

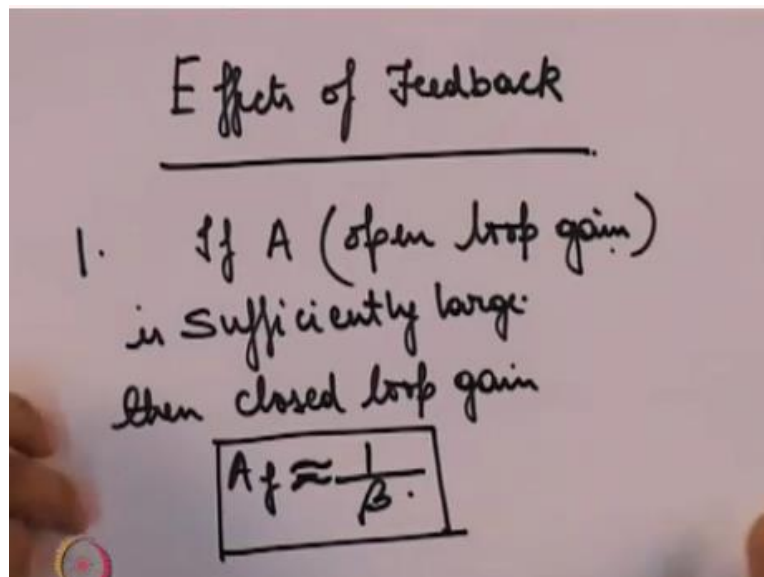
Analog Circuits
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Week- 03
Module -04
Effects of Feedback

Hello, welcome to another module of this course analog circuits. In the previous module we covered about we discussed about feedback, so as I mentioned that feedback is very important aspect of any analog circuit design we can control many of the variables in any analog circuit using feedback.

So one of the variables that we discussed in the previous module was that of the gain whereas the forward or open loop gain of any amplifier cannot be controlled unless we go inside the amplifier using feedback we saw that if the gain of the amplifier is sufficiently large then the overall gain on what we call the closed loop gain is dependent can be made dependent only on the feedback factor beta and the forward loop gain does not have any role to play.

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So in this lecture we will discuss some more effects of feedback, so one, effect that we already saw in the previous module was that if A that is open loop gain is sufficiently large then closed

loop gain A of f is approximately 1 upon beta so here we have made this closed loop gain dependant only on the feedback parameter let us see some other effects.

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2. Gain desensitivity

$$A_f = \frac{A}{1 + A\beta}$$

Fractional change in A

$$\frac{dA_f}{dA} = \frac{-1 \times A}{(1 + A\beta)^2}$$

is reduced by a factor

$$\frac{dA_f}{A_f} = \frac{dA/A}{1 + A\beta} \cdot \left(\frac{1}{1 + A\beta}\right)$$

if $\beta = 0$, then $\frac{dA_f}{A_f} = \frac{dA}{A}$

We saw that the feedback gain for a negative feedback system is given by this formula, now if we take the derivative on both sides that is if we perform this operation then what do we get or it is = ok now it can be shown from here that this dA_f upon A of f is given by dA upon A upon $1 + A\beta$ okay so what you see is that fractional change in A is reduced by a factor 1 upon $1 + A\beta$.

So in the overall the feedback gain A_f the fractional change in the feedback gain is 1 upon $1 + A\beta$ times the fractional change in the open loop gain so this is an advantage what we are seeing is that suppose for some reason there is a change in this A you know due to some factors like its internal circuitry or something like that say there is some change in this open loop gain A then in the feedback gain the same change will be reflected by a much smaller factor as you can see if $\beta = 0$ then dA_f upon A_f is = dA upon A .

So then the fractional change becomes the same that is the close of course if $\beta = 0$ then the closed loop system no longer remains a closed loop system it becomes an open loop system so this phenomenon where the impact of any change in the open loop gain is transmitted to a much smaller extent to the at the output of the closed loop system is called gain de-sensitivity, so de-

sensitivity means the output becomes less sensitive to any changes, so here since the output that is the feedback gain or closed loop gain is getting less sensitive to any changes in the open loop gain we call this de-sensitivity gain de-sensitivity.

