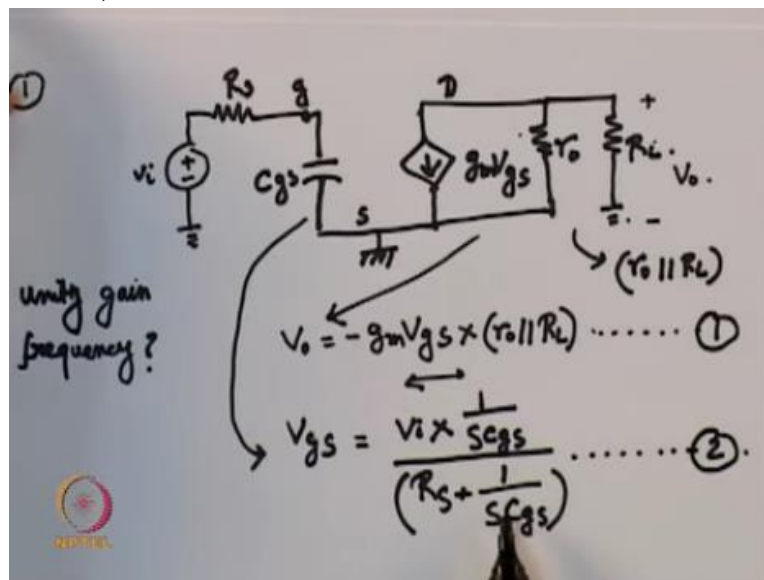


Analog Circuits
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Week – 03
Module - 05
Tutorial No. 1 & 2

My name is Basudev Majumder, I am the TA for the course of analog electronic circuits, today we will be discussing some of the problems that the problems mainly have been picked up from the ideas of op-amps, simple op amp circuit related problems feedback problems and some problems are on gain and phase margin, now let us start with the first problem let us draw the circuit.

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So, we have given V_i which is a source and it is grounded in 1 terminal and other terminal is connected with gate terminal with a series source resistance actually this is a internal circuit model for the MOSFET and this is our capacitor C_{gs} , this is the source terminal which is grounded, this is a dependent ground source $g_m V_{gs}$ and here it is 1 r_o and this is R_L this both are grounded r_o is grounded and R_L is also grounded.

Now this $g_m V_{gs}$ is actually a current source and it is controlled by the voltage which has been applied to the gate and source terminal, this is our gate terminal, this is source terminal and this

is the drain terminal, let me tell you R_S is the source resistance and r_o is the output resistance of that MOSFET op amp without the load resistance and R_L is the load from where of across which we are taking the output.

Now what we are ask to find out, we are asked to find out the unity gain frequency, now unity gain frequency is what, unity gain frequency is the frequency point at which the gain of the amplifier becomes 1, so what we are asked to find out? We have to find out the voltage gain that is V_o by V_i and then we have to take the magnitude of this V_o by V_i and we have to find out at which omega, voltage gain is coming out to be 1.

Now suppose output voltage we are taking across R_L , so V_o is coming - of $g_m V_{gs}$ because this current is going this current going to the parallel combination of r_o and R_L so this both parallel combination is coming r_o parallel R_L and the $g_m V_{gs}$ current is going across the parallel combination of r_o parallel R_L , now we are taking the voltage drop across R_L and our positive polarities is upper side and negative polarities is the down side.

But if you see the direction of the current the current is going from bottom to top, so as the direction of current is opposite to what the polarity that we have assume across the R_L so we will get the output voltage before that the negative side so $V_o = - g_m V_{gs}$ in to r_o parallel R_L , now this V_{gs} this V_{gs} from where it is coming, it is coming from our input, so V_{gs} this equation we are writing from the output and this equation we are writing from our input site.

So, V_{gs} actually it is coming V_i into across this S_{cgs} by $R_s + 1$ by S_{cgs} , so this is the simple voltage divider rule we have assumed and this is coming from the Laplacian 1 by SC operator, because this is a capacitor C_{gs} and 1 by S_c we are taking because we are working in the Laplacian domain or in the frequency domain and this circuit is working for an AC signal, so were s equal $j \omega$ we can simply assume that will see later.

