Computational Electromagnetics and Applications

Professor Krish Sankaran
Indian Institute of Technology Bombay

Lecture No 29

Finite Volume Time Domain Method-II

Welcome back! In the last week we had the basic introduction to Finite Volume Time Domain Method where we briefly introduced various aspects of this method we discussed about what are the important ingredients when it comes to the formulation? What are the aspects of which we are talking about it for modelling electromagnetic problems. So in this week what we are going to do is we are going to look into the main aspects of computing the fluxes because the flux function is one of the main factor which defines the accuracy of this method for various electromagnetic problems. So depending on how we compute the flux this is going to impact the accuracy. So with that we will start with this week's second part.

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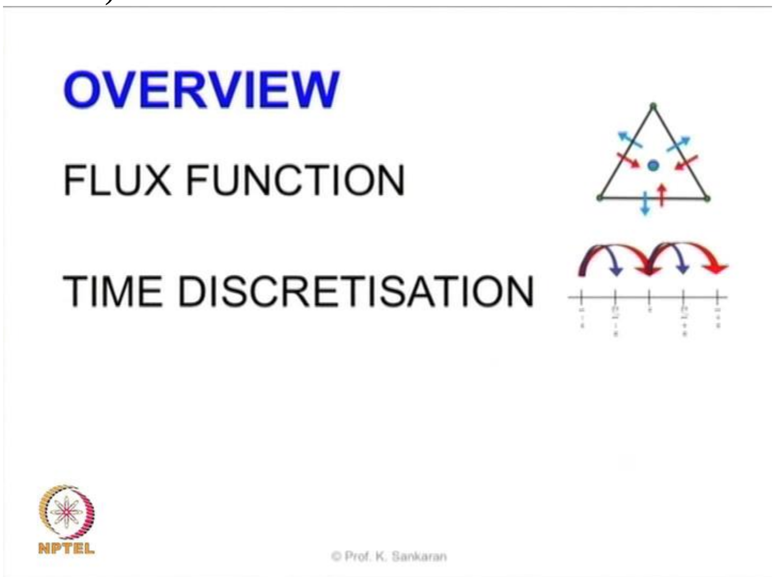

So where the first part will be the discussion of flux function. Remember that I discussed in the earlier module that the flux will be something that we compute across the surfaces if you are talking about the 2D this will be the edges which are covering this area. If it is a 3D problem it will be a surface that is covering the volume control volume and then with that flux function we will discuss quite a bit about what are the different ways in which we can compute that flux. In addition to that we will also look into the time discretisation aspect because we have not covered so far how we are going to discretise the time. How we are going to go from time step n to n plus 1, to n plus 2, to n plus 3 and so on and so forth. So time discretisation is something that we will cover in this module. because this is going to be slightly different to Finite difference method. But still it has a lot of similarities so this one we will look into it in this lecture.

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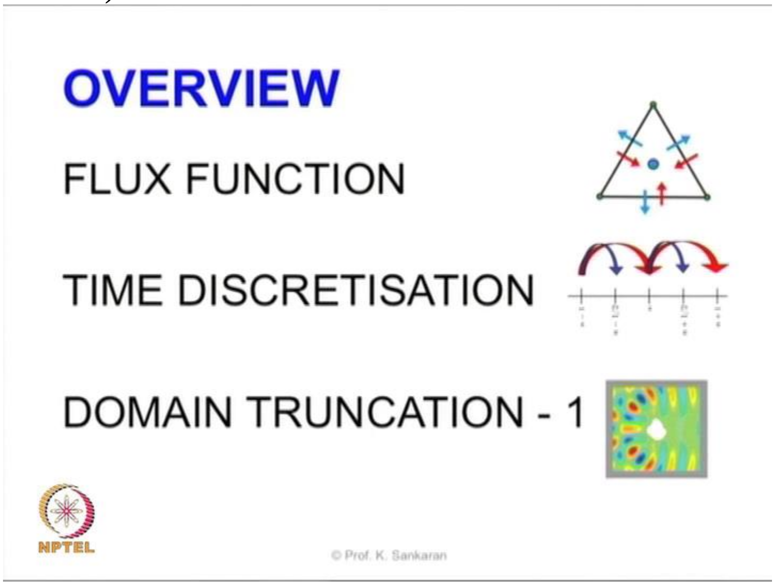

And also we will talk about some basic ideas about domain truncation I will cover one of the types and of course in the next lectures we will see more advanced domain truncation techniques with that we will start with todays lecture.

