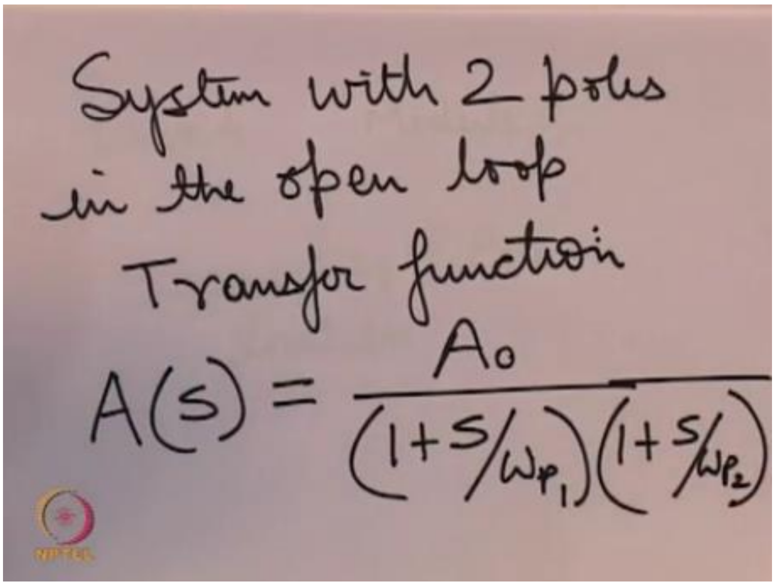
Analog Circuits
Prof. Jayanta Mukherjee
Department of Electrical Engineering
Indian Institute of Technology-Bombay

Week -04
Module- 04
Stability and pole location continuation

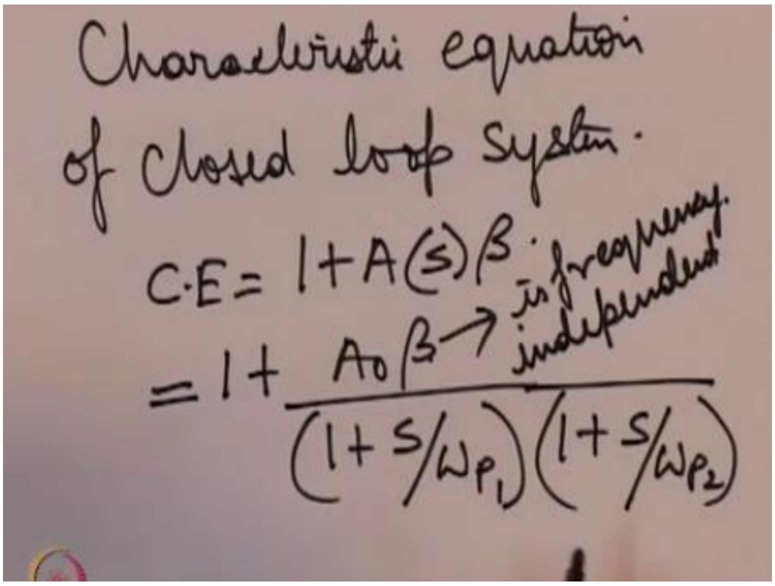
Hello, welcome to another module of this course analog circuits, so in the previous module we had see what is the effect on the stability for a system whose open loop transfer function has just a single pole and we had seen that the system will continue to be unconditionally stable that is the feedback does not affect its stability.

(Refer Slide Time: 00:53)



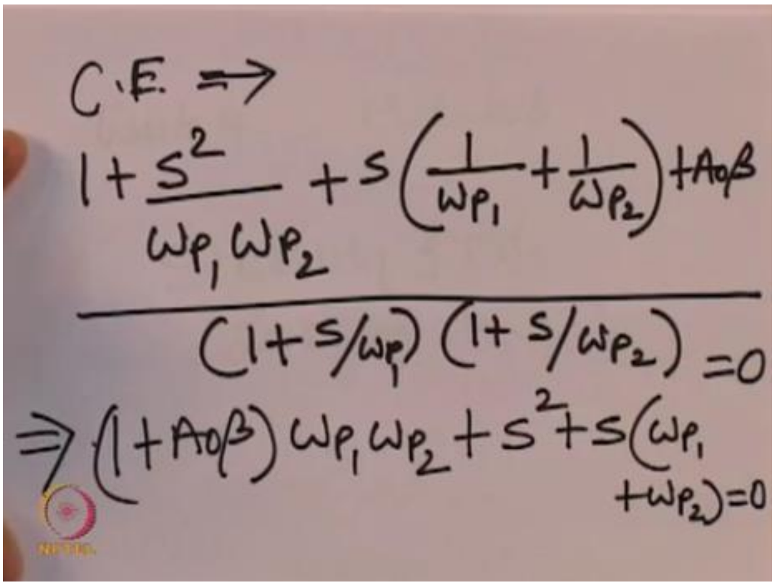
let us now do the same analysis for a system which has more than one poles let us first see a system that has 2 poles in the open loop transfer function, so such a system will have an open loop transfer function like this the characteristic equation will be given like this.

