

Analog Circuits
Prof. Jayanta Mukherjee
Department of Electrical Engineering
Indian Institute of Technology - Bombay

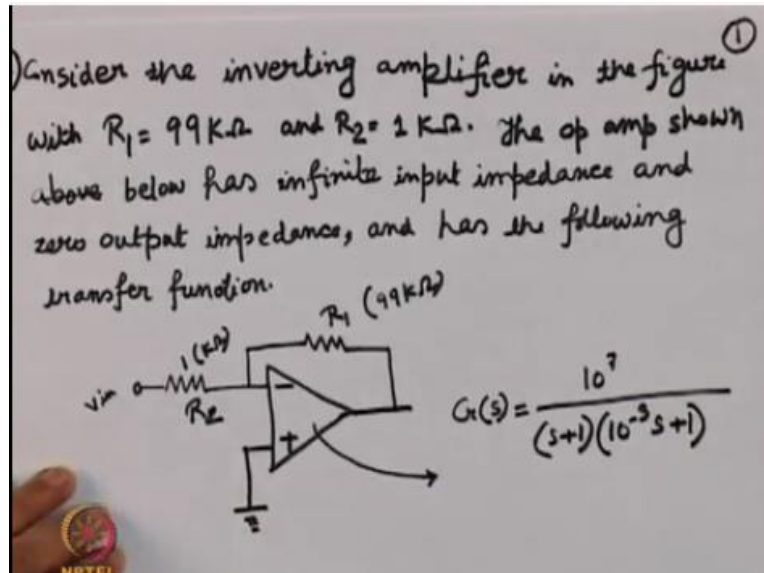
Week – 05
Module - 05
Tutorial No.4

Welcome everyone my name is Basudev Majumder, I am the TA of the course of analog circuit, last time we have seen that we have solved rather some problems which are related to the unity gain magnitude.

Where the gain of the any transfer function or the circuit given is becoming unity and also, we have seen that if any problem is given in op amp how to simplify that problems circuit and how ultimately, we can achieve the magnitude and from there how we are calculating the pole frequency is the 0 frequencies and also how we have drawn the bode plot from them and we have also learned that if any circuit given from there the concepts of drawing the bode plots or will remain same.

Just we have to find out the transfer function or the output Laplace theorem divided by the input Laplace theorem and from the that we have to find out the necessary information to draw the bode plots, today we will see in our second part some of the problems which are related to the gain margin and phase margin and also while discussing the gain margin and phase margin the concept of the unity gain frequency will frequently come that we have discussed in our previous tutorial let us start with the problem.

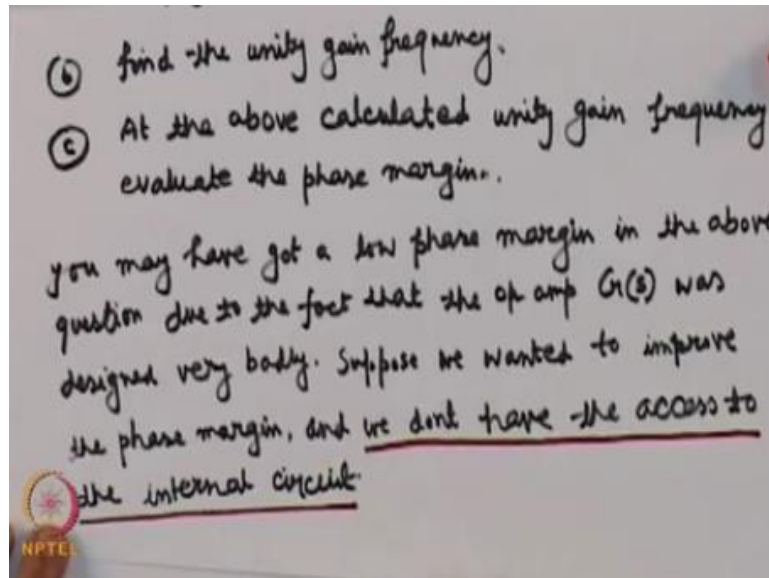
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Consider the inverting amplifier in the figure with $R_1 = 99$ kilo ohm and $R_2 = 1$ kilo ohm, the op amp shown below has infinite input impedance and 0 output impedance and has the following transfer function, so this is page number 1 and this is continuation of our previous tutorials say this is the problem number 4 and this is today's problem number 1 the circuit is - the inverting non-inverting terminal is grounded.

