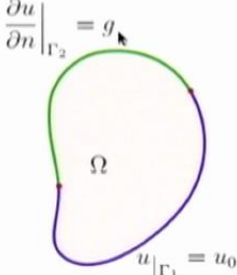
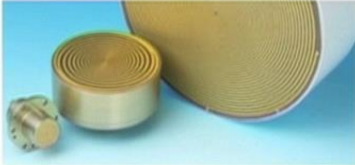




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## MOTIVATION

COMPLEX COMPUTATIONAL DOMAIN  
MIXED-TYPE BOUNDARY CONDITIONS  
(BCs)



$\frac{\partial u}{\partial n} \Big|_{\Gamma_2} = g$

$u|_{\Gamma_1} = u_0$

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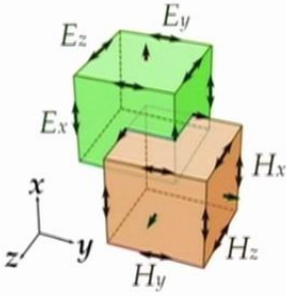
Highlighting some of the limitations and challenges of analytical methods.

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## BACKGROUND

**A. Thorn** (1920s) “the method of squares” for non-linear hydrodynamics equations

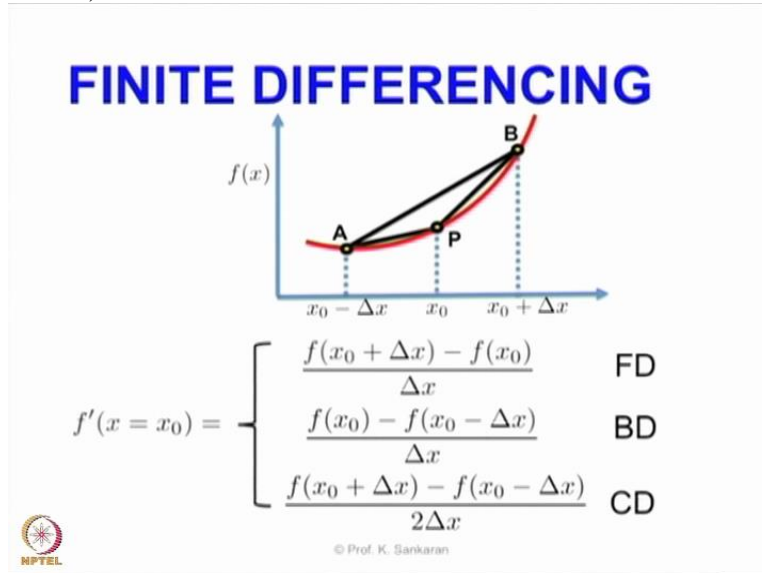
**K. S. Yee** (1966) used two staggered Cartesian grids for Maxwell equations



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As the first numerical method we introduced the finite difference method getting its historical background.

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We discussed some of the basic finite differencing schemes like the Forward, Backward and Central Differencing schemes.

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## FINITE DIFFERENCING

### Finite difference eqn using Taylor series

$$f(x_0 + \Delta x) + f(x_0 - \Delta x) = 2f(x_0) + (\Delta x)^2 f''(x_0) + \mathcal{O}(\Delta x)^4$$

$$f''(x_0) = \frac{f(x_0 + \Delta x) - 2f(x_0) + f(x_0 - \Delta x)}{(\Delta x)^2} + \mathcal{O}(\Delta x)^4$$




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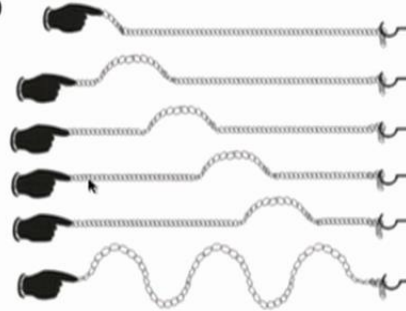
Building on these basics the lecture 2 showed how one could use Taylor series to derive finite difference approximations for first and second order differentials of a function.

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## FINITE DIFFERENCING

Now look at a 1D  
wave equation

$$k^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$$



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We briefly introduced the concept of order of truncation error in Taylor series approximation.

