

Analog Circuits
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Week -03

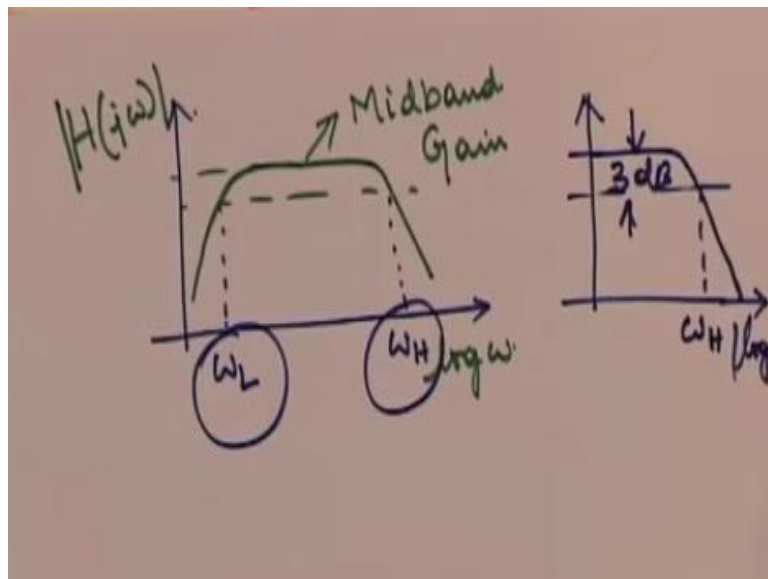
Module- 01

Frequency Response (High Frequency Response)

Hello, welcome to another module of this course analog circuits. We are now in week 3, in the last module in the past week we had covered we are just started with frequency response and we had discussed the low frequency response of an opamp and how to find the lower 3 db frequency in terms of the dominant force.

In this module, we shall be continuing on the same discussion but for the higher frequency response, so I would like to just remind you once again what the characteristics were high and low frequency response.

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So they were the general response of an opamp can be written something like this, now this is the midband gain ok where the gain is relatively flat and for both when the frequency decreases as well as increases, we find that the gain decreases. Now usually this as I also mentioned in the last module that this ω_L is not always present hence in most cases the response is like this where only there is an upper 3db frequency present.

Now in the past module we had covered the derivation of this omega L, in this module we will study how to find out this omega H if the transfer function is given to you, so the transfer function that we discussed in the last module was of this form.

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$$H(s) = \frac{(s + w_{z1})(s + w_{z2}) \dots (s + w_{zn})}{(s + w_{p1})(s + w_{p2}) \dots (s + w_{pn})}$$
 for low freq response

$$H(s) = \frac{(1 + s/w_{z1})(1 + s/w_{z2}) \dots (1 + s/w_{zn})}{(1 + s/w_{p1})(1 + s/w_{p2}) \dots (1 + s/w_{pn})}$$
 for high freq resp. max.

At $\omega = 0$ $|H(j\omega)| = 1$

Now as you can see that when $S = \text{infinity}$ or tends to infinity, the magnitude of H will be $= 1$ and for when the value of S is less that is when $S = 0$ depending on the values of omega z's and omega p's, the magnitude of we can make the magnitude of H of J omega lesser so this kind of transfer function will not is not suitable for high frequency response because there does not appear to be any dip of the magnitude of H with increase in frequency.

But what if I write my transfer function in this form if you see this transfer function then what happens now at $S = 0$ at $S = J$ omega = 0 magnitude of H of J omega is $= 1$ isn't it and when S tends to high value. So this is my transfer function when $S = J$ omega tends to infinity the magnitude of H of J omega becomes $= 1$ upon omega z1 1 upon omega z2 omega zn upon 1 upon omega p1 1 upon omega p2 1 upon omega pn, you see that depending on the values of omega z's and the omega p's we can make H of J omega tending to be lower.

Now if you see fundamentally if you go back to the 2 transfer functions once again, so you see that so this is the for low frequency response and this is for high frequency response, is there anything fundamentally different between the two, no see actually they have the same structure so then how is it that this is giving or rather apparently giving a low frequency response with an omega L clearly defined and omega H not clearly defined, whereas here omega H is clearly defined but omega L cannot be defined, why is this difference?

The difference arises due to the difference in the values of the omega z's and the omega p's, that is it depending on how you design your omega z's and omega p's, you will get a different transfer function because see this is also we can just take omega z1 here and then solve for this equation and we will get a similar transfer function like this or in or we can also reduce this transfer function to a form like this so the difference is just the values of this omega z's and omega p's that is what makes the difference between high frequency response and a low frequency response.