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$$V_o = -g_m \times (r_o \parallel R_L) \frac{V_i}{(sC_{gs}R_s + 1)}$$

$$\frac{V_o}{V_i} = \frac{-g_m (r_o \parallel R_L)}{(sC_{gs}R_s + 1)}$$

$$\left| \frac{V_o}{V_i} \right| = 1 \text{ at } \omega_u = ?$$

Now so we have 2 equations we have got one is this equation and other one is this equation, now putting the equation number 2 in to equation number 1, - gm in to Ro parallel RL Vi if we simplify this portion It is directly coming Vi SCgsRs + 1, so Vo by Vi it is coming - gm Ro parallel RL SCgsRs + 1, so this is the gain we have find out, now for the unity gain frequency we have to find out the magnitude of Vo by Vi = 1 at what frequency? This is called a unity gain frequency and that we have to find out.

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$$\left| \frac{g_m (r_o \parallel R_L)}{(sC_{gs}R_s + 1)} \right| = \left| \frac{V_o}{V_i} \right|$$

$$\Rightarrow \frac{g_m^2 (r_o \parallel R_L)^2}{(1 + \omega_u^2 C_{gs}^2 R_s^2)} = 1$$

$$1 + \omega_u^2 C_{gs}^2 R_s^2 = g_m^2 (r_o \parallel R_L)^2$$

$$\omega_u^2 C_{gs}^2 R_s^2 = g_m^2 (r_o \parallel R_L)^2 - 1$$

$$\omega_u = \sqrt{\frac{g_m^2 (r_o \parallel R_L)^2 - 1}{C_{gs}^2 R_s^2}}$$

So, we will be equating with this magnitude to be 1 and as we are taking the magnitude so the negative sign will not be there, so this was our early equation Vo by Vi, now mod if we take so

mod if we are taking so negative sign will not be there mod of this thing, $g_m R_o \parallel R_L \cdot 1 + \omega^2 C^2 R^2$ this will be 1.

So, just what we have done we have taken the magnitude, magnitude we have taken root over of $X^2 + Y^2$ and then we have done the square of that root over of $X^2 + Y^2$ or root over of that denominator positions so taking the square on the all numerator and denominator both we are getting this relation from here we can easily write 1 by $\omega^2 C^2 R^2 g_m^2 R_o \parallel R_L$ whole square.

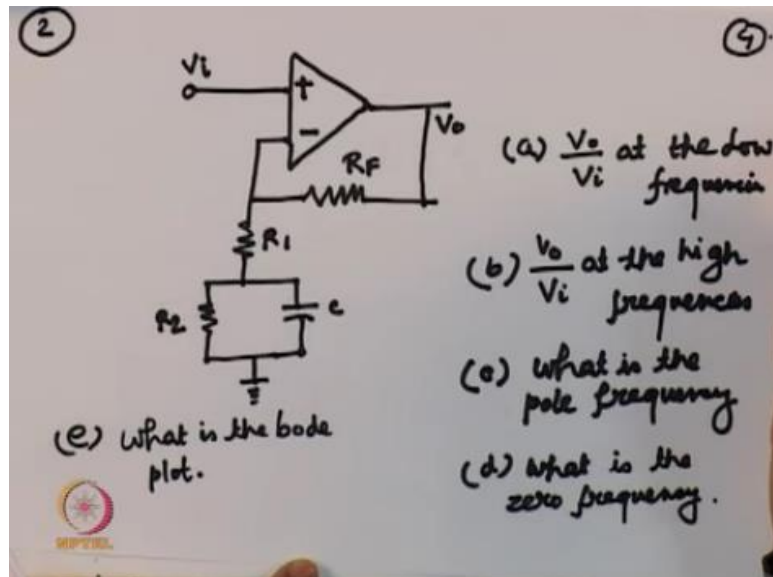
Now $\omega^2 C^2 R^2 g_m^2 R_o \parallel R_L^2 - 1$, so $\omega^2 = \frac{g_m^2 R_o \parallel R_L^2 - 1}{C^2 R^2}$, so let we tell you that the ω is coming root over of $g_m^2 R_o \parallel R_L^2 - 1$ total under root divided by $C^2 R^2$ or it is simply C by R so this is our ω , so this is the unity gain frequency point, so this was the circuit given and this was the common search configuration.

So, like that if any amplifier configuration has been given which is in common drain or common gate terminal and we are asked to find out the unity gain frequency or any circuit if we are given and we have to find out the unity gain frequency of that circuit, the concept is that we have to find out at first the V_o by V_i that is the output voltage that has been taken across the load and the input being applied to the system, so that too we have to first select and then V_o by V_i we have to calculate and from that magnitude of the V_o by V_i we equate it with 1 and from that whatever frequency we will get that frequency will be our unity gain frequency.

