Analog Circuits

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Week -01
Module -02
Poles and Zeros

Hello, welcome to another module of this course analog circuits. In the last module, we covered only the introduction where we introduced you to why analog circuits are important in this module we are actually going to start with the details of the course first topic in this course is about poles and zeros. So, what happens is that poles and zeros are related to the concept of transfer function of a system.

Now transfer function is a frequency domain technique for finding out the solutions to the or rather finding out what the output of a system will be to various inputs transfer function is only applicable for linear systems.

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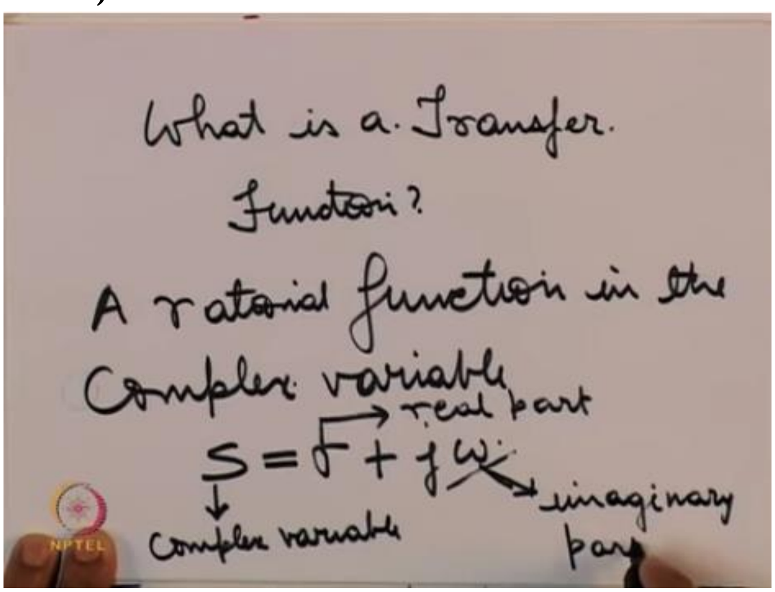

So, let us first let us start with the what is a transfer function so transfer function can be defined as a rational function okay in the complex variable S where S is given as Sigma plus J Omega so this is the complex variable S this is the real part this is this part is the imaginary part we know what is a transfer function but what is its advantage.

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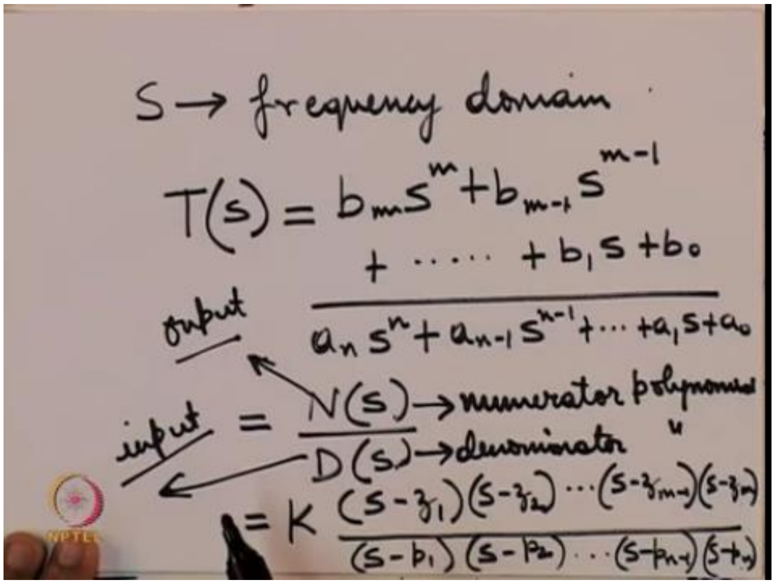

The first thing is this variable S this is a frequency domain variable okay so this entire transfer function suppose as I said if TS represents the transfer function of our linear system and I mentioned that TS transfer function is a rational function so I might be able to write it like this okay. So, a rational function is basically a function which involves a numerator and a denominator an alternative way of writing this transfer function could be so we have a numerator function and a denominator function and instead of writing it like a polynomial.