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The image shows a handwritten derivation on a whiteboard. It starts with the closed-loop gain formula: $A_f = \frac{A}{1+A\beta}$. This is then rewritten as $= \frac{1}{\frac{1}{A} + \beta}$. The next step is to find the derivative of A_f with respect to A : $\frac{dA_f}{dA} = \frac{-1}{\left(\frac{1}{A} + \beta\right)^2} \times \left(-\frac{1}{A^2}\right)$. The final result is $\frac{dA_f}{dA} = \frac{1/A^2}{\left(\frac{1}{A} + \beta\right)^2} = \frac{1}{(1+A\beta)^2}$.

Let us see some other parameters I just want to correct one thing in this derivation this will be + let me write this formula correctly I this so we had A of f is = A upon 1 + A beta which is = 1 + 1 upon A + beta so then dA of f by dA is = - 1 upon 1 upon A + beta whole square into - 1 upon A square which is = 1 upon K square upon 1 upon A + beta whole square which is = 1 upon 1 + A beta whole square okay from this if we now write dAf is = dA upon 1 + A beta whole square okay.

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$$\begin{aligned}
 dA_f &= \frac{dA}{(1+A\beta)^2} \\
 dA_f &= \frac{\frac{dA}{A} \times A}{(1+A\beta)^2} \\
 &= \frac{dA/A \times \frac{A}{1+A\beta} \times \frac{1}{1+A\beta}}{1+A\beta} \\
 \Rightarrow dA_f &= \frac{\frac{dA}{A} \times A_f}{1+A\beta}
 \end{aligned}$$

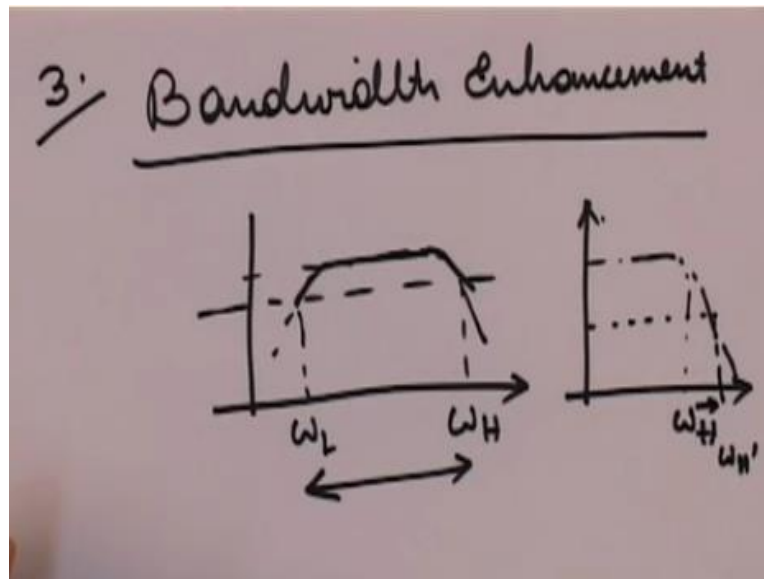
Proceeding further what we see is that so we have so we have d of Af is = d of A upon 1 + A beta whole square then from here we have we can write this dAf is = dA upon multiplied by A upon 1 + A beta whole square so this from here we can write this as dA upon A times A upon 1 + A beta times 1 upon 1 + A beta so this is = A of F so therefore this dA of f will be = dA upon A into Af this whole upon 1 + A beta and from which we can finally write the N formula.

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$$\begin{aligned}
 dA_f &= \frac{A_f \times \frac{dA}{A}}{1+A\beta} \\
 \Rightarrow \frac{dA_f}{A_f} &= \frac{dA/A}{(1+A\beta)}
 \end{aligned}$$

Which is so we have d of Af is = A of f into dA upon A this whole on 1 + A beta so from here we have dA of f on A of f dA upon A so this is the proof of that formula that formula which I gave here when discussing then this incident, so here it is not properly clear how this we were getting this formula so this is the derivation for that okay.