So, this is true for any circuit which will work in the Laplacian domain or in the frequency domain so one point to be remember here actually this S what we are assuming here this is an Laplacian operator and generally S is $= \sigma + j\omega$, so for the Laplacian domain if this is σ and this is $j\omega$ as we are giving only AC excitation it has no decaying amplitude part e to the power - σt this is not that.

So, we are only tracing the Y axis so $j\omega$ axis, so while calculating in or taking in to the magnitude domain in the ω domain we have simply putting $S = j\omega$ there is no question of σ coming in the circuit and for all the SC excitation circuit we will assume this convention because we are not having any decaying amplitude part in our SC excitation as we do not have the e to the power $-\sigma t$.

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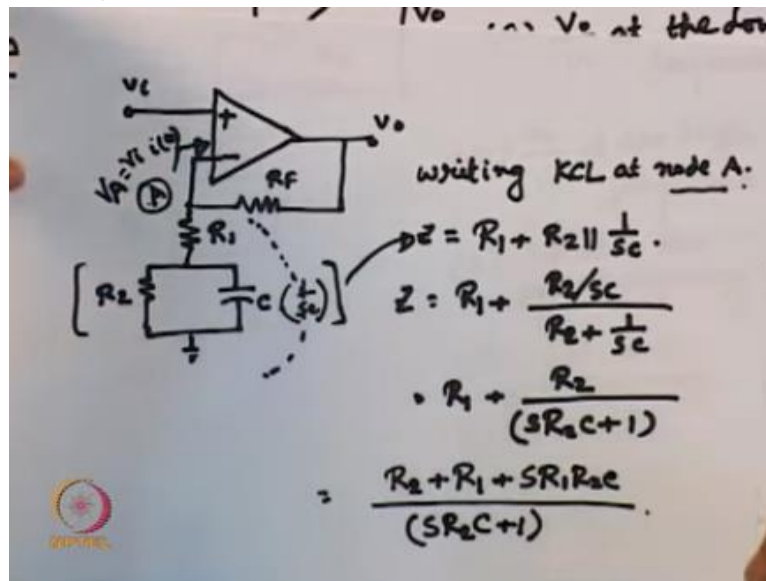
Circuit is given, this is an op amp circuit problem number 2, this is an op amp circuit so at first let me draw the circuit first then I will explain what we have to find out from the circuit, so this is + this is - so input is applied at the non-inverting terminal that is at the + terminal from - is connected 1 resistance which is R_F , this is R_1 , this is R_2 and this is C , so what are the things we have to find out?

We have to find out the V_o by V_i at the low frequency, next V_o by V_i at the high frequencies then what is the pole frequency and what is the zero frequency and next we have to from this all information from all this information we have to find out what is the bode plot, so comparing with the first problem here V_o there also we have to find out the V_o by V_i and this case also we have to find out the V_o by V_i with some extra information.

We need to explicitly determine the regions of the V_o by V_i which have been covered or which have been in the low frequency domain and the high frequency domain and from that also we

have to find out what is our pole frequency and Zero frequency and with from all of these information's we have to draw the bode plot of this circuit, so as can be seen from the circuit so there also 1 output voltage is there V_o and input it is been applied V_i , so here also we can easily calculate or compute V_o by V_i from the simple circuit related concept.

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Solution question 2, so this C can be written by $1/sC$, now for simplification of the circuit or to calculate the V_o by V_i let us assume that this node is named as A and the potential of this node suppose it is V_A , now we all know that the op amp given in the circuit, so let us assume this op amp is an ideal op amp sorry this op amp is an ideal op amp so if it is an ideal op amp we can assume that there is a virtual short in between and there is node current drawn by this op amp circuit, so the current drawn is 0, so there is no current drawn.

So, this is virtual short and there is no current drawn from this 2 terminal of the op amp is V_A actually becomes V_i , now at node A writing KCL node A, so we can write before that we have to simplify little bit for this portion because this R_2 and $1/sC$ these 2 portions are in parallel, total combination from A to ground here the total impedance is coming $Z = R_1 + R_2$ parallel $1/sC$ so I am simplifying only this portion so it is coming $R_1 +$ like that, so it is coming $R_2 + R_1 + sR_1R_2C$ by $sR_2C + 1$ so this is only the Z of this portion.