(Refer Slide Time: 01:48)



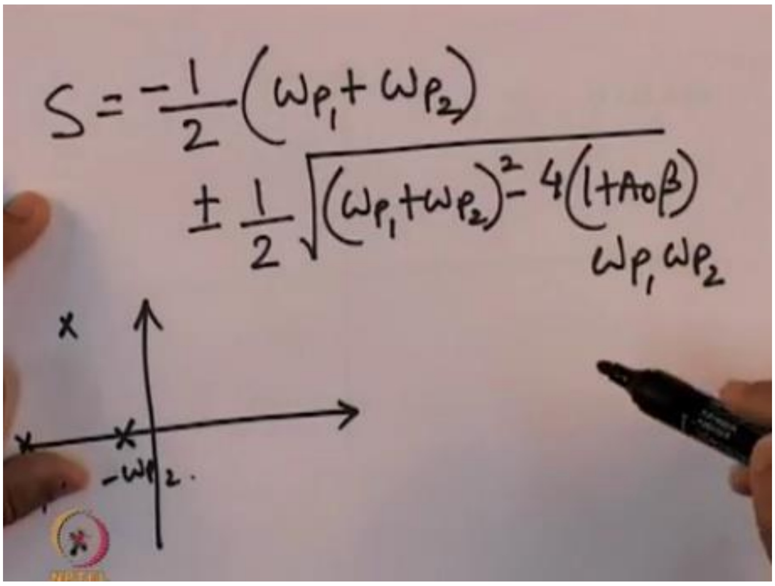
The characteristic equation of the closed loop system that is which is = 1+A0 beta upon 1+S upon omega p1 S upon here I should mention about this beta this beta is frequency independent which means that the beta is not a function of S it is remains the same for all frequencies now such types of feedback networks are all of course possible if for example if we have a purely resistive feedback network then beta indeed will be independent of frequency now simplifying this equation further.

(Refer Slide Time: 03:12)



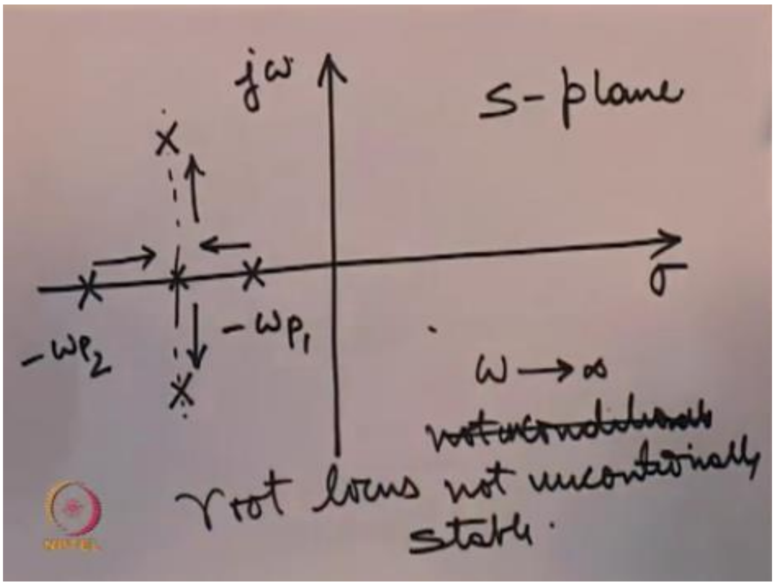
The CE becomes so the CE will then become by will be formed by equating this = 0, now from this we can simplify this as ok the roots of this equation so this is my C, now the roots of this equation comes out to.

(Refer Slide Time: 04:49)



So what do we see if we plot the roots of the characteristic equation they are somewhat like this, so these were my initial roots -omega p1 and -omega p2, let me let me use a fresh.

(Refer Slide Time: 05:57)



So this is my S plane my initial roots were at -omega p1 or the yeah and -omega p2, now from this equation we see the roots of the equation that we have just derived what do we observe, first thing we observe is that even though the coefficients of the characteristic equation where or the characteristic polynomial were all real after solving, we see that depending on the value of A0 beta the roots of the characteristic equation can be complex, of course there will be in complex conjugate quantities if they are complex then what does it mean.

