Analog Circuits
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Week -04
Module- 02
Stability

Hello, welcome to another module of this course analog circuits, in this module we are going to cover another important aspect about analog circuit design which is stability, now stability is important because the output of any analog circuit should be stable that is the output should neither decrease with time not should it go on increasing with time it should remain at the value that we predicted or that we designed it for.

So now, it often happens that the circuit itself can undergo certain changes when the circuit is in operation so the circuit should be resistant enough to so that even if something changes in the circuit still the output is controlled that is the output does not go out of bounds neither does it decrease too much neither does it increases too much.

Now under this circumstances, of course depending for each circuit analog circuit the kind of techniques that we employ to make it stable will be different but still there are some general mathematical principles which can be formulated to give an idea first of all to see whether the system is stable or not and secondly if the system is unstable then can we do some changes in the circuit so as to make it stable.

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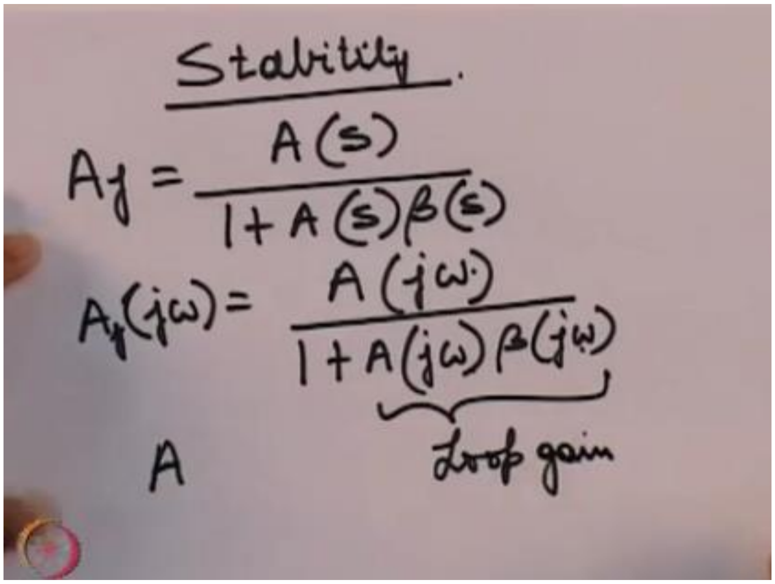
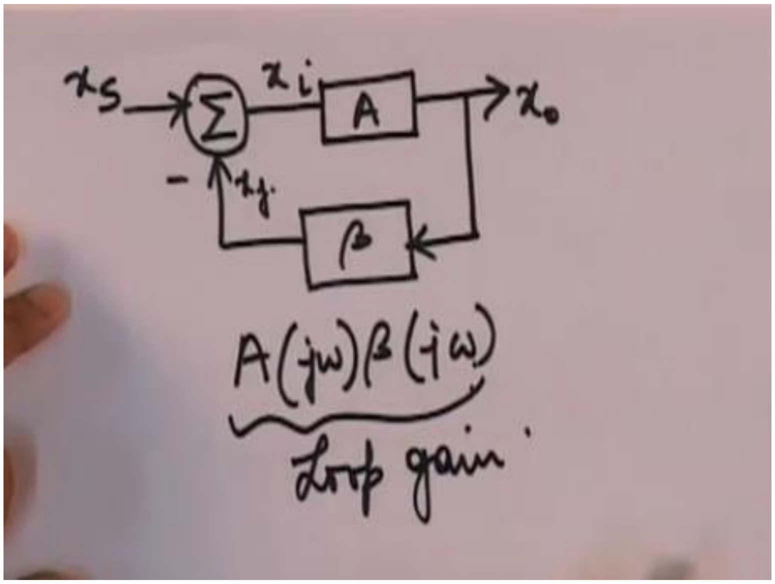


So, let us see how we can formulate the criterion, so we can go back months again to our closed loop system gain in frequency domain instead of S domain, if we change it to j omega then A of S A of F j omega will be given by A of j omega upon 1 + A of j omega beta j of omega now this terms A of j omega B of j omega is often called the loop gain, why is it called the loop gain? **(Refer Slide Time: 03:06)**



Because, if we go back to the closed loop system that we were describing in this system this A of this product A beta represents the total gain that a signal undergoes when starting from this point and coming back to this point therefore the total gain that the signal undergoes while traversing the loop is A beta or Aj of Omega Beta J of omega so that is why this term is called the loop gain.

