Computational Electromagentics and Applications
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Lecture No 39
Algebraic Topological Method (ATM II)

The last modules we have looked into introduction for algebraic topological methods and we also introduced some of the basic ideas what relates to the topological dimensions and what parameters are connected to what kind of topological aspects and based on that we kind of created two types of variables primarily we looked into source variable and configuration variables source variables are the ones which are directly related to electric charges and magnetic poles.

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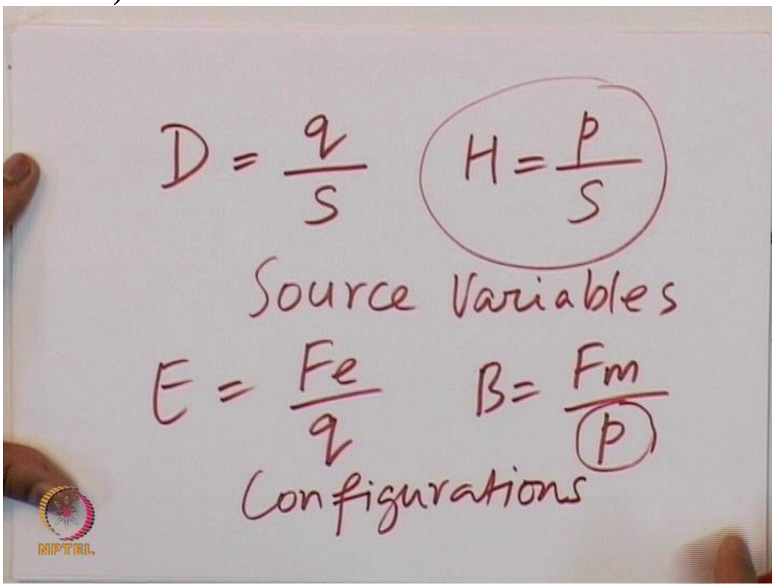

In this case they are going to D which is related to q and the value of h which we said we can write it as P divided by S of course you have to exercise certain caution in doing that because this is only magnitude voice equivalent obviously if you take the vector science and direction you have to be sure about the right direction for that so if you only considered the magnitude D and H are the source variables likewise the configuration variables are E which is related to the electric force per unit charge and the magnetic force on the unit pole doesn’t exist but you are using it too mathematically say E and B called as configuration variables.

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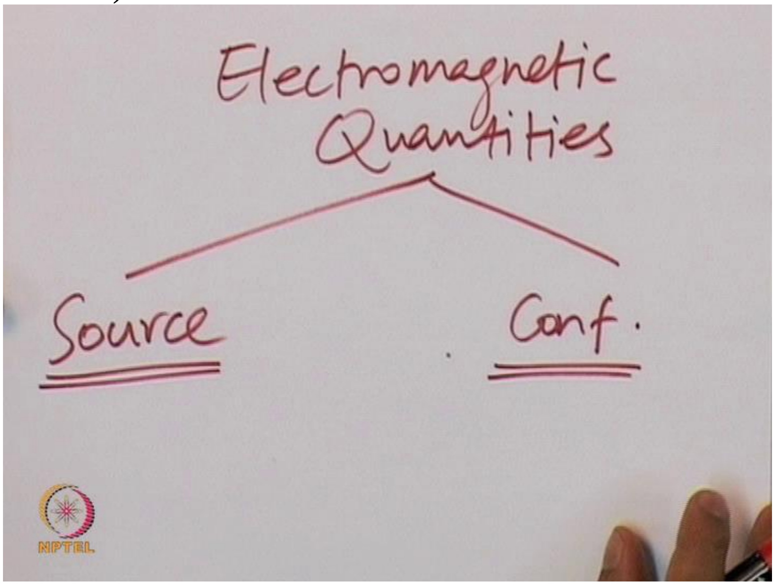

So based on that we were able to create two sets of electromagnetic quantities they are source and configuration the combination of source and configuration will give you also energy variables but but we are not interested in talking about it now it's enough to focus only on source and configuration variables the next thing what we looked into is there are geometrical objects and they are called simplexes and depending on that dimensions of the simplex we call them as 0 simplex 1 simplex 2 simplex 1 and so forth and we also looked into the chain of simplexes is called as k chain so if it is a chain of points it's called as zero chain chain of lines is 1 chains on and so forth so in this module.

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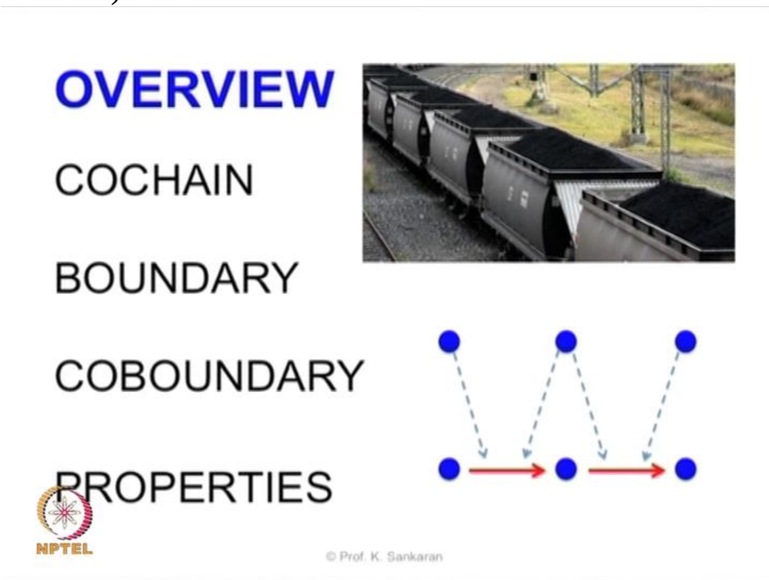

We will look into are the concepts related to algebraic topology namely, we will look into the idea of Cochains here I have used a metaphorical figure containing a Boogie with the materials inside charcoals in each of the bogies I'll explain you why it is metaphorically related to Cochains. Then I will discuss the concept of boundary operator and we will also look into the concept of co boundary operator and discuss some properties this will be the focus for this module. Ok let's go into the topic of cochains.

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Look at this figure here this figure basically consists of a series of compartments or bogies consisting of materials inside in other words.

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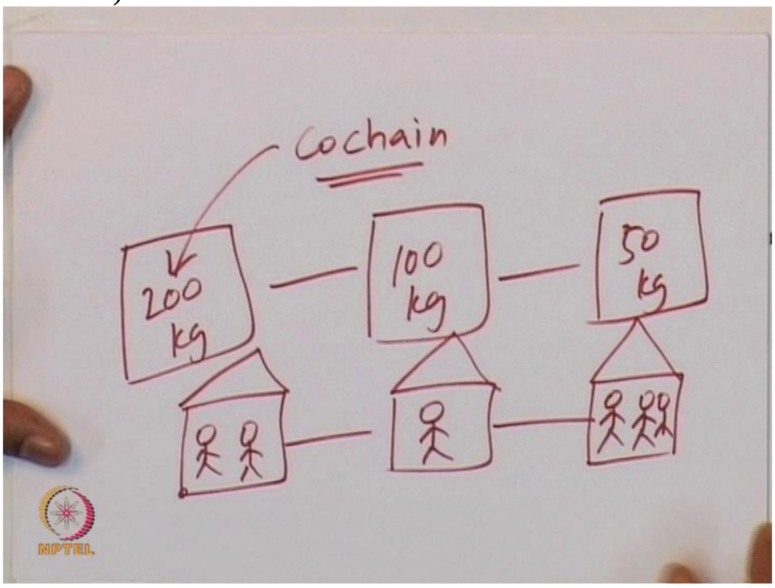

What we are seeing here is we have certain boxes that are tied together let's say here there is 200 kilograms of material here 100 kilograms of material the same material and 50 kilograms of the same material so we can say it's a chain of like in this case that's a volume so it's a chain of volumes tied together but whatever is inside the chain they are called as cochains.

