Analog Circuits

Prof. Jayanta Mukherjee
Department of Electrical Engineering
Indian Institute of Technology-Bombay

Week -04
Module -03
Stability and pole location

Hello, welcome to another module of this course analog circuits, so in the past module and the last module we had covered a Nyquist plot and how to determine stability of a system from the Nyquist plot, but then the Nyquist plot is a frequency domain technique for determining whether the system is stable or not, what I mean is that only when you know how the open loop transfer function behaves for various frequencies you can understand or determine whether the system is stable or not from the Nyquist plot.

Now there is another method for determining whether the system is stable or not and that is from the roots of the characteristic equation, so in this module first we will cover what kind of response we can expect depending on the various root positions depending on the various positions that the roots of the characteristic equation can take. So, recall what we mean by characteristic equation we had covered a few modulus back in fact enough in the first module actually when we were discussing the transfer function.

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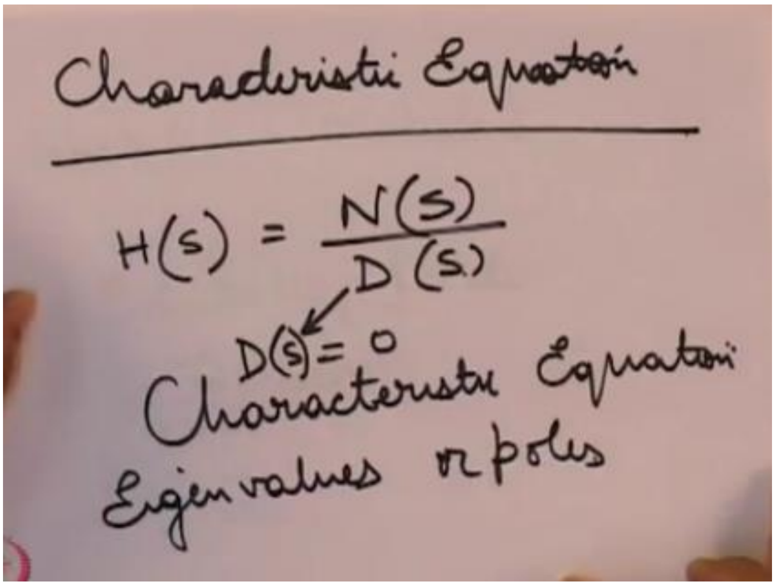

So characteristic equation recall, that if we have a transfer function of this form a numerator polynomial and a denominator polynomial then this denominator polynomial is known as the characteristic equal, so Ds =0 if I said then this is known as the characteristic equation and the roots of this these characteristic equation are also know as Eigen values or poles in general the root of characteristic equation.

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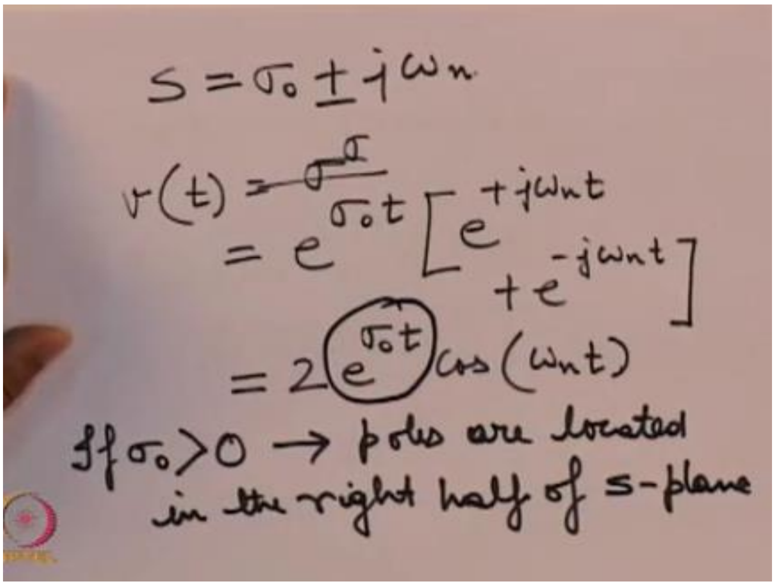

The roots of the characteristic equation will have the form like this, the roots are the poles can be written like this that is if the coefficients of the characteristic polynomial is real then the all the roots will be either real or if they are complex then they will be in conjugate pairs, suppose the root is of this form then the output that we can expect will be sorry will be of this form and this comes out 2.

