Computational Electromagentics and Applications
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Lecture No 42
Algebraic Topological Method (ATM III)

So you have been able to stay this longer the Algebraic topology. I know that the kind of material we are covering is quite unusual to a Engineering literature but still I think once you see the entire Maxwell equation in the way I promised without vector calculus or differential equation will be a good relief in itself. With that promise I am going to take you to the of the Algebraic topology which will be on the time orientation and the concept of Maxwell equation itself model using the different tools which we have created.

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So with that let us start the Time orientation.

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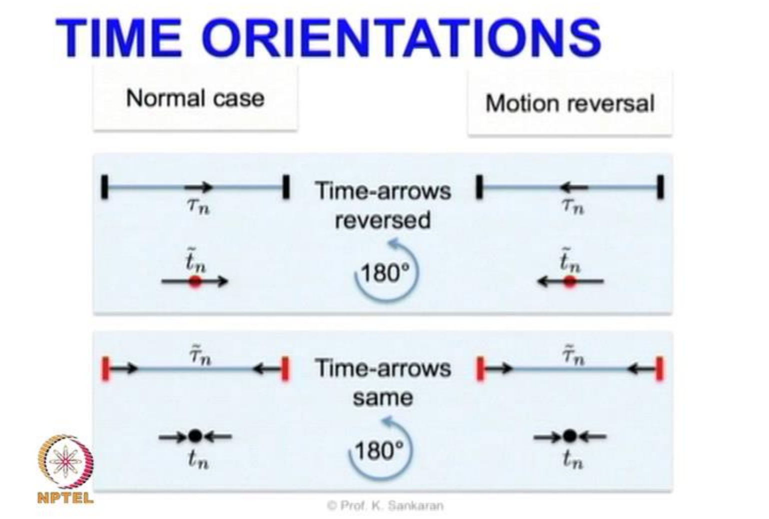

So what do I mean by time orientation. This slide might not make any sense before I give you a little bit of a story from mechanics. So let us take an example.

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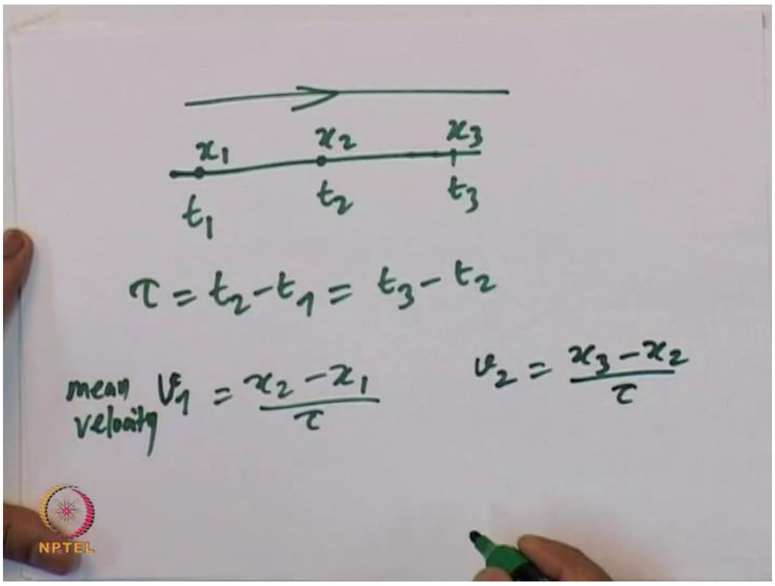

Let's assume that we are talking about a body that is moving from left to right and let's say that it is at time instant at time t is equal to 1 at point x 1 at time T2 it is at X 2 and at time T3 it is at X3 so what we have got is time duration let's say we call it Tou is equal to T2 minus T1 and let's also assume is equal to t 3 minus T2 in other words we are talking about the time duration that is equal between these two observations.

So based on that we can compute the mean velocity while the body is moving in this direction at this point between X2 and X1 so the mean velocity we write V 1 is equal to X 2 minus X 1 divided by Tou so this is the mean velocity. Let's say we are computing the mean velocity between point X2 and X3 so it will be given by this is V1 and this is V2. V 2 is equal to X 3 minus X 2 divided by Tou.

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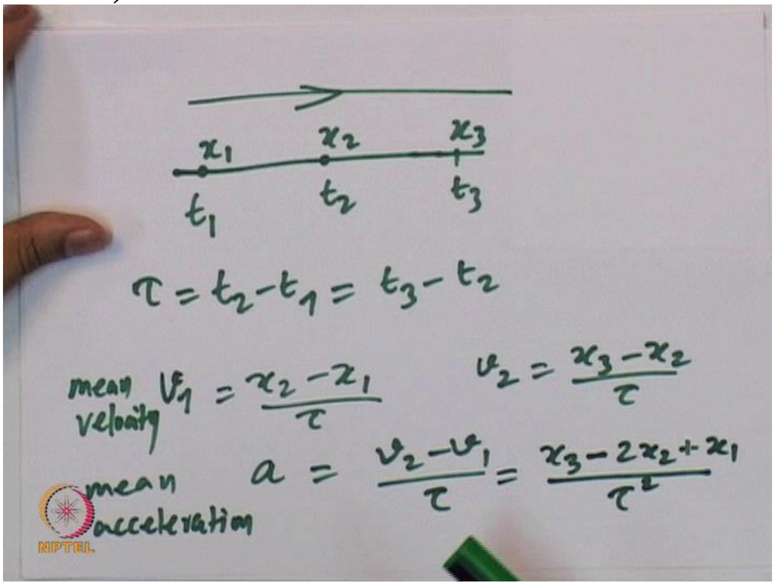

So for these two points we can also compute the acceleration based on these two things because we have got to velocities so we can compute the acceleration in other words we can say mean acceleration is equal to V2 minus V1 divided by Tau. If you substitute the value of V2, V2 and V1 you will get an expression for acceleration which is X3 minus 2 X 2 plus X 1 divided by Tou square. So this is the basis to for us to talk about something very unique about time which normally in engineering literature we don’t make use of . So what we are going to look at is we have got an expression for mean acceleration and mean velocity.