So co chains are nothing but the components that we put inside or the components that we associate to certain objects let's take a much more easy example let's say we have an apartment house aunties house are three dimensional objects and you can say there are people in the house let's say there are 2 people in this house one person in this house and 3 person in this house so what we are telling is its volume chain it's a chain of three simplexes and what we are saying here is the people are associated to this volume or we can think about it is we can't associate the people to a.

We can't associate the people to a line so we can't live in the line are we can't live in the. What we can live is in a three dimensional space so it make sense to say the co chains are associated to a particular geometrical object that is what we are going to talk about in the case of electromagnetics

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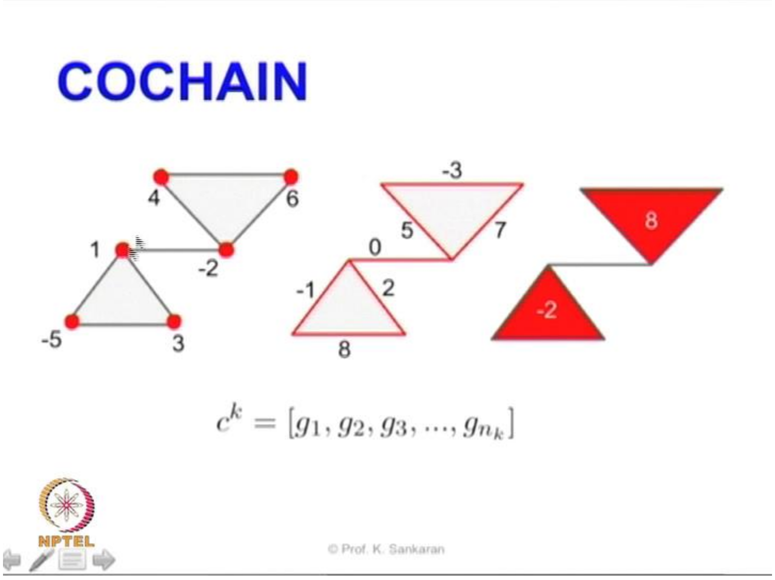

So in this slide what you are seeing here is you have points that are tied together so it's a chain of zero simplex but each of the. Has certain value its minus 5 ,1,3,4 minus 2,6 and so on and so forth what you are seeing here is nothing but in the case of electromagnetics voltages the voltages or the scalar potential are the values that we have associated to certain points so the scalar electric potential is nothing but zero cochain associated with zero simplex 0 chain similarly what we see here is we see the potential difference in other words the electric motors the electric motors is always associated to a line whether it's a straight line or curve line is always associated to a one dimensional object.

So what we see in this example is direct remote and son nothing but 1 co chains associated with 1 chain or 1 simplex similarly what you see here is fluxes are nothing but to co chains associated with certain 2 chains or 2 simplex show the most simplex co chain will be values associated to the simplex let me explain this further.

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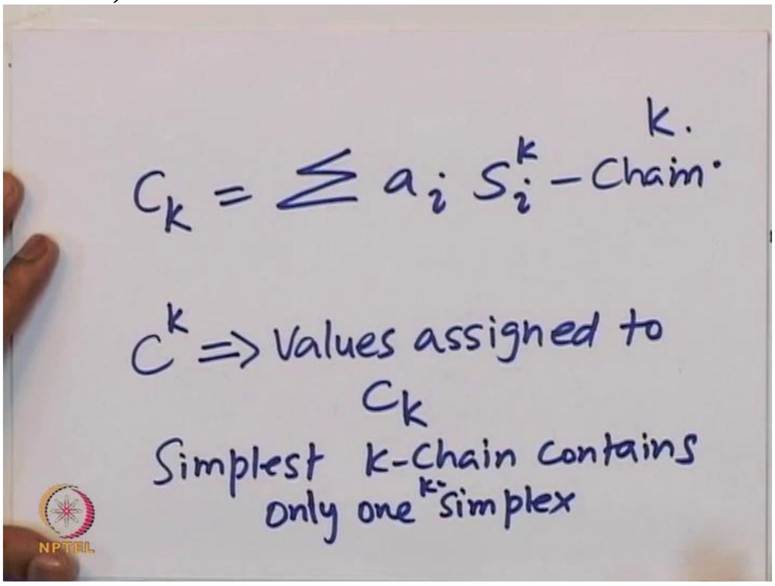

We said that chains are when you use a subscript we are calling it as a chain so this is a k chain what we call as co chain we use the word superscript C superscript and these are values assigned to k chains the question is let's say I take the simplest chains. The simplest chains are the simplex itself it has only show the simplest K Chain contains only 1 simplex let say k simplex so if I say the values of assign 2 CK values of assign 2 CK means these are the values of assigned to the K simplex the simplest k Chain so that is what you see in the slide

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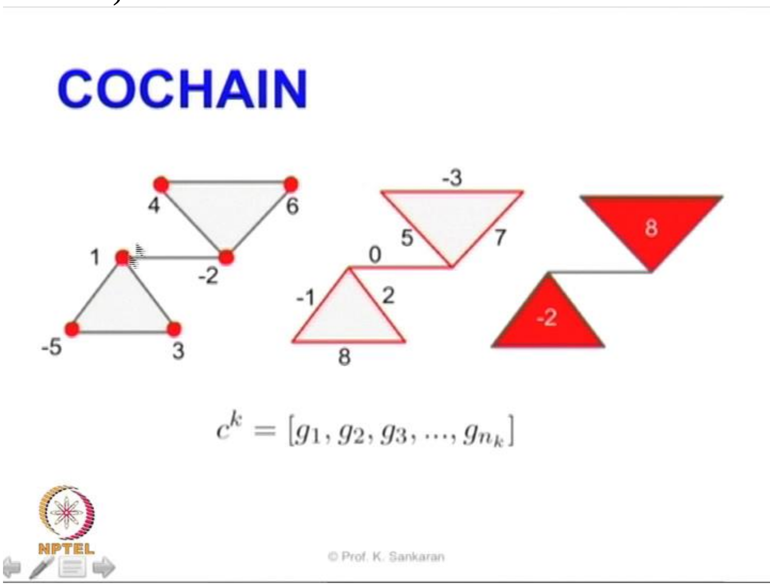

So when I say potential associated to this particular chain what I am talking about the individual points so we say these are the values return as G1 G2 G3 G4 until g n k . So G1 could be minus 5 if it is a zero coaching if it is a 1 Co chains G1 can be the electro mountains or the potential difference or in the case here if you are talking about 2 Co chains G1 G2 G3 can be minus 2, minus 8 and so on and so forth. So now we have discussed the concept of Co chains. The Co chains are nothing but the physically measurable quantities that are assigned to the chain in other words the simple simplexes.

