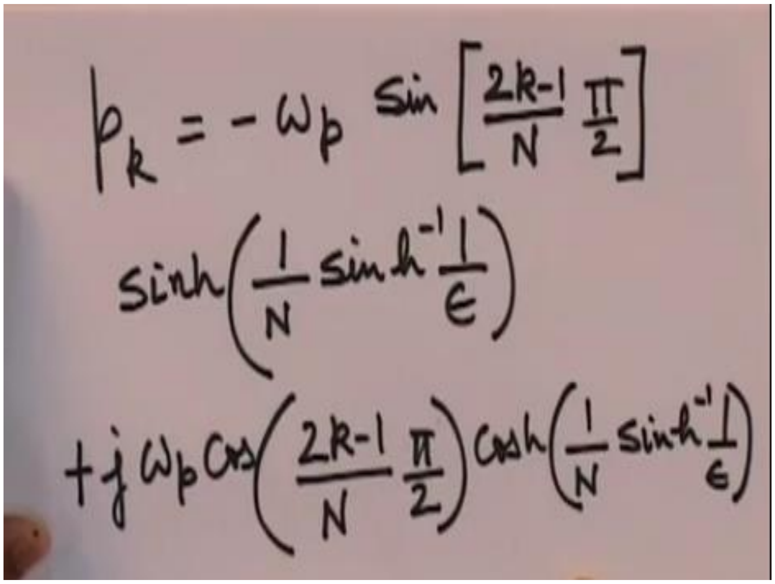

So the K-th pole, first of all for the N, N pole system N pole transfer function or an N when the order of the filter is N the number of poles will be N and K-th pole is given by this formula, it is a somewhat long formula ok, it can be shown that these poles now this might look like a very complicated formula but actually graphically you know this like for Butterworth case graphically we saw that the poles lie on the left half of the S plane and along the edges of a circle the poles for this product Chebyshev prototype will like along the left half of the S plane but along the edges of a ellipse.

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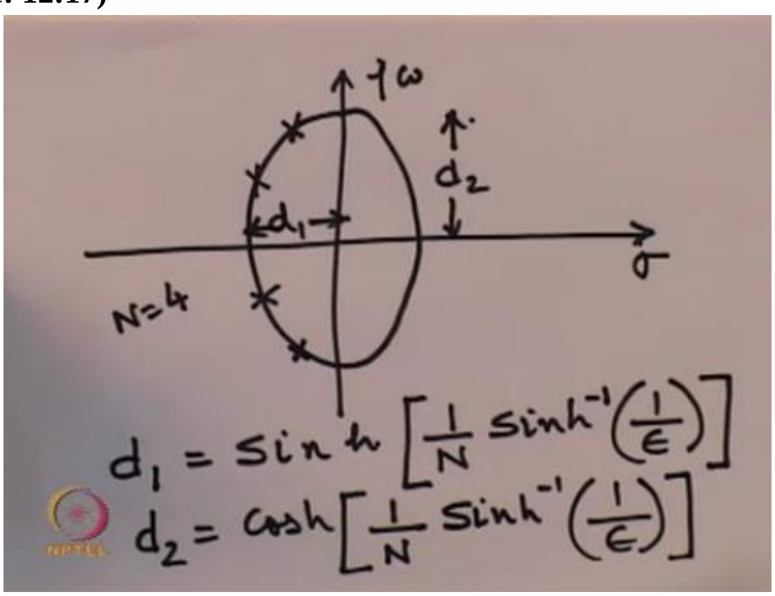

So we can graphically represent the poles like this, this is my complex S plane suppose we have a N = 4 where this say d1 represents the semi minor axis and d2 represents the semi major axis this is the way the semi major and the semi the semi minor I beg your pardon and the semi major axis is are given so once we you know like find out the poles then we can find out the from poles once we get the poles we can obtain the transfer function and we can get all the parameters of the filter just like we did for the Butterworth case.

So that was all about this Chebyshev prototype filters, now there is one thing that I not had not mentioned so far now all this realization both for the Chebyshev as well as for the Butterworth now we have been doing a low pass realization isn't it but then filters that you see around you that you use commonly they may be high pass they may be band pass they may be band stop so what is then the realization rule for such filters isn't it because it seems as if the Butterworth and Chebyshev realizations are applicable only for the low pass prototype.

So the way to design the high pass prototype suppose you want to I give you a task to design a Butterworth prototype for a high pass filter, so the way to do it is first to realize a basic low pass prototype using Butterworth achieve chef polynomials and then transform that low pass prototype to a high pass prototype with the correct cutoff frequencies.

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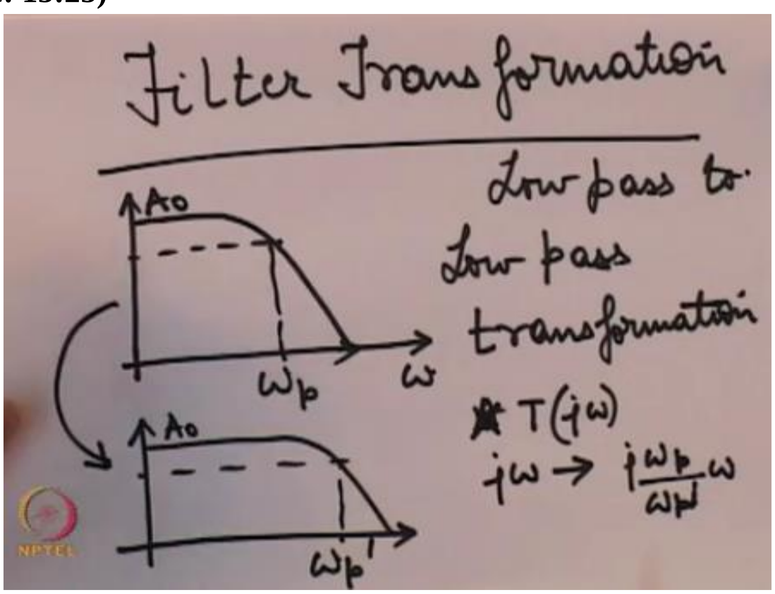