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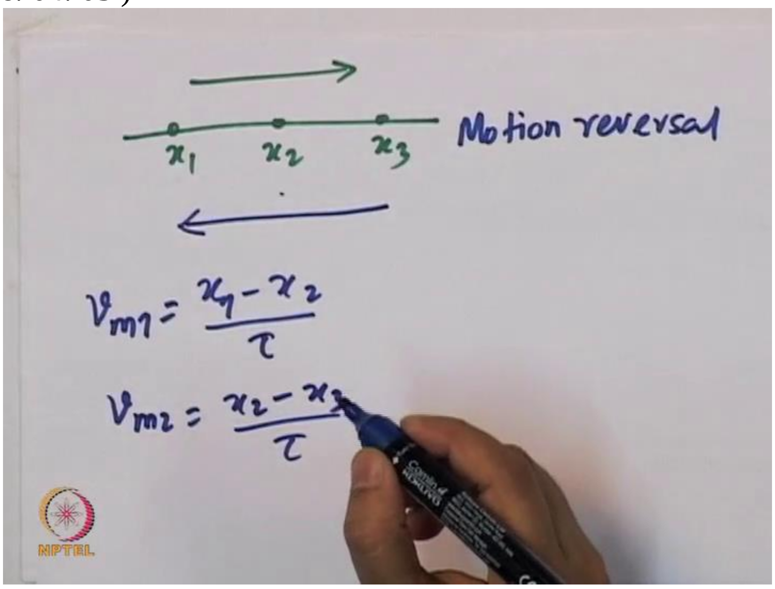

We said we are going from point X1 X2 and X3. So we are going like this so now what I am going to do is I am going to reverse the motion so motion is going to be reversed so in other words we are going to change the order in which the points are going to be seen in the order the objects are moving in other words X2 will be our starting point and we are moving towards X1 so this is the concept of motion reversal so when we do the motion reversal what we are doing is we are kind of going in the opposite direction to that of the formerdirection88 so the order in which the object is going to see the things will be different so it will see at X3 T2 it will cx2 and T1 it will see X 1.

So then that is been happened let's see what are the values of v and acceleration so when we compute that let's say we are computing under the motion reversal I am using the word motion reversal and the first one will be x 1 minus X 2 if I say this is going to be the motion reversal velocity so it will be X 1 minus X 2 divided by Tou because in this case the object is going to go fromX2 to X1. similarly we will have V M 2 is equal to here in this case initially we call this side the object is going from here to here in computed the VM2 . Now we are going to compute the VM to when the object is going to go from here to here so in other words we will get X2 minus X3 divided by Tau .Nothing has happened to Tau. Tau remains the same.

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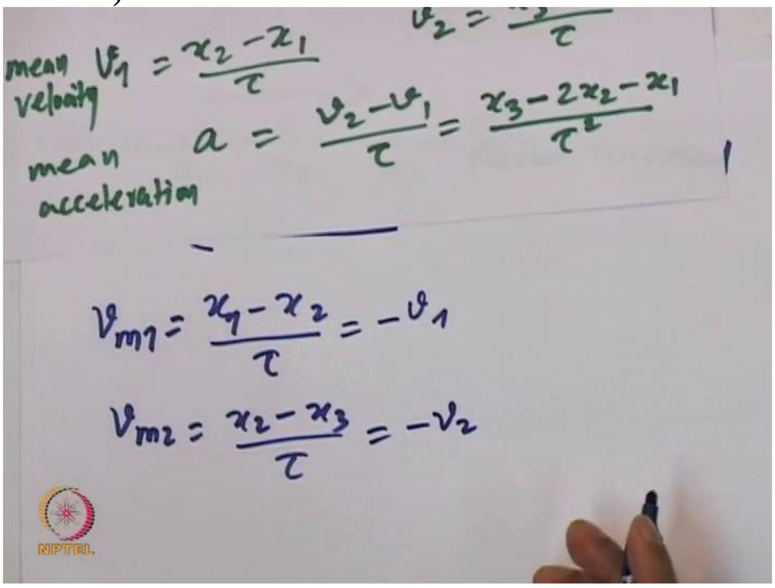

So what we are getting here is when you compare this one to the value that we have computed here we are able to see that this value and this value are opposite to each other. We have X1 minus X2 whereas here we have X2 minus X 1. So this is we can write is equal to minus V1 and this is equal to minus V2.

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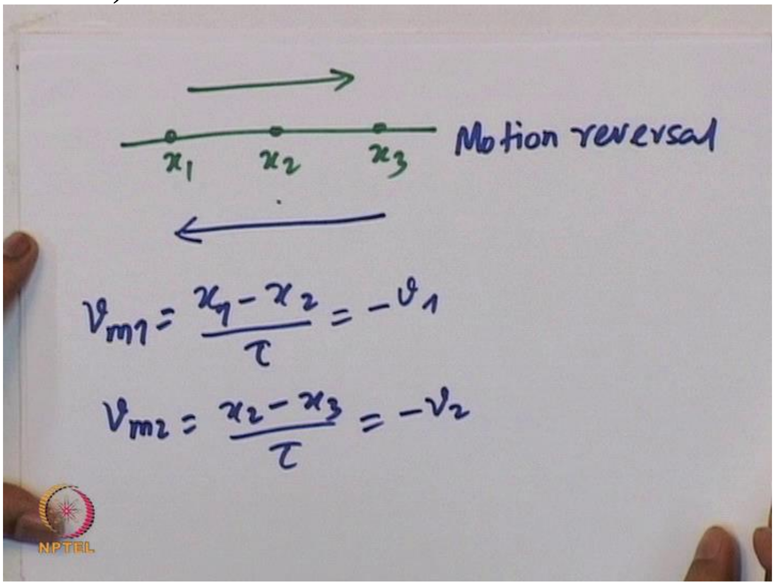

So this is about motion reversal. so let's consider what happens when we don't do motion reversal and we do just time reversal so time reversal is nothing but just purely mathematical thing it has nothing to do with any physics motion reversal that is a physical meaning instead of going in One Direction you are going in the opposite direction where is time reversal is purely mathematical where you are putting T instead of minus T. So let's put the thing and see what is happening to velocity and acceleration.

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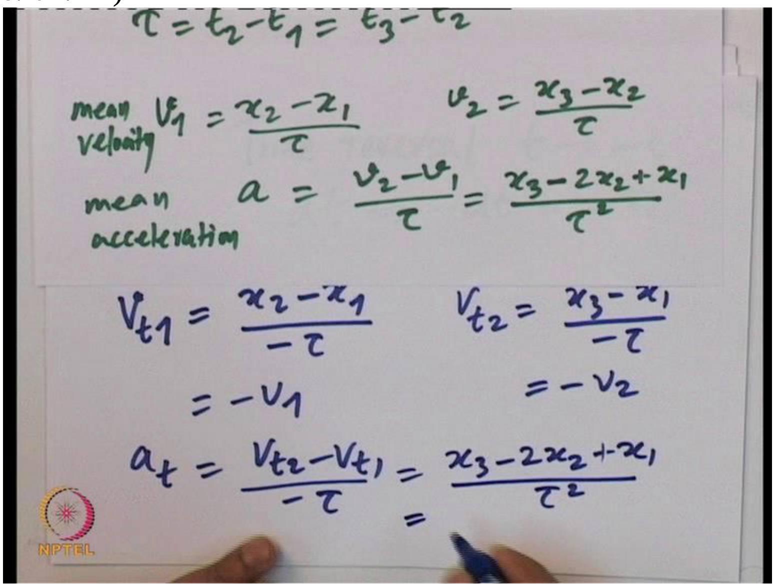

So what we have got now is we said we called Time reversal is T becomes minus T so when you have Delta T where you have to use the word minus Delta T in other words what we will have is minus how so what's happening now is the value we call velocity t is for time reversal will be equal to X2 minus X1 like before divided by minus 2 instead of Tau we will have minus Tau. similarly VT to time reversal is equal to this we can see from here instead of Tau

I am using minus Tau so similarly for VT2, I put X 3 minus X 1 instead of Tau I use minus Tau. So these are nothing but minus of V1 this is equal to minus of V2 let's see what's happening to acceleration so acceleration under time reversal is equal to VT2 minus VT 1 divided by minus Tou. As you can see this minus Tou will cancel off with the minus Tou we have on the numerator so what we will get is nothing but X3 minus 2X2 plus X1 divided by Tou square which is nothing but the same expression we have got in the initial case without any time reversal which is a.