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## FINITE DIFFERENCING

Role of aspect ratio

$$r = \left( \frac{k\Delta t}{\Delta x} \right)^2$$

$$0 < r \leq 1$$

Case 1  
 $r < 1$

Case 2  
 $r = 1$

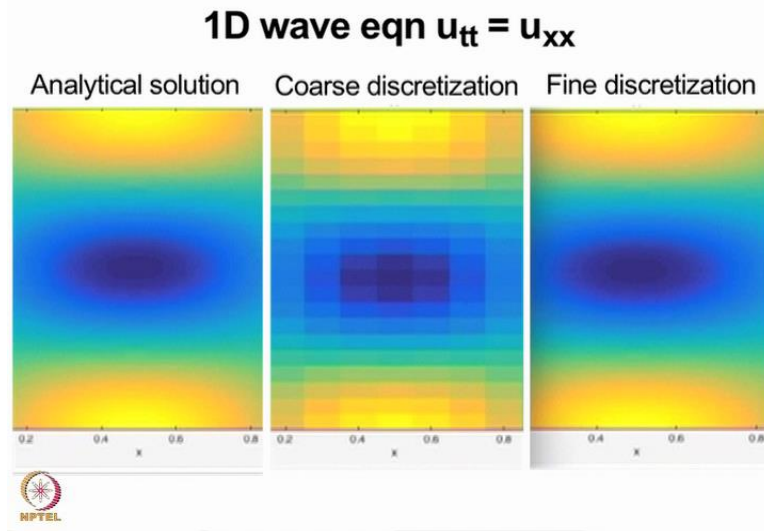
$$u(i, j + 1) = 2(1 - r)u(i, j) - u(i, j - 1) + r[u(i + 1, j) + u(i - 1, j)]$$



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In lecture 3 using the example of one dimensional wave equation we discussed the role of aspect ratio and the effect of special discretization in the numerical solution.

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Later we introduced some general problems to study the efficacy of finite difference schemes that we have introduced earlier.

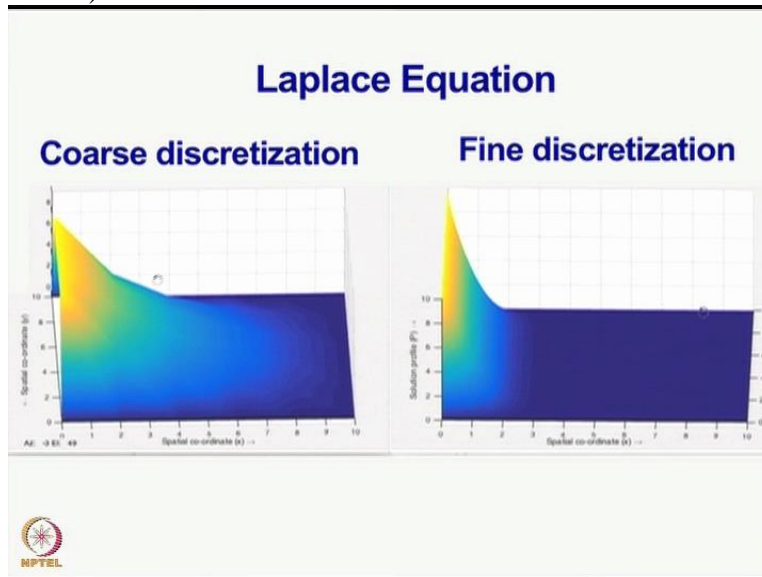
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**Laplace Eqn with 5-point CD scheme**

The figure shows handwritten notes on a whiteboard. At the top, it says 'Laplace Eqn with 5-point CD scheme'. Below this, there are several equations and a diagram. The first equation is  $\nabla^2 \phi = 0$ . The second equation is  $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$ . To the right of this equation is a diagram of a 5-point stencil, showing a central point  $(i, j)$  and its four neighbors:  $(i-1, j)$ ,  $(i+1, j)$ ,  $(i, j-1)$ , and  $(i, j+1)$ . Below the diagram, there are two equations for the second derivatives:  $\frac{\partial^2 \phi}{\partial x^2} \approx \frac{\phi(i+1, j) - 2\phi(i, j) + \phi(i-1, j)}{\Delta x^2}$  and  $\frac{\partial^2 \phi}{\partial y^2} \approx \frac{\phi(i, j+1) - 2\phi(i, j) + \phi(i, j-1)}{\Delta y^2}$ .

We started with the 2D Laplace equation using the 5 point central differencing scheme.

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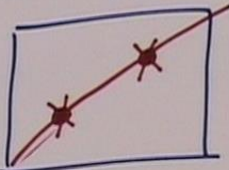
We simulated this problem and investigated the apex of iteration and special discretization on the numerical solution.

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### Poisson Eqn with two point sources

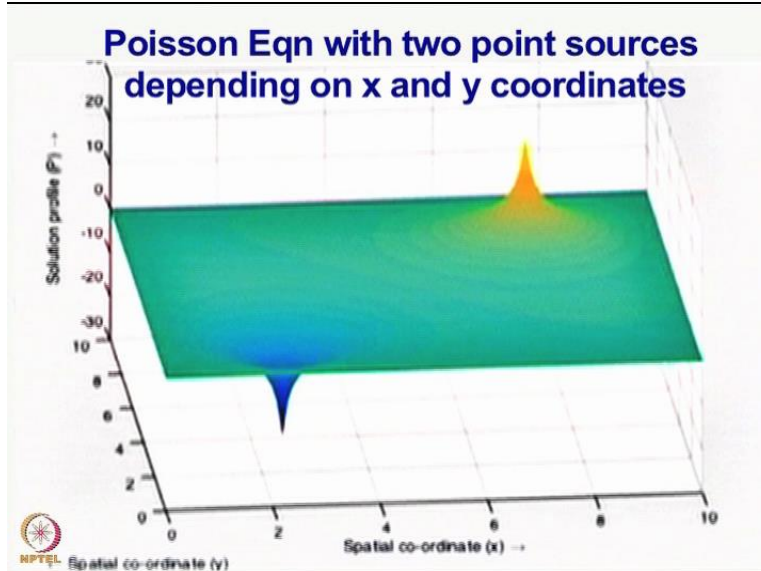
$$\nabla^2 \varphi = \frac{\rho}{\epsilon} = f(x, y)$$
$$f\left(x = \frac{n_x}{4}, y = \frac{n_y}{4}\right) = 3000 \frac{\text{V}}{\text{m}^2}$$
$$f\left(x = \frac{3n_x}{4}, y = \frac{3n_y}{4}\right) = -3000 \frac{\text{V}}{\text{m}^2}$$