This is R_2 this R_1 the value given is 99 kilo ohm and this value is 1 kilo ohm and V_{in} is applied here and we have told that the G_S that is open loop gain of the op amp given is 10^7 divided by $S + 1$ into $10^{-3}S + 1$, now what are the things we have to find out?

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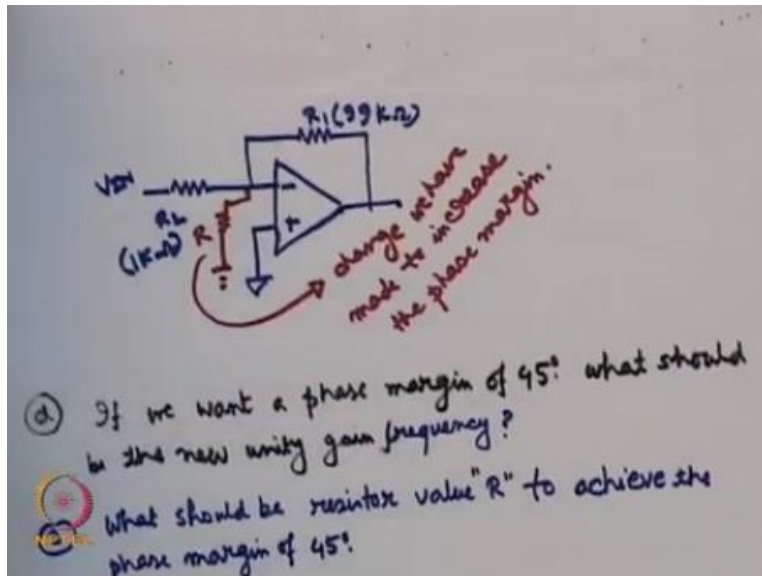


So this is the circuit and the basic problem description, now from here what we have to calculate let us see the first we have to calculate is derive an expression for the overall loop gain part b is the same way find the unity gain frequency actually this part we have covered in our previous tutorials but the while solving our next problems we require this part to be calculated that is why we again have inserted this one to calculate the unity gain frequency, now at the above calculated unit gain frequency evaluate the phase margin.

Now the problem has a little bit trickier extension so they are telling you may have got a low phase margin in the above question due to the face that the op amp GS was designed very badly we are assuming so we will see what the phase margin it is coming and we are expecting that the phase margin is what we are getting is not up to the mark or up to desirable so we want to improve the phase margin so that low phase margin problem has been improved by some techniques that we are explaining in this trickier extension.

Suppose we wanted to improve the phase margin and we do not have the access to the internal circuit, so this part is important because any op amp generally we are not allowed to change in our internal circuit what we have to do we have to adjust our the external register connections and from that we have to achieve what it has been asked in the problem.

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so to achieve that, so this was our earlier given 1 this was R_2 this was V_{in} this one is R_2 , so this one was 99 kilo ohms this one was 1 kilo ohm now what we have done this time to improve the phase margin the circuit has been little bit changed and from the inverting terminal actually have connected a resistor of value R that was missing earlier, so this is the change that we have made change we have made to increase the phase margin.

So, why the phase margin is increasing? Because by which is one of the way though it is not a desirable way to do but it is a very crude way of increasing the phase margin it attenuates the gain and so we are getting a good phase margin or by this so we can improve the phase margin so this is a problem actually so we are solving it so we are not going in detailed to design how a phase margin should be improved ideally that will not discuss.

We will just tell that if some problem comes and one of the way of improving the phase margin is just connecting the resistances in your inverting terminal like this practice whatever we are showing now, so how to solve that problem or what change it will come in our basic circuit if this type of problems are given that is our main aim to solve this one and what we have been asked from here.

So this is the suppose if we want a phase margin now for this circuit if you want a phase margin of 45 degree what should be the new unity gain frequency, so earlier we have calculated the unity gain frequency for the circuit when that resistance was missing.

Now we have added that resistance in view that we are trying to improve the phase margin and then what effect it will have on the phase margin then what will be its new unity gain frequency that we have to find out and the last part is the value of the new register what should be the register value R to achieve the phase margin of 45 degree, so these 5 problems we have to solve, so let us start with 1 by 1.