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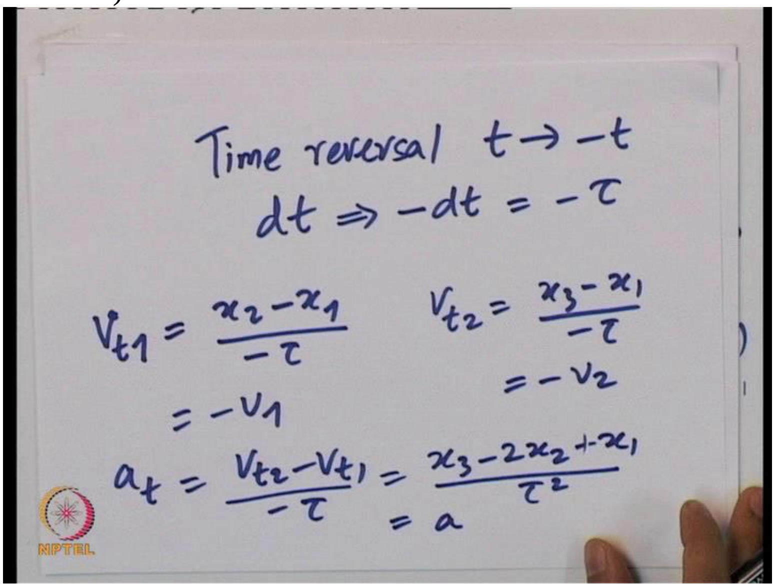

So what is important to see is under time reversal and motion reversal the value of the velocity changes to minus whereas the acceleration doesn't change its sign under Motion and Time reversal. So that being said there is something unique about the time that we are associating to velocity and acceleration.

So we cannot associate the same time for acceleration and velocity because we saw when we do motion reversal the value of velocity before motion reversal and after motion reversal they are exactly opposite similarly when you do time reversal they are also opposite where as acceleration doesn't care. It doesn't change when you do motion reversal or when you do time reversal . So there is something unique about the way in which acceleration is associating itself with time so that gives as a kind of a motivation or that time we are talking about is not the same time when it comes to velocity it is associated with one particular time when it comes to acceleration it is associated with one particular time and that is what we call it as inner and Outer orientation of time .

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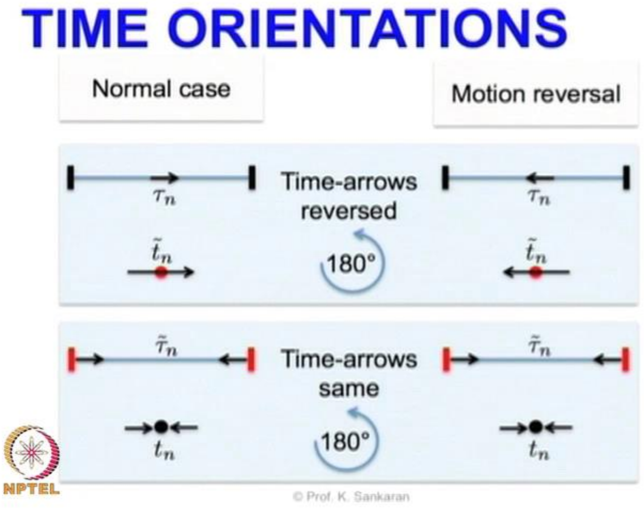

So here you will see in this case in the slide I have a normal case where I have a time duration and I do the motion reversal so my duration is oriented in the opposite direction but still the time duration will be in the same limit orientation is going to change whereas when you see the inner orientation in the time duration this is the inner oriented one and this one if the outer orientation show the inner oriented time duration changes direction so it goes from this side to outside whereas outer oriented time duration is pointing inside and then when you turn 180 degree it will still be the arrows will still point inside so this is what we call it as inner and Outer orientation of time. So the inner orientation of time will be always oriented towards that point.

So for example if we call tn as the time instant the inner orientation of time when you rotate the time to be the same so it always be the same point so for example if we call tnf the time instant the inner orientation of time when you rotate the time it will be the same so it always will be at the same point the arrows doesn't change direction where as in the case of the outer oriented time the time duration is going to change direction the time instances at the point along which it acts along which it acts is also going to change direction.

So this is the motivation that we have to associate the velocity to a kind of time orientation and the acceleration to other type of orientation. In this slide what you can see velocity will be associated with this time orientation which we call it as outer orientation. And the acceleration is going to be oriented to the inner orientation where even if you change the direction it doesn't make a big difference so this is the motivation for us to understand that time also has two different orientations.

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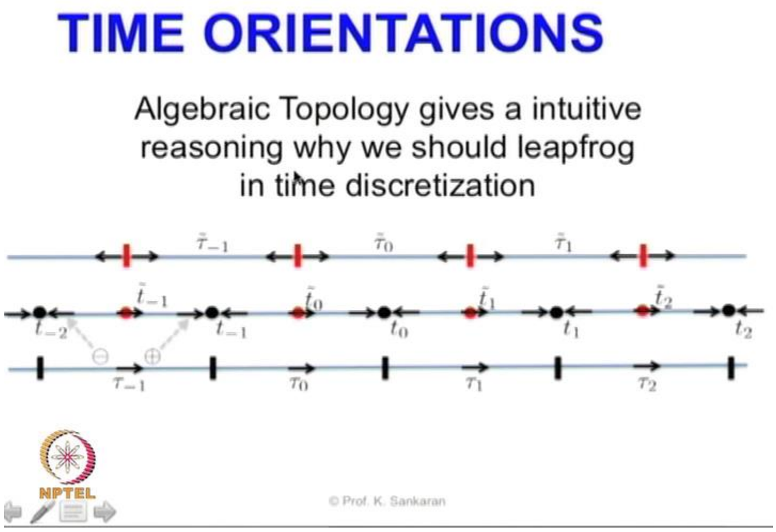

Now we will see the one of the main reason why we are doing leapfrogging you remember you remember in the case of doing finite difference time domain we did leapfrogging in time we never asked the question why are we doing leapfrogging. The leapfrogging is basically having to set of time differences time durations that are called as inner and Outer orientations one of them will be called as Primal time the other one will be called as the dual time. So the Primal time where the component can change even if you change the direction the components will not have any fact or the outer orientation when you change the directions are obtained will have certain changes like we saw in the case of velocity versus acceleration .