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So we are going to discuss about flux function as to begin with.

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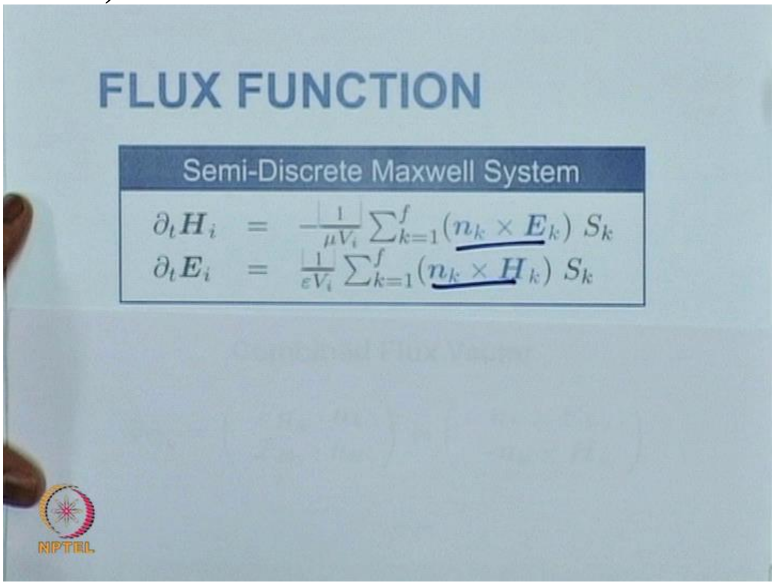

So remember in the slide we discussed about a flux function in the semi discrete form. So we have a semi discrete form for the definition of the flux where the partial derivative with respect to the partial derivative with respect to t is equal to the 1 by Mu over V i , V is the volume. And then we have the sum of all the flux components that are coming. So we said that these are the flux components, so this 2 things are the flux components. And of course the S k is the value of the surface area in a 3D problem. In a 2D problem this volume will change into area and then the surface area will change into length of the side.

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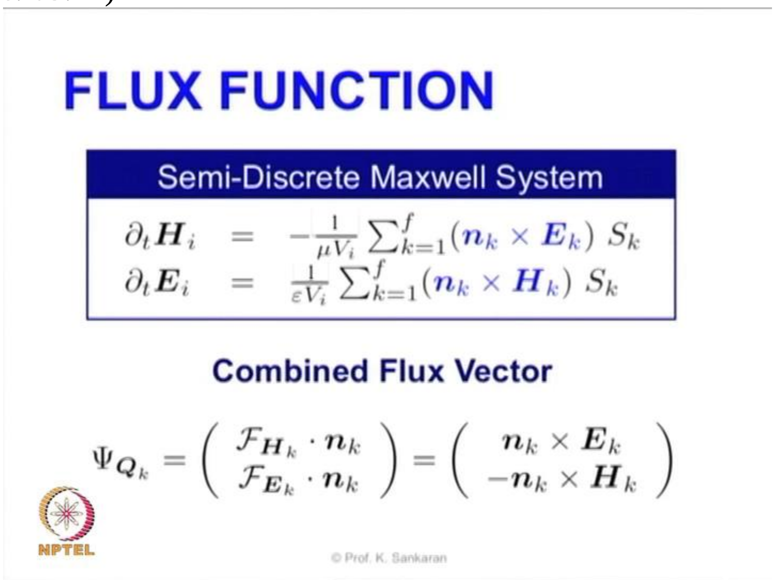

That being said the combine flux is something that we can write it as 1 vector. So which is given by the value here. So this thing is the first component will be the flux that is related to the partial derivative with respect to time for the magnetic field and the second one will be with respect to the electric field. So this is straight forward. So this is the basic thing that we will be looking into and we have to see how we can go on to compute this one in this lecture