Now we are writing the KCL at node A, direction of the current let us assume the current is flowing one this direction this is arbitrary according to our equation the sign will change another current suppose it is flowing in this direction and we do not we know that there is current drawing by the op amp, now the equation becomes V_i by Z actually this is the current magnitude that we are writing so V_i by Z , Z we have calculated in the just in our previous page.

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$$\frac{V_i(1+SR_2C)}{(R_1+R_2+SR_1R_2C)} + \frac{V_i-V_o}{R_F} = 0.$$

$$\cdot \text{or } \frac{V_o}{R_F} = V_i \left[\frac{1}{R_F} + \frac{(1+SR_2C)}{R_2+R_1(1+SR_2C)} \right].$$

$$\boxed{\frac{V_o(s)}{V_i(0)} = 1 + \frac{R_F(1+SR_2C)}{R_2+R_1(1+SR_2C)}}$$

So, it is coming $1 + SR_2C R_1 + R_2 + SR_1R_2C$, so this is the outgoing current, so one current we have written that is the outgoing current which is going in this direction total incoming current = total outgoing current we know, so the outgoing current next outgoing current which is in this direction so this also is an outgoing current, so we are writing + the magnitude of the current will be $V_i - V_0$ by R_F so it will be $V_i - V_o$ by $R_F = 0$.

Now so here from this equations we have to find out the relation between V_o by V_i so V_o by R_F that we have taken on the other side and here V_i , if we take common 1 by $R_F + 1 + SR_2C R_1$ sorry $R_2 + R_1$ suppose we are taking it common $1 + SR_2C$, now V_o by $V_i R_F$ it is cross multiplying, so it is $1 + R_F 1 + SR_2C R_2 + R_1 1 + SR_2C$ so up to this much it was like our earlier problem where just we have to find out the gain of the given circuit for which is V_o nothing but V_o by V_i or rather more specifically if we have to say actually this is V_o by S and V_i by S because this is a function of S .

So, if it is a function of S and we have seen from our previous case that S actually is a coming because of the SC excitation and so if we put it j omega so depending on the omega we are having 2 cases we are either operating in the low frequency region or we are operating in the high frequency region, so suppose for the time being we are only interested in the low frequency region what is the value of Vo by Vi.

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$$\frac{V_o(s)}{V_i(s)} = 1 + \frac{R_F(1 + sR_2C)}{R_2 + R_1(1 + sR_2C)} \quad \omega \gg$$

or high frequencies \rightarrow

$$= 1 + \frac{R_F \left(1 + \frac{1}{sR_2C}\right)}{R_1 + \frac{R_1}{sR_2C} + \frac{R_2}{sR_2C}} \quad \omega \gg$$

$$= 1 + \frac{R_F \left(1 + \frac{1}{j\omega R_2C}\right)}{R_1 + \frac{R_1}{j\omega R_2C} + \frac{R_2}{j\omega R_2C}} \quad \frac{1}{\omega} \approx 0$$

$$= \boxed{1 + \frac{R_F}{R_1}}$$

So, for low frequency, so we are interested in the low frequency at first, Vo by S Vi by S $1 + \frac{R_F}{R_1 + sR_2C}$ for low frequencies omega is very less than, so when omega is very less than we can from this equation we can write $1 + j\omega R_2C$ $R_2 + R_1 + j\omega R_2C$, so when omega is very-very less than, this portion $j\omega R_2C$ and this $J\omega R_2C$ this can be negligible, because omega is very less compared to R_2 and C value here when omega is very less than at this point if we assume it is coming out to be $1 + \frac{R_F}{R_2 + R_1}$

So, this will be our Vo by Vi at omega very less than, so this is our first portion, so now looking at our problem part B, so part B similarly we have to find out that the Vo by Vi at the high frequency regions, so just it will be like the similar like what we have done earlier let me write the transfer function or the gain.

Now for the high frequencies, so from this we will assume that omega is very greater than in that case for this part $1 + \frac{R_F}{sR_2C}$ lets divided by $1 + \frac{R_1 + R_2}{sR_2C}$ with of from this numerator or

denominator so it will come $1 + 1$ by SR_2C similarly denominator also we have to do that the same way so it will be coming $R_1 + R_1$ by $SR_2C + R_2$ by SR_2C , now as we are assuming that ω is very greater than so by putting $j\omega R_2C$ $R_1 + R_1 j\omega R_2C + R_2 j\omega R_2C$ when ω is very greater than 1 by ω will be negligibly small.