So this slide you can see algebraic topology algebraic topology gives you a intuitive reasoning why we should do leapfrogging in time discreditisation because the electric field and the magnetic field they are not the same kind of variables one of them when you reverse the direction changes time where as the other one does not. So that is why you have to use them in 2 different time instances so I have given you now almost everything that you need to know about algebraic topology except one thing we have not yet modelled the Maxwell equation the way I promised you before I said we are not going to need any vector calculus and partial differential equations and that is exactly what we are going to do now so let’s go into the Maxwell equation with the theoretical framework what we have got.

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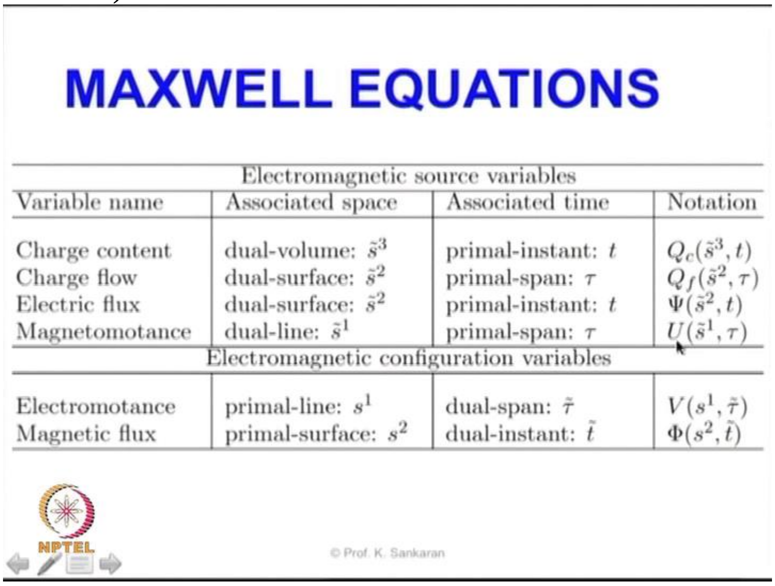

Electromagnetic source variables			
Variable name	Associated space	Associated time	Notation
Charge content	dual-volume: $\tilde{s}^3$	primal-instant: t	$Q_c(\tilde{s}^3, t)$
Charge flow	dual-surface: $\tilde{s}^2$	primal-span: τ	$Q_f(\tilde{s}^2, \tau)$
Electric flux	dual-surface: $\tilde{s}^2$	primal-instant: t	$\Psi(\tilde{s}^2, t)$
Magnetomotance	dual-line: $\tilde{s}^1$	primal-span: τ	$U(\tilde{s}^1, \tau)$
Electromagnetic configuration variables			
Electromotance	primal-line: s^1	dual-span: $\tilde{\tau}$	$V(s^1, \tilde{\tau})$
Magnetic flux	primal-surface: s^2	dual-instant: $\tilde{t}$	$\Phi(s^2, \tilde{t})$

So we will see we have got various variables charge content written as Qc charge flow written as Qf electric flux written as Psi magnetomotance written as capital U electromotance written as capital V and magnetic flux written as capital Phi. So what we are seeing now is all these variables are scalar quantities like I said before and they have to be associated with certain special orientation and a time orientation .

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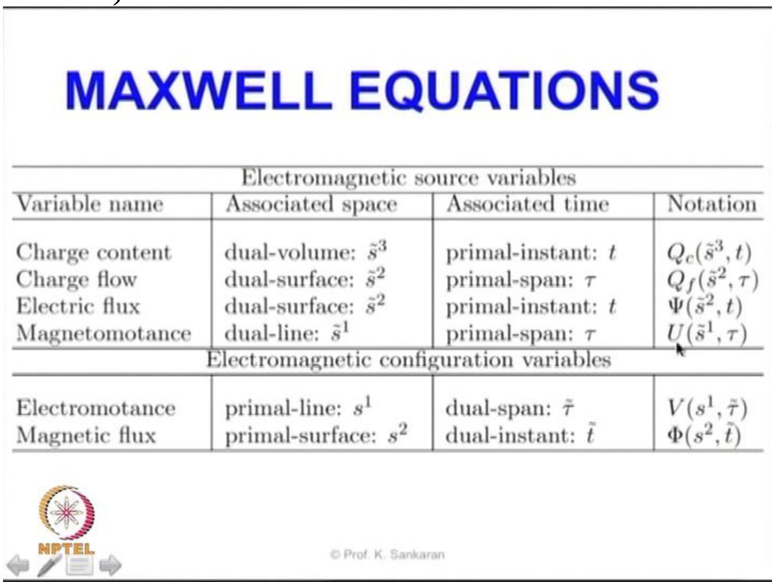

Electromagnetic source variables			
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Charge flow	dual-surface: $\tilde{s}^2$	primal-span: τ	$Q_f(\tilde{s}^2, \tau)$
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As you can see age charge will be oriented with duel volume and the Primal time instant likewise the charge flow will be oriented with dual surface and the Primal time duration show all the things we call as core variables related to either electric or magnetic forces charge content charge flow magnetic flux Magnetomotance they are all connected to the dual volume and the Primal time the volume the surface And The Line aspect are in the dual phase for the source variables whereas the time for current are all in the Primal stage there are no fielder time T is a time instant Tau is a time span of the time duration likewise we have got the configuration variables where we call them electro motors and magnetic flux they are related to the Primal space and the duel time coordinates. So V is measured on the Primal line and the dual time span. Similarly the magnetic flux is is calculated on the Primal surface and dual instant. With this as a motivation we are going to do now the Maxwell equations one by one.