So let us consider this equation once again a little closely (Refer Slide Time: 07:03) we see that as the feedback increases till to a certain point, this term within this square root bracket will continue to be greater than 0 till certain value of beta then when beta increases further this term within the square root sign or bracket will become negative and when this becomes negative S will become complex so S will continue to be real for certain values of beta then when beta increases further S will become complex.

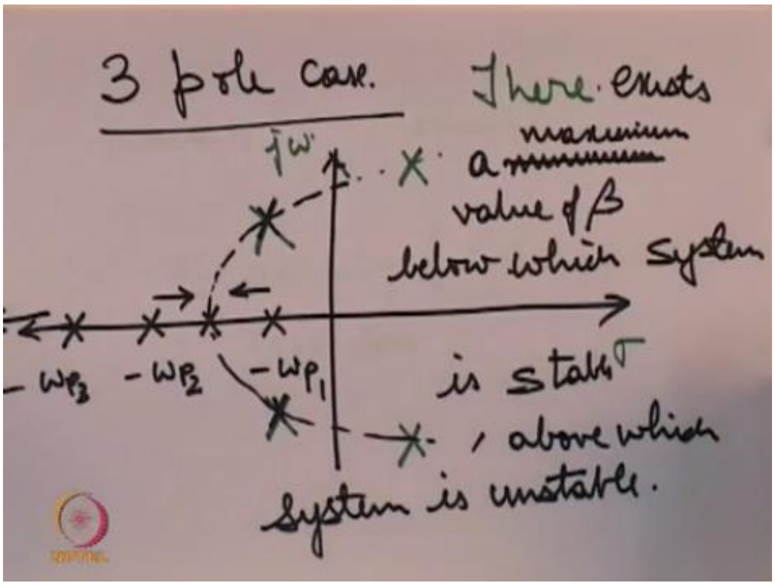
So if we plot the root locus or the locus that the roots of the characteristic equation of the closed loop system follows for increasing beta it will be something like this then so to some extent they will converge then when the term within the square root bracket becomes 0 from there onwards it will diverge so it will be something like this however note that even after feedback the roots continue to be in the left half of the S plane and even with increasing.

If the even if beta is increased further they will continue to follow this vertical line and so what does it mean that it is for a system whose open loop transfer function contains only 2 poles that system will continue to be stable except when omega is omega tends to infinity when omega tends to infinity the roots will converge with the imaginary J omega axis and thus the system will become oscillatory.

So it is not unconditionally stable first thing isn't it the roots do not enter the right half or I should say the imaginary axis until when omega tends to infinity so it is not unconditionally stable like the single pole case, however for most practical frequencies it does remain stable because for most practical frequencies the root locus will never cross the J omega axis neither will it merged with the J omega axis.

So this is a bit different from the single pole case where for the whereas for the single pole case even for when the frequency approached infinity the system was unconditionally stable for the dual pole case it is unconditionally stable for all practical frequencies except when frequency tends to infinity now what happens when we have more than 2 poles let us see the 3 pole case.

(Refer Slide Time: 10:46)



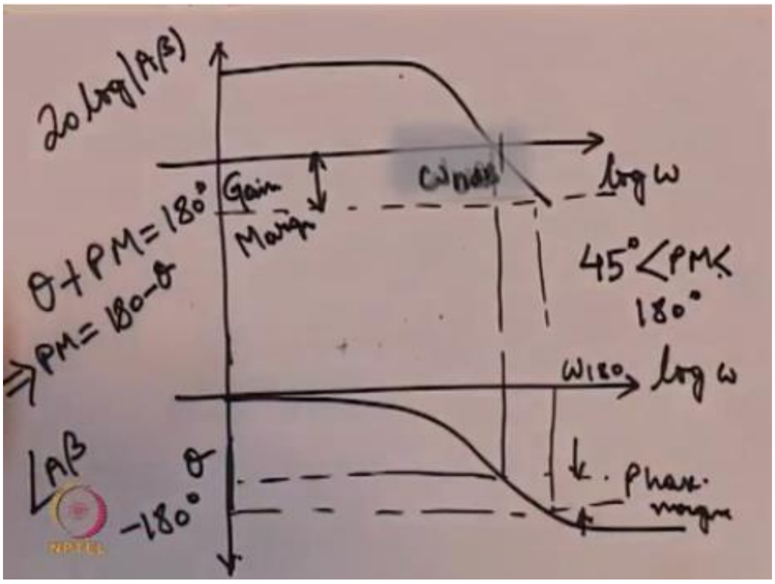
So for the 3 pole case also we can you know make a similar analysis of the of the root locus by solving first writing down the characteristic equation and then solving for the poles and then seeing how the position of the poles depends on the feedback factor beta, so for that a typical cases like this suppose initially your poles are like this I call this -omega p1 -omega p2 and -omega p3.