$n_x = \# \text{ steps in } x$   
 $n_y = \# \text{ steps in } y$



Building on this exercise we introduced Poisson equation in the second example. Instead of having a constant source term or the right hand side we made it slightly more interesting by making the source dependent on x and y coordinates.

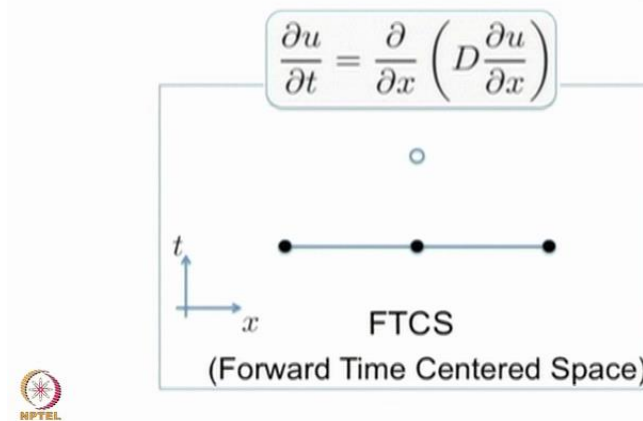
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Here again we saw the impact of special discretization on the numerical solution.

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## HEAT DIFFUSION PROBLEM




Example 3 was a more general physics problem involving Heat Diffusion Equation. The purpose of introducing this particular problem in this course is to illustrate how one can combine different finite differencing scheme to model any practical problem here we used forward in time and centered in space scheme to model the heat equation.

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### Matlab code for heat diffusion eqn

```
ExplicitMethodAdvection_Eqn.m | LaxMethodForAdvectionEqn.m | HeatDiffusion1DExplicitMethod.m
n=50; % Number of space steps
nint=50; % The wave-front:intermediate point from
% which u=0(nint<n)!!
dx = L/n;
cond = 1/4 %Conductivity
b=2.*cond*dt/(dx*dx); % stability parameter (b<=1)
% the stability condition depends more on solution propagating
% from the point (n,j), hence we use b = 2r for the stability
% parameter.

%Initial temperature of the wire- a sinus
for i = 1:(n+1)
    x(i) = (i-1)*dx;
    u(i,1)= sin(pi*x(i));
end
```



We briefly discussed the role of stability parameter in the numerical simulation. Of course we will discuss more about the stability parameter in the coming weeks.

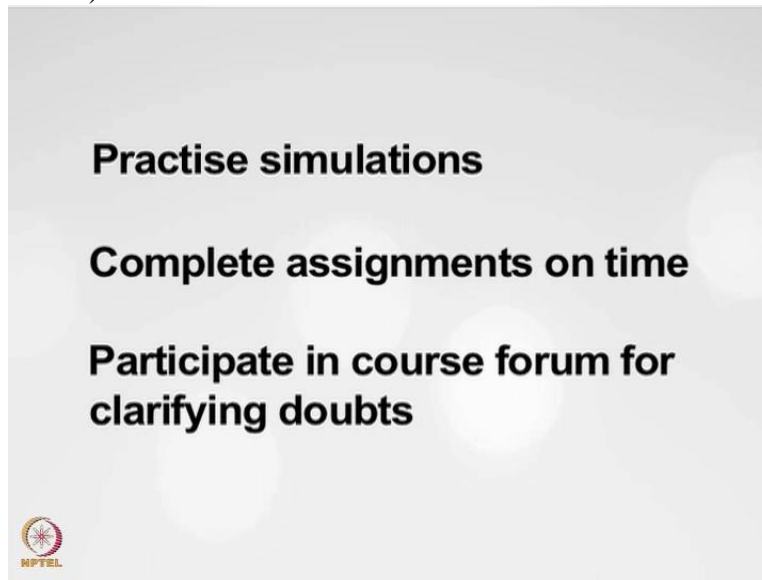
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Last but not the least this week we visited different labs to introduce some basic Electromagnetic devices. Some student might have not seen even an antenna or a wave guide, so we took this opportunity in this lecture to introduce these devices which we shall be modeling the course.



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So please look into the exercises which we have discussed in this week and also try simulating these problems for yourself to gain some experience in modeling. Also do not forget to complete the assignments and submit them in time. If you have particular questions or comments please use the course forum to post your questions and my teaching assistants will do their best to support you. There are lot of interesting topics to cover in the coming weeks. So we hope you are excited and motivated as much as we are.

So see you next week. Good Bye!