So now let us see how we can obtain an equation or the obtain the value of omega H that we had described previously, so omega H once again is given like this it is the upper 3 db frequency okay so just like the previous case if we have a transfer function with just 2 zeros and 2 poles and then find out the magnitude square of the transfer function.

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$$H(s) = \frac{\left(1 + \frac{s}{\omega_{z1}}\right) \left(1 + \frac{s}{\omega_{z2}}\right)}{\left(1 + \frac{s}{\omega_{p1}}\right) \left(1 + \frac{s}{\omega_{p2}}\right)}$$

$$|H(j\omega)|^2 = \frac{\left(1 + \frac{\omega_H^2}{\omega_{z1}^2}\right) \left(1 + \frac{\omega_H^2}{\omega_{z2}^2}\right)}{\left(1 + \frac{\omega_H^2}{\omega_{p1}^2}\right) \left(1 + \frac{\omega_H^2}{\omega_{p2}^2}\right)} = \frac{1}{2}$$

At $\omega = \omega_H$
 $|H(j\omega)|^2 = \frac{1}{2}$

So we will get something like this, in that case okay now at omega = omega H this becomes equal to half just like the previous case in other words this becomes half at omega = omega H and then solving, so we can simply write this as instead of we can substitute omega H here and then solving for omega H what we will get is an expression like this.

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$$\omega_H \approx \frac{1}{\sqrt{\frac{1}{\omega_{p1}^2} + \frac{1}{\omega_{p2}^2} - \frac{2}{\omega_{z1}^2} - \frac{2}{\omega_{z2}^2}}}$$

ω_{p1} should be lesser than $\frac{1}{4}$ th the value of the other poles & zeros

ω_H is nearly equal to now the concept of dominant pole for a high frequency case is slightly different than that for the low frequency case in the low frequency case we saw that ω_{p1} should be greater than other poles and zeros here the dominant pole should be lesser than one fourth the value of the other poles and zeros that is if suppose this is ω_{p1} then all other poles should be at least four times away like this now this is the formula when we have 2 poles and 2 zeros we can actually generalize it for any number of poles and zeros present in the transfer function.

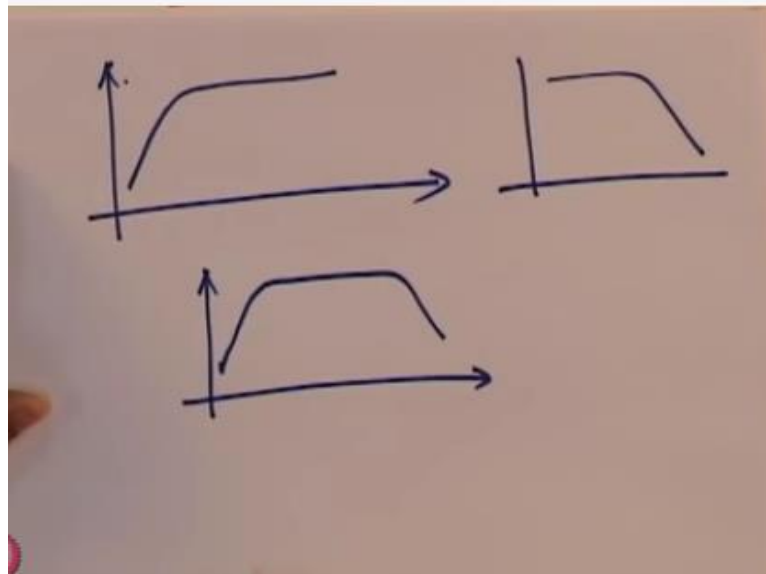
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$$\omega_H \approx \frac{1}{\sqrt{\frac{1}{\omega_{p1}^2} + \frac{1}{\omega_{p2}^2} + \dots + \frac{1}{\omega_{pn}^2} - \frac{2}{\omega_{z1}^2} - \frac{2}{\omega_{z2}^2} - \dots - \frac{2}{\omega_{zn}^2}}}$$

If ω_{p1} is the dominant pole.
then, $\omega_H \approx \omega_{p1}$

So the general formula is if this is ω_{p1} is the dominant pole then this ω_H is nearly = ω_{p1} so in other words when a dominant pole is present the upper 3 db frequency is = the dominant pole just like the case for the low frequency response when the dominant pole itself is the lower 3db frequency.

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Now taking this a bit further how to so we saw our response transfer functions for the case when our response is like this or our response is like this, so how to obtain the transfer function for the case when our response is like this.