We can write it in a factorial form, so then we will have S- z1, S-z2, S-zm-1, S-zm upon S- P1, S-P2, S- and so on till S- Pn-1, S-Pn, so you see here we have written the same transfer function in a factorial form, so this S-z1, S-z2 and so on. These are the factors of the numerator and S-P1, S-P2, these are the factors of the denominator, now this Ns is the numerator polynomial, Ds is the denominator polynomial.

Now before going into the definition of what is the poles and zeros, I would like to point out something that we are operating in the frequency domain as I said this S is a frequency domain variable, usually this transfer function is found out you know we can this numerator represents the output and the denominator represents the input and the way to find out the input and this N(S) and D(S) is often by first finding out the Laplace transform of the output and then dividing that by the Laplace transform of the input okay.

So that is how it is done that is how we find out the transfer function of a linear system. So, before as I said before going on to the formal definition of the poles and zeros let us discuss the advantages of operating in the frequency domain.

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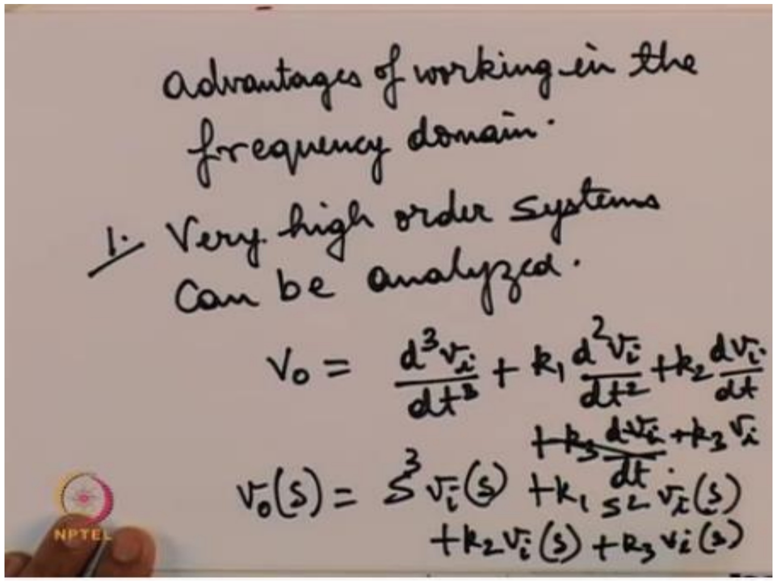

So first is, once you are in the frequency domain very high order systems can be analyzed easily. So now what do you mean by high order system? High order system basically the order of a system is that degree of differentiation, for example say if your V out is Vo represents the output of a system and say the input, the output is related to the input like this where Vi is the input.

So here in this case the order of the system will be 3 this is a linear system by the way and since the highest derivative order is 3 the order is also 3 so the order of a system is the highest order of the derivative that relates the output to the input so in the time domain as you can see if we had tried to analyze the system in the time domain we would have to find out the solution of this complex differential equation whereas in the frequency domain they this entire input and output relationship is reduced to a simple polynomial.

For example this becomes equal to this sorry here this will be k3Vi just note this change plus k3Vi of S, so you see this entire input and output relationship once we take the Laplace transform on the left hand side in the right hand side and we go to the S domain which is the frequency domain we see that this input and output relationships are now described by means of a polynomial.

So the analysis of a polynomial is much easier than the analysis of a derivative terms okay, differential equation gets transformed to a simple linear equation, so this is the one advantage of the of working in the S domain or the frequency domain which is that very high order systems can be simplified and analyzed easily, what is the other advantage? Now when we will see later on that when we try to analyze the stability of a system stability analysis.