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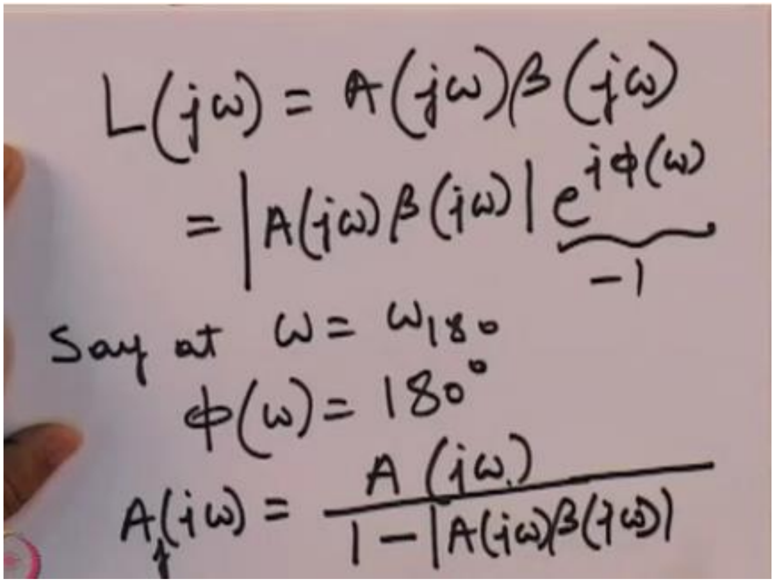

Now so this loop gain I am I can also write it in this form suppose I used L of j omega to represent the loop gain, where Phi of omega is the argument of this Aj of omega B beta j of Omega now say at a particular frequency say at when the frequency is omega 180 this Phi of omega is equal to 180 degrees so then the feedback so 180 degree means this term becomes equal to - 1.

So, in other words at this frequency A of j omega is equal to Af of j omega is equal to A of j omega upon 1 - like this so in other words the feedback has become positive now so we started with a negative feedback system but at this particular frequency omega equal to 180 omega 180 the feedback is positive now depending on what the magnitude of this loop gain that is this magnitude of A beta is we can obtain three distinct cases.

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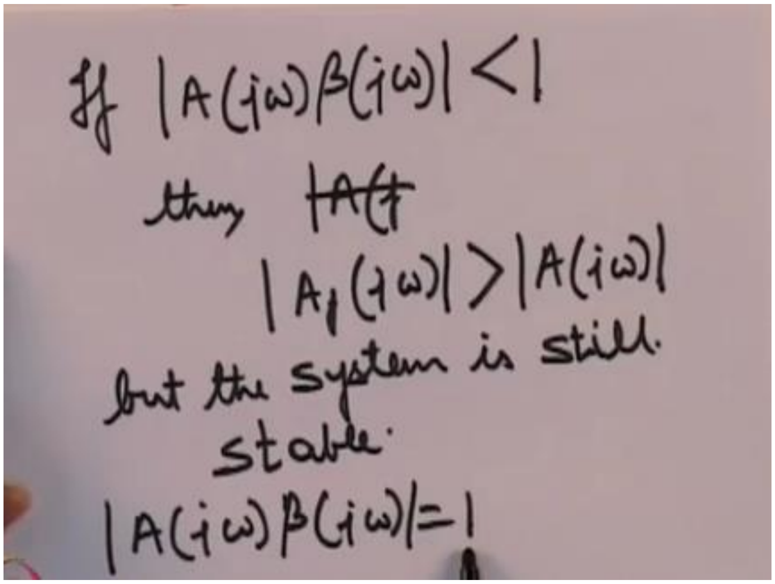

The first case is, if the loop gain magnitude is lesser than 1 then what do you see that A of f omega so A of f j omega magnitude is greater than A of j omega magnitude but the system is still stable, if modulus of A of j omega beta j omega is equal to 1 then the denominator becomes 0 and therefore A of j omega will become infinite and therefore the system is unstable in this case as long as the denominator polynomial the magnitude of the denominator remains positive it can be shown that A of j omega still represents a stable system.

So in the first case, where a magnitude of Aj omega beta j omega was lesser than 1 the denominator was still positive and therefore the system was still stable for this case when the magnitude of A of j omega beta j of Omega is equal to 1 we see that the denominator is 0, hence the system is not stable.