Now if sigma 0 is positive that is poles are located in the right half of S plane then what happens then from what we had discussed earlier that if the poles are on the right half of this plane then what we can see from this equation is that if sigma 0 is positive then as time increases this value increases exponentially and therefore the output will keep on increasing exponentially hence that is a unstable, so it will be you know as we had discussed earlier it will be something like this.

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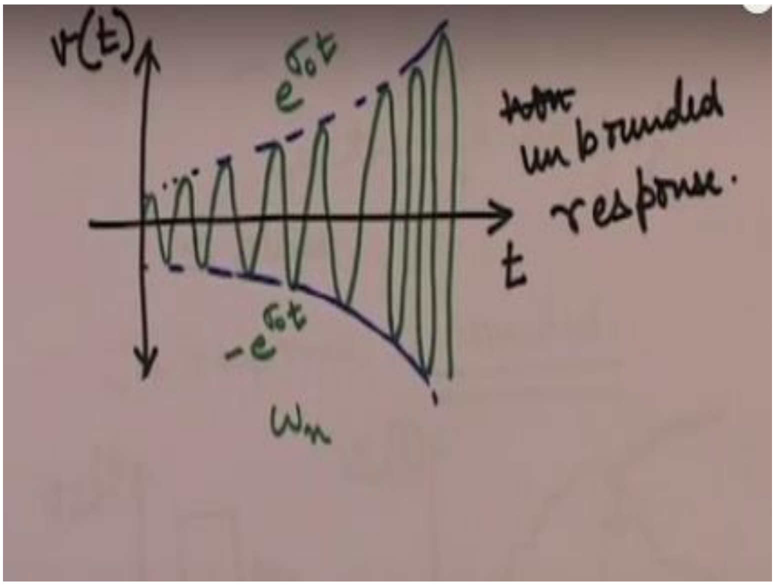

So there will be oscillation, this is the envelope of the output and output will be something like this the frequency will remain same actually here from it appears as if the frequency is decreasing as the time probably know the frequency remains the same which is omega N but this envelope as you can see the amplitude of oscillation usually goes on increasing so this will be your this envelope upper envelope will follow this line and the lower envelope will follow this line.

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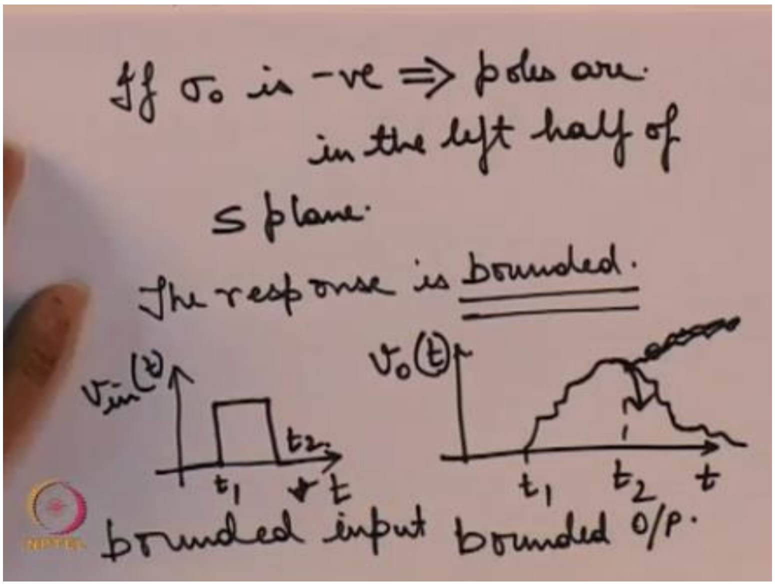

If on the other hand sigma 0 is negative then what happens, this implies that poles are in the left half of S plane and then from the equation that we derived earlier this equation if sigma 0 is negative then you can see that as time increases this response will decay and finally for a very large time interval the output will be negligible not 0 but with negligible so what does that mean actually what it means is that the response is bounded.