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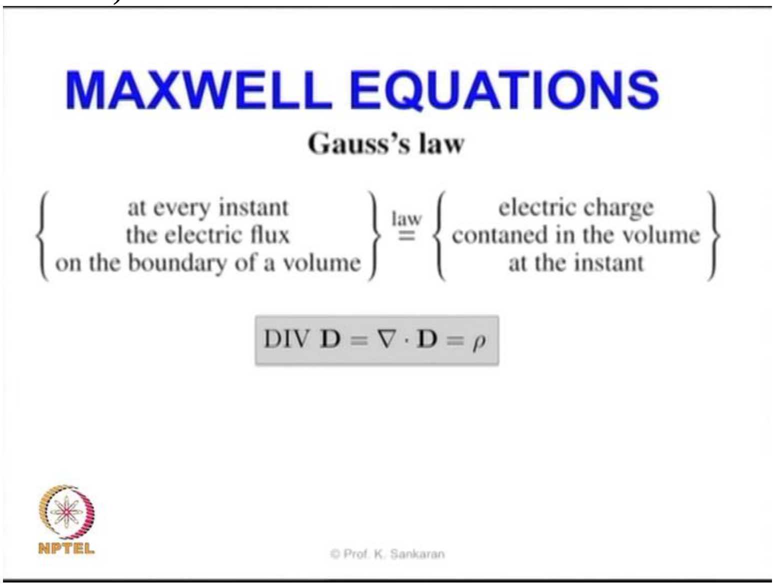

So let's start with the easiest one the Gauss's law. We have got now the divergence of d is equal to Rho.

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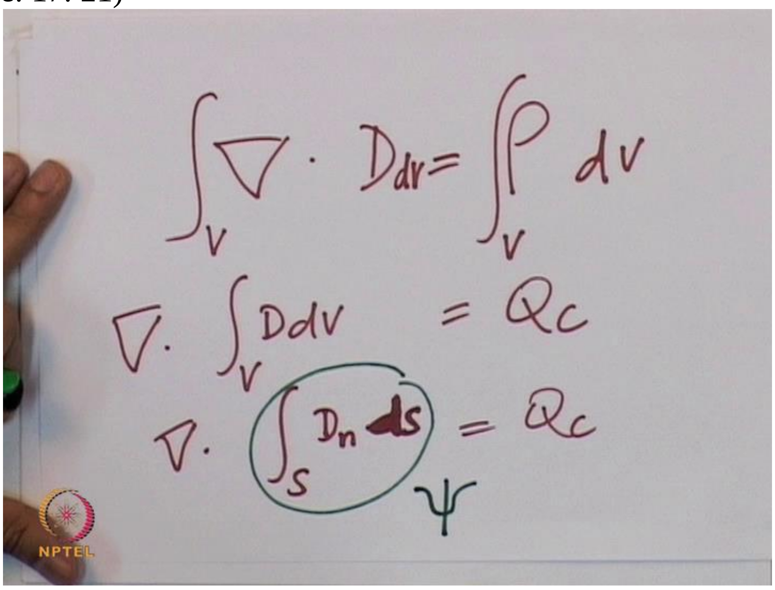

So let's write this one I am going to do the volume integration on both sides so I do so I am just integrating using the volume inside which interested in doing this so by doing this I know from the previous case this thing the integral Rho vdv is nothing but the charge content so this is what we did in the first module on algebraic topology this is the definition of the charge content which is integral Rho EV so if you see row and DVR related to this side on the left hand side what we have got something that we need to do a little bit more of thinking so what we are going to do now is we take the divergent outside and we have got volume integral DV we know that we can change this side from a volume integral to surface integral using the spokes theorem so what we are getting now is the surface and the normal component of the flux that are passing through the surface so this is the normal component and we are interested in the surface so this is nothing but we can write this as equal to QC and we can see how this is related to because this term is something that we already saw we know that this term is our definition for the electric flux.

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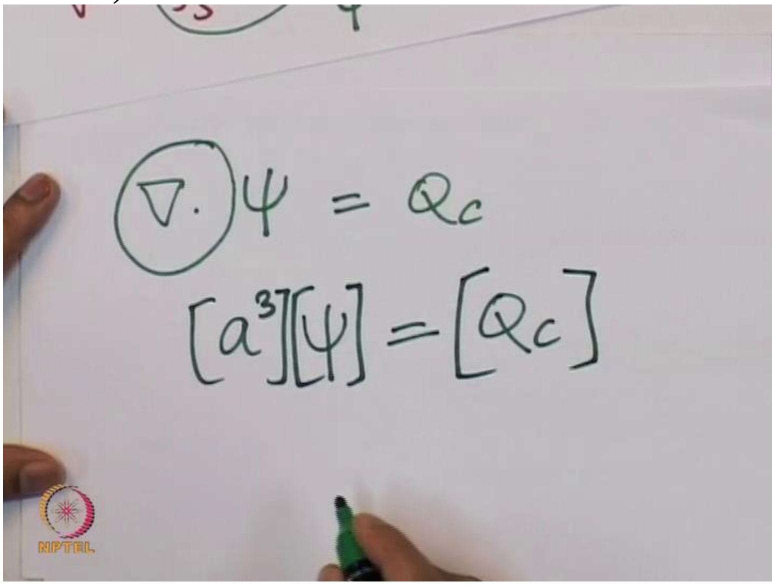

So we can write this term in a much more simpler form that is divergence of the flux is equal to QC but using our earlier understanding that this term can be represented using the core boundary operator. we said the divergence will be the third core boundary operator so what we have got is A 3 so these are all matrices.

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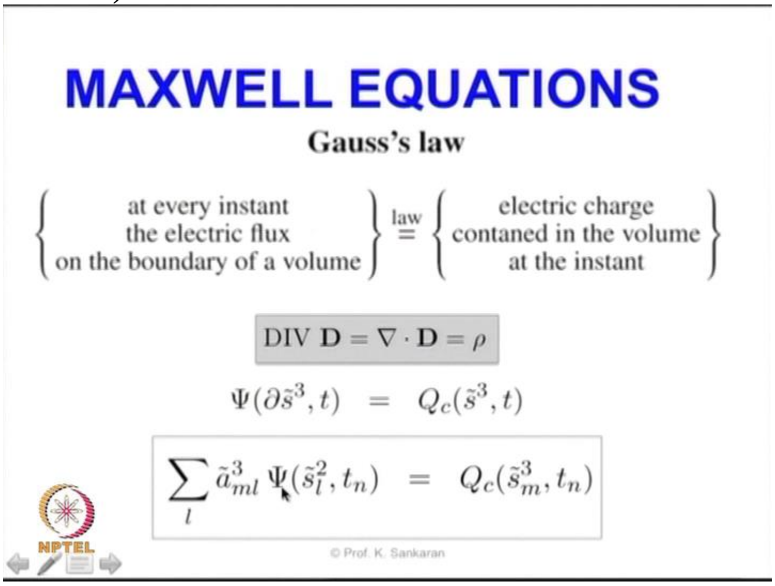

So that is what we have got in this case can you see the slide here what we have got here is This term is the A3 matrix and since the electric flux what we are calling here electric flux .