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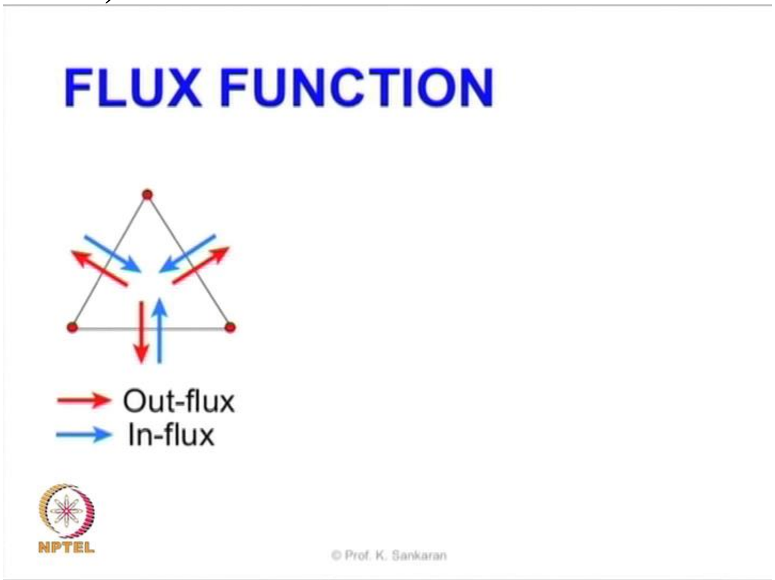

So I have said in the past that flux is both ingoing and outgoing.

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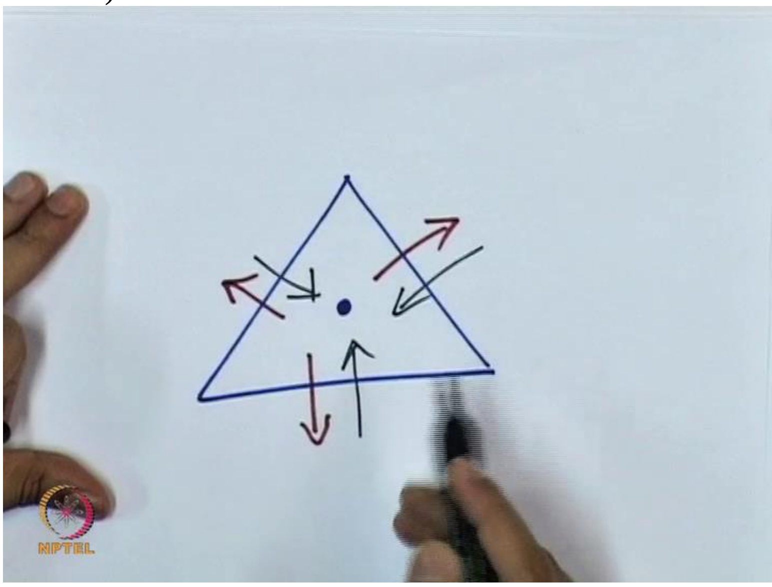

So when you consider a particular problem for example you have a triangle and this is your control volume and the values are in the centre and this control volume will have fluxes that are going in and out. So whatever is going out I mark it in red. And there is also fluxes that are going inside. So we said that the net flux will be the algebraic sum of those ingoing and outgoing flux.

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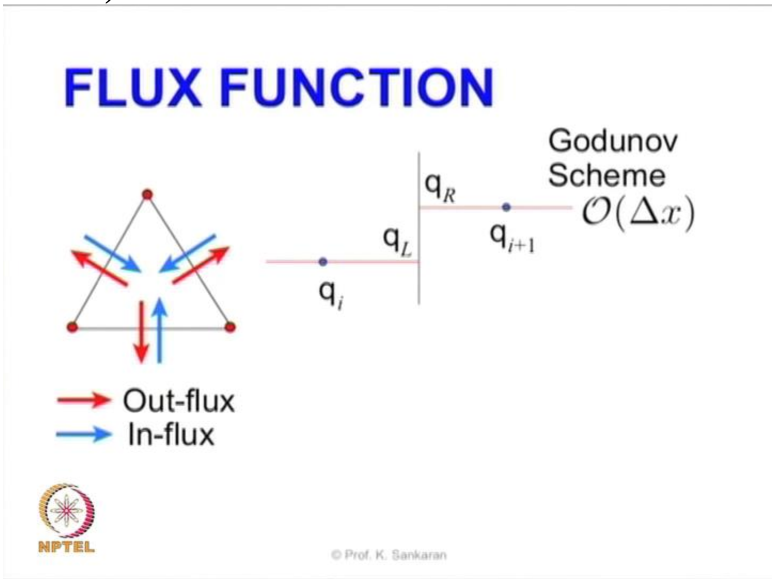

So this is what we have been pictorially mentioned here and one way to do that is to use a scheme that is first order approximation. The first order approximation is called as Godunov scheme. Let me explain that a little bit in further.

