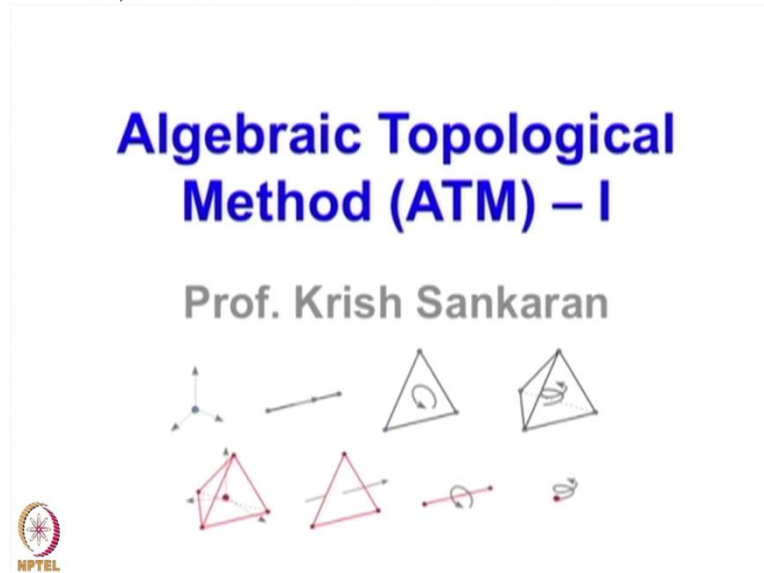


Computational Electromagnetics and Applications
Professor Krish Sankaran
Indian Institute of Technology Bombay
Summary of Week 11

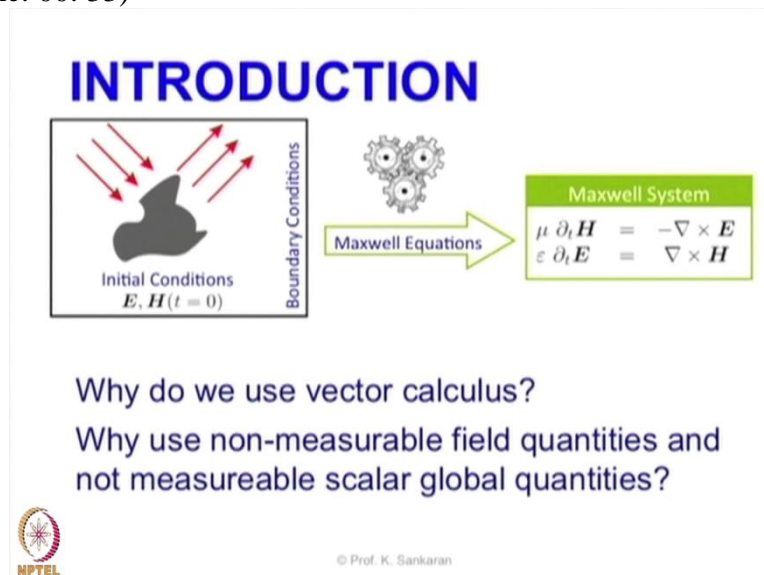
This week is one of the most interesting week in this entire Course work .

(Refer Slide Time: 00: 09)



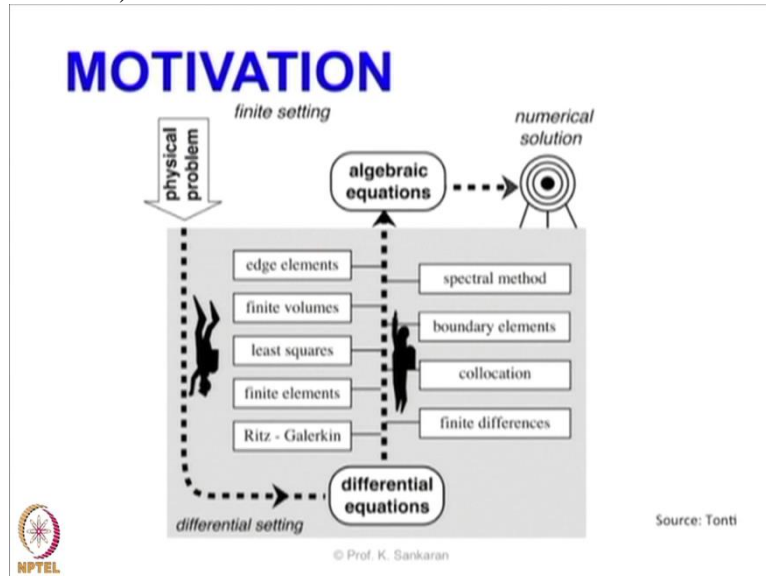
We will be introducing a very unique method which is completely different with the earlier methods we have discussed in this lecture.