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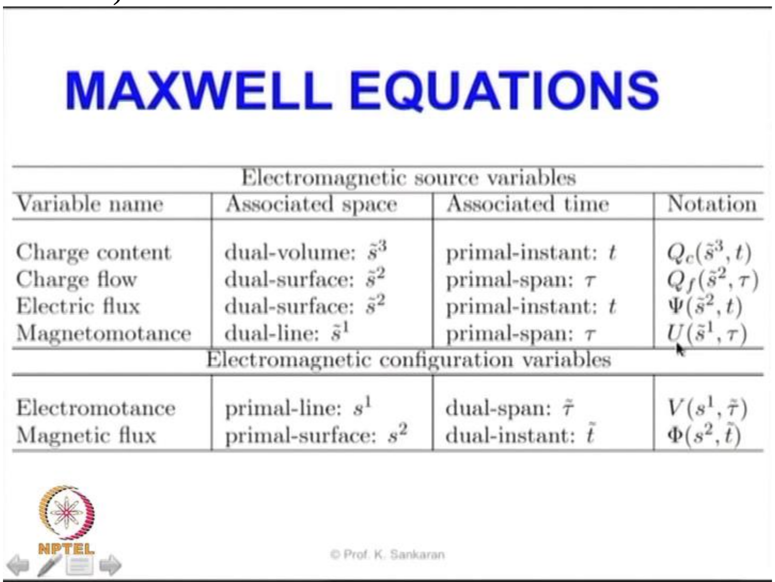

In the previous slide we said it is associated to the electric flux is associated to the duel surface since it is in the dual space we are going to have a tilde on the top and that's what we are having here.

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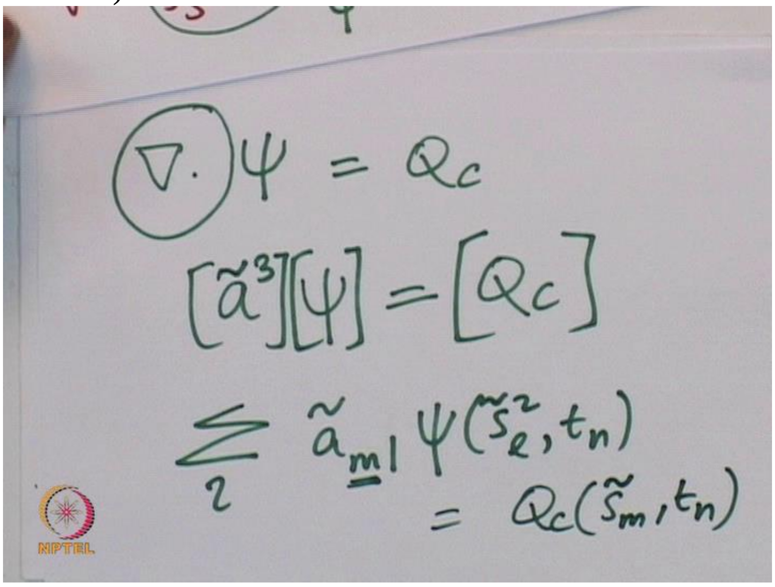

So we look here in the slide this is the dual space because this is related to the dual space and they are all related to the dual space and that's what you have if you write it in the individual farm this is the Matrix form and the individual form will be the sum of over all the sides of the volume so l will be a chef let's say MLA M is the volume we are talking about and then we will have 3 that is defined on that particular surface since it's a dual I put a dual sign and is it is associated with the Primal time instant so what we are talking about here is this is the right hand side is the same one what we have got here is these are the sum of all the surfaces that are in closing the volume so when we do this we are getting the same equation and this is equal to QC it is defined on that control volume and this control volume is in the duel space and is calculated with the Primal time so from the divergence equation here we are able to see how we have come to this form and this is the Maxwell equation for the Gauss law only using algebraic topology without vector calculus without any differential equations.

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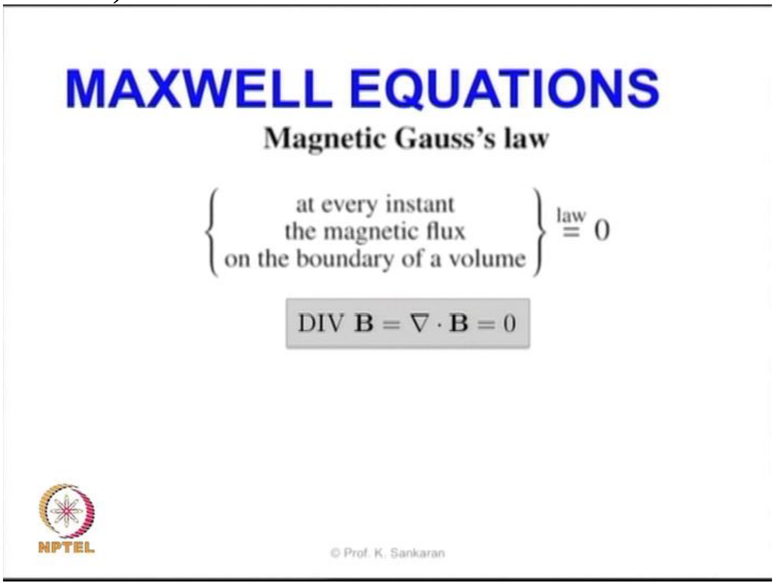

So now let us look at what we have got for the magnetic Gauss' law so right now we are looking to the electric component.

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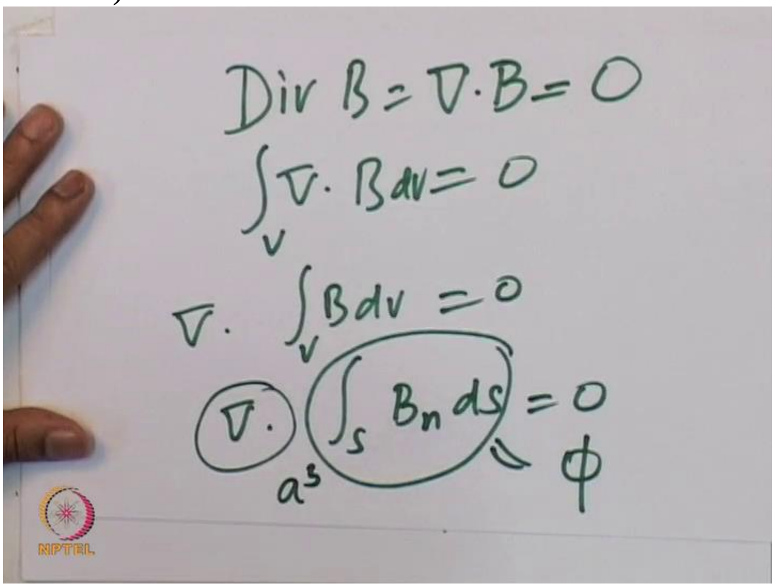

So let's look at the magnetic Gauss' law it's very simple and straight forward we have got like before the divergence of B so when you do the volume integral on both sides we have got so I am going to take out the divergence outside and then I am going to talk about integral bdv on the volume will be equal to zero and I can change this from a volume integral to a surface integral normal component of the b on the surface is equal to zero and this term is my definition for the magnetic flux and this one I know is the definition for my A3'