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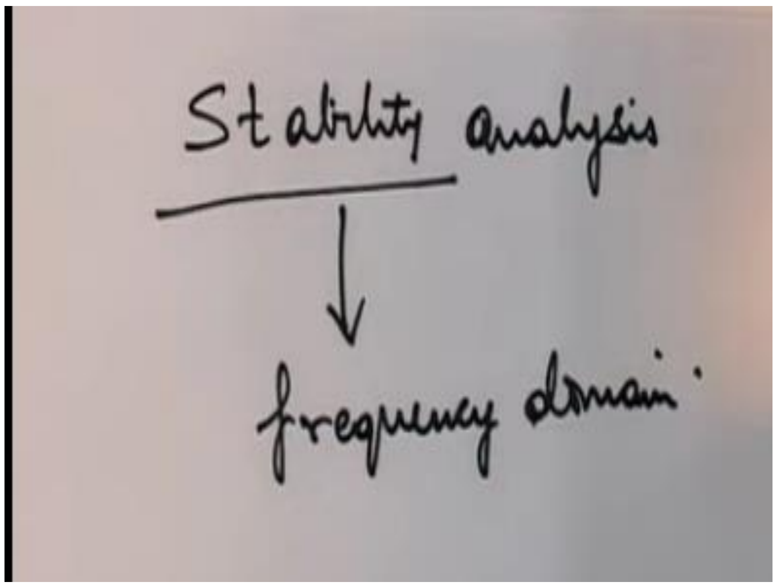

We will see is much easily done in frequency domain as compared to time domain one reason is because in the time domain to understand the stability of a system we have to let the system run for a long time, so if we you know a system whether a system is stable or not it will ultimately be reflected in the time, that is if the output collapses or becomes null or if the output becomes is too high then we say the system is unstable.

But then to actually see in time domain whether a system is stable or not we have to go on analyzing the system over very long time perhaps may be the system becomes unstable after a very long time, some systems maybe become unstable after a very short time whereas there are systems which may become unstable after a very long time and initially it may not be obvious that the system is actually unstable.

A frequency domain analysis on the other hand considers the overall response of the system in the frequency domain and therefore we do not have to wait or analyze the system over every time instance till it becomes unstable a frequency domain analysis is therefore much easily gives a more comprehensive view of the stability of a system and that is why working with transfer functions. We will see that it is much easier to find out the stability of a system.

Now coming back to the equation for the transfer function which I mentioned earlier, so we see that our numerator and denominator can be factorized the Ns and Ds, which represent the numerator and denominator polynomials they can be factored like this, NS is factored as S-z1, S-z2 and so on, DS is factored as S-P1, S-P2 to S-Pn-1 and so on.

Now if you see, if you consider the denominator polynomial which is now in a factorized form if S is equal to any one of these P1, P2, or Pn then we see that the output will blow up if we had an input and if we put S equal to, if you had an input to the system and if we make S equal to any one of these values the output will simply blow up because the transfer function itself becomes very high or infinite, for S equal to P1, P2 and so on so these points where S equal to P1, P2, P3 and so on till say Pn these points P1, P2, P3 are called the poles of the system.

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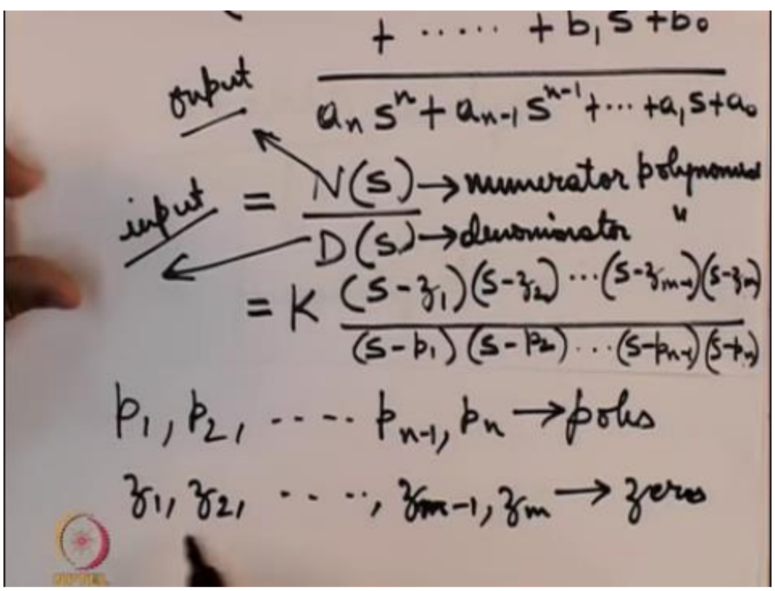