Now what does this word bounded mean? Bounded mean means you know suppose you suppose this is your input, so it is t this is say Vin t you introduce an input like this see this input is bounded why because it does not continue say this is t1 and this is t2 but it exists only between t1 and t2 after t2 it no longer exists so for a stable system what we need is that the output okay which will start somewhere after t1.

So this is I say V0T the output voltage of some system so this is the input voltage is the output voltage it starts somewhere but it should not go on expanding like this it should fall so the expected response should be like this need not be at t2 that is becomes 0 suddenly it may not follow the input closely but it should fall after some time.

So this should not happen this part should not be like this it should go up and go down, now this kind of system is called bounded input bounded output system and this is one of the characteristics of a stable system that if such a system is present then we say the system is stable why because the output does not exist indefinitely after the input has been cut off

In this case when sigma 0 was positive what did we see our output is going on increasing not only is it not decreasing it is actually going on increasing hence this is unbounded response now when your sigma 0 is negative on the other hand what do we see the response drawing on the same axis let me use a different sheet of paper when sigma 0 less than 0 response will be like this and this is following this curve ok.

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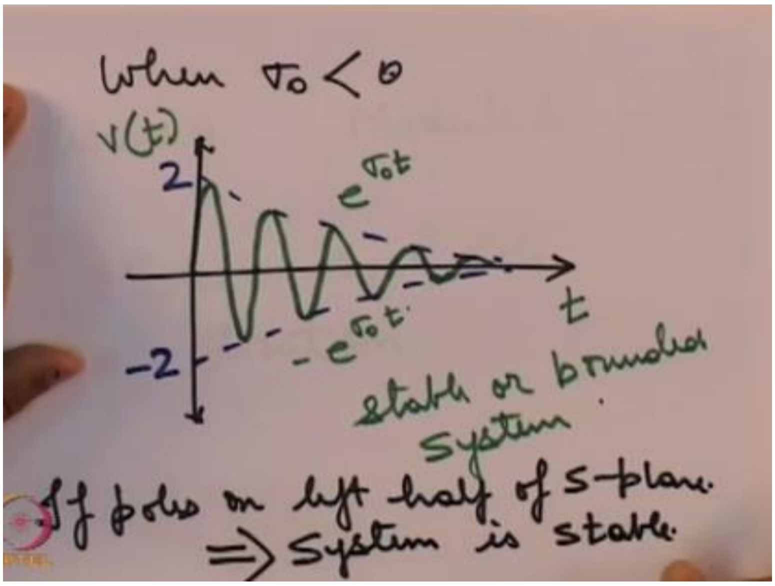

So what we observe here is that as time increases the output will decay or go down and for some very large time interval the output will be negligible so this is the characteristic of a stable or bounded system so what do we conclude from here then that if poles on left half of S plane implies system is stable.

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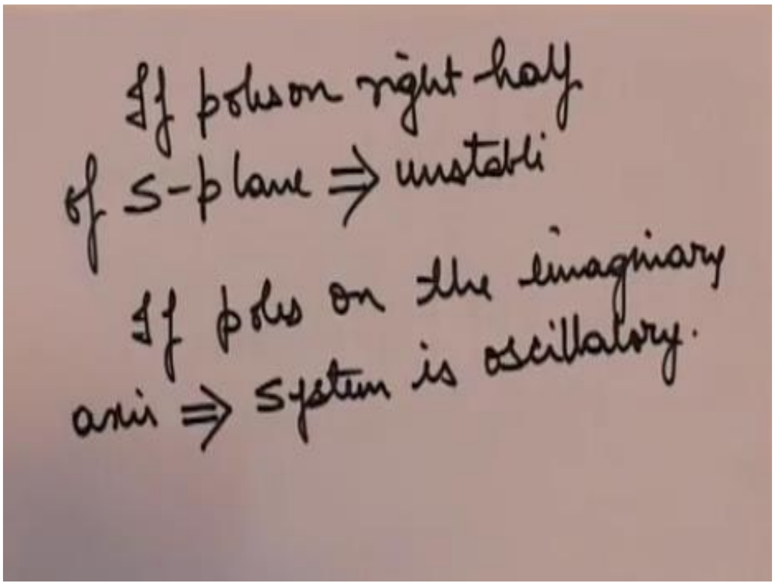