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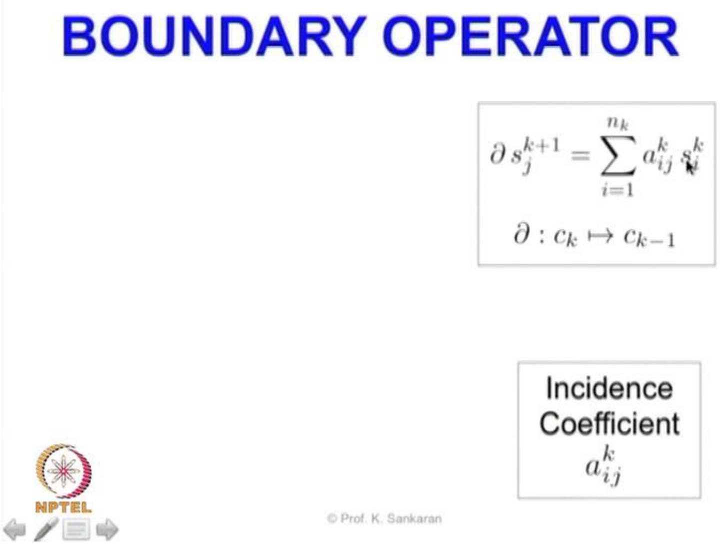

So let's not discuss the concept of boundary operator so boundary operator is a very very fundamental operator that is going to give us the boundary of a certain object that you are interested in so what this equation says here is so this is ok plus one simplex so what it does s when I am interested in finding the value of the boundary of certain simplex what I am interested as I am trying to get the value according to this equation which says the boundary of K plus 1 simplex is nothing but certain parameter what we call it as incidents coefficient X the k simplex so we are going from k plus 1 to k this will become more easy to understand if you understand the physical meaning of the boundary operator.

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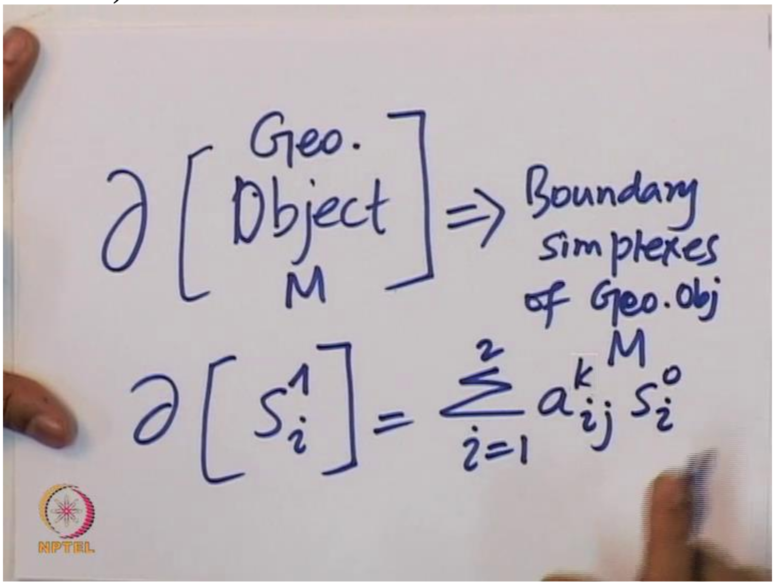

So the boundary operator , which be represented as the symbol partial derivative it takes an object that say a geometrical object and it gives the boundary simplex of that geometrical objects that’s why we call it as m so let’s take a simple example I gave the boundary operator or simplex A1 simplex let’s say I am interested in an i th 1 simplex so what it will give me is it will give me Sigma I equal to 1 to 2 because for a one simplex there are only two points some of this a k i j s i here I go from 1 to 0. So what I am doing is I am taking an object I am going from K plus one to k so if K plus 1 is equal to 1 what I will get k as k equal to zero .

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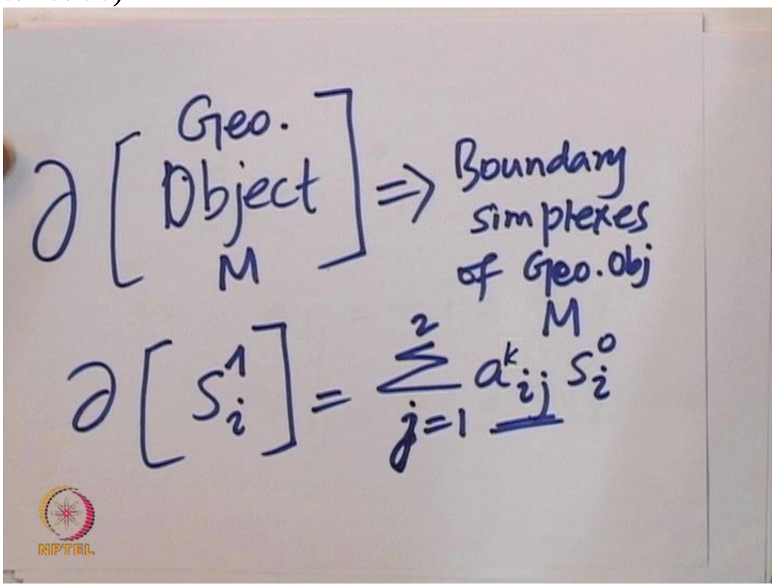

But the important thing is what is the value of this a i j k for a simplex with points let’s say P0 and P1 and we say the orientation is this direction the first points a i j will be equal to minus one and II points a i j will be equal to plus one show the node from which line the arrow points out will be minus one and the not where the line arrow points to will be plus 1

So this is the way we define plus one and minus one for a S1 of eye so here we are talking about i which is the element number and J is a point. So j is 1; J is 2 here so we can say this is a i 1 this is a i 2 the first point a i will be equal to minus1 and second point a i will be plus one so this is what we are saying in this equation so J goes from 1 to 2 and a i j k is the I coordinate so J here is 1 to 2. So let's take the case where we are interested in looking at 2 simplex the two simplex boundary operator

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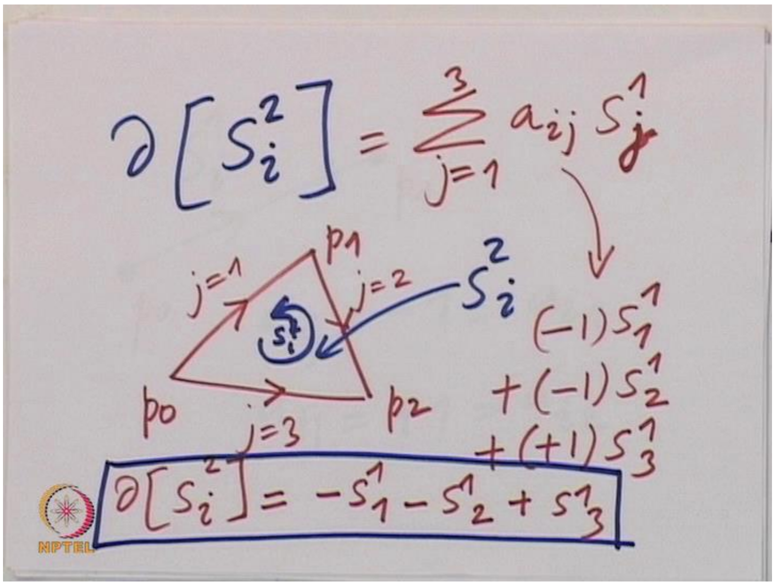

So let's say we have an operator boundary operator operating on to simplex written as like this and let's say the two simplex has coordinates like this so P0 P1 P2 and let's assume that the orientations are like this and we have certain directions that you are choosing let's say our surface orientation is in this direction so this is S 2 of i. So let me write it down again this is s 2 of I so in this case what I am looking at is I am looking at likewise is equal to Sigma J goes from so here there's a will be the boundary elements which are nothing but the lines themselves J goes from 1, 2, 3.