(Refer Slide Time: 00: 33)



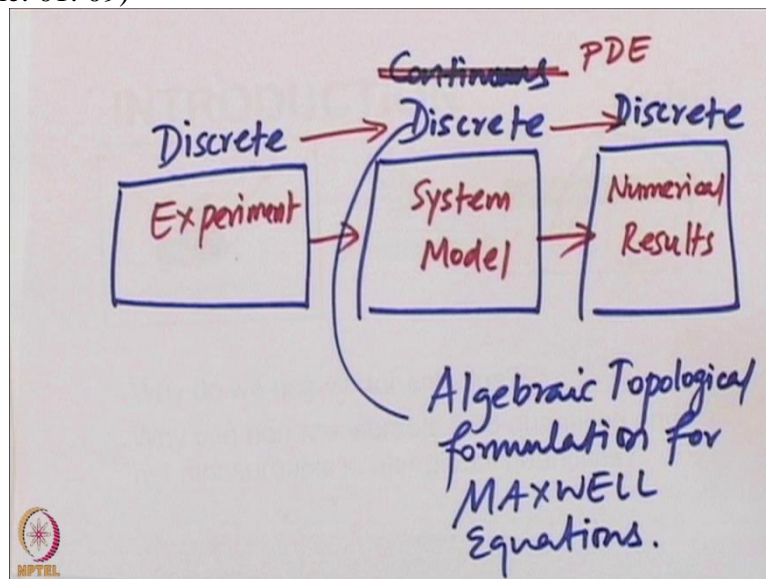
We started this week with a very proactive question about the need for using vector calculus and differential equations in modelling any electromagnetic problem we have been taught since school and college base that electromagnetic problems start with the vectorial description of electric and magnetic base described by the Maxwell partial differential equation.

(Refer Slide Time: 00: 58)



We showed how and why this is a roundabout day for modelling any physical phenomena in particular electromagnetic problems.

(Refer Slide Time: 01: 09)




We discussed how one can model electromagnetic problems without the need of differential equations and explain our approach involving discrete algebraic formulation instead of continuous differential formulation.

(Refer Slide Time: 01: 25)

THEORY – 1

Lorentz Force	$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$
Configuration Variables (\mathbf{E}, \mathbf{B})	$\mathbf{E} = \frac{\mathbf{F}_e}{q}, \quad \mathbf{v} \times \mathbf{B} = \frac{\mathbf{F}_m}{q}$ $\mathbf{B} = \frac{\mathbf{F}_m}{p} \quad m_m = pl$
Source Variables (\mathbf{D}, \mathbf{H})	$D = \frac{q}{S}, \quad H = \frac{p}{S}$ Magnitude only!




© Prof. K. Sankaran

We introduced the theory of algebraic topological method starting with the Lorentz Force equation and we define the configuration and source variables involved in electromagnetic problems.