So, this portion will go out this portion also will go out and this portion will also go out so what we are having then $1 + RF$ by R_1 , so at high frequency this is the V_o by V_i so till this point what we have learned we have seen that if the circuit is given and it is in the frequency domain or the capacitors and inductors are there and it is driven by a SC excitation.

We know how to calculate the V_o by V_i and from that actually we have learned that if we have to separate it for the different frequency zones that also we can separate and the tricky is that we have to simplify it for $S = j\omega$ conditions without considering the sigma because we do not have any decaying amplitude in our SC excitation.

Simply we will put $\sigma = j$ sorry $S = j\omega$ and then depending on the conditions of ω we will get the high frequency gain and similarly the low frequency gain now what we have to find out we have to find out the 2 cases where the pole frequency we have to find out and we have to find out the 0 frequency for that so let we write the same transfer function.

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$$\frac{V_o(s)}{V_i(s)} = 1 + \frac{R_F(1 + SR_2C)}{R_2 + R_1(1 + SR_2C)}$$

$$= \frac{R_2 + R_1(1 + SR_2C) + R_F(1 + SR_2C)}{R_2 + R_1(1 + SR_2C)}$$

$$= \frac{(R_2 + R_1 + R_F) + S(R_1R_2C + R_FR_2C)}{R_2 + R_1 + S(R_1R_2C + R_FR_2C)}$$

$$= \frac{(R_2 + R_1 + R_F) + j\omega(R_1R_2C + R_FR_2C)}{[R_2 + R_1 + j\omega R_1R_2C]}$$

$\frac{V_o}{V_i} = \frac{K(1 + \frac{s}{\omega_z})}{(1 + \frac{s}{\omega_p})}$
 $T = \frac{1}{\omega_p}$

$S = j\omega$

Now to find out the pole frequencies and zero frequencies we have to take it in the form $K \frac{1 + S \text{ by } \omega Z}{1 + S \text{ by } \omega P}$ actually $T = \frac{V_o}{V_i}$ is = 1 by ω , so this will be our zero frequency and this is our pole frequency, so we have to simplify it in such a way so that we will get $1 + \omega Z$ and $1 + \omega P$ and then it should look like the similar expressions then we can able to determine that which will be our pole frequency and which will be our zero frequency.

So, let us we simplify this, so you see this $\frac{R_2 + R_1 + R_F}{R_1 + R_2}$ and here $\frac{R_1 R_2 C}{R_1 + R_2 + R_F}$ both are S related terms, so directly I am taking common S and it is coming out to be $\frac{R_1 R_2 C}{R_1 + R_2 + R_F} + \frac{R_1 R_2 C}{R_1 + R_2}$. Now let us put $S = j\omega$, so we have to actually simplify it in this way so after this to make it 1 at the starting of the numerator we just have to take common one at this term and below also we have to do the same thing, and so by doing this we will be actually calculating the K which is DC gain $S = j\omega$ and ω if you put 0 this part will go and what we are left it, we will left it with DC gain.

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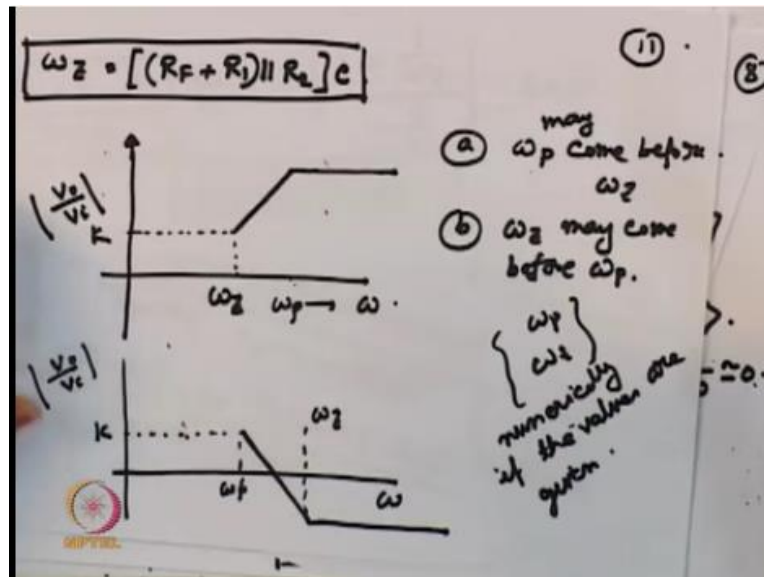
$$\begin{aligned}
 &= \frac{(R_2 + R_1 + R_F) + j\omega(R_1 R_2 C + R_F R_2 C)}{(R_1 + R_2 + j\omega R_1 R_2 C)} \\
 &= \frac{(R_2 + R_1 + R_F) \left(1 + \frac{j\omega R_2 C (R_F + R_1)}{R_1 + R_2 + R_F}\right)}{(R_1 + R_2) \left(1 + \frac{j\omega R_1 R_2 C}{R_1 + R_2}\right)} \\
 K &= \frac{R_2 + R_1 + R_F}{(R_1 + R_2)} \cdot \frac{V_o}{V_i} = K \left[\frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}} \right] \cdot s = j\omega \cdot \\
 \omega_p &= (R_1 || R_2) C.
 \end{aligned}$$