So I will have a i j s k plus 1 is equal to 2 so we have k which is equal to 1 so now we are interested in knowing what are the incidents coefficients for individual 1 simplex so let's take this example say we are going from J equal to so this is J equal to 1 the side J equal to 2 this side is J equal to 3 so what we are getting here is the first component will be minus of the direction in which my orientation will be there.

So my orientation is in the anticlockwise but this is pointed in this other direction so this will be minus 1 S of so this is sorry J. So the first component will be equal to S11 plus the second component will be the second component is in this direction so it is also in the opposite direction to the orientation so this will also be equal to minus one S12 and plus the third component is in the same direction so we will have plus one S13 so far this example what we have is we have minus s so the boundary operator operating on S i 2 is equal to the value that we have got here minus S11 minus S12 plus S13 so this is what we have seen in this slide.

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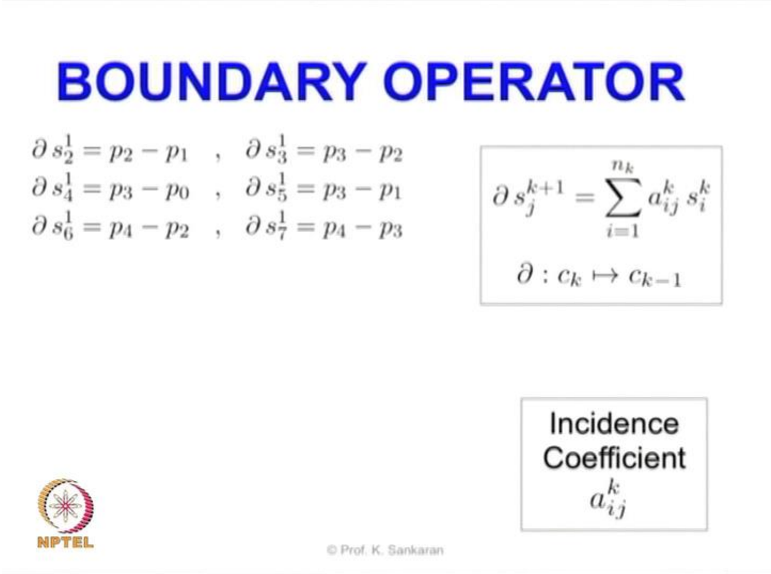

Remember our old example where we had a set of values given by the problem what we had so I am going to show the problem one more time.

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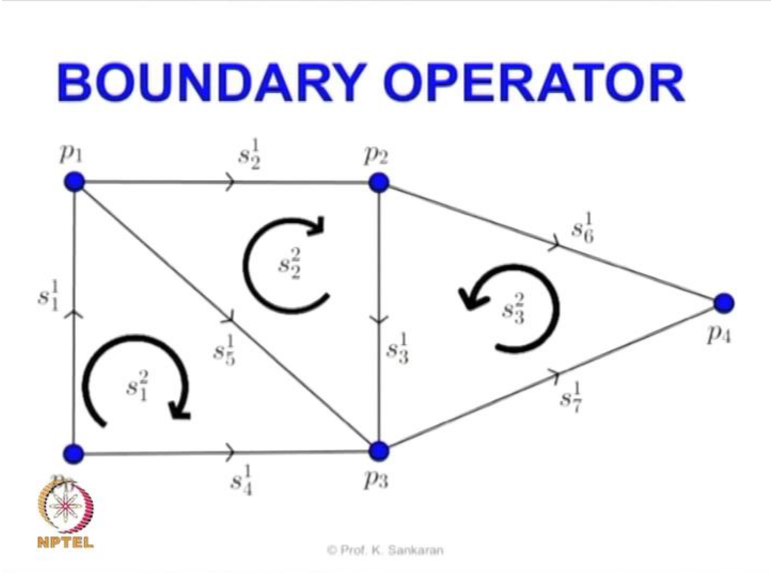

So this is the problem we are talking about. So we had certain values of direction we have chosen already and we have also computers the value of various triangles so let's quickly see what will be the value of the boundary operator for one or two cases and we can based on that we will reduce the other values so let's take the first example where we are interested in finding out the incidents Coefficient for a particular set of values so for that I will keep this bigger Triangle so using this we will see that we have to compute the value of various boundary operators so the first boundary operator that we are interested in is the value that is given by a one.

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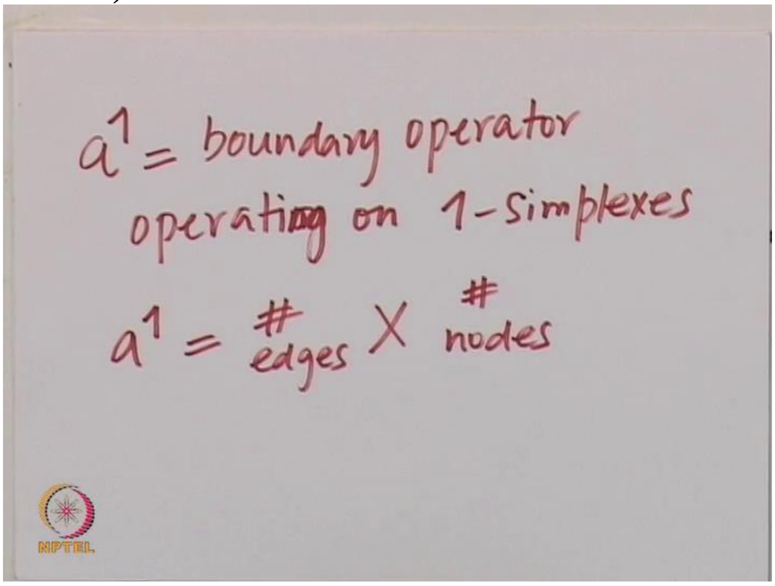

So A1 is equal to the boundary operator operating on so operating on 1 simplex so one operates on one so what you are getting is a one will be a matrix that has number of edges X number of nodes send this example what we see is will get certain A1 matrix so I am going to compute the even matrix for this example right now so that you can get a physical sense of how your computing it.

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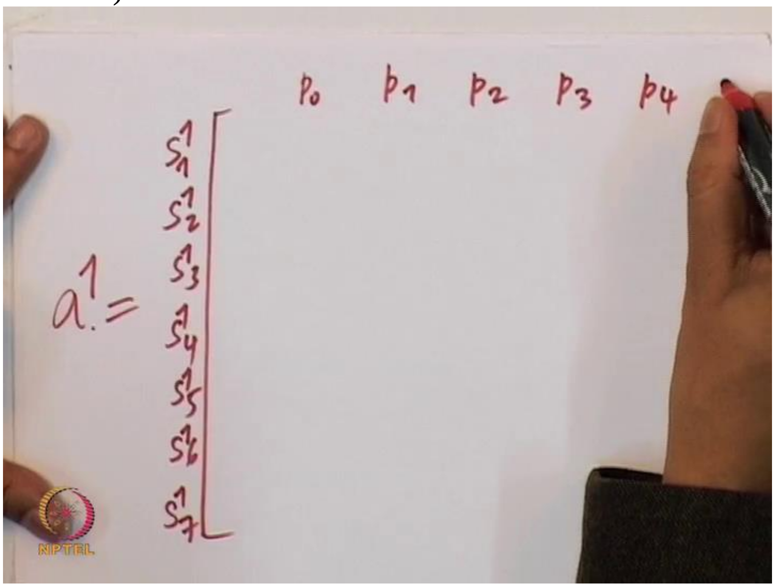

I said a one will be having number of edges in this case as you can count there are totally Seven Ages show the edge number 1 as Number 2 as Number 3 as Number 4 as Number 5 as Number 6 Energy number 7. So what we are saying now is we write down all the ages so these are S11, S12, S13, S14, S15, S16, S17 so these are the seven ages and we set rows will be the number of edges and the columns will be the number of nodes so we have totally fine arts these are given by P0, P1, P2, P3 and P4.