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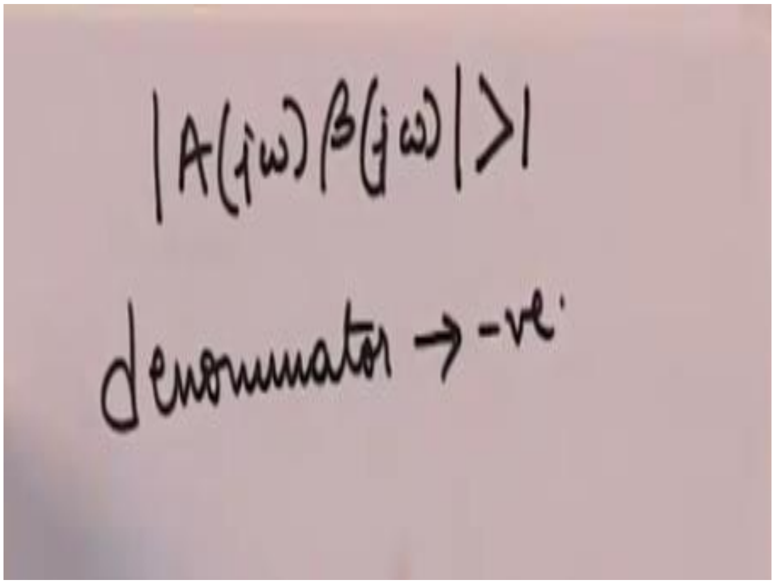

Now what about the case where this A of j omega B of j beta j of omega magnitude is greater than 1, then the new denominator becomes negative it can be shown week it is not readily obvious from this equation that why if the denominator magnitude is negative or the denominator is negative but take it from me that if the denominator is negative then A of j omega will be unstable.

In fact, when the denominator is negative such a system is said to be unbounded system that is the output keeps on increasing when the denominator is 0 that represents what we call an oscillatory system that is the output does not go on increasing the magnitude of the output but it does it keeps on oscillating so it does not go for a bounded input.

If my input is finite my output will still be infinite, because it is an oscillatory system hence the term unstable when the denominator is negative the for a bounded input that is for a finite input the output not only does it not decline not only does it not go down after some time but it actually goes on increasing when the denominator on the other hand is positive then it can be shown that if the input is bounded that is if my input declines or becomes 0 after some time then the output will also decline or be bounded what we call. So coming back to our equation therefore as we saw 3 distinct cases.

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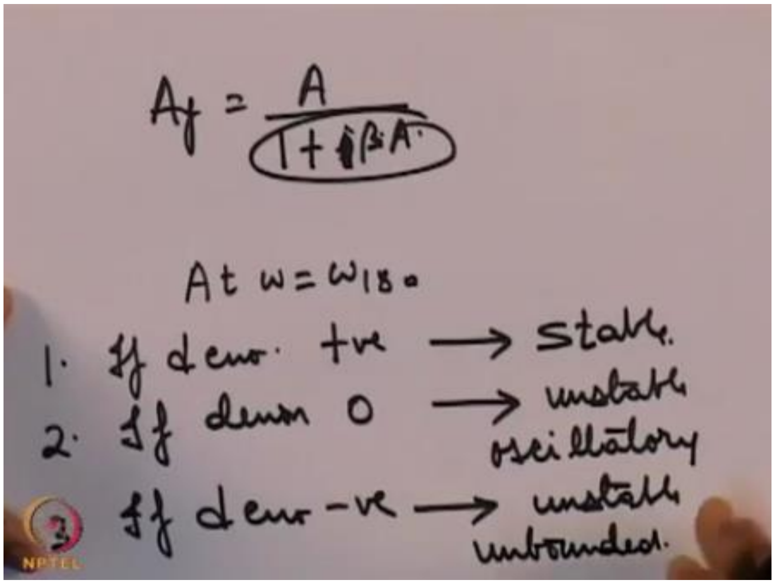

Now later on you know when we shall be discussing about oscillated design we shall be using this concept that is when A of f equal to A upon 1 + j beta sorry 1 + A beta we shall be specifically designing our circuit so that this denominator is becomes 0 and we get an oscillator.

Now one criterion for knowing you know whether the system is unstable so we you know we have discussed 3 distinct cases that is one when the denominator is positive 1 when so at omega equal to omega 180, number one if denominator I am just writing it as denominator positive that represents a stable system if denominator 0 that represents a unstable oscillatory system and if denominator is negative that represents an unstable unbounded system.