On the other hand, if poles on right half of S plane implies unstable and if system and the third option is if poles on the imaginary axis implies system is oscillatory okay so what do we mean by oscillatory so we saw what happens when sigma 0 is positive and when sigma 0 is negative.

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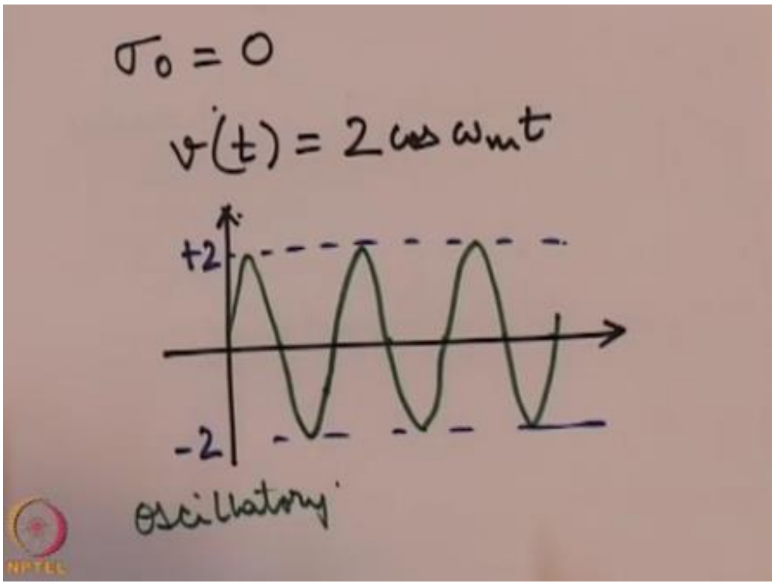

Now if suppose, your sigma 0 is =0 if that is 0 then our Vt will simply be = 2 cos of omega mt, so the output response will be like this then again this type of system is called oscillatory, now on the one other issue that frequently comes up is how should the system behave depending on the number of poles that is have that it has I mean in the feedback system so depending so we saw how the closed loop system behaves when we have the open loop transfer function using the Nyquist plot in the frequency domain.

But can we make such a similar analysis depending on how many poles the open loop transfer function has, so while discussing the frequency domain response we saw that if the number of circulations of the point -1 0 on the Nyquist plot was non 0 then the system is unstable let us see whether we can make a similar analysis based on the number of poles that the open loop transfer function has so suppose your open loop transfer function.

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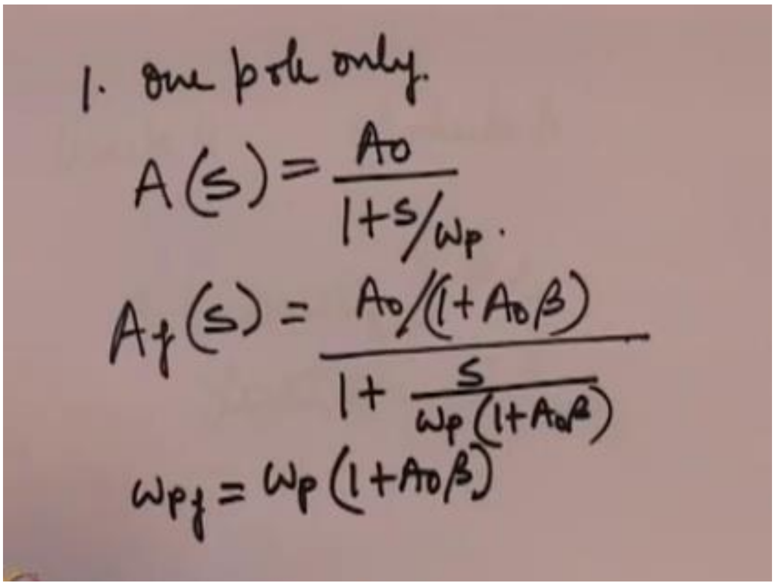

