Analog Circuits
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Week- 03
Module -02
Frequency Response example

Hello, welcome to another module of this course analog circuits, in the past module we had covered about the high frequency response and also given a circuit how to find the upper and lower 3db cut off frequencies, now in this module we are going to take up an example because all the concepts that we introduced in the last modules it may be a little difficult to understand in the absence of an example.

So, let us consider an example, however before going on to an example I will just introduce a concept called the Miller's theorem which is an important tool used for determining the frequency response of many circuits.

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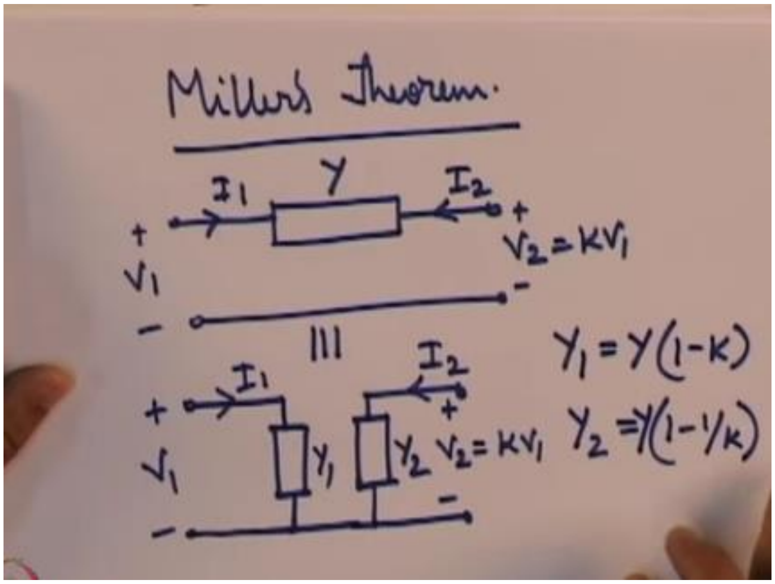

Suppose you are given a circuit like this, a series element with admittance why we are suppose that the input the voltage is V1 and I1 at the output the current is I2 voltage is V2 given = KV1 and it can be shown that this whole circuit can be reduced to this form where this Y1 is given by Y times 1-K and Y2 is given by Y times 1-1 upon K, So we are given a circuit like this admittance series elements having admittance Y and output voltage V2 which is K times the value of input voltage V1 then that circuit can be reduced to this the proof if fairly simple the proof is somewhat like this, so for the first circuit the current I1 is given by V2 so I am here is given like this which in turn is like this which in turn can be written as okay.

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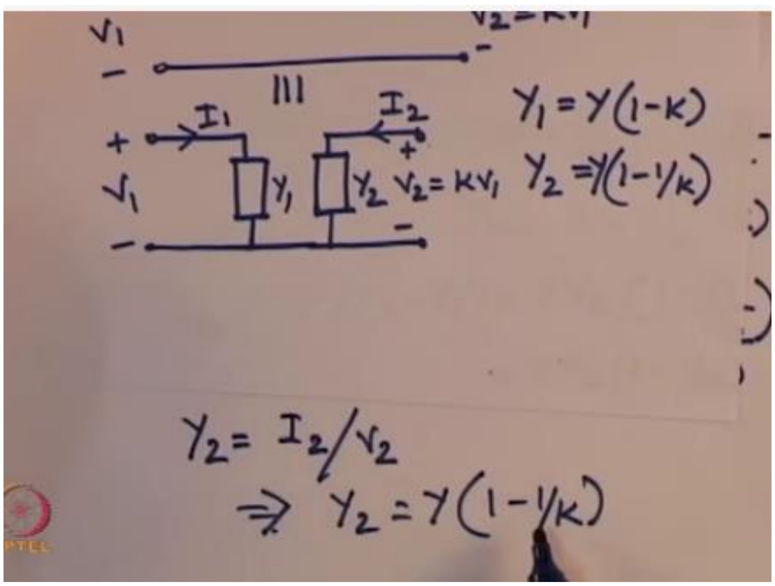

Now let Y1 be = I1 upon V1 then from this equation where I1 upon V1 is given by YX-K then I should be able to write Y 1 is = Y multiplied by 1-K similarly I2 is given by Y V2-V1, which in turn is = Y, if I take V2 out then this is V1 upon V2 which is = YV2 1-1 upon K, so here also what we see that if we have Y2 given by I2 upon V2 then that Y2 is = Y into 1-1 upon K.