I can say that, let me write it down here clear P1, P2, till Pn-1, Pn poles. z1, z2, till zm-1, zm represent the zeros they are called zeros, because if we make S equal to any one of z1, z2 till zm, any one of them we will find that the output collapses or becomes 0 therefore they are called as zeros okay.

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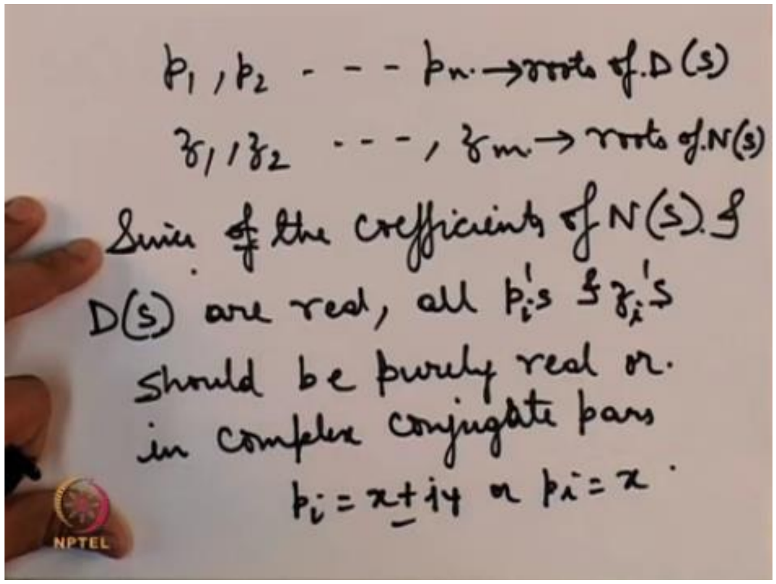

Now all these values this P1 and P2 till Pn or z1, z2 till zm, so these are the roots of NS and these are the roots of DS. Since the coefficients of since the coefficients of NS and DS are real all Pi's and zi's and zi's should be purely real or in complex conjugate pairs so we should have say Pi is equal to X+J+-J Y or Pi should be simply equal to X okay so either Pi's and zi's should be purely real or there should be in complex conjugate pairs.

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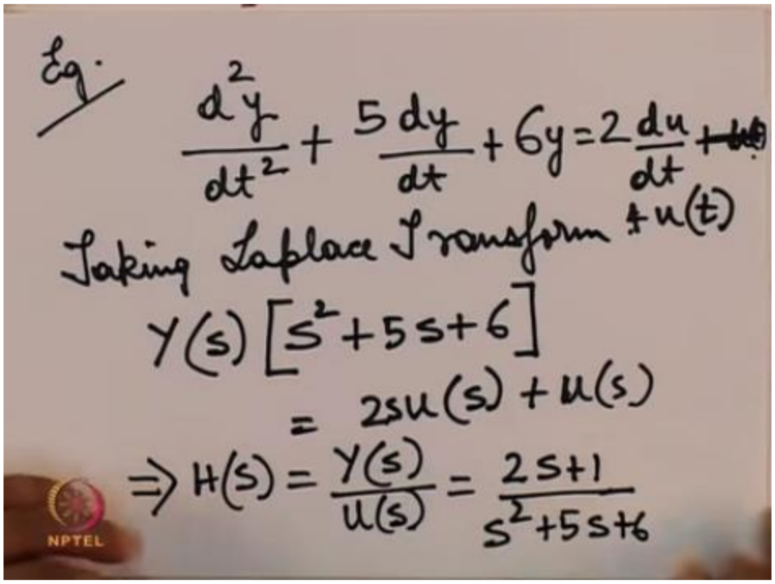