So if your system has only one pole then I can write my open loop transfer function like this and this is of course of the type of a low pass filter with a single upper 3 db frequency now the feedback transfer function will be given by ok so then first thing we note that the bandwidth with the of the feedback system is given by this and this we had seen while studying the effects of feedback that bandwidth enhancement take place and the poles of this system will be where.

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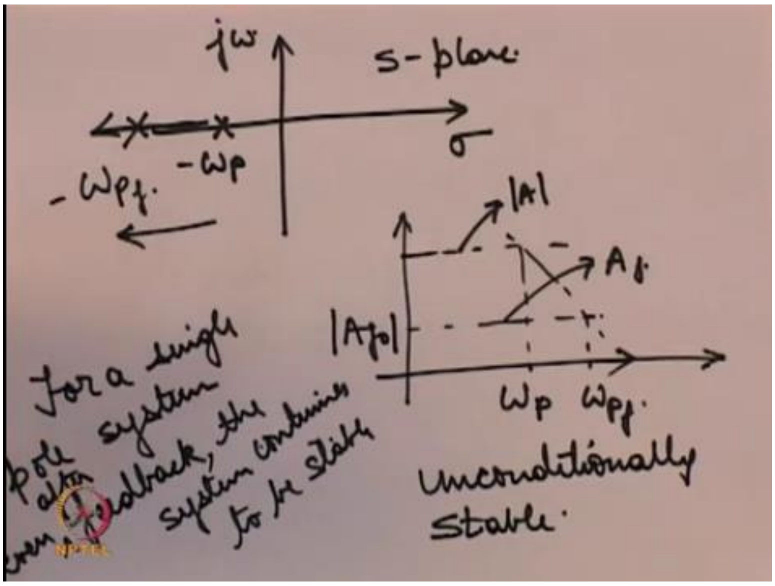

So omega p was the pole of the open loop system so this is our S plane this is Sigma this is j omega initially for the open loop system the single pole of the open loop transfer function lied at -omega p okay and so this is -omega p of course the transfer function that we have chosen here in this transfer function the pole is on the left half of the S plane for stability so that is why we have this + in the denominator and not a -.

So the pole therefore lies at -omega p and after applying the feedback what we see is the pole shifts left wards isn't it this is -omega pf I beg your pardon so the new pole of the closed loop transfer function is that -omega pf and this is this has a lower value or it shifts much farther away from the imaginary axis j omega which basically means that the stability of the system actually improves.

Because now this pole of the closed loop system is much farther away from the imaginary axis as compared to the pole of the open loop system and we all know on a frequency graph how this looks, so here these are of course the asymptotic plots this is my Af0 or DC gain with feedback this is A of f this is ADC gain so my initial pole was at omega P the new pole with feedback is that omega pf.

So what do we observe here that on application of feedback the whole can shift only in the negative sigma direction so what it means is that feedback after applying feedback for a system with a single pole under no circumstances can the system become unstable because even after feedback the pole is still lying on the left half of the S plane not only is it lying on the left half of the S plane this further away from the j omega axis.

Hence for a single pole system feedback even after feedback of course here by feedback we mean negative feedback the system continues to be stable hence for a system whose open loop transfer function has a single pole is said to be unconditionally stable which means that whatever you do the stability of the system cannot be altered it will continue to remain stable even after feedback.

So in this module we covered the various types of response that we can expect depending on the location of the poles that is in the left half of the S plane the right half of their S plane or on the imaginary axis.

We also saw what the response will be for a single pole and what we conclude is that for a system whose open loop transfer function contains just a single pole such a system even after applying feedback will continue to remain stable and any form of negative field whatever form of negative feedback we apply for any frequency it will retain its stability hence as such a system is said to be unconditionally stable, in the next module we shall see what happens when we have more than one poles, thank you.