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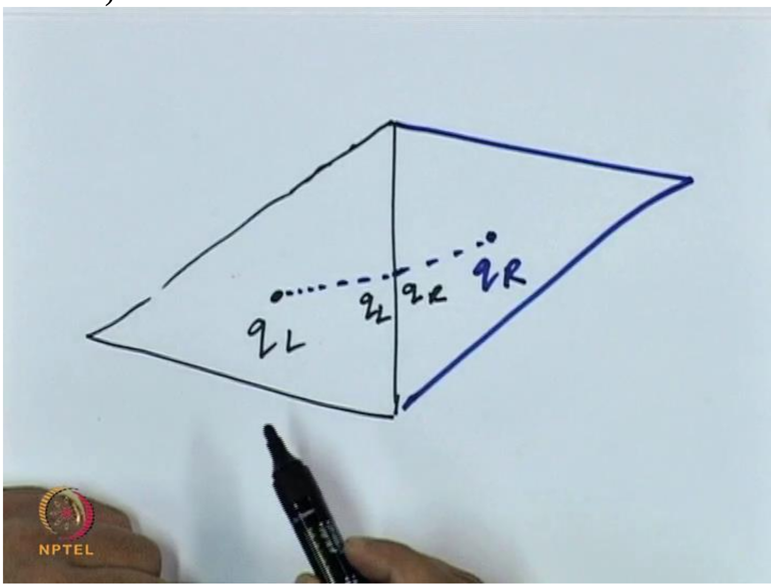

What I have is I have two triangles and these triangles are let us say the left triangle with value q L the bary centric value and the right triangle I draw it with blue colour whose value bary centric value is written as q R. So this is the face centre; so we are interested in the projection way. So one easy way of doing that is we can say whatever is on the right will be the value here. So we say Q on the right will be the right value here and q L on the left will be the left value here. This approximation is very crude as you can see obviously the value on the bary centre and the value on the face centre will not be the same but still we can say somehow they are the same.

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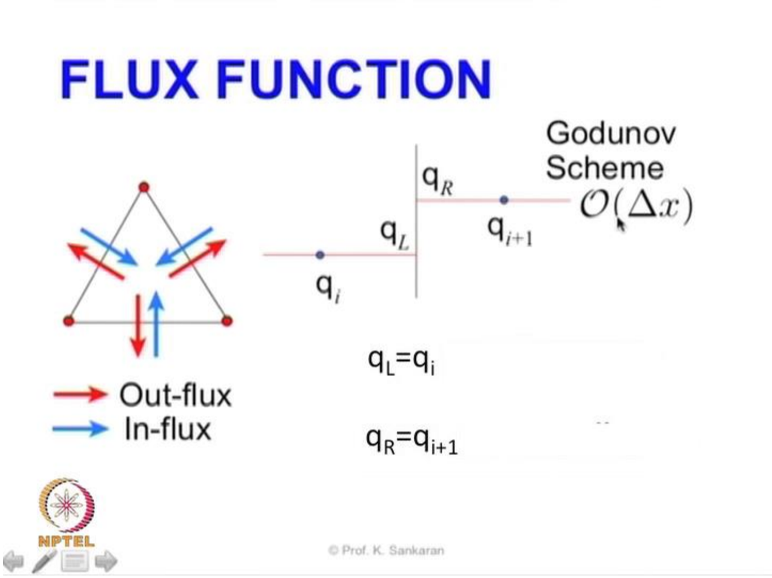

So the consequence of this is basically what I have shown here in the slide. So what is happening is we can say that it is a piece wise constant. We will see in the 2D how it will be in the next slide. But this is a kind of a constant approximation. Whatever is on the left so here qL is equal to q i and q R is equal to q i plus 1, where i and i plus 1 are nothing but bary centric values on the left and right hand side. And as I marked here this is of first order approximation.