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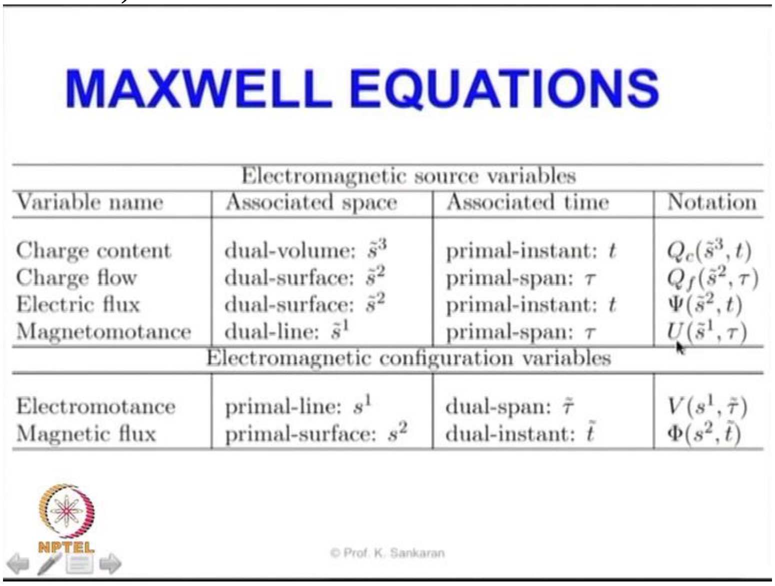

MAXWELL EQUATIONS

Variable name	Associated space	Associated time	Notation
Electromagnetic source variables			
Charge content	dual-volume: $\tilde{s}^3$	primal-instant: t	$Q_c(\tilde{s}^3, t)$
Charge flow	dual-surface: $\tilde{s}^2$	primal-span: τ	$Q_f(\tilde{s}^2, \tau)$
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And since they are belonging to the primal grid you remember in this case the magnetic flux is in the Primal surface so I am not going to use tilde there it is going to be without a tilde.
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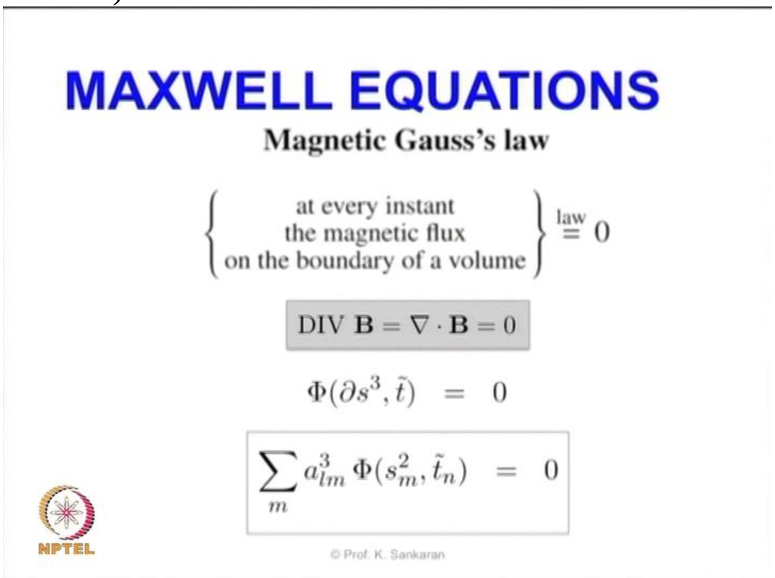

And the time instant is always with the tilde they're sore that's why we will get now the counterpart will be a l m and without a tilde and then free which is also without a tilde the counterpart will be the time duration which is in the dual space so this will give you the Maxwell equation for the magnetic Gauss's law without using vector calculus and as I promised without using differential equations.

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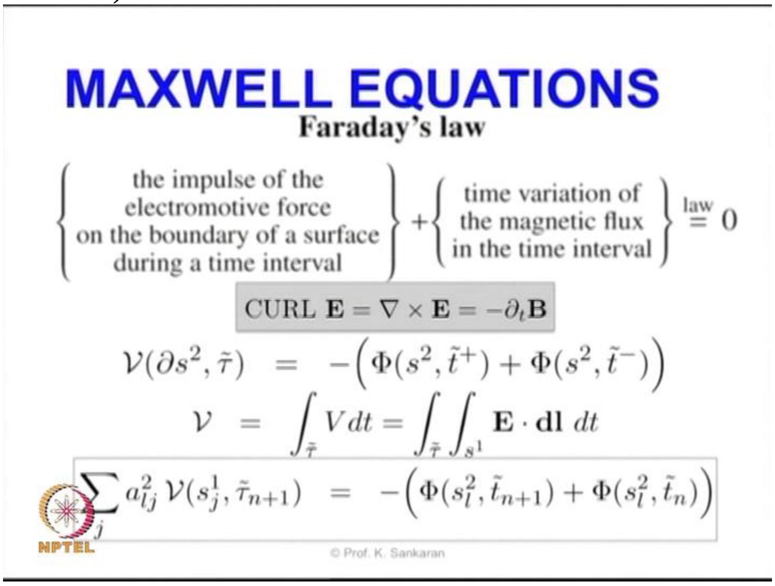

So what we have got now is we have got the two equations namely the Faraday's law the case of ampere's law.

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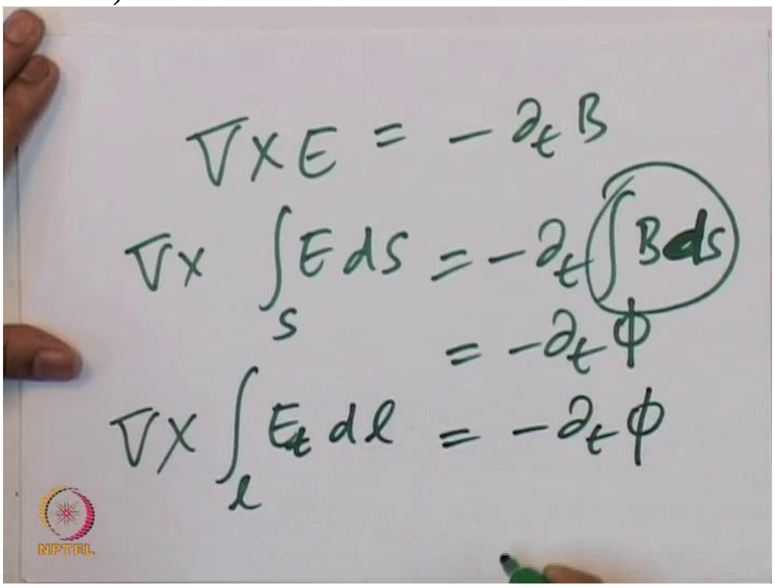

So let's now look at the Faraday's law so the Faraday's law is the curl of E is equal to minus Db by dt. I am going to do on either side a surface integration so I'll take the call out I do e DS around the surface m i n u s DT by DBds.