(Refer Slide Time: 01: 40)

THEORY – 1

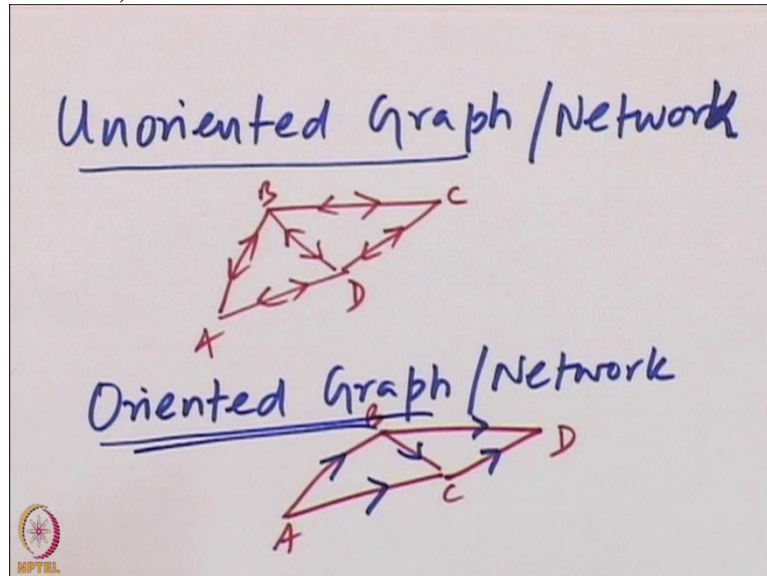
Point association	φ
Line association	$V = \int \mathbf{E} \cdot d\mathbf{l}$ $U = \int \mathbf{H} \cdot d\mathbf{l}$
Surface association	$\Psi = \int \mathbf{D} \cdot d\mathbf{s}$ $\Phi = \int \mathbf{B} \cdot d\mathbf{s} \quad I = \int \mathbf{J} \cdot d\mathbf{s}$
Volume association	$Q_c = \int \rho dv$



© Prof. K. Sankaran

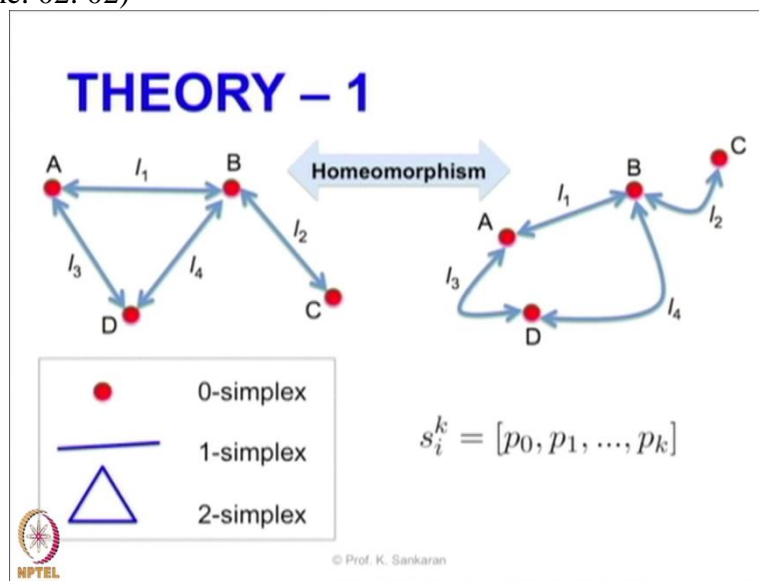
We also discussed how these Global variables are related to certain topological entities such as points lines surfaces and volumes.

(Refer Slide Time: 01: 53)



Using a simple electrical circuits we introduced the concept of oriented and an oriented graphs or networks.

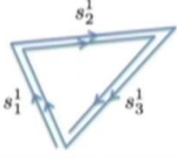
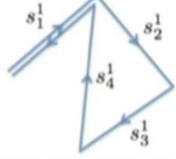
(Refer Slide Time: 02: 02)



We explained the basic notion of homeomorphism which is normally seen in algebraic topological models we also introduced using examples basic terminologies such as 0 1 2 and 3 simplices.