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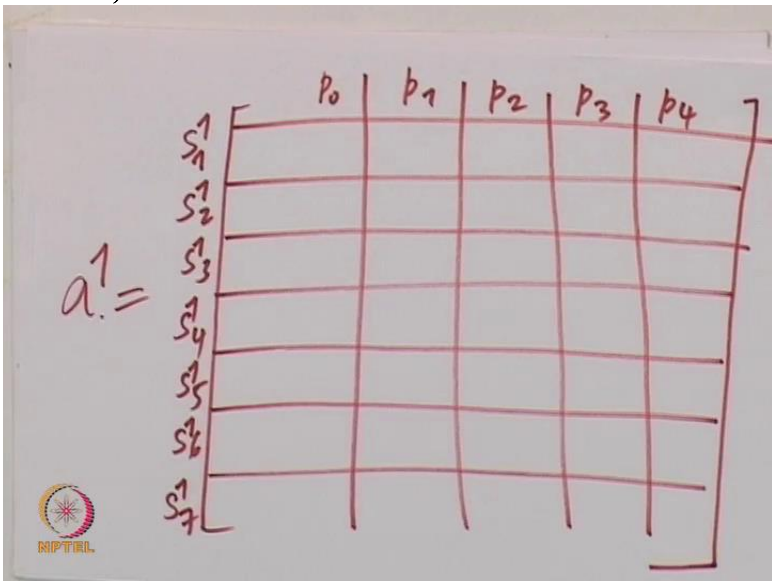

So let us write them down we have P1, P0, P1, P2, P3 and P4 so now we are going to populate this matrix purely using the information that we have got in this light so let us do that one by one so what we have got here is let's look at the slide.

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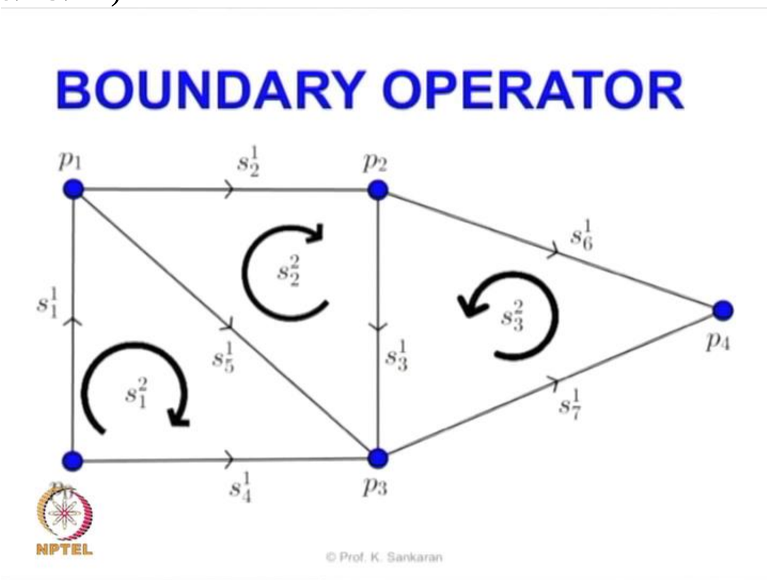

So what we have is for this node there are only P0 and P1 that are connected and all other things are zero so we can safely put 0, 0, 0 for all other things P0, P1 we need to know whether it is plus one or minus one so it is going away I said going into I said it is Plus 1 in the same way let's look at the case of P2 nodes that are associated are P1 and P2 so I can put zero for all other things.

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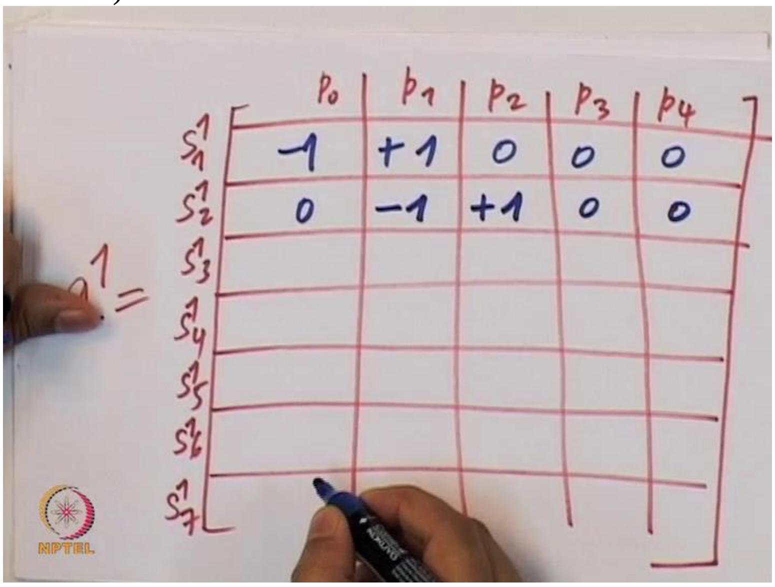

I am putting 0 for all other things except for P1 and P2 and I see the note which is going away is P1 so it is minus1 and the know where it is going into is plus one

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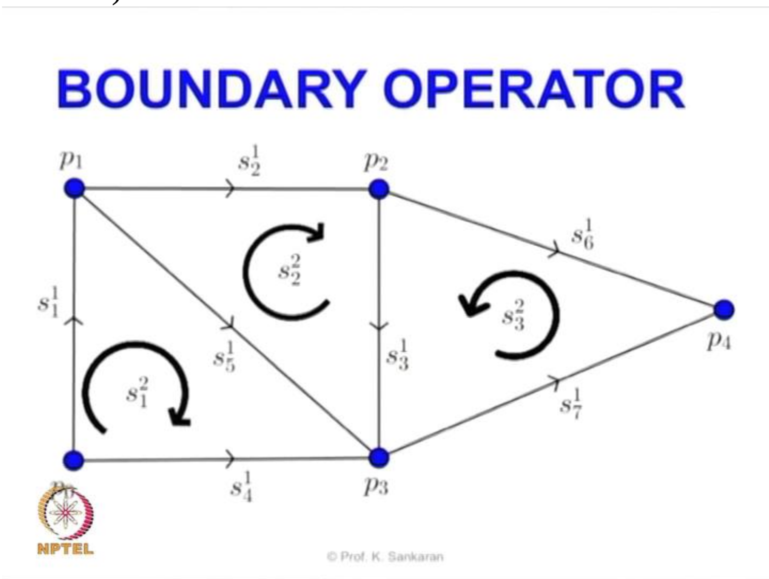

Similarly for P3 what we see in this graph is P2 and P3 for all other things I can put zero.