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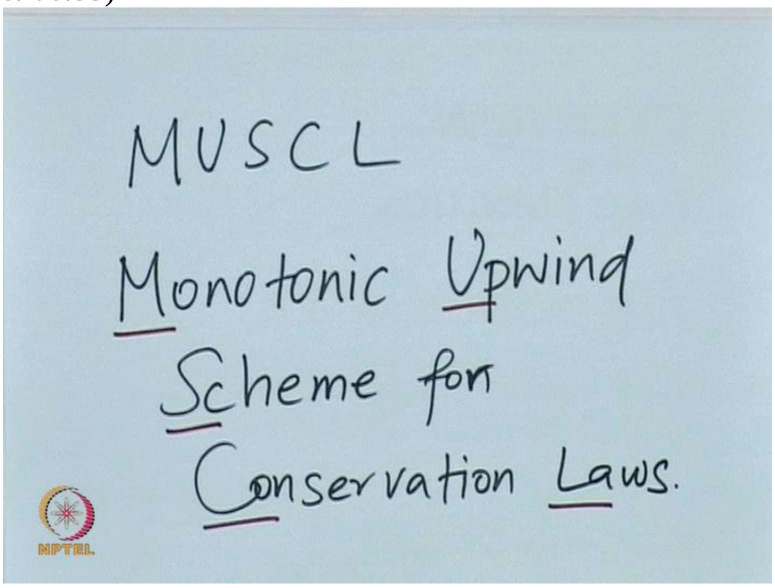

Another way of doing that is we call it as Muscle scheme is monotonic upwind scheme for conservation law. The word Muscle comes from there. This is a kind of a scheme that comes from computational fluid dynamics. As I mentioned to you the idea of Finite Volume Method itself is coming from computational fluid dynamics and they have been using this for several applications in that domain. So this Monotonic Upwind Scheme for Conservation Law or other words called as MUSCL scheme is an approximation and this is a second order of approximation. So we will see how it is second order.

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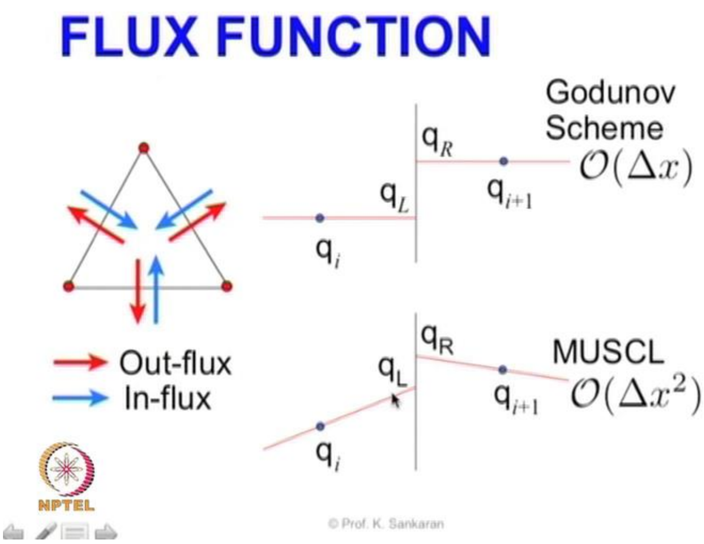

So what is happening here is what ever is on the left hand side of this interface will give you certain value but this value will not just be the value itself but the gradient. What I mean by this? Let me explain this further.

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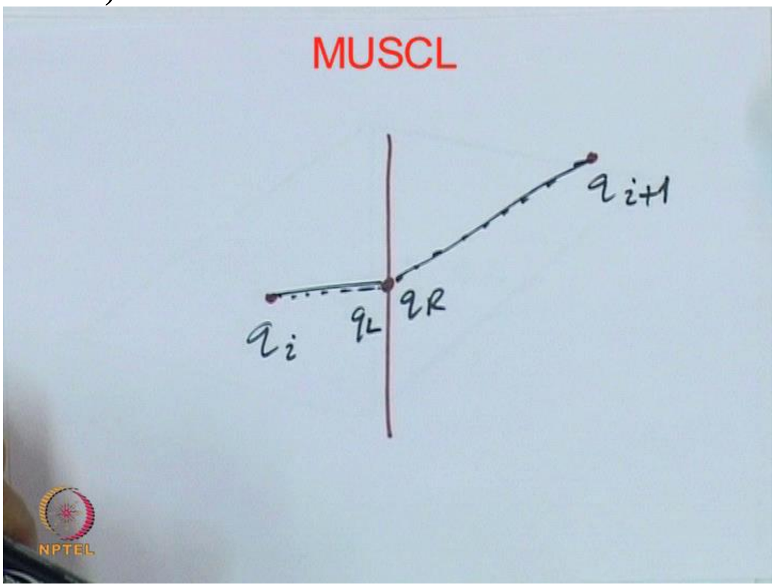