If suppose we keep on increasing the feedback factor then the root locus will be something like this way again just like the previous case they start converging this one, however will continue to move in the left half for the left so it does not pose any problem as far as stability is concerned, however now with after converging the 2 poles after converging they follow a locus which is somewhat like this so for certain value of beta the pole positions might be like this okay.

But then so these are the new pole position values, so let me it represent the new pole with this green signs okay and then for even larger values of beta the pole positions might be like this till the time they are the poles are in the left half of the S plane the closed loop system will continue to be stable once they cross the J omega line and move on to the right half of the S plane the system will become unstable.

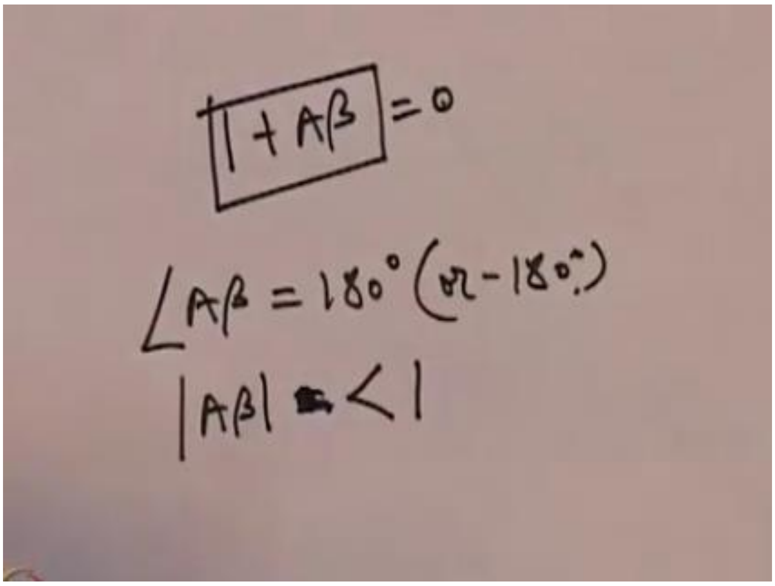
So in other words there let me use the black marker exists a minimum value or rather I should say maximum value of beta below which system is stable above which system is unstable, 2 terms which frequently crop up while discussing the stability of a system is what are known as gain margin and phase margin so let us see what is gain margin and what is phase margin.

(Refer Slide Time: 14:06)



Now while discussing the feedback of a system, recall that we saw that the new denominator I mentioned that the stability of the system is dependent on the characteristic equation 1+A beta = 0.

(Refer Slide Time: 15:10)



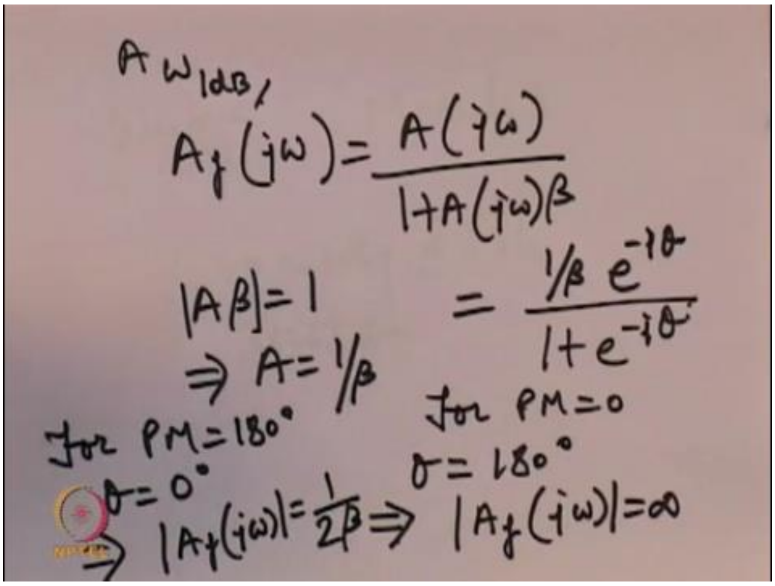
So there are 2 critical features you know, one is that when this angle A beta becomes = 180 degree or -180 degree then we have a potentially unstable case the other issue is that not only should it be 180 degree but the magnitude of A beta should also be less than 1 isn't it only when the magnitude of A beta is less than 1 even if the phase of A beta is = 180 degree the system will continue to be stable on the other hand.