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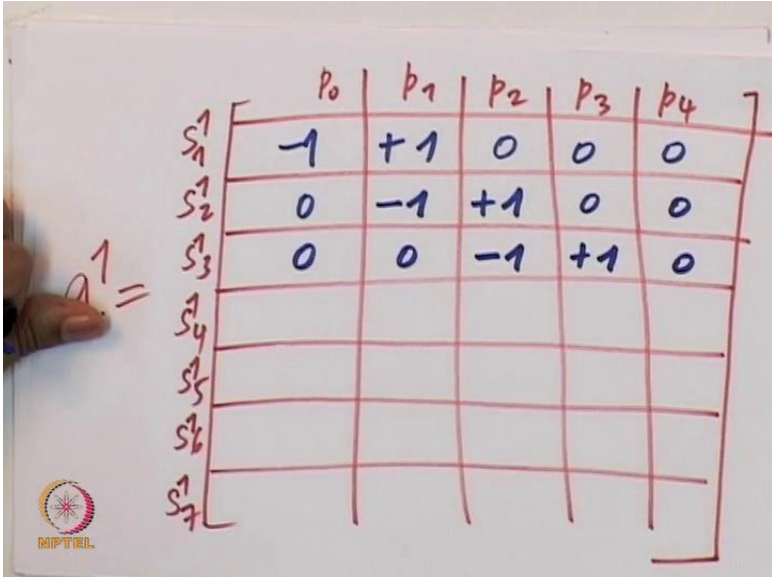

I will put zero for P0 P1 and P4 for P2 and P3 it's going away from P2 so it is minus1 it is going in to p3 so it is plus 1.

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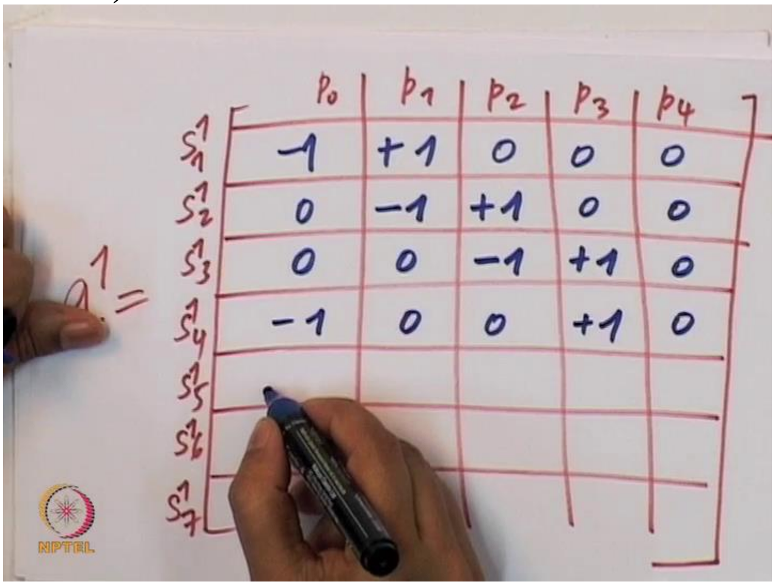

Similarly for IV line what is see is only points are P1 and P3 the rest of the things I can put 0 0 0 it is going away from P0 so its minus1 its going into P3 so its plus one.

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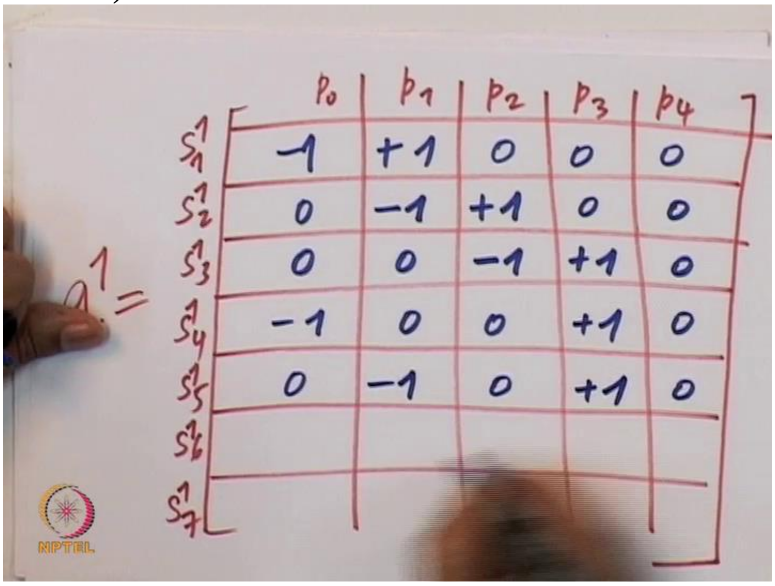

For S 1 5 likewise in the graph what you see the value for S5 it's P1 and P3 for the rest of the things I can put zero so it is going away from P1 set is minus one it is going into P3 so it is plus 1.

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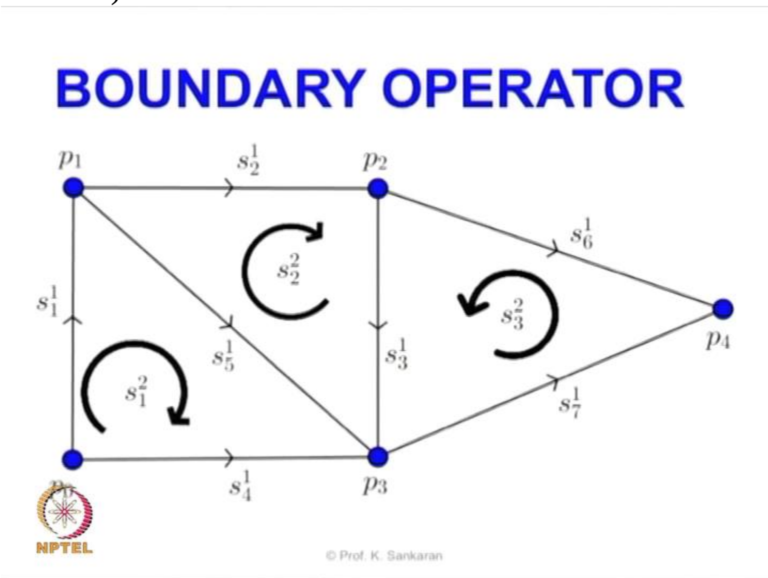

For S 1 6 it's B to P4 they are connected so the value is P4 other nodes so P0 is zero P1 P2 is not zero P4 is not zero but other things are zero so I am putting zeros for other things except for P2 and P4 so it is going away from P2 so it is minus1 it's going into P4 similarly for S17 the last one P3 is going away from P3 so it is minus1 and it's going into P4 it is plus 1 rest are zero that's what I am putting in the graph here.