So let me show you what I mean you suppose let us start first with a I will go to the high pass transformation later but suppose let us first start with a low pass to low pass transformation suppose this is what we have realized using our Butterworth or whichever transformation we use but say this is not the actual cutoff frequency that we want to achieve say the actual cutoff frequency that we want to achieve is like this.

So we have to transform our response from here to here okay so the way we do it is that suppose our transfer function is some function of omega then we simply transform that omega the j omega 2 j omega p upon omega p dash omega if you do this transformation from j omega to j omega p upon omega p dash omega then you will realize that so you know just to show you what this transformation happens in real life and why it is happening.

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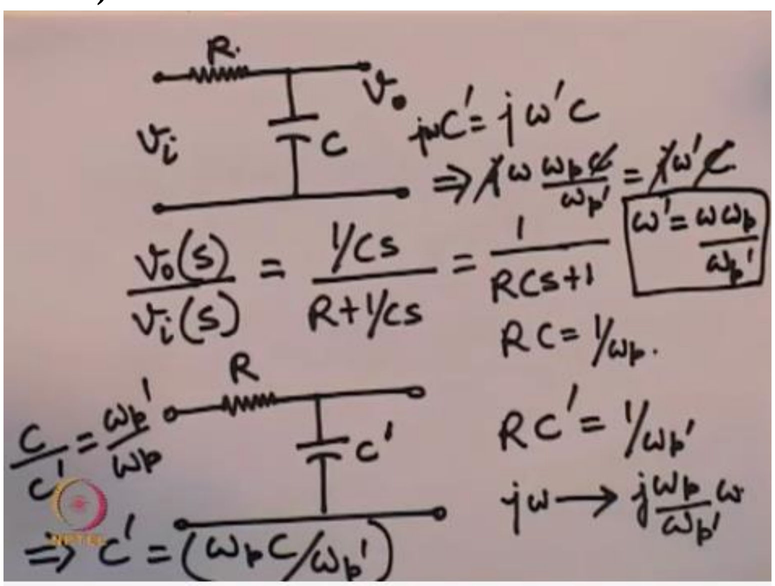

Suppose you have a circuit initially like this okay so here in this case Vo upon Vi in the Laplace domain will come out to and this in turn comes out to 1 upon so here this RC is the 3 db or cutoff frequency is RC is equal to 1 upon omega p where omega p is the 3 db or cutoff frequency.

Now suppose there is a alternate circuit like this where the cutoff frequency is 1 upon omega p dash then out C dash should be equal to 1 upon omega p dash isn’t it so this implies that this C upon C dash is equal to omega p dash upon omega p from which we get C dash is equal to omega p C upon omega p dash so this is my transformed capacitance which in turn translates to the transformation that we have done which is j omega is transformed to j omega p upon omega p dash omega.

So what you see that new capacitance is can be given in terms of the C dash can be given in terms of the old capacitance like this so suppose this new C dash is equal to j omega or say j omega C dash is equal to j omega C then this will imply j omega, omega p C upon omega p dash is equal to j omega dash C okay so if you cross out then this omega dash will be equal to omega, omega p upon omega p dash this is how this transformation is coming okay.

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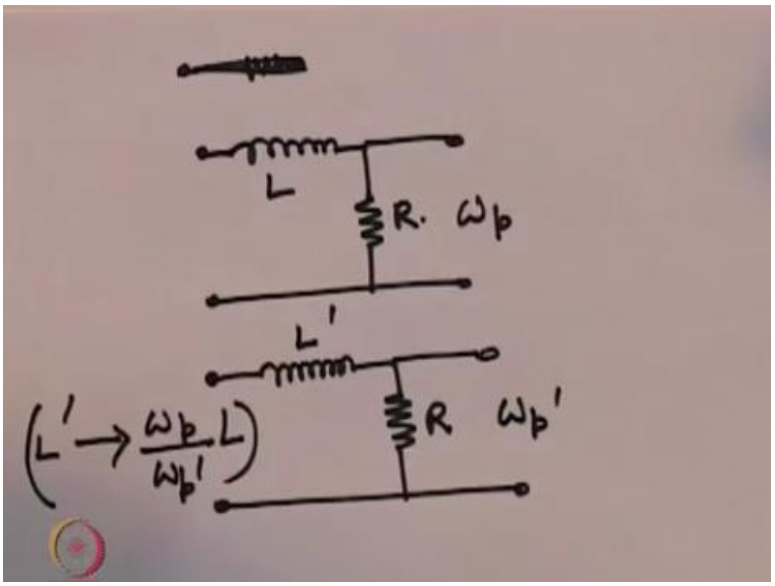

If you had in place of in place of this capacitor if you had an inductor say your original circuit was like this or let me say original circuit was like this okay and the modified circuit is like this, suppose this had a 3 db frequency of omega p and this has a 3 db frequency of omega P dash then this L dash will be given by omega p upon omega p dash L okay, so this is about low pass to low pass transformation in the next module we shall be covering the other transformations and also see some simple implementation of the filters, thank you.