So let us consider an example, suppose we have a linear system whose input and output are described by this differential equation. Now taking Laplace transform on both sides we get Y of S multiplied by S square +5S+6 is equal to 2U of S+1 okay let me this a slight correction it is du+dt so then we you have u of s here just note this correction this is let me write it down at the bottom this is u of t.

So then after taking Laplace transform on both sides we have this equation so now the transfer function HS which is equal to Y of S upon u of s will be equal to 2S+plus 1 upon S square +5S+6, so what are the factors in the numerator? The factors are 2S+1 which is a single factor.

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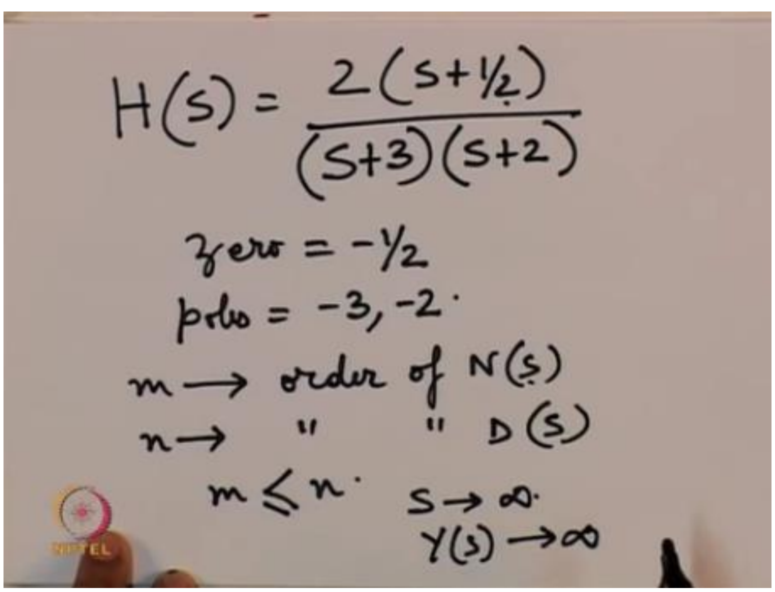

What are the factors in the denominator? So this transfer function I can rewrite it as, so the 0 as single 0 is at minus half and the poles are at -3, and -2, now one thing you must note that order suppose m is the order of NS that is the numerator polynomial and if N is the order of the denominator polynomial DS and for physical systems you always find that m is lesser than n and the reason it is so is because we see that had m been allowed to be higher of n.

Then what would happen is that when S tends to infinity that is when the frequency is very high the output say Y of S also tends to infinity which is not observed in physical systems in physical systems it is always observed that as the frequency goes on increasing the output of any system will tend to decline or remain the same it will not exceed the value and therefore for physical system that is why this limitation for this physical systems now any based on this transfer function we see that every linear system has a number of poles in it.

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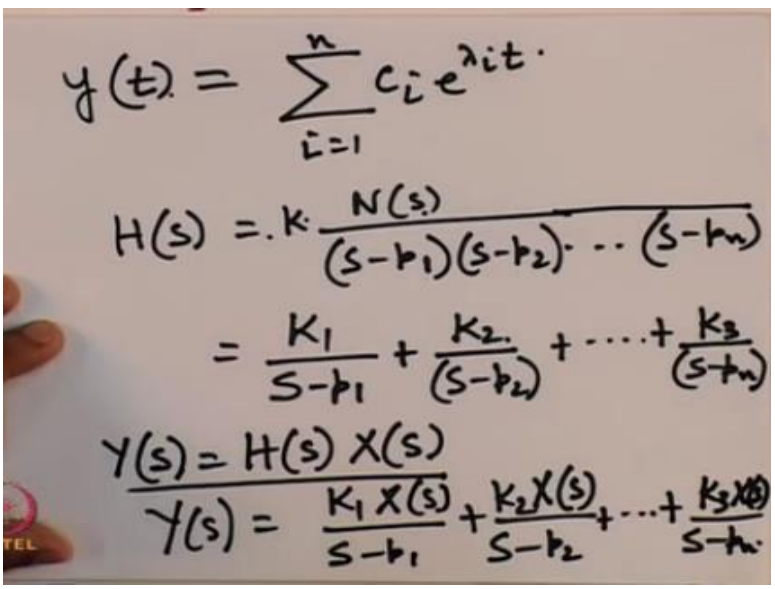