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(a) Overall loop gain.
 $L(s) = -A(s)\beta$
 $\beta = \frac{R_2}{R_1 + R_2}$
 $A(s) = G(s) = \frac{10^7}{(s+1)(10^{-3}s+1)}$
 $L(s) = -\frac{10^7}{(s+1)(10^{-3}s+1)} \cdot \frac{R_2}{R_1 + R_2}$
 $L(s) = -\frac{10^5}{(s+1)(10^{-3}s+1)}$

Circuit diagram: An op-amp with a feedback network. The feedback network consists of a resistor $R_2 = 1\text{ k}\Omega$ in series with a resistor $R_1 = 99\text{ k}\Omega$ connected to ground. The input is V_{in} . The transfer function is $G(s) = \frac{10^7}{(s+1)(10^{-3}s+1)}$.

So first what we have to find out, we have to find out the overall loop gain so the circuit is for the first part the circuit is this is 1 kilo ohm R_2 this is R_1 99 kilo ohm this is ground, this V_{in} so the overall loop gain determines we are determine it $LS = -AS$ in to beta, so if you see in this case in this circuit beta is R_2 by $R_1 + R_2$ and this AS actually is GS that is the open loop gain and the value of GS is 10 to the power 7 by $S + 1$ 10 to the power -3 $S + 1$ so $AS = G2 = 10$ to the power S by $S + 1$ into 10 to the power -3 $S + 1$.

So simply loop gain becomes 10 to the power 7 $S + 1$ 10 to the power -3 $S + 1$ this is GS - and beta and your beta is R_2 by $R_1 + R_2$ so if we simplify it R_2 value is your 1 this one is $99 + 1$ so 100 so it is coming 10 to the power 5 by $S + 1$ into 10 to the power -3 $S + 1$, so this is our loop gain we have to find out only the S into beta now find the unity gain frequency, now to calculate the unity gain frequency we will start with this relation.

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(b) Unity gain frequency is -

$$L(s) \Big|_{s=j\omega} = - \frac{10^5}{(1+j\omega)(1+10^{-6}j\omega)}$$
$$\left| L(s) \Big|_{s=j\omega} \right| = 1$$

$\omega = \omega_u$

$$\frac{10^5}{(1+\omega^2)(1+10^{-6}\omega^2)} = 1$$
$$(1+\omega^2)(1+10^{-6}\omega^2) = 10^5$$
$$\omega_u = 10^4 \text{ rad/sec}$$

So this is the loop gain so unity gain frequency is so this is the solution of part B unity gain frequency is, we will calculate $L(s) = -10^5 / (1+j\omega)(1+10^{-6}j\omega)$ we are putting, so $-10^5 / (1+j\omega)(1+10^{-6}j\omega)$ divided by 10^5 we have just replaced the s by $j\omega$ because it is a $(j\omega)$ (15:11) and for a single frequency ω we are not having any decaying amplitude in the input part now mod of this value so mod of $L(s)$ at $s = j\omega$ will be 1 at frequency that will be $\omega = \omega_u$ that is the unity gain frequency.

So what we need to find out here? So we have to take the magnitude of this portion, so it is coming 10^5 we are directly squaring it so $1 + \omega^2$ $1 + 10^{-6}\omega^2 = 10^5$ so just we have taken the mod of this one and we have squared it up and if you split this $1 + \omega^2$ $1 + 10^{-6}\omega^2$ into 10^5 so from here if we calculate that ω_u is going to come 10^4 rad per second, so this is our unity gain frequency.

So, what you have seen till now, that if the problem is given like that is any op amp problem it is inverting configuration or non-inverting configuration or any op amp circuit which is you know related with capacitance or inductors connected in its terminal and we have to just to find out the overall loop gain we have to find out the open loop gain which is $A(s)$ which is given and which is and we have to determine what is the beta or what is the feedback.

So, in our problem the beta we have calculated that this is $R2$ by $R1 + R2$ and we have directly from there we have put the value and we have calculated the overall loop gain LS this overall loop gain is very important in the feedback stability so once we have got the open loop overall loop gain we can find out that what will be the unity gain frequency from that, the C part what we have been asked to do at the above calculated unity gain frequency.

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the phase margin.

$$L(s) = - \frac{10^5}{(1+j\omega)(1+10^{-3}j\omega)}$$

$$\angle L(s) = \phi(j\omega) = -\tan^{-1}(\omega) - \tan^{-1}(10^{-3}\omega)$$

at unity gain frequency pt.

