

Computational Electromagnetics and Applications
Professor Krish Sankaran
Indian Institute of Technology Bombay
Summary of Week 10

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We started this week's lecture building on the previous week's introduction to finite volume method

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DOMAIN TRUNCATION: 2

$$\begin{aligned} \partial_t H_x &= \frac{-1}{\mu_0 |V_c|} \sum_{k=1}^J (\mathcal{F}_{H_{xk}} \cdot n_k |S_k|) - \frac{\sigma_x}{\epsilon_0} K_x \\ \partial_t H_y &= \frac{-1}{\mu_0 |V_c|} \sum_{k=1}^J (\mathcal{F}_{H_{yk}} \cdot n_k |S_k|) - \frac{\sigma_x}{\epsilon_0} H_y \\ \partial_t E_z &= \frac{-1}{\epsilon_0 |V_c|} \sum_{k=1}^J (\mathcal{F}_{E_{zk}} \cdot n_k |S_k|) - \frac{\sigma_x}{\epsilon_0} E_z \\ \partial_t K_x &= \frac{\sigma_x}{\epsilon_0 \mu_0 |V_c|} \sum_{k=1}^J (\mathcal{F}_{H_{xk}} \cdot n_k |S_k|) \end{aligned}$$

Standard FVTD equations with PML losses

Reuse flux from the 1st equation

This is the uniaxial PML model from previous lecture

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We further developed the idea of Perfectly Matched Layer technique introduced in the earlier lectures for advanced applications.

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DOMAIN TRUNCATION: 2

$$\begin{aligned} \partial_t H_x &= \frac{-1}{\mu_0 |V_i|} \sum_{k=1}^f (\mathcal{F}_{H_{xk}} \cdot n_k |S_k|) - \frac{\sigma_x}{\epsilon_0} K_x \\ \partial_t H_y &= \frac{-1}{\mu_0 |V_i|} \sum_{k=1}^f (\mathcal{F}_{H_{yk}} \cdot n_k |S_k|) - \frac{\sigma_y}{\epsilon_0} H_y \\ \partial_t E_z &= \frac{-1}{\epsilon_0 |V_i|} \sum_{k=1}^f (\mathcal{F}_{E_{zk}} \cdot n_k |S_k|) - \frac{\sigma_z}{\epsilon_0} E_z \\ \partial_t K_x &= \frac{\sigma_x}{\epsilon_0 \mu_0 |V_i|} \sum_{k=1}^f (\mathcal{F}_{H_{xk}} \cdot n_k |S_k|) \end{aligned}$$

Standard FVTD equations with PML losses

Reuse flux from the 1st equation

Material Matrix


$$\alpha = \text{diag}[\mu_0, \mu_0, \epsilon_0, \mu_0]^T$$

Field Vector

$$Q_i = [H_x, H_y, E_z, K_x]^T$$

Simplified U-PML update equation

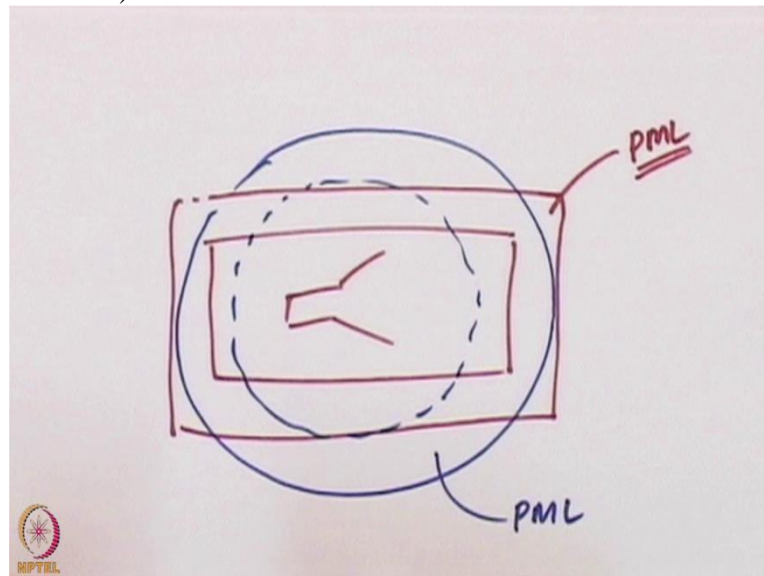
$$\partial_t Q_i = \frac{-1}{\alpha |V_i|} \sum_{k=1}^f \Psi_{Q_i} - \mathcal{L}_i$$



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We looked into a simply kind uniaxial perfectly matched layer formulation for Finite Volume Time domain simulation.