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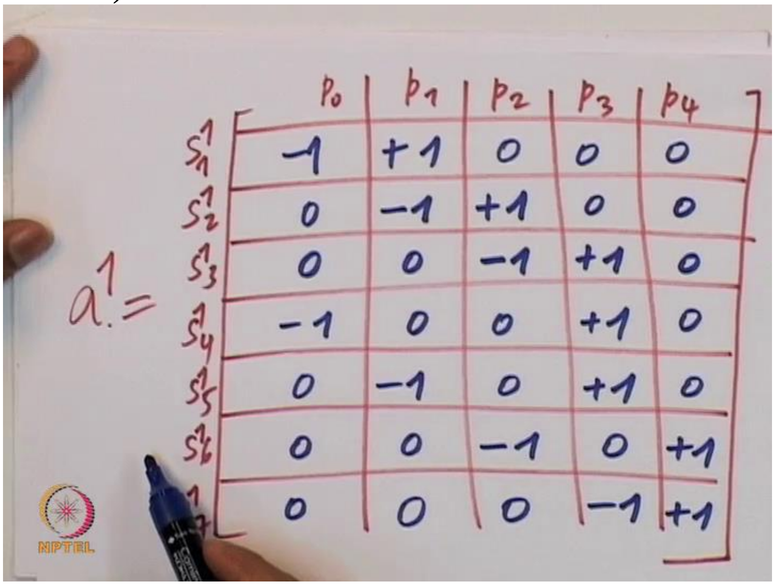

So we can see here in the slide the final Matrix what we have is as follows so as you can see in the slide here, so this will be the final matrix of a 12.

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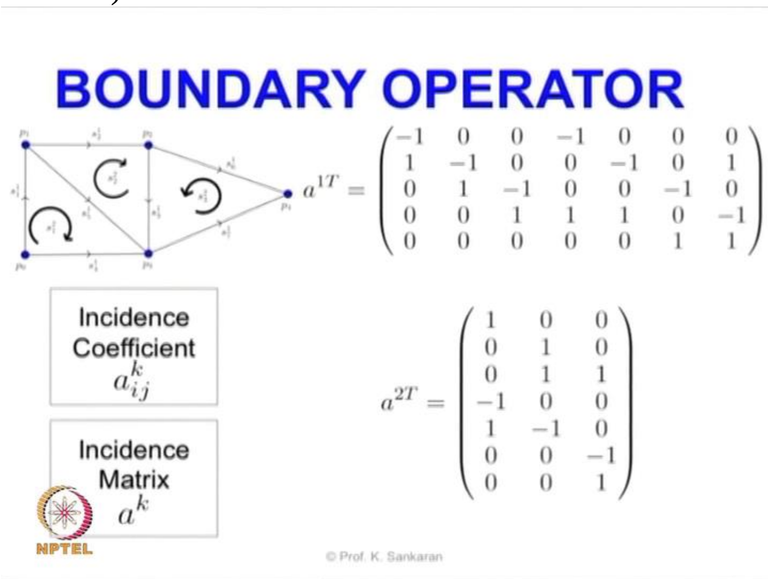

And that one is what we have represented here in the next slide as a transpose matrix so we have taken a and we have taken the transpose of this similarly we can do the same thing for the a to transpose where we will have a 2 will be the number of surfaces to the number of edges and a transpose of this is given here so let’s do the case of a two matrix for us to get certain confident on what it is and then you will take it from there on.

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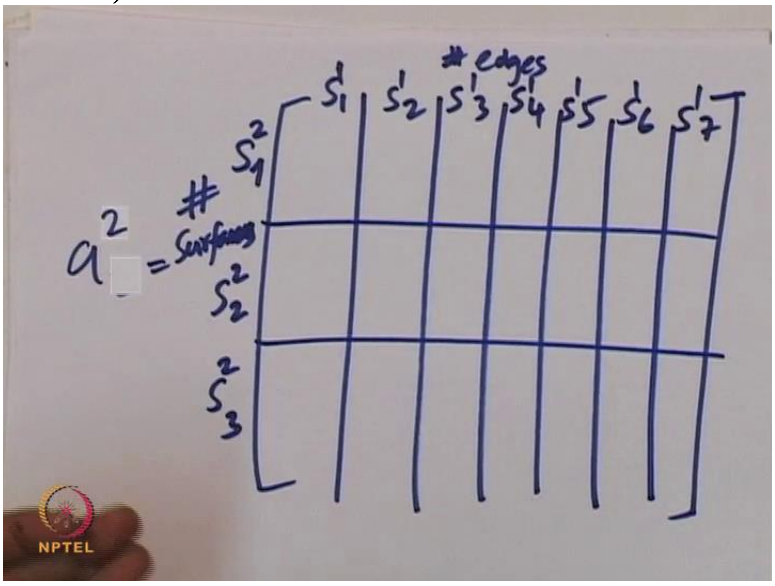

So let us look at the slide here one more time so we are interested in A2 matrix whose rows are given by the number of surfaces so we will have 3 surfaces they are written as S 2 1 S 2 2 and S 2 3 and we will have the column as number of edges so what we have here is S 1 1 S 1 2 S 1 3 S 1 4 S 15 S 16 and S 1 7 so we have 7 columns and we have three rows.

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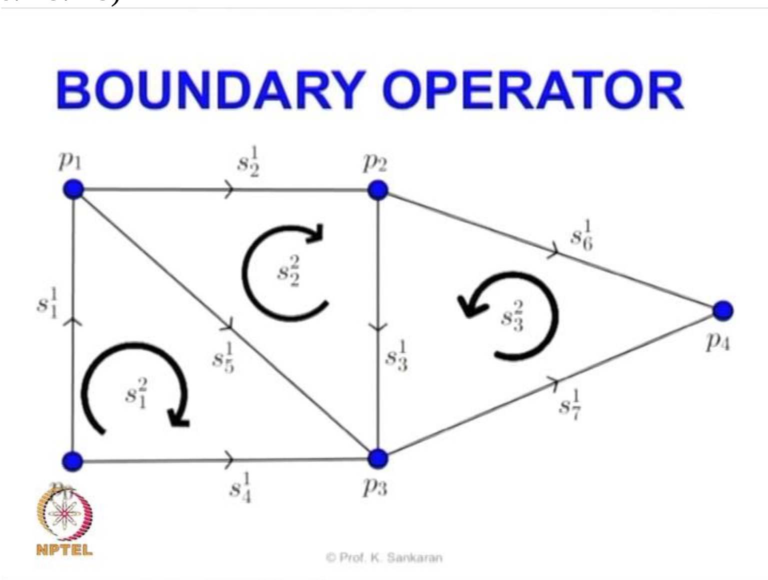

So when we look at the first graph what we see here is so the first for the S 12 the first one is in the same direction the second one S1 so there are 3 edges so we have one we have 5 and 4 so 1 5 4 are non zero and the rest are zero so we can already do that in or graph.

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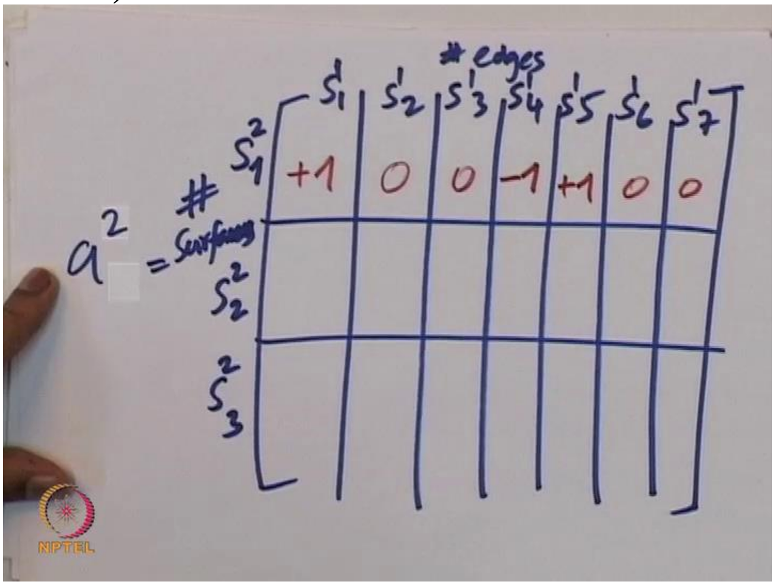

Except for one 5 and 4 I put zero for others and for one it is in the same direction so it is plus one for 4 it is in the opposite direction so its – 1 and for 5 it's in the same direction so its plus one so what you see here is basically from the from the graph here.