When A beta magnitude becomes more than 1 at phase = 180 degree we have the system becoming unstable hence these 2 are the markers or the kind of danger points that is when A beta crosses the 1 magnitude value then we have a potentially unstable case or when A beta phase is = 180 degree we have a potentially unstable case.

Now here, in these plots if suppose for some value of omega the magnitude of A B becomes = 1 or on a db scale that is 20 log AB becomes = 0 and I label that frequency as omega 0 db similarly say for some frequency omega 180 the phase of A beta becomes = -180 degree or +180 degree both are the same then the gain margin refers to the margin available between or the amount of gain that is available between this omega 0 db frequency and omega 180 degree.

So this is my gain margin and phase margin represents the total margin available bit phase or total amount of phase that is present between this omega 180 and omega 0 db so this is my phase margin so if suppose at a particular at omega 1db my phase is theta then I can write theta +phase margin is = 180 degree in other words phase margin is = 180 -theta now phase margin for good design should be between 45 degree and 180 degree.

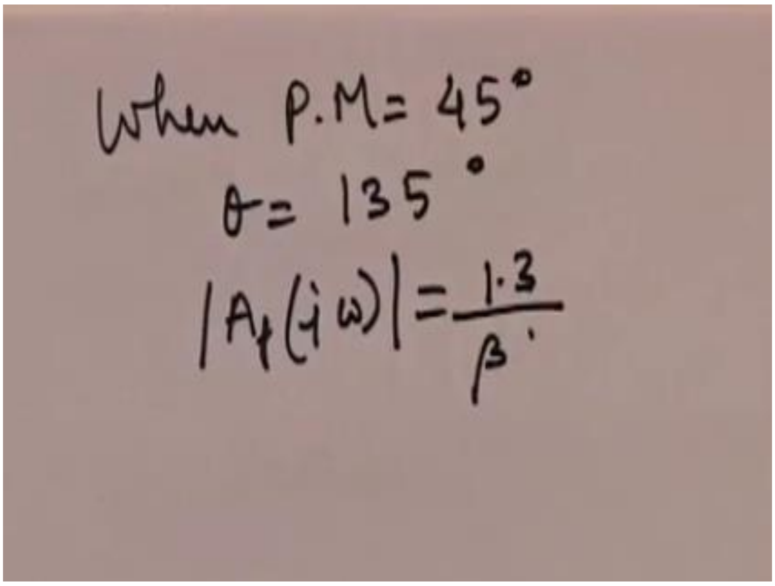
(Refer Slide Time: 18:50)



Let us use a sample calculation and see what should be the limit on my magnitude so at omega 1 db ok we have Af of j omega is = A of j omega upon 1+A of j omega beta now A of j omega is A beta = 1 at omega 1 db this implies A is simply = 1 upon beta therefore this equation becomes 1 upon beta e raise to -j theta okay where theta is the value of the phase at omega actually I should have written this way okay.

So where theta is the phase of A beta at omega 1 db upon 1+e raise to -J theta right now for phase margin = 0 theta should be = 180 degree this implies what? so if theta is = 180 degree and what does that mean it means that see when theta = 180 degree my denominator vanishes so this implies that Af of j omega will be = infinity which is a clearly an unstable system for phase margin = 180 degree theta is = 0 now these 2 we are deriving from the formula that we just derived.

(Refer Slide Time: 21:13)



So in that case when theta is = 0 you are closed loop gain magnitude will be = 1 upon 2 beta which is clearly a stable system and when your phase margin is = 45 degree theta is = 135 degree and A of f j omega is = 1.3 over beta which is again a stable system.

So in this module we covered the response or the root locus of a system and its potential stability when the open loop transfer function of the system has 2 3 or more poles we discussed for 2 and 3 poles and we saw that when the number of poles of the open loop system is to it is stable for all practical frequencies except when the frequency approaches infinity for 3 poles for the open loop transfer function of a system having 3 poles.

We saw that the system will be stable for certain values of beta, but then it when beta crosses a certain threshold value it will the poles will move on to the right half of the S plane and thus closed loop system will become unstable, now even though I did not mention this but for higher number of poles like 4, 5, 6 we observe similar patterns that is for a certain values of beta the system will be stable when the beta value crosses a threshold the system will become unstable.

And finally we also studied about what is the gain margin and phase margin and we saw that for good designs the phase margin should be between 45 degree to 180 degree and be sure 2 examples where when the phase margin is 0 the system becomes unstable when the phase margin is 180 degree the system is highly stable. In the next module we will be covering the topics of frequency compensation, thank you.