The input you will have an admittance whose value is given like this Y1 and on the output you will have another admittance Y2 whose value is given like this that is the proof now we will come to the application of this theorem in a moment but first let us go back to the example I was mentioning.

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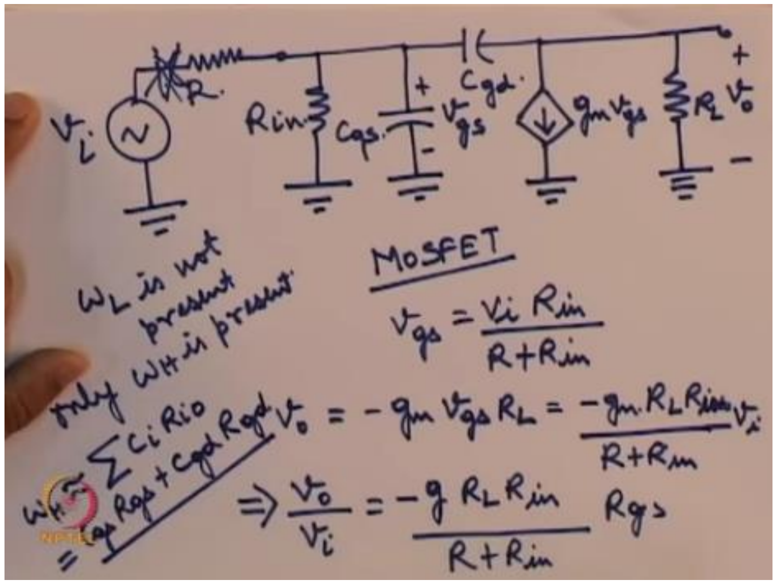

So suppose you have a circuit like this is the equivalent linear model of a device called a MOSFET the first thing that you notice that there is no DC blocking capacitor and that is why omega L is not present only omega H is present because I mentioned that omega L is present for those circuits that that have an input blocking capacitor so if you have would have a capacitor like this here then you would have an omega L since that is not present hence this does not have an omega L so then what is the so case now that it only omega H is present.

What is the mid band gain or what is the DC gain for calculating the DC gain what you do is you open all the capacitors as if the capacitors are not present and then find out VO upon VI so in the absence of any capacitor Vgs this voltage is given by VI times Rin upon R+Rin, VO is given by -GM Vgs RL which is = -GM RL Rin upon R+Rin VI from which VO upon VI is = -G RL Rin upon R+Rin so this is the DC gain of the mid band gain.

Now Rgs or okay now since we have to find out only omega H we know that omega H is given by sigma Rio so this in turn we can write for this particular circuit Cgs Rgs+Cgd Rgd, since we have only 2 capacitors present so therefore how to find out the values of this Rgs and Rgd let me go back to the circuit once gain so for finding out Rgs.

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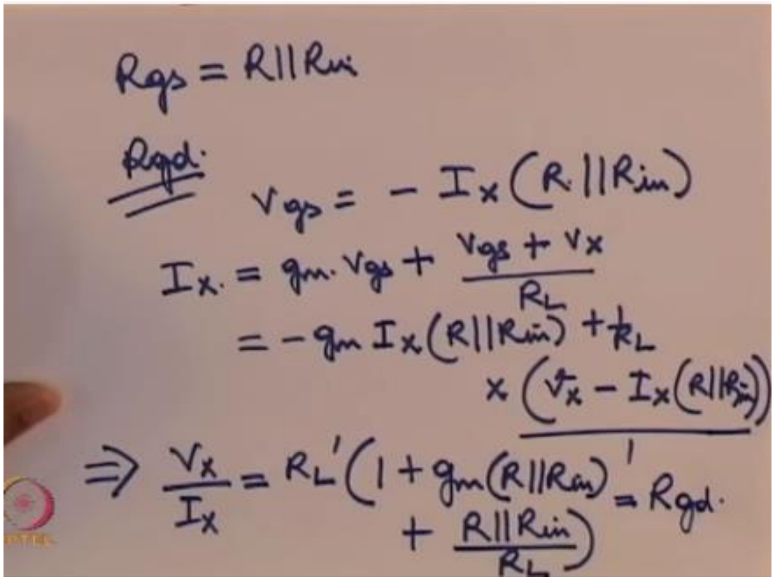