So, these 3 things can be we can devise a graphical means to depicts this these 3 cases and also when we are given a transfer function that graphical method will help us to identify whether the system is stable or not just stable so that system is called, what we call the Nyquist plot.

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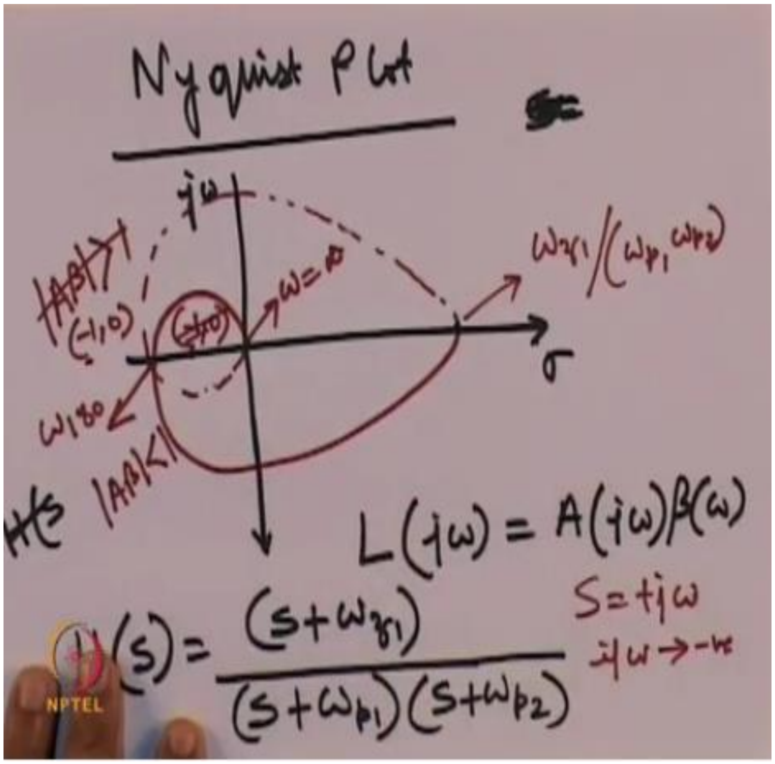

So Nyquist plot is a plot on the complex plane so on the complex plane we have Sigma and J omega we have S equal to R rather we are trying to plot this L of j omega is equal to given by A of j omega beta j omega now suppose we take a transfer function in say a given transfer function is like this H of S let me write it here ok so we see that when S is equal to 0 that is at 0 frequency we will start with a point.

Let us say that point is here and the value at this point will be okay now as the frequency keeps on increasing we will get a plot like this it can be shown that if this S is made negative that is or rather S equal to + j omega if omega negative then for such cases also will get a plot which is the mirror image of this.

Now other thing to note is that this omega 180 is that frequency where this plot touches the X-axis or the sigma axis so the plot touches the X-axis at omega equal to 0 and also at omega equal to omega 180 when it touches omega equal to omega 180 the position of this plot at omega equal to omega 180 is significant.

So, for example suppose it encloses the point - 1, 0 okay now usually of course I want to say something one more thing that it touches again the X-axis or the origin as you can see at omega equal to infinity this H of S will be 0, so this point usually corresponds to omega equal to infinity.

Now coming back to discussion about encirclement of this point - 1, 0 if this plot in circles the point - 1, 0 then that means that the magnitude of this H of S is greater than 1 at omega equal to omega 180 so this case represents the case when the denominator of that closed loop transfer function is lesser than 1 is lesser than 0 I beg your pardon okay so once again at omega equal to 1 omega 180.

If there is an encirclement or this omega the curve lies to the left of this point - 180 then this signifies that modulus of A beta is greater than 1 and therefore the denominator is negative which in turn means it represents an unstable system on the other hand if the point - 1, 0 lies somewhere here then that means at omega equal to omega 1 it is 180 A beta is lesser than 1 and therefore it represents a stable system.

So, this magnitude this Nyquist plot is as an important tool for determining the stability of a system in fact the general rule is that if this Nyquist plot net if it does not encircle the point - 1, 0 then the system is stable, if it encircles then the system is not stable.

So, in this module we covered 2 points, one was what is stability and how to determines the stability from the loop gain and then how to use the Nyquist plot to which is a representation of the loop gain on a complex sigma omega plane and finally based on the Nyquist plot how we can determine the stability of a system, thank you.