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Coming to the next effect which is what we call bandwidth enhancement, so coming to the next effect which we call bandwidth enhancement now recall in a few modules before we had introduced the concept of bandwidth so bandwidth is like a you know it is that there are 2 3db frequencies and this range of frequencies between ω_H and ω_L is called the bandwidth.

Now already when discussing the unity gain frequency we had seen that an opamp which has characteristics like this initially after feedback the characteristics become like this and the upper cut off frequency shifts in this case we had already seen that the presence of feedback will increase the bandwidth but for a more general case where both ω_L and ω_H is present let us see what happens.

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$$A(s) = \frac{A_M}{1 + s/\omega_H}$$

$$A_f(s) = \frac{A(s)}{1 + \beta A(s)}$$

$$= \frac{A_M / (1 + s/\omega_H)}{1 + \beta \frac{A_M}{1 + s/\omega_H}}$$

$$= \frac{A_M}{1 + s/\omega_H + \beta A_M}$$

The suppose we have a transfer function of a system given like this as you can see this is the high frequency response because when $S = 0$ gain is A_M and when S tends to infinity the gain decreases, now with feedback the transfer function gain or the gain with feedback becomes like this if you simplify this becomes like this ok.

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$$A_f(s) = \frac{A_M}{1 + s/\omega_H + \beta A_M}$$

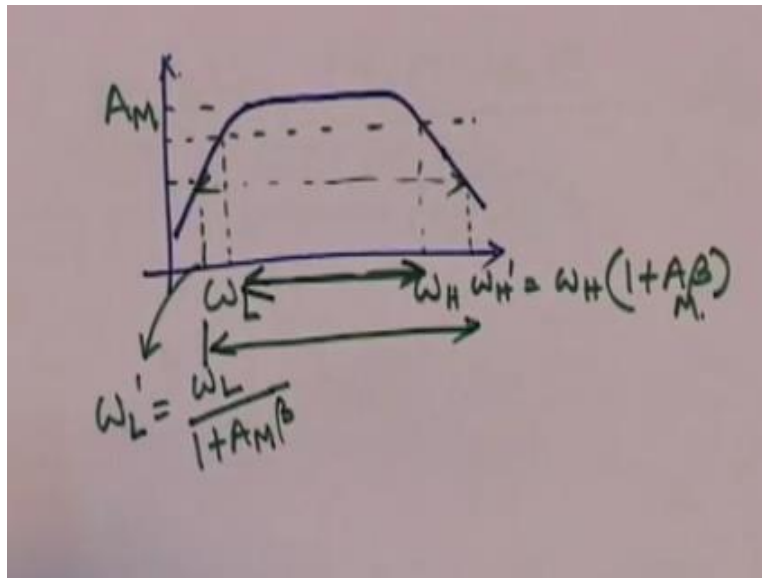
$$= \frac{A_M}{1 + \frac{s}{\omega_H (1 + A_M \beta)}}$$

$$\omega_H' = \omega_H (1 + A_M \beta)$$

So rewriting just extending this equation we have so we have A of f whole of S is $= A$ of M $1 + S$ upon $\omega_H + \beta A_M$ which is $= A_M$ upon $1 + S$ upon $\omega_H (1 + A_M \beta)$ here what can we infer from this equation so this new 3 db frequency or upper new upper 3 db frequency ω_H dash is now $= \omega_H$ times $1 + A_M \beta$ okay so clearly since β is A this $A_M \beta$

is a this product $M\beta$ is usually a large quantity the new 3 db frequency is significantly larger than ω_H .

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So we have extended the upper 3 db frequency, so the fresh page for this so initially suppose our frequency characteristics looked like this now with feedback present my ω_H dash is here like this AM so clearly the upper 3 db frequency has increased now what about the lower 3 db frequency so going back to the same type of derivation that we did for the upper 3 db frequency we can obtain something similar for the lower 3 db frequency as well.