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We showed some applications using rectangular geometry using this PML formulation

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DOMAIN TRUNCATION: 2

x-Axis Oriented U-PML

Radially U-PML

Basic Idea

Radial U-PML Anisotropy is "locally" defined in the radial direction

(x, y, z) – Global coordinates

(x_r, y_r, z) – Local coordinates

Radially Uniaxial Behaviour

- * Approximation perfect at infinite PML radius of curvature
- * Accurate enough for most engineering applications!

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We further extended this idea to include a more general radial uniaxial PML

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DOMAIN TRUNCATION: 2

Loss-Vector Definition: Radial Versus Rectangular U-PML

$$\vec{\mathcal{L}}_i = \begin{pmatrix} K_{x_r} \cos \varphi + \frac{\sigma_x}{\epsilon_0} (H_x \sin^2 \varphi - H_y \cos \varphi \sin \varphi) \\ K_{x_r} \sin \varphi + \frac{\sigma_x}{\epsilon_0} (H_y \cos^2 \varphi - H_x \cos \varphi \sin \varphi) \\ \frac{\sigma_x}{\epsilon_0} E_x \\ 0 \end{pmatrix} \xrightarrow{\varphi=0} \mathcal{L}_i = \begin{pmatrix} K_x \\ \frac{\sigma_x}{\epsilon_0} H_y \\ \frac{\sigma_x}{\epsilon_0} E_x \\ 0 \end{pmatrix}$$

One generalized formulation for all regions

Lower number of update equations

Radial U-PML

Trade-off: Corner-reflection Vs. Focusing-effect

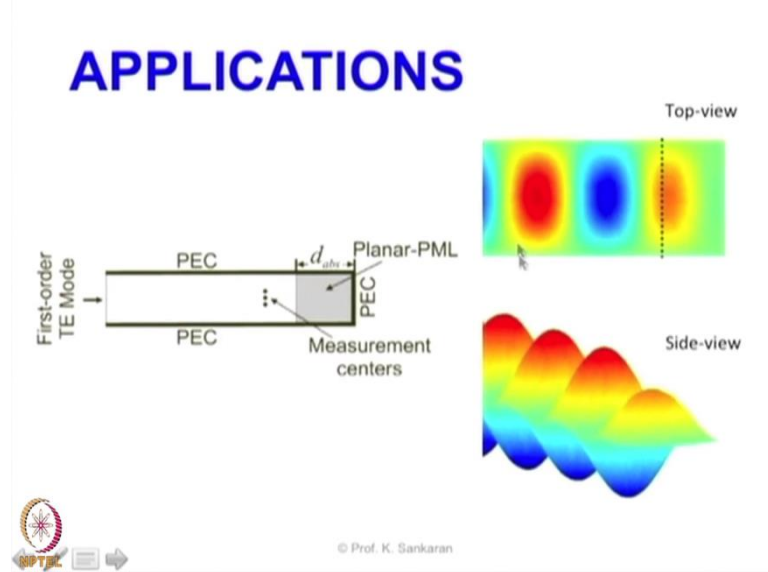
Geometry-dependent terms computed only once