As I said, what you do is you open the capacitor I think it is not visible yeah so for finding out Rgs you open this capacitor Cgd and then find the equivalent resistance across these 2 terms, if you open this capacitor then this capacitor is disconnected is basic this part of the circuit is disconnected from this part of the circuit, so in that case Rgs will simply be = R parallel to Rin of course here another point is this VI has to be shorted the (()) (10:43) voltage sources should be shorted and all current short sources should be opened okay.

Now what about Rgd? So find out Rgd what we do is we open Cgs and then find out what is the equivalent resistance across the Cgd terminals, so to do that let us try to solve it you know this Vgs okay in order to find this Vgs suppose the current flowing is Ix so Vgs is = -Ix suppose the current is flowing in this direction sorry VGs is = -Ix R parallel to Rin okay Ix this current is = Gm vgs+vgs+Vx so here you know what are assuming is that this is open that Ix here and the difference between the 2 voltage is = so this is + this is - this Vx okay upon RL.

So this current Ix is = this current flowing through this dependent current source and the current flowing through this RL which is so the current flowing through this RL is the voltage at this point divided by RL and the voltage at this point is VX which is the voltage across the capacitor terminals + the voltage across the Cgs terms that is from this point to ground.

So now solving this so if we substitute in place of Vgs this formula what we get is -Gm Ix R parallel to Rin+1 upon RL times Vx-Ix, so this Vx comes here and Vgs Vgs is simply Ix multiplied by the parallel combination of this voltage you know Vgs voltage at this point is Ix the negative of Ix multiplied by the parallel combination of R and Rin upon 1 okay.

So solving this what we get is and this is = Rgd is not it, Vx upon Ix is the equivalent is = the equivalent resistance across Cgd with Cgs open and all voltage sources shorted all current sources open all independent not the dependent current sources so then this Rgd comes out to be we simplify the previous expression a bit ok.

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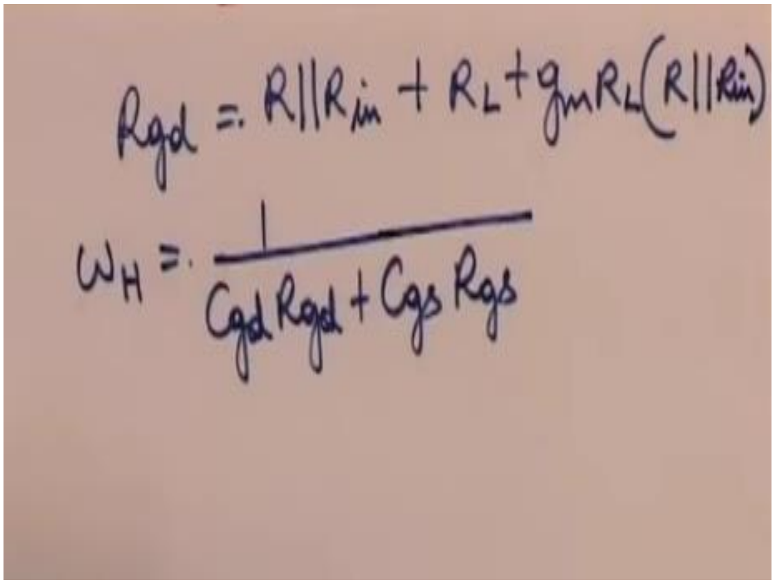

Now this upper 3db frequency is as I was mentioning earlier is = Cgd Rgd+Cgs Rgs. So now we have the value for Rgd we also have the value for we have the value for Rgs, we know the values of Cgs Cgd substituting them we can get the value of omega H.