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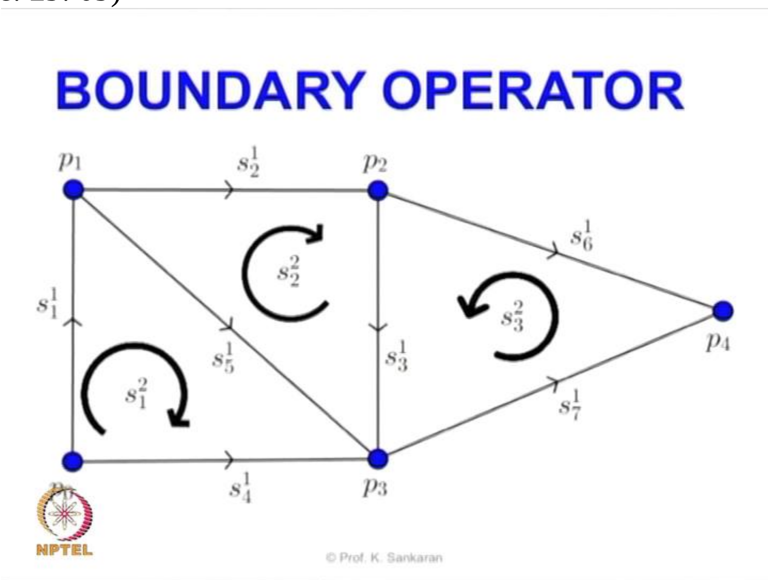

One is in the same direction 5 is also in the same direction 4 is in the opposite direction. Similarly for S 22 it's the subscript to 3 and 5 are non zero except for that I put zero on other things.

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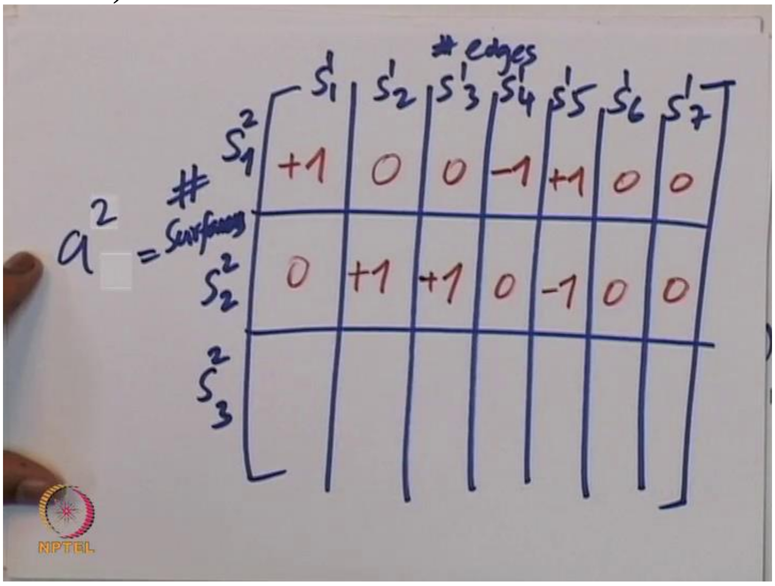

2 3 5 r non zero and the rest are zero SO2 is in the same direction as that of the curve so you can see that here in the graph SO2 is plus 1 3 is also in the same direction as you can see from the graph so I put plus one and 5 is in the opposite direction so I put minus 1

Similarly for the last Triangle 376 are responsible what we have is 3 6 and 7 except for that I put zero on other things 36 and 7 are non zero So 1 is 0, 2 is 0, 4 is 0, so 3 and also 5 is zero 3,6and 7 are non zero.

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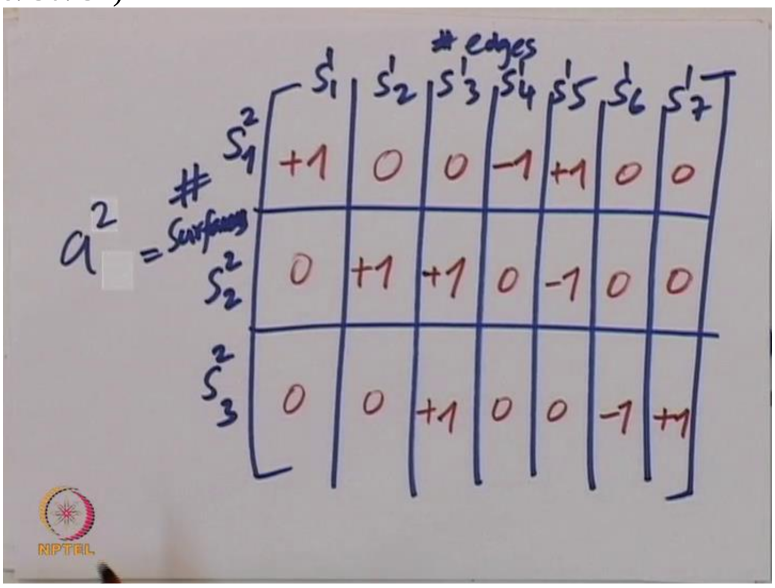

So let’s look at what is 33 is in the same direction so we put plus 1 6 in the opposite direction as you can see in the graph so we put minus one and 7 is in the same direction so we have plus one so if you take this one and we do the transpose of that we will get here as the a two matrix.

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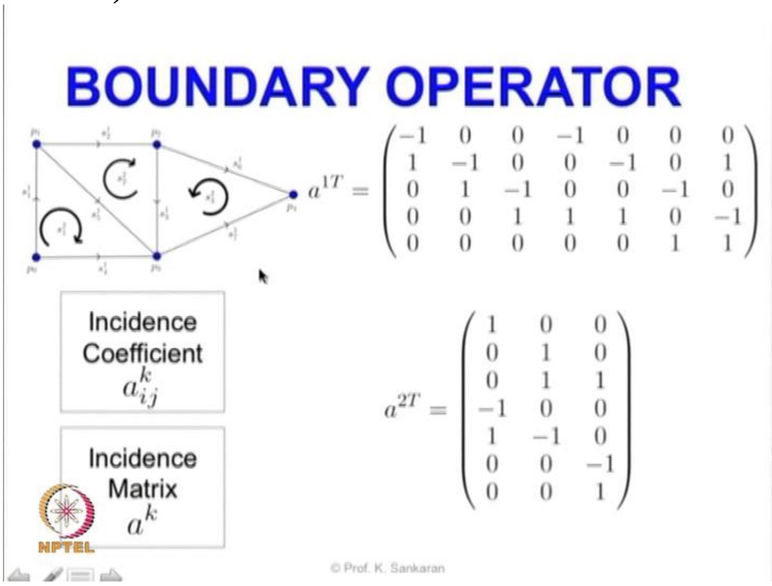

So what we have done now is we have looked into the boundary operator how we are defining the boundary operator we have taken this from a simple example in the next module we will look into the topic of boundary operator which is the counter part of the boundary operator and also we will discuss certain properties so with this evil and this module see you again in the next module thank you.