(Refer Slide Time: 02: 26)

THEORY – 1

c_1^1	$a_1 = a_2 = a_3 = a_4 = 2$
---------	-----------------------------

c_1^1	$a_2 = a_3 = a_4 = 1, a_1 = 0$
---------	--------------------------------

c_1^2	$a_1 = a_2 = a_3 = a_4 = -2$
---------	------------------------------

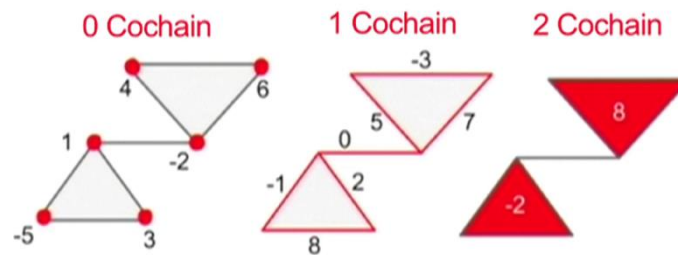
c_1^2	$a_2 = a_3 = a_4 = -1, a_1 = 0$
---------	---------------------------------

NPTEL © Prof. K. Sankaran

And respective 0 1 2 and 3 chains.

(Refer Slide Time: 02: 32)

COCHAIN



$$c^k = [g_1, g_2, g_3, \dots, g_{n_k}]$$



© Prof. K. Sankaran

Building on these basics we introduced advanced concepts of co chains.

(Refer Slide Time: 02: 38)

BOUNDARY OPERATOR

$$\begin{aligned} \partial s_2^1 &= p_2 - p_1, & \partial s_3^1 &= p_3 - p_2 \\ \partial s_4^1 &= p_3 - p_0, & \partial s_5^1 &= p_3 - p_1 \\ \partial s_6^1 &= p_4 - p_2, & \partial s_7^1 &= p_4 - p_3 \end{aligned}$$

$$\begin{aligned} \partial s_1^2 &= \sum_{i=1}^3 (-1)^i [p_0, p_1, p_3] \\ &= (-1)^0 [p_1, p_3] + (-1)^2 [p_0, p_3] + (-1)^3 [p_0, p_1] \\ &= [p_1, p_3] - [p_0, p_3] + [p_0, p_1] \\ &= s_5^1 - s_4^1 + s_1^1 \\ \partial s_2^2 &= s_2^1 + s_3^1 - s_5^1 \\ \partial s_3^2 &= s_3^1 - s_6^1 + s_7^1. \end{aligned}$$

$$\partial s_j^{k+1} = \sum_{i=1}^{n_k} a_{ij}^k s_i^k$$

$$\partial : c_k \mapsto c_{k-1}$$

Incidence Coefficient
 a_{ij}^k

© Prof. K. Sankaran

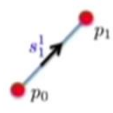
And also the most important operators in algebraic topology namely the boundary and core boundary operators.

(Refer Slide Time: 02: 48)

COBOUNDARY OPERATOR

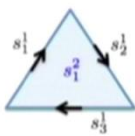
δ^1

φ



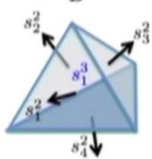
δ^2

V



δ^3

D



$\delta^0 = \nabla \equiv \text{GRAD}$	$\delta^1 = \nabla \times \equiv \text{CURL}$	$\delta^2 = \nabla \cdot \equiv \text{DIV}$
--	---	---

$$\sum_{i=1}^{n_p} a_{ji}^1 \cdot \varphi(s_i^0) = \varphi(p_1) - \varphi(p_0)$$

$$\sum_{j=1}^{n_t} a_{ij}^2 \cdot V(s_j^1) = V(s_1^1) + V(s_2^1) + V(s_3^1)$$

$$\sum_{l=1}^{n_s} a_{ml}^3 \cdot D_n(s_l^2) = D_n(s_1^2) + D_n(s_2^2) + D_n(s_3^2) + D_n(s_4^2)$$

© Prof. K. Sankaran

We explain the parallel connections existing between the core boundary operators operating on different co change and the vector calculus operators namely divergence curl and gradient these relationships are the most important connections for modelling physical problems using algebraic topology without the need for vector calculus.

(Refer Slide Time: 03: 17)

THEORY – 1

Lorentz Force

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Configuration Variables
(E,B)

$$\mathbf{E} = \frac{\mathbf{F}_e}{q}, \quad \mathbf{v} \times \mathbf{B} = \frac{\mathbf{F}_m}{q}$$

$$\mathbf{B} = \frac{\mathbf{F}_m}{p} \quad m_m = pl$$

Source Variables
(D,H)

$$D = \frac{q}{S}, \quad H = \frac{p}{S} \quad \text{Magnitude only!}$$



© Prof. K. Sankaran

Please carefully go through the concepts and examples that we have discussed in this week.