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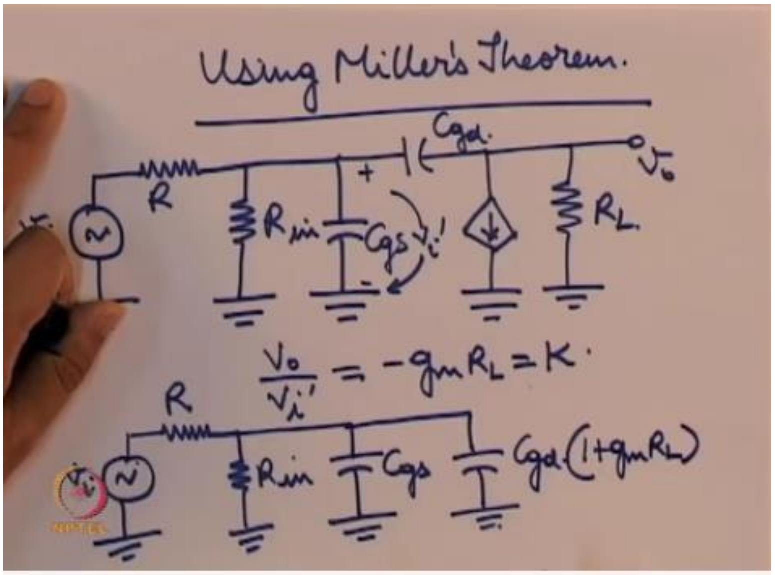

Let us try another method which is using the Millers theorem, so once again we will draw the circuit, if you find out the DC gain VO upon Vi so our Vi of course Vi dash let me say Vi dash is of course at this point so this VO upon Vi dash for DC comes out to -gm RL and which is = the value of K, so then now doing the Miller decomposition that is if we just like you know while discussing the Millers theorem we had split our circuit in this form because were this Y is now split into 2 admittances Y1 and Y2.

Now one thing I would like to mention for this Miller’s theorem is that this theorem is valid only when the input and output conditions are kept the same that is the V1 and I1 and V2 and I2 are the same for both cases only the internal structure is changed, so if that is the case then it is not a good method for finding on the output impedance, because when you are trying to find out the output impedance you short all your sources but then formulas theorem to be valid no change in the input and outputs force voltages and currents can be done hence output impedance cannot be measure using this theory.

However input impedance if you want to find out the input impedance then this is a good method, because while finding out input impedance you do not make any changes in the output or input that is you do not have to short or open the output or short or open the input, therefore Miller’s theorem is only valid for finding characteristics at the input port not so much at the output port with this in mind the input part that is the Y1 part of this circuit the equivalent y1 part becomes something like this.

So here the Cgd is that Y part that is now split into 2 parts and the one which appears at the input will be 1-K so here K is -gm RL so this will be 1+gm RL, so then if this is my equivalent input circuit then the input the value of Omega H or the dominant pole is very simply written as the input total capacitance multiplied the input total resistance.

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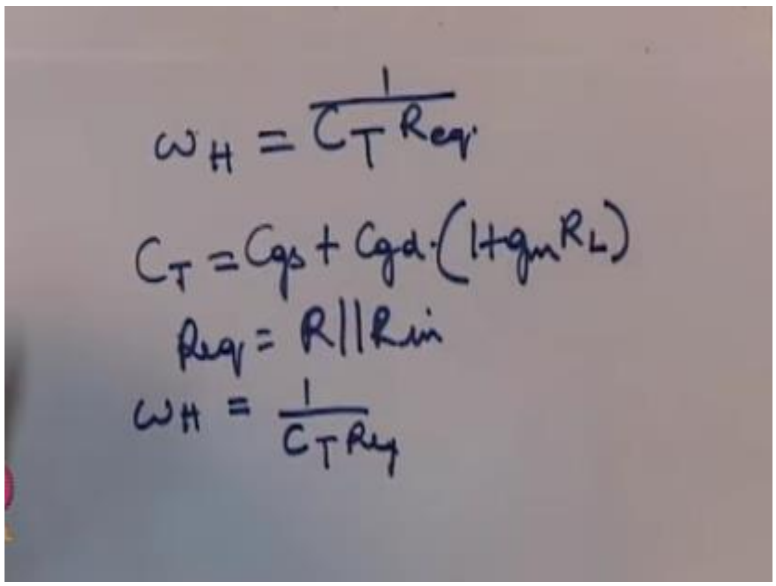