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$$A(s) = \frac{A_M}{1 + \frac{\omega_L s}{s}}$$

$$A_f(s) = \frac{A(s)}{1 + \beta A(s)}$$

$$= \frac{A_M}{1 + \frac{\omega_L s}{s}} \cdot \frac{1 + \frac{\omega_L s}{s}}{1 + \beta \times \frac{A_M}{1 + \frac{\omega_L s}{s}}}$$

$$= \frac{A_M}{1 + \frac{\omega_L s}{s} + AM\beta}$$

So for calculating the change in the lower 3 db frequency the kind of transfer function that we have to use is now something like this as you can see when $S = 0$ the magnitude of the transfer function is low and when $S = \infty$ the gain is AM so this does represent the low frequency response and this ω_L is the lower 3 db frequency so now in a feedback system what we will get is of course a negative feedback system, a note in some places I am ignoring the S that is usually associated with β that reason I am ignoring it is because this β often is frequency independent okay.

So then this becomes what this is $= AM \text{ upon } 1 + \omega_L \text{ upon } S \text{ upon } 1 + \beta \text{ into } AM \text{ upon } 1 + \omega_L \text{ upon } S$ which in turn is $= AM \text{ upon } 1 + \omega_L \text{ upon } S + AM \beta$ and then proceeding in the same way that we did previously just one small correction I want to make in the previous one the I divided both numerator and denominator by $AM \beta$ so the correct formula should be this of course that does not affect the numerator does not affect the upper 3 DB frequency but still it should have been shown properly.

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The image shows two handwritten equations on a whiteboard. The first equation is the closed-loop transfer function:

$$A_f(s) = \frac{AM / (1 + AM\beta)}{1 + \frac{\omega_L}{(1 + AM\beta)}s}$$

The second equation shows the derivation of the new lower 3 dB frequency ω_L' by dividing the numerator and denominator of the first equation by $1 + AM\beta$:

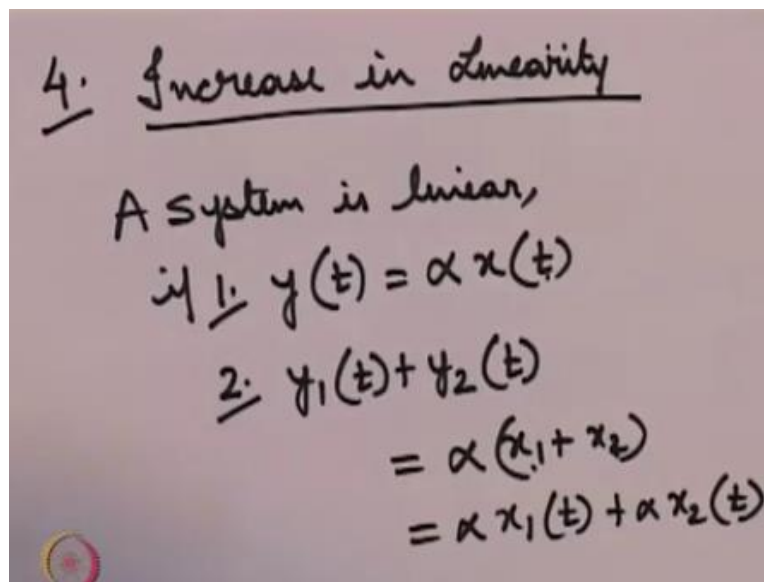
$$\omega_L' = \frac{\omega_L}{1 + AM\beta}$$

So now dividing both the numerator and denominator by $1 + AM \beta$ what we get is $A \text{ of } f \text{ } S \text{ } 1 + AM \beta \text{ upon } 1 + \omega_L \text{ upon } 1 + AM \beta \text{ } S$ so you see the new lower 3 DB frequency is given by $\omega_L \text{ upon } 1 + AM \beta$ okay so coming back to the graph that we had used previously (Refer Slide Time: 19:26) we see that upon applying feedback the lower cut off

frequency becomes this, so the overall bandwidth is now this much as compared to the previous bandwidth of here.