So if we can then write the output of any system in this form okay this follows because we know that H of S is given by NS upon the denominator polynomial which say the constant S-P1, S-P2 till S-Pn, now it is possible to factor the this H of S like this it is a possible to obtain the partial factors of this H of S in and write this whole transfer function in this form.

Now any output YS will be given by H of S times the input X of S, so then we see that after this equation we will have YS like this K1X of S upon S-p1+K2X of S upon S-p2 till K3X of S S-pn, okay so then you see that if we try to find out the time domain solution of this equation how will it be the time domain solution of this equation will indeed be is what I wrote it like this where lambda i represents the various poles of the system.

So here we are using the lambda i interchangeably with this P is okay so if we find out the time domain or the reverse Laplace inverse Laplace transform of this right hand side of the equation and it will look something like this, so in other words the poles of a system completely determine the response okay.

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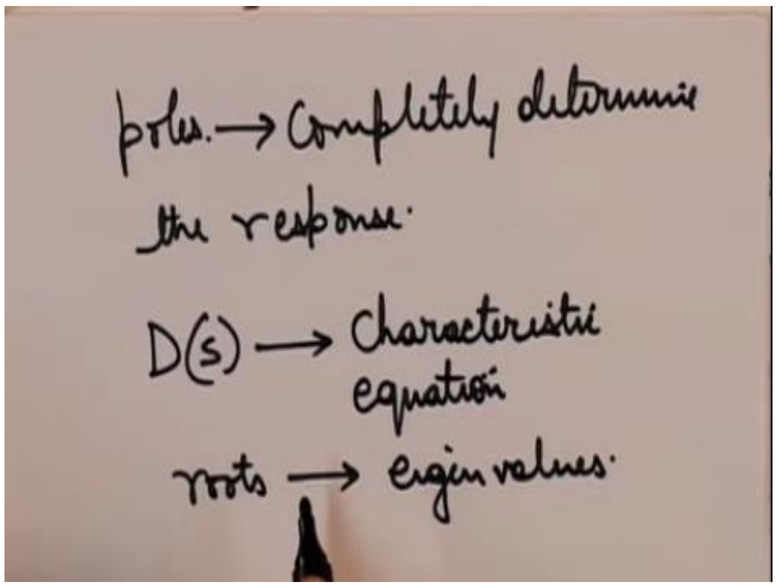

They are more important than the numerator as you can see because the output will be dependent on what the values of the poles are, now the denominator polynomial is the source of the poles right, so that is why and since the poles play an important role in determining the overall response of the system for this denominator polynomial is also sometimes know as the characteristic equation and the roots of the denominator polynomial which are also known as the poles are also know as Eigen values.

So Eigen values are the same as the poles of a system except that Eigen values are a term used for characteristic equation, whereas poles are a term used for the transfer function so depending on what the values of the poles are, you will get various kinds of responses.