So, from this V_o by V_s we will be able to find out the DC gain the zero frequency and the pole frequency, let me write the equation once again now this part we are taking common so $1 + R_2 C$ is here also present here also present so we are taking $j\omega R_2 C R_F + R_1$ and because of this one we are getting one $R_1 + R_2 + R_F$ similarly from the denominator we take common $R_1 + R_2$ $1 + j\omega R_1 R_2 C$ $R_1 + R_2$.

So, now let us see that how it involves to the which we are want to compare it, so V_o by $V_i = K$
 $1 + S$ by ωZ $1 + S$ by ωP , so our K is K will be what, K is this thing the DC gain $R_2 +$
 $R_1 + R_F$ divided by $R_1 + R_2$ and what is our pole frequency? Our pole frequency will be so this
 is S by ωP so S is $j\omega$ nothing but $j\omega$, so S by ωP means it is coming out to
 be R_1 parallel R_2C because R_1R_2 by $R_1 + R_2$ is actually R_1 parallel R_2 and for ωZ .

So, we have to find out the ωZ by this equation so this S by ωZ so ωZ actually
 is coming you can say $R_F + R_1$ is a series combination and with that it is R_2 so similarly we can
 write that $\omega Z = R_F + R_1$ parallel combination of R_2 and this is C , so this is our zero
 frequency and this is our pole frequency, so this is our pole frequency, this is our zero frequency
 and our DC gain is this one, now let me draw the last part or the bode part

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So, what is the bode plot? So, with the information given above we should be able to draw the
 bode plot of this given circuit, so to draw the bode plot what are the information's required? The
 first information required is the DC gain, so abruptly I am just giving the concepts so I am just
 drawing what we have to do for bode plot we are having ω in this one and the magnitude we
 are V_o by V_i mod we are having in the Y axis we can see that there are 2 cases may come out
 one case is ωP .

So, basically the bode plot is something that will give you how the magnitude of that system or the transfer function varies with the frequencies, so what actually we have found out in mathematically or analytically in our previous pages that is a pictorial version of that, so to find out that thing we are ending up with 2 cases, 1 case is we may have ω_P which may come before ω_Z ω_P are the pole frequency may come before ω_Z or ω_Z may come before ω_P .

So, at first it will start with the DC gain suppose some DC gain value we are calculated that is K , after that if we are having ω_Z so suppose if this is the ω_Z , so from this as it is a 0 so it will have a decayed of 20 db and it will go up and till that point it will go up until it is facing the ω_P , suppose this is the point where it is ω_P , so from here it will go 20 db down so it will look like that but for all real system ω_P comes first that is the pole is coming first for the stability of the system for all real system if we consider and then ω_Z will come.

In that case the V_o by V_i will look like so this is the same DC gain V_o by V_i from here it is ω_P then 20 db fall it will come then it is ω_Z then 20 db straight, so 20 db falling then ω_0 comes so again 20 db it will go up so it will go straight, so depending on the conditions of ω_P and ω_Z we will have 2 bode plots and the problem given we have to see if any numerically any values are given what is the R_2 R_F R_1 and C from that actually we have to calculate this ω_P and ω_Z numerically, if the values are given and from there we can easily see that the earliest of the other and we can easily draw the bode plot.

So, up to first problem and second problem we have seen that we know how to draw the bode plot and we have seen that from that bode plot if we simplify our ω for low frequency and high frequency how we can split it for 2 different zones and if you have to find out the ω_P ω_Z or the corner frequencies that also how it should be done that also we have seen and finally we have drawn the bode plot for that circuit.