So it is now between these 2 points as compared to it previously being between these 2 points so the overall bandwidth has shown an increase okay so this is the major conclusion from this discussion that upon applying feedback the overall bandwidth of a system is much larger than its open loop bandwidth also you know one other effect that is frequently seen with feedback is we can you know express it very simply.

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4. Increase in Linearity

A system is linear,

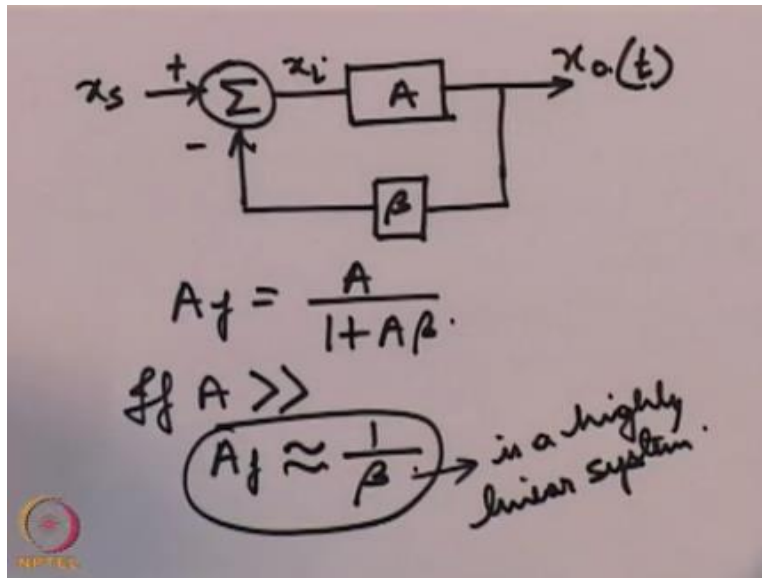
if 1. $y(t) = \alpha x(t)$

2. $y_1(t) + y_2(t)$
 $= \alpha (x_1 + x_2)$
 $= \alpha x_1(t) + \alpha x_2(t)$

So the we can so this is the fourth effect increase in linearity so linear system you know as is well know that a system is linear as we had discussed previously if y of t 2 conditions one is y of t is equal proportional to the input and second the principle of superposition applies that is overall impact of a sum of 2 signals is the same as the summation of the responses if each signal would have been applied individually.

So this is what it means, so if suppose y1 and y2 are the responses for x1 and x2 inputs and the overall impact for x1 + x2 will be the same if x1 and x2 were individually apply, now how does feedback then improve the linearity of a system for understanding this.

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Let us go back to the negative feedback model that we were discussing previously now one thing which I did not mention previously is that this negative feedback has a tendency to reduce the output whereas positive feedback has a tendency to keep the output increasing keep the increase in the output or in fact increase the rate of increase in the output, so positive feedback you know often leads to saturation of a system that is the system tends to produce an output which reaches its maximum limit.

But negative feedback tends to pull the system back from its maximum output its tends to stabilize the system as we shall see later when we study about stability but if consider this system where a negative feedback is present as we know A of f is $= \frac{A}{1 + A\beta}$ here we are ignoring the laplacian S terms in parentheses and we also saw that if A is quite high then this A of f will be nearly $= \frac{1}{\beta}$.

Suppose this β is a highly linear system then the overall gain of the closed loop system since it depends only on β now hence the closed loop system must also be linear isn't it so what happens is it does not matter if this gain if this open loop gain of this system or this open loop system itself is non linear as long as feedback is present the overall gain can be made linear so this is another major impact of feedback.

So in this module we covered 3 major aspects, one was gained the sensitivity that is the gain or any change in the open loop gain tends to affect the close loop gain much less than it would do for the open loop system, second the bandwidth of the closed loop system increases and thirdly the system becomes more linear especially for negative feedback, in the next module we shall see an example of a practical circuit with feedback on it, thank you.