Trade-off: Corner-reflection Vs. Focusing-effect

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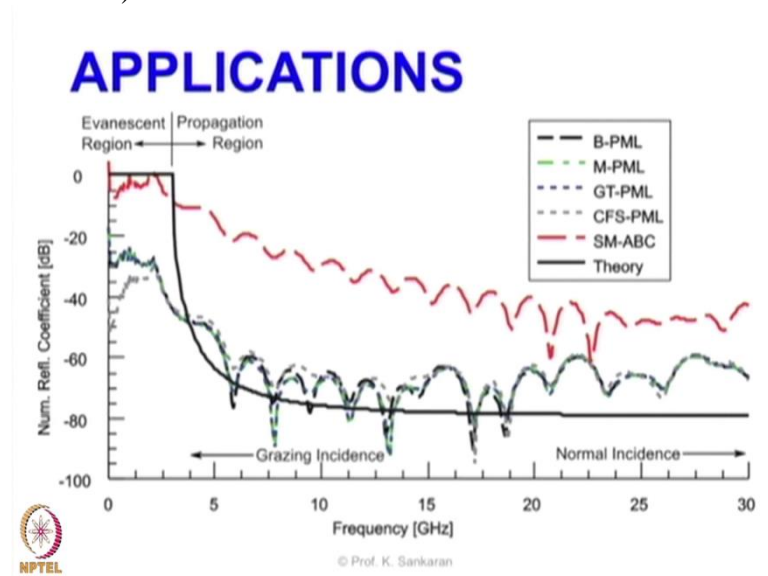
Showcasing pros and cons of this generalized formulation for 2D applications.

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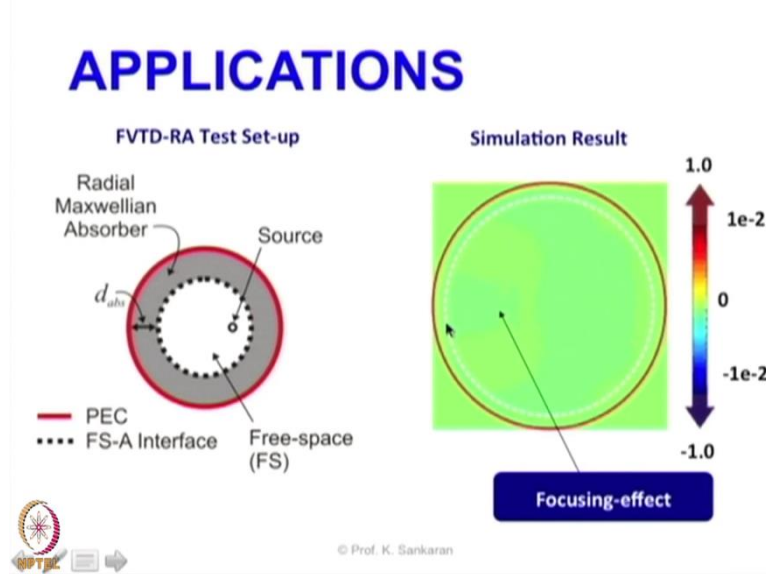
We later studied using a simple wave guide problem. The efficacy of radial uniaxial PML for finite volume time domain applications

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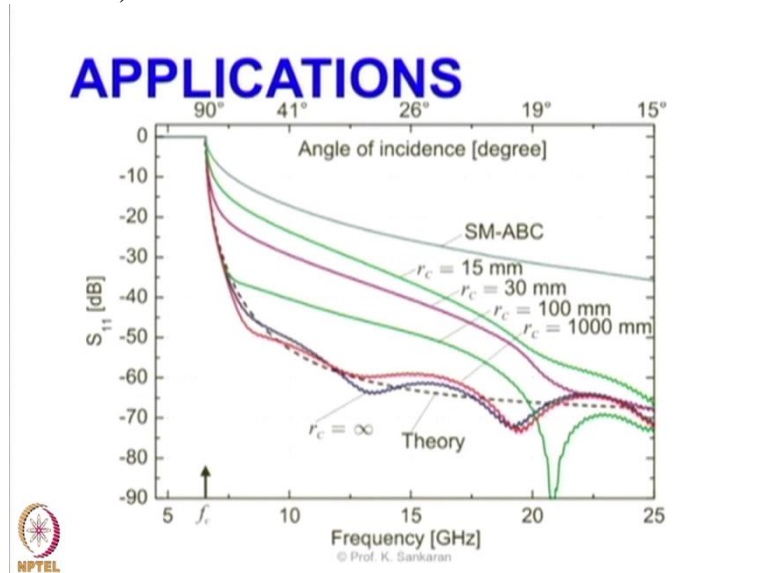
We also compared the performances of a standard silver Muller absorbing Boundary condition and uniaxial radial PML in the finite volume framework.