Now what is the CT? Now looking at this circuit can we write CT if we go back to this circuit can I right Cts simply the parallel combination of this Cgs and Cgd times 1+gm R is = Cgs+Cgd 1+gm RL and R equivalent is the parallel combination of Rin therefore my omega H is simply = 1 upon Ct Rin equivalent, (Refer Slide Time: 22:56) I would just like to make a correction in this formula omega H is 1 upon Ct are equivalent and similarly for the previous case.

Also I think the formula which I gave for this omega H in this case when discussing the previous method the formula that I gave was this will be 1 upon CiRio, please make this correction this slight error if omega H is = 1 upon sigma CiRi0 sorry for that and here also omega H will be = 1 upon CT are equivalent ok, and finally the best way to find out the value of omega H is to do a rigorous solution of the you know using traditional Kirchhoff's current and voltage law's.

We can rigorously find out the VO upon VI formula so the third method will be the rigorous derivation and if we do that what we will find is that this omega H, so here we are not using any Millers theorem or any you know using those dominant pole derivation techniques that I discussed earlier here we are simply doing a V0 upon Vi and based on the value of this V0 and Vi we can find out what does dominant poles are so if we do that this value of this V0 upon Vi comes out to be this.

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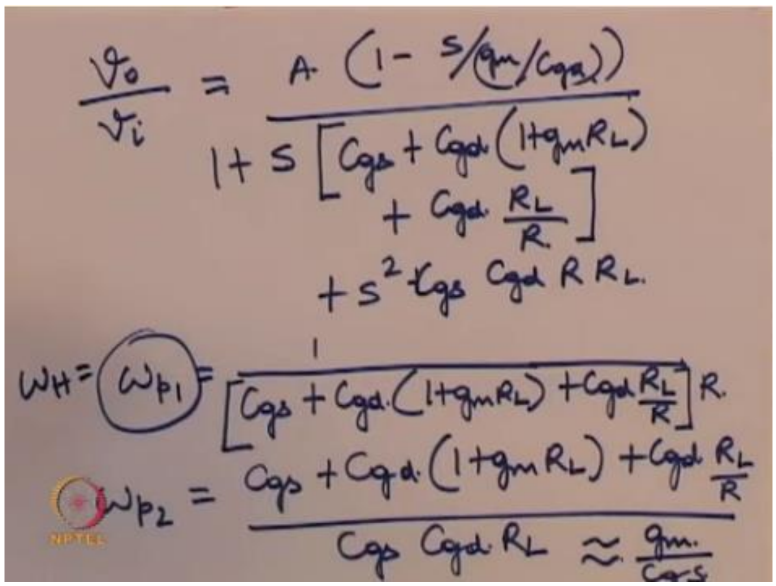

So first thing you see that the denominator is a polynomial of second order therefore we will have 2 poles the first pole being and the second pole if you solve this polynomial it will be given like this usually this gm RL is a somewhat is quite a high value there for all the other terms will be negligible the Cgs and Cgd terms will be negligible compared to this middle terms and this usually reduces to Gm upon Cgs which is also pretty high so the definition of dominant pole where omega P2 should be much higher than omega P1.

So that omega P1 becomes the dominant pole is valid here and what we see is that this omega P1 is exactly = the first method to that value of Omega H that we derived using the first method and nearly = the value we derived using Miller’s theorem.

Hence, the method that is by submitting capacitances with open opening up the capacitances and then finding out the equivalent resistance is a valid method and it is proved that the method is applicable for finding out the dominant poles of us of a given analog circuit.

Also to note that the Millers theorem is also somewhat convenient it is appears to be a little easier than the other method in finding out the dominant poles and of course the final proof that there is a dominant pole what is its value or whether it is indeed the dominant pole or not is by a rigorous method which we showed just now that is the best method but the other methods are also equally applicable so with this we come to an end to this module in the next module we shall be covering the topic of feedback, thank you.