(Refer Slide Time: 03: 21)

THEORY – 1

Point association

$$\varphi$$

Line association

$$V = \int \mathbf{E} \cdot d\mathbf{l}$$

$$U = \int \mathbf{H} \cdot d\mathbf{l}$$

Surface association

$$\Psi = \int \mathbf{D} \cdot d\mathbf{s}$$

$$\Phi = \int \mathbf{B} \cdot d\mathbf{s}$$

$$I = \int \mathbf{J} \cdot d\mathbf{s}$$

Volume association

$$Q_c = \int \rho dv$$



© Prof. K. Sankaran

This is one of the Unique opportunities for you to learn and master advanced methods such as algebraic topology.

(Refer Slide Time: 03: 29)

THEORY – 1

Diagram 1 (Left): Triangle with edges s_1 , s_2 , s_3 .
 Box 1: c_1^1 $a_1 = a_2 = a_3 = a_4 = 2$
 Box 2: c_1^2 $a_1 = a_2 = a_3 = a_4 = -2$

Diagram 2 (Right): Triangle with edges s_1 , s_2 , s_4 .
 Box 1: $a_2 = a_3 = a_4 = 1$, $a_1 = 0$, c_1^1 c_1^1
 Box 2: $a_2 = a_3 = a_4 = -1$, $a_1 = 0$, c_1^2 c_1^2

NPTEL © Prof. K. Sankaran

The method of algebraic topology is normally couched in complex terminologies.

(Refer Slide Time: 03: 35)

THEORY – 1

$$c_k^i = \sum_{i=1} a_i s_i^k$$

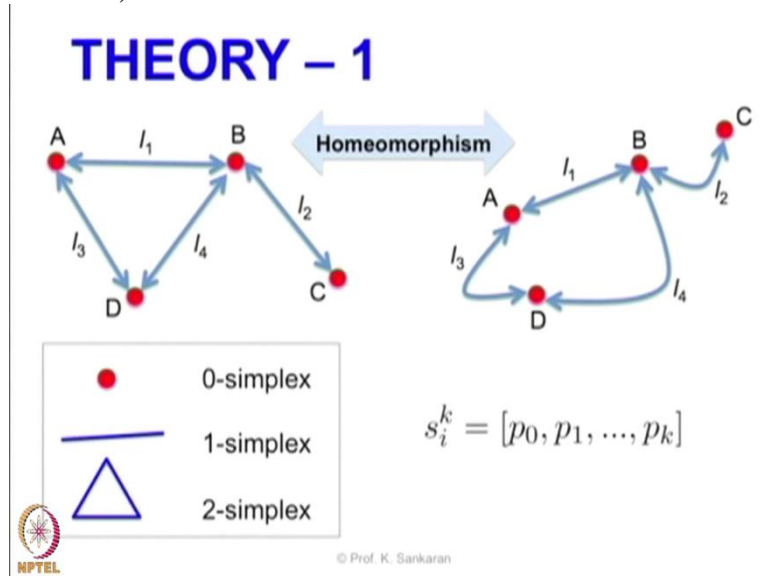
Diagram A: Square with diagonal s_1 and edges s_2 , s_3 .
 Diagram B: Square with diagonal s_1 and edges s_2 , s_4 .
 Diagram C: Square with diagonal s_1 and edges s_2 , s_3 , s_4 .

Row c_0 : Red dots at vertices.
 Row c_1 : Red edges.
 Row c_2 : Red faces.

NPTEL © Prof. K. Sankaran

And hence it is always been a distant method do engineers and applied thysicist.

(Refer Slide Time: 03: 45)



We have tried our best to reduce this complexity.

(Refer Slide Time: 03: 48)



By giving simple examples to explain Complex terms please post your questions in the forum clarify your doubts about the terms and terminologies that we have used in this lecture and get ready for the next week until then goodbye.