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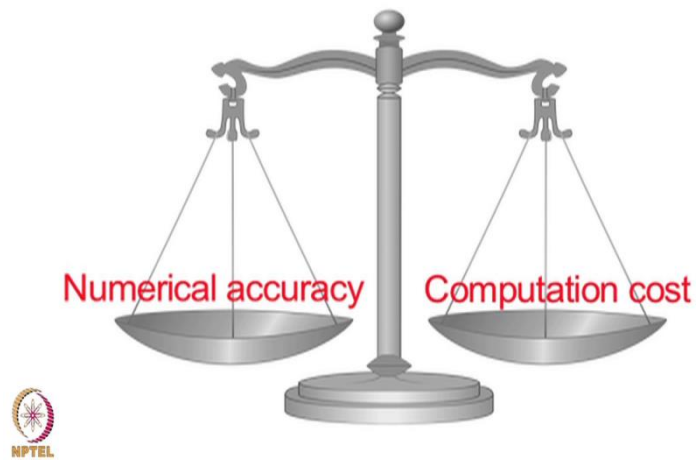
We demonstrated the impact of source locations in the PML absorption behaviour

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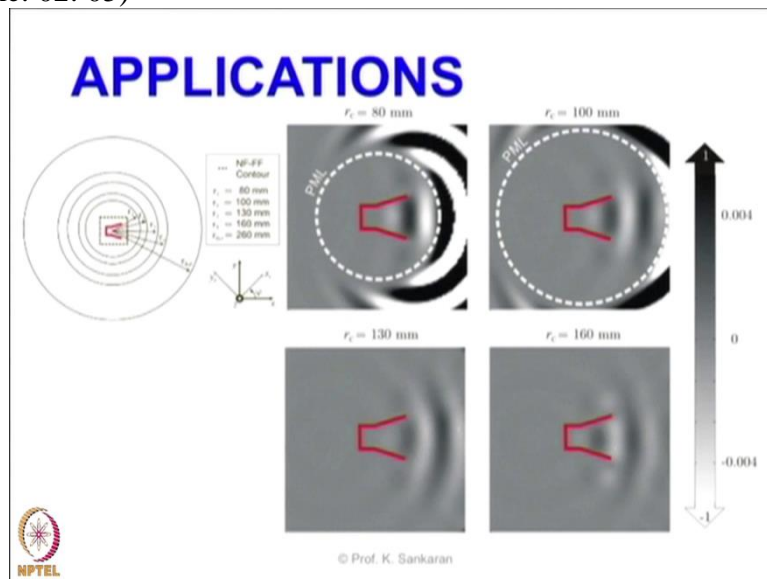
Later we discussed a critical role where radius of curvature of radial PML placed in the PML absorption performance.

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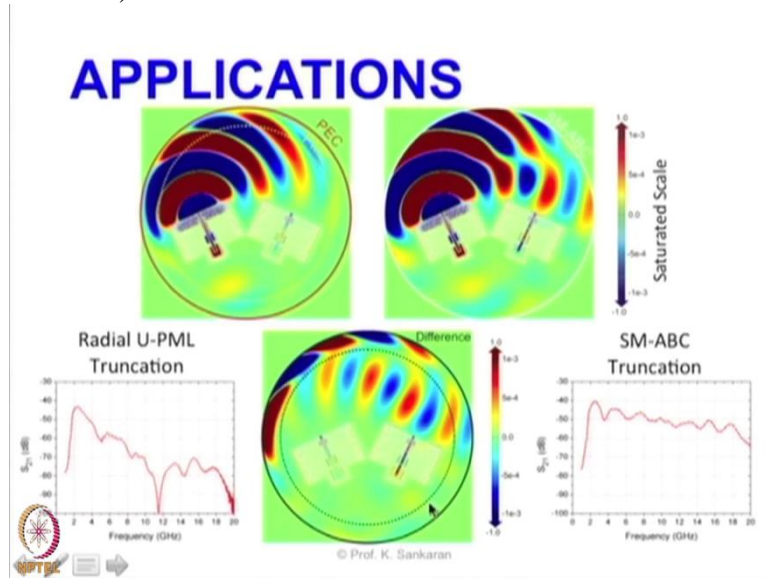
We later discussed some practical tips for making some trade off between accuracy and computation cost while using these advanced methods.