$$= -\tan^{-1}(\omega_u) - \tan^{-1}(10^{-3}\omega_u)$$

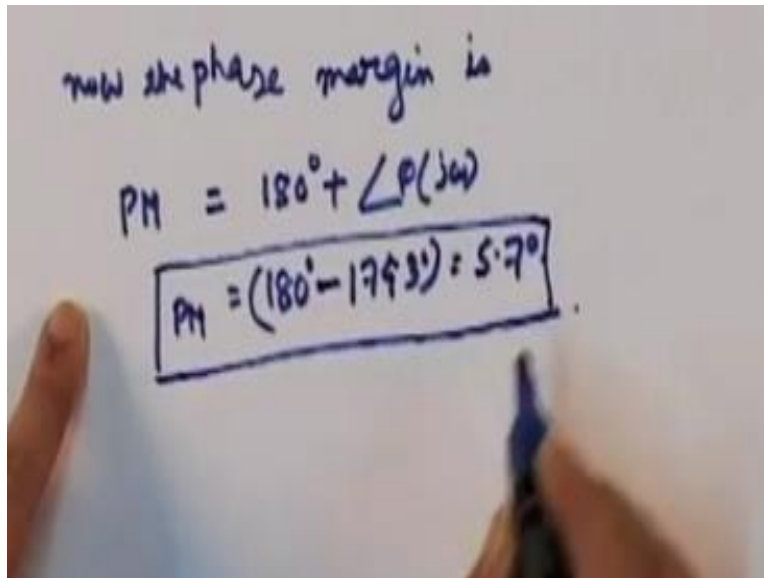
$$\phi(j\omega) = -\tan^{-1}(10^4) - \tan^{-1}(10^4)$$

$$\phi(j\omega) = -174.3^\circ \text{ (approximately)}$$

So this is our unity gain frequency at the above calculated unity gain frequency evaluate the phase margin so our LS was we will start from the LS 10 to the power 5 by $1 + j\omega$ into $1 + 10$ to the power $-3 j\omega$, now the phase margin how we will calculate at first we have to determine the ϕ of $j\omega$ so the angle of LS the angle of LS so it is coming -10 inverse of ω - ten inverse of 10 to the power -3ω at unity gain frequency point.

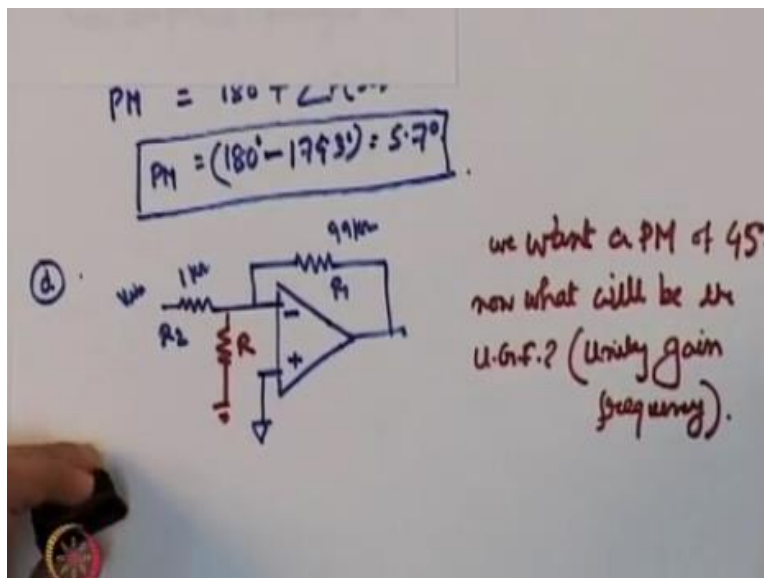
We will put ω_u - of 10 inverse 10 to the power $-3 \omega_u$ and putting this $\omega_u = 10$ to the power 4 - of 10 inverse this will come only 10 , 10 to the power -3 into 10 to the power 4 so it will come only 10 so if we calculate this ϕ of $j\omega$ it will give you -174.3 degree approximately it is in degree and our ω_u we have go that is in rad per second, so this conversion should be noticed while calculating the value of $\phi j\omega$.

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Now the phase margin is difference between the this to 180 degree + phi of j omega so it is coming - 174.3 degree so it is 5.6 this is our phase margin which have been asked in the part C so here comes the proper introduction of the problem part d so we have noticed that the phase margin in the above question is going to come low because the problem has been chosen like that, so because of that we have introduced one more register, so let us see the part D how to solve?

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So this is 99 kilo ohm R1 1 kilo ohm Vin and we have connected one more register which is R, part D what we have to find out we want a phase margin of 45 degree now what will be the unity

gain frequency note one thing that this unity gain frequency point only we will calculate our phase margin the phase margin will always been calculated where the unity gain frequency has been found out, so for this one to solve as we can see that now the because of the introduction of this new resistance the open loop gain or the closed loop gain will have an impact.