So what I have got here is the flux from this I know this is the definition of flux magnetic flux on the right hand side I can use the Spoke's theorem to convert the surface integral into a line integral so what I got here is a curl will become the line integral and I will be talking about the tangential component of the and DL is equal to minus DT by D and I know this one is my definition for the electromotance.

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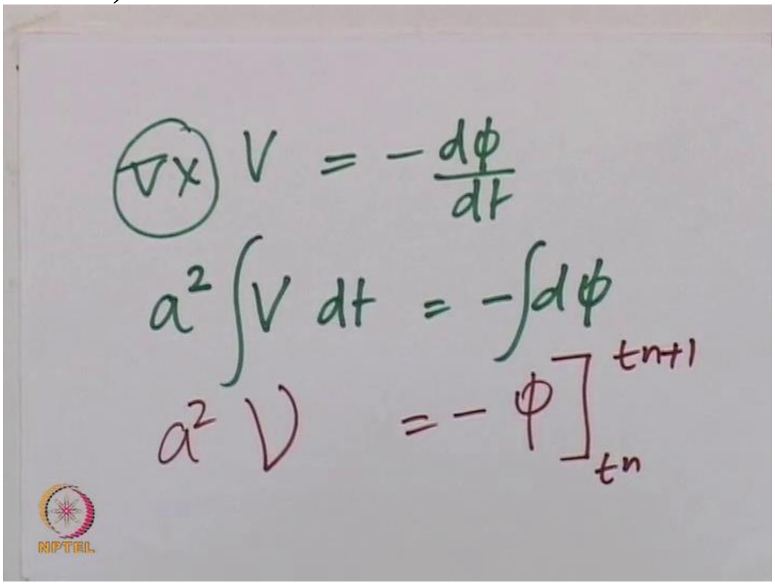

So what I am going to have now is I am going to have simple equation that will give me that the curl of the value v is equal tominus DT by DB so I can write this one as d phi d t. And This term will be the counterpart of the curl. Curl will be the second co boundary operator and we have got v and I'll take the DT on the other side so I will have DT here which will be equal to minus DPhi. So What you have got now is I have to integrate it on both sides to get the equation only in terms of fee so I do the integration so when you do the integration what you have got is you have got a volume integral of V DT so the volume integral let's write it as v with a inclind v and this is a 2 and this is equal to the value that we are computing at two different points for time TN and time TN + 1 so long we will be integrating this is what you get.

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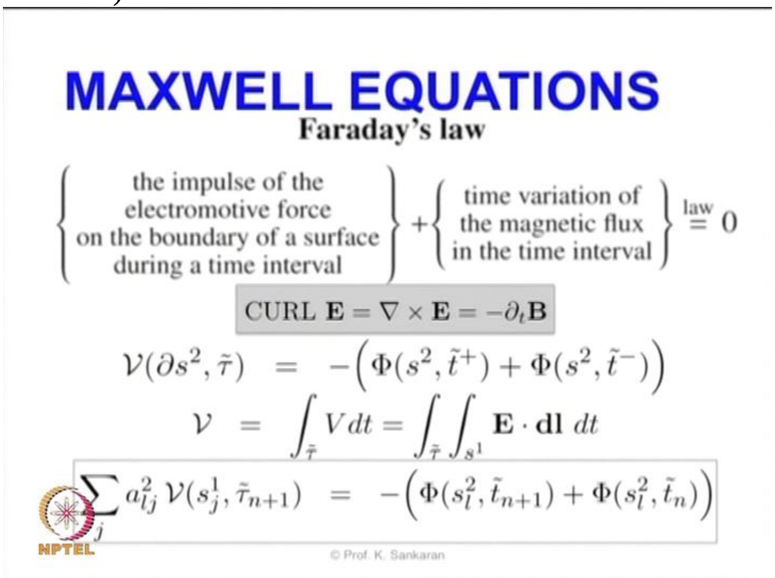

And that what you are got in this equation that you have got the right hand side which is basically the curl formulation and then you have got the time integration of V which is here and the minus sign Phi magnetic flux that are calculated at two different time instances . Time instance Tn plus 1 and Time tilda so this is the equation that has no again no vector and only using scalar quantities and no differential equations.

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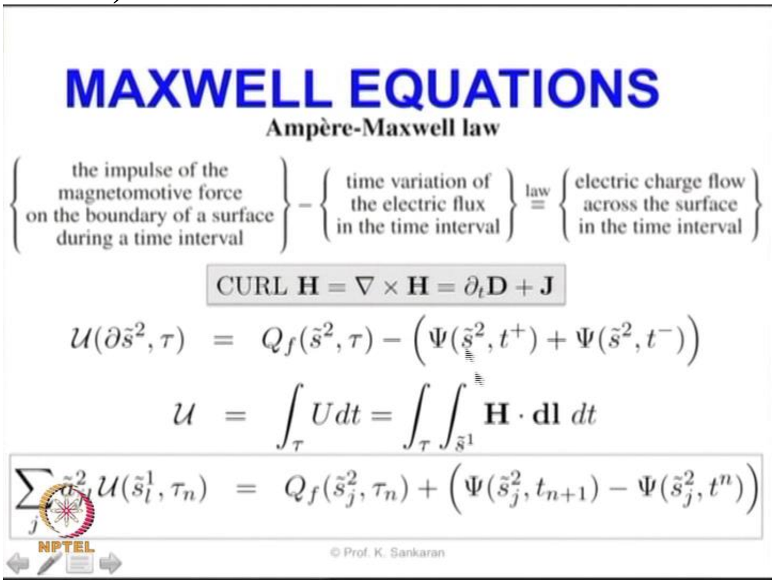

The last equation will be very similar one in the case we call it ampere Maxwell law where you see the curl of H is equal to dt D plus J.

And when we do the same way when we do the space surface integration what we have got is the J surface integration will become the charge flow and that’s what we have got here so that J will become the charge flow component here and the dt by DE will become the electric flux component like we did for the earlier equation and on the right hand side we will have the time integrated value of the Magnetomotence of is given here and we have got the value for the curl which is the boundary operator on the duel equation with that we are coming to the end of the algebraic topological model what we have done so far is we have basically used our understanding of topological parameters to model the entire set of maxwell’s equations only using scalar quantities which we can tangible measure and also get a sense of their Association with certain topological quantities.

With that being said we have covered the entire gamut of all the tools that you will need to model Maxwell equation using algebraic topological method with that being said I thank you for your attention let’s see in the next modules .

Thank you.