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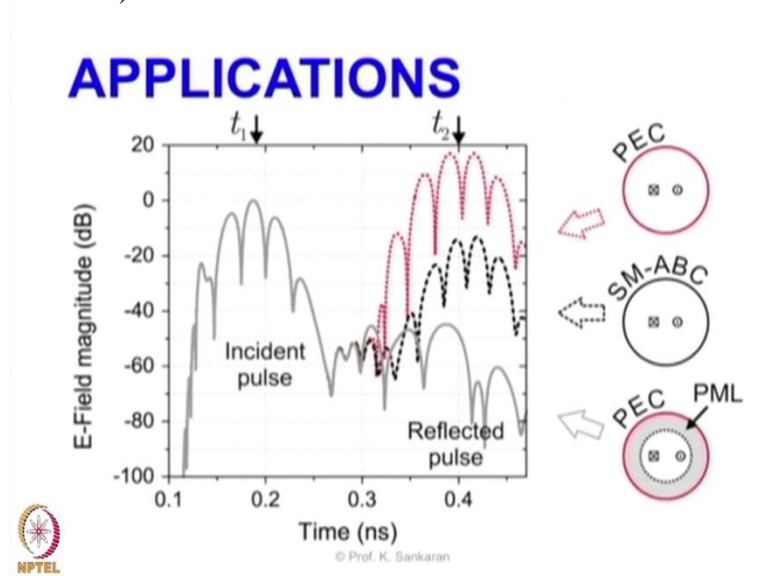
We also simulated some advance applications like a horn antenna.

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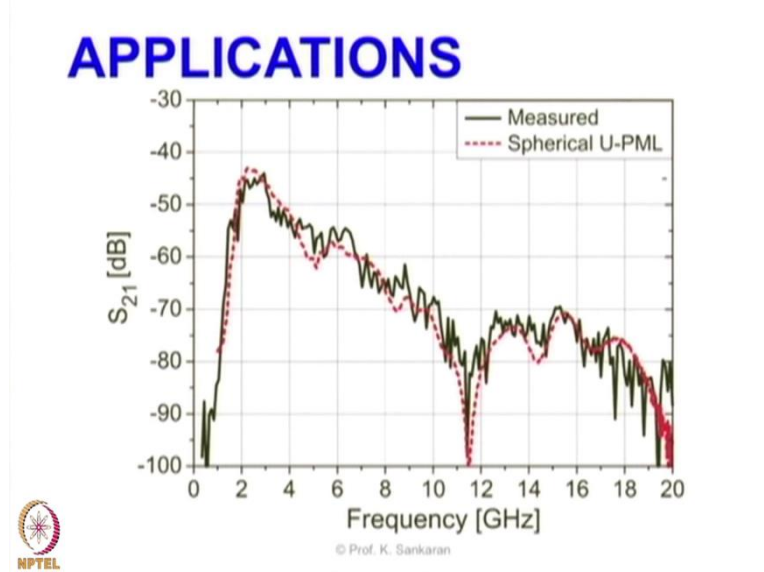
And coupling between 2 Archimedean spiral antennas using radial PML and Silver Muller absorbing boundary condition in the finite volume framework.