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Handwritten notes on a whiteboard:

$$L(s) = -G(s)B$$

$$\beta = \frac{R_2 \parallel R}{R_1 + R_2 \parallel R}$$

$$L(s) = \frac{10^7}{(s+1)(10^{-3}s+1)} \cdot \frac{R_2 \parallel R}{R_1 + R_2 \parallel R}$$

now we want the PM $\rightarrow 45^\circ$

$$180^\circ - \tan^{-1}(\omega) - \tan^{-1}(10^{-3}\omega) = 45^\circ$$

$$\tan^{-1}\omega + \tan^{-1}(10^{-3}\omega) = 135^\circ$$

$$\omega_u = 10^3 \text{ rad/sec}$$

So both will have an impact, so now the LS becomes for part D LS becomes GS by beta so ultimately what is the change the change will be in beta because earlier beta was R_2 by $R_1 + R_2$ now the beta is R_2 parallel R divided by $R +$ sorry R_2 parallel R divided by this $R_1 + R_2$ parallel R now the new LS becomes 10 to the power 7 that is our earlier GS what we have considered that is remained same $S + 1$ into 10 to the power -3 $S + 1$ into the new beta $R_1 + R_2$ parallel R .

Now we want the phase margin to be 45 degree so whatever knowledge we have of the definition of the phase margin that then 180 degree - this part this is our new LS considering the connected resistance which is R this is our new R , so it has no impact on that phase part so it will be - of tan inverse omega - of tan inverse so this is required to be 45 degree what have been asked in the problem.

So from here we can write tan inverse omega + tan inverse 10 to the power -3 omega this will be 135 degree and then from here unity gain frequency will come 10 cube rad per second, so

after connecting this resistance if we want 45 degree phase margin then the unity gain frequency at which this phase margin has been calculated that value is coming 10 cube rad per second.

Now let us have a look at the last part or the part E the question is what should be the register value R2 achieve the phase margin of 45 degree because we have seen that now the new LS is involved with some new resistance R, so what will be the value of R to achieve the phase margin of 45 degree so we are achieving a phase margin which is 45 degree we have found out the unity gain frequency, now they are told to find out what will be the value of R.

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To find out the value of R.

$$L(s) = -\frac{10^9}{\sqrt{1+s^2} \sqrt{1+10^{-6}s^2}}$$

$$|L(j\omega)| = \frac{\frac{RR_2}{(R+R_2)}}{R_1 + \frac{RR_2}{(R+R_2)}}$$

at $\omega = \omega_u = 10^3 \text{ rad/sec.}$

$$\frac{7071 \times RR_2}{RR_2 + RR_1 + R_1 R_2} = 1.$$

$R_1 = 99 \text{ k}\Omega$ $R_2 = 1 \text{ k}\Omega.$

$$R = 14.20 \Omega.$$

Now to find out the value of R, so again we will see the LS which is or rather we can directly take the mod L j omega, because we know this phase margin has been calculated in the unity gain frequency point so that 45 degree phase margin will correspond to unity gain frequency for that omega so 1 + 10 to the power - 6 omega square R into R2 R + R2 divided by so this is the numerator and this total portion is in denominator R1 + R2 divided by R + R2.

So at omega = omega u 10 cube rad per second this part if we calculate only this part this value is going to come after putting this 7071 so 7071 into this portion if you simplify it, it will come RR2 divided by RR2 + RR1 + R1 R2 this will be = your 1 now from there we know that we have been given for this problem R1 = 99 kilo and R2 = 1 kilo so R1 = 99 kilo R2 = 1 kilo, so if we

simplify this we will see that the value of R is going to come 14.20 ohm, so this is the value of R that we are interested in.

So this problem what we have seen in this circuit we are asked to find out what is the gain margin what will be its phase margin and how to calculate it from the unity gain frequency what is the trick to find out the phase margin and if some change comes in the resistances or if anything we employ from the outside the how the circuit will behave in terms of phase margin and gains margin and specially if we want to improve that gain margin and phase margin part where we have to connect the resistances and what impact it will have on its on its frequency response, that also we have seen.

So whatever problems or tutorials we have covered till now we have seen simple circuits related to op amps some MOSFET circuits how we have learned how to calculate the bode magnitude, we learn how to calculate or how to find out the pole frequency zero frequency and also how to calculate the unity gain frequency and we have to play something with our gain margin and phase margin then how to do that, also we have learned in the next session we will cover from the problems on some Butterworth and Chebyshev filter and some diode related problems, thank you.