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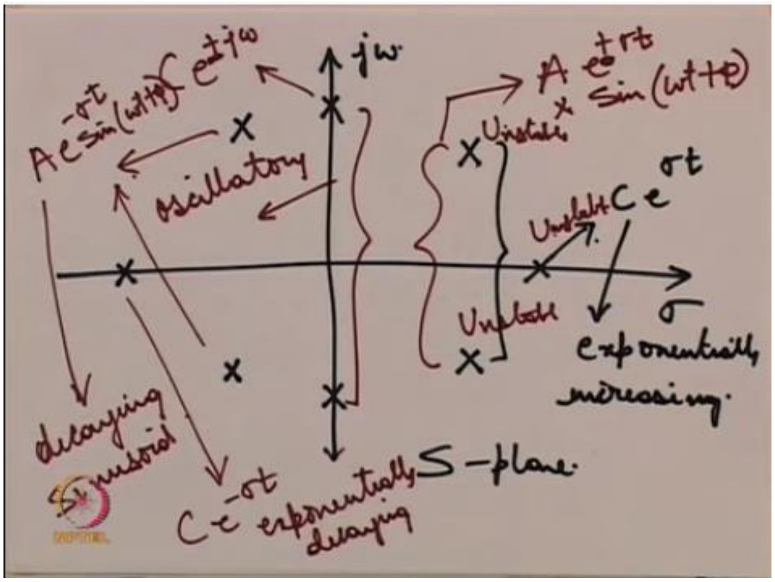

For example, if we plot on the complex S plane poles our complex S plane is like this ok the poles can be in either in one of these regions only and since we said that the poles will always be in conjugate pairs, so these two are conjugate pairs these two are conjugate pairs is suppose the poles are on the X axis that is on the sigma axis but on the right hand side of the S plane then the output will be something like this.

So this is a case of exponentially increasing and therefore does not represent the stable system, this is the case of an unstable system similarly here also these 2 if we see here the output response will be of this nature A+Sigma, T multiplied by sin of Omega T+Phi, this is also exponentially increasing except that in this case the exponentially increasing function is a sinusoid whereas here it is straightaway exponential in it was straight away exponentially increasing.

So here also since the output goes on expanding unchecked these 2 all these 3 represent unstable, okay so I will just write unstable this is unstable this is also unstable what about the case when the poles are on the Y axis, so when they are on the Y axis the output will be something like this, this + only not -, so they hear when the poles are on the Y axis the output will be C E raised to +-J Omega.

So here it is -+ and – both, so this represents what we called as oscillatory system what it means is that here the output neither increases not decreases but simply keeps oscillating, the amplitude remains the same but the amplitude does not decrease or increase with time for this particular case the amplitude goes on increasing with time what about these 2 cases now? In this 2 cases also we see that let us see what kind of suppose your poles are at these 2 positions then your output will be of this nature.

Now this represents a sinusoid which whose amplitude progressively goes on decreasing with time here the amplitude will go on decreasing with time, so this is what we call a decaying sinusoid and therefore represents a stable system this output for a bounded input, we will get an output which will gradually decrease with that this is what we want in this case when you have any input which may be bounded or unbounded if you have a unbounded then anywhere the system will be the output of the system will go on increasing, but even if we have a bounded input that is an input which after some time decreases or become 0.

Even then for this particular system we will have the output which goes on increasing bitten, but these 2 cases also for a bounded input that is an input which becomes 0 after some time the output will keep on oscillate and therefore is not a stable system and finally for this particular case we will get an output which is of this nature so this is exponentially decaying this also as we can see for a bounded input.

Since the output becomes 0 after some time therefore this particular system a pole placed at this region also represents a stable system, so with this you know we conclude our discussion on poles and zeros, we will take up this part of poles and zeros later on well while discussing the stability and frequency response of systems of filters especially.

But this is not the actual analog circuit part of the course this was just an introduction to a tool that we often use while discussing linear systems and this is a very important tool this is the frequency domain tool and as I said it is important because it gives an overall idea of the system without having to analyze the system for every time instance.

So it can as I mentioned earlier it can give the response of a, it can be used to analyze the response of a very high order system and also the stability which is an important aspect of any analog circuit. So in the next module, we will start with opamps which is the where we actually get to introduce some of the analog components that are frequently used, thank you.