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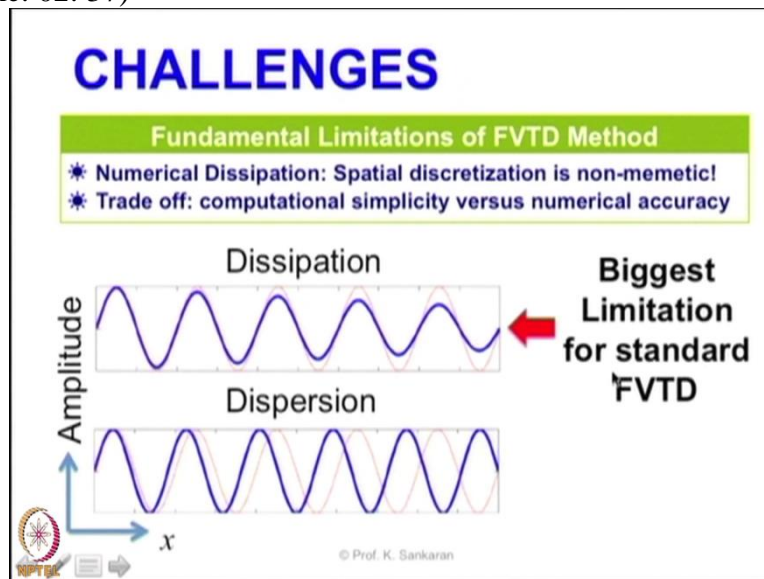
We compared the computed radiation pattern and;

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Cross coupling to discuss the performance of different domain truncation techniques in the finite volume time domain method

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Finally we also discussed one of the biggest challenges while using Finite difference time domain method namely the numerical dissipation.

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CHALLENGES

Spatial Discretisation Challenges
Collocating doesn't follow duality between E and H
Difficult to verify: $\nabla \times (\nabla \cdot) = \nabla \cdot (\nabla \times) \neq 0$

Classical FVTD Method
Doesn't take into account the "geometrical" aspects of underlying physics

point line surface volume
P L S V

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We also discussed source of such dissipation while computing the flux function and how it is inherently tied to the manner in which we do the spatial discretisation and compute the flux function.

We also briefly alluded how and why co locating electric and magnetic fields in space and time affect the duality relationships that exists between them. We mentioned the coefficient between space and time while using the finite volume formulation does it consider the underlined topological aspect of the electromagnetic quantities

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$$u_i^{n+1} = u_i^n - \frac{\Delta t}{h} [\psi(u_i) - \psi(u_{i-1})]$$

2D

$\psi(u_i) = c u_i$

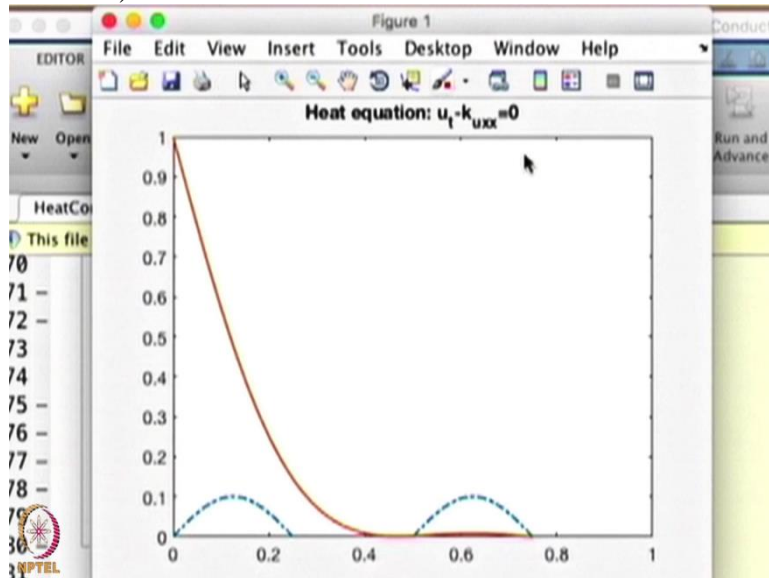
CFL stability $\rightarrow 0 < \frac{c \Delta t}{h} \leq 1$

$$u_i^{(n+1)} = u_i^{(n)} - \frac{c \Delta t}{h} (u_i^{(n)} - u_{i-1}^{(n)})$$

Explicit Formulation.