So what is happening here is? I have a value which is on the left hand side, and I have a value which is on the right hand side. And I am interested in this point which is the centre of the space. So there is going to be a gradient that we are going to compute between these two points. What I mean by gradient is the slope. The value of the function on this one will be taking into account this particular geometrical gradient so whatever is there so it will change. So if there is a slope here, so based on that we can compute a gradient between this point and this point, this point and this point so the slope will affect on the computation of whatever is going to be computed on the left and right hand side. For example q i will take q l will take into account the slope between these two points and slope between these two points.

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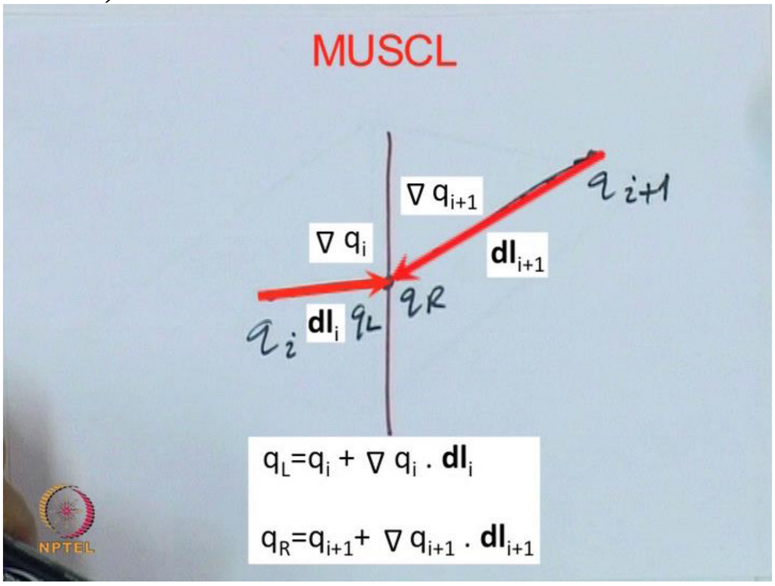

So when we go in the next slides we will discuss some improvements to this particular thing But its important to know that it is not just equating directly to the left hand side itself or right hand side itself. But we are going to talk about the slope of the thing.

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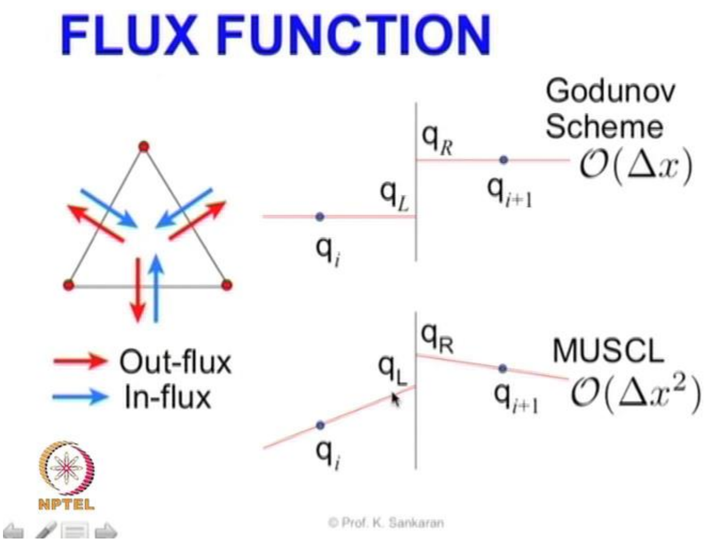

So as you can see