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$$\begin{aligned}
 H(s) &= H_1(s) \times H_2(s) \\
 &= \frac{(s + \omega_{z1})(s + \omega_{z2}) \dots (s + \omega_{zL})}{(s + \omega_{p1})(s + \omega_{p2}) \dots (s + \omega_{pL})} \left\{ H_1(s) \right. \\
 &\quad \times \left. \frac{\left(1 + \frac{s}{\omega_{z,L+1}}\right) \left(1 + \frac{s}{\omega_{z,L+2}}\right) \dots \left(1 + \frac{s}{\omega_{z,L+N}}\right)}{\left(1 + \frac{s}{\omega_{p,L+1}}\right) \left(1 + \frac{s}{\omega_{p,L+2}}\right) \dots \left(1 + \frac{s}{\omega_{p,L+N}}\right)} \right\}
 \end{aligned}$$

So for that, we can simply have our overall transfer function as the product of 2 transfer functions one taking care of the low frequency response and the other taking care of the high frequency response so if the final transfer function will be something like this, this whole thing multiplied by okay so this is the part caring for the high frequency response and this is the part taking care of the low frequency response.

Of course final values will depend on how carefully we choose the values of the omega z's and the omega p's how can we find out the values of these poles by the way suppose you are given a transfer function or you are given a circuit, what is the easy way to find out these poles and these dominant poles?

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$$H(s) = \frac{1 + a_1 s + \dots + a_n s^n}{1 + b_1 s + \dots + b_n s^n}$$

$$|H(j\omega)| = 1$$

$$b_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \dots + \frac{1}{\omega_{pn}}$$

$$= \sum C_i R_{i0} \approx \frac{1}{\omega_{p1}} = \frac{1}{\omega_H}$$

$$\Rightarrow \omega_H = \frac{1}{\sum C_i R_{i0}}$$

So one method which is popular is that suppose you have a transfer function given like this, now it can be shown that this b1 is given by the poles omega p's like this okay and this in turn is given like this, now here this Ci suppose your circuit consists of a number of capacitors like this suppose this is your circuit then this Ri0 is so for each of these C the value of the corresponding Ri0 will be the value of the equivalence.

Suppose let us consider this C2, if you want to find out say R2o then R2o is the equivalent resistance across the 2 terminals of the capacitor with all other capacitors opened so you just open all C you open C1 or just remove them C1 and C3 then the equivalent resistance and of course C2 is also removed then the equivalent resistance between these 2 terminals will be the value of R2o.

Now if there is a dominant pole present now here you can see by the way that this represents the high frequency response because when S = 0 the magnitude of H of J omega is = 1 so here if a dominant pole is present which is at least 4 times lesser than the other poles and zeros then we can reduce this can be written as 1 upon omega P1 which is = 1 upon H hence omega H will be simply = 1 upon sigma Ci Ri0 ok preceding similarly for the low frequency response also we can obtain a similar formula.

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$$H(s) = \frac{s^n + d_1 s^{n-1} + \dots + d_n}{s^n + e_1 s^{n-1} + \dots + e_n}$$

for $s = j\omega \rightarrow \infty$
 $|H(j\omega)| = 1$

$$e_1 = \sum \omega_{p_k} = \frac{1}{\sum C_i R_i s}$$
$$\approx \omega_{p_1} = \omega_L$$

Suppose your transfer function is given like this, you can see that this is the low frequency response because for $S = j$ omega tending to infinity this magnitude is = 1, now we can write this e_1 is = the summation of the all the poles and this e_1 is also given by this formula.

Where if we go back to this circuit the CR2S or CRiS this value so here this CRiS which is a new constant that we found is the equivalent resistance across any capacitances with all other capacitor sorted out, if you want to find out the R2S, then what we do is we short C1 and C3 and then find what is the equivalent resistance at (()) (21:12) is the equivalent resistance across the terminals of this capacitor C2 with all other capacitors that is capacitor C1 and C3 shorten and then if omega p1 there is a dominant pole present then that will be at least four times larger than other poles and zeros this will be nearly = omega p1 which is = omega L.

So this is how we find the lower cut off frequency from the circuit and this formula this formula we use for finding out the upper cut off frequency. In this module we covered the high frequency response of an opamp, we also found out the formula for obtaining the values of the upper cutoff and lower cutoff upper 3db and lower 3db frequencies and how if we are given a circuit, how we can find out based on the resistance and capacitance values. In the next module we will be covering an example that will illustrate these concepts better, thank you.