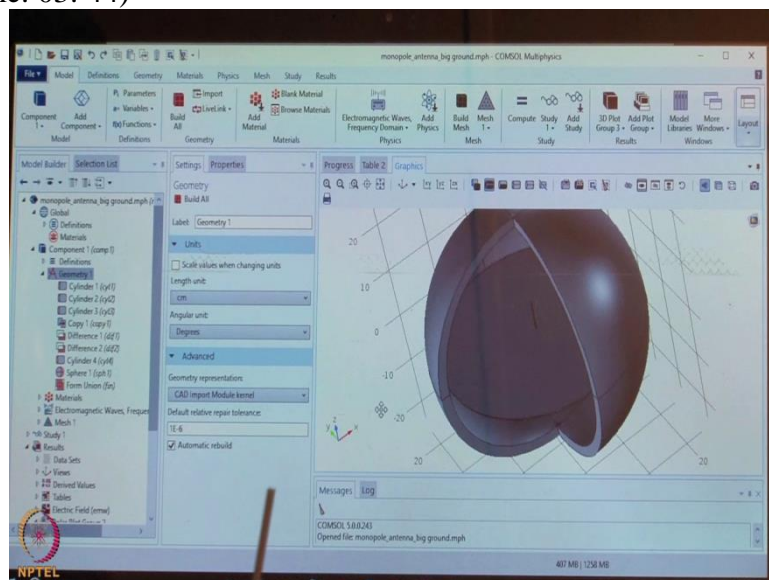
Towards the end we also introduced a simple heat conduction problem using finite volume method.

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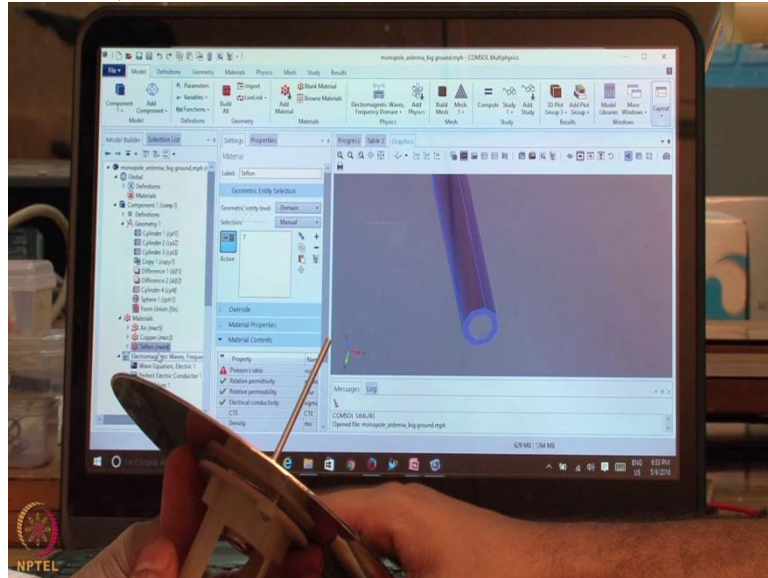
And also explained how we can model such problems using matlab.

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In this week's lecture during the lab tour we also modelled and simulated the permanent bar magnet and monopole antenna problems.

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We carefully learned the theoretical aspects and applications we studied this week

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BACKGROUND

Net Flux into volume [a,b] \longleftrightarrow i \longleftrightarrow Net Flux out of volume [a,b]

$x = a$ $x = b$

Rate of change of total-field inside the control volume is **ONLY DUE TO FLUX FLOWING INTO AND OUT** of the control volume.

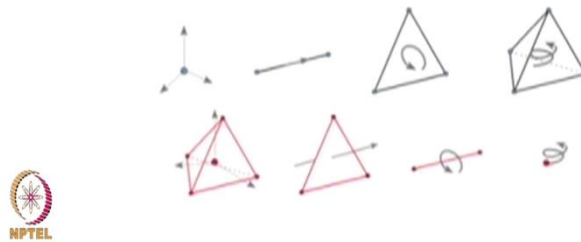
Finite volume time domain method is not normally discussed in any graduate course work in computational electromagnetics

Hence take this opportunity to learn as much as you can during the course work and also disclose your work

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Algebraic Topological Method (ATM) – I

Prof. Krish Sankaran



In the next week we would be introducing one of the most beautiful methods in computational electromagnetics there is not Algebraic topology. This is one of the rarest methods that you will learn in any course work or computational electromagnetics. I am very much excited and I hope you are equally excited as well. So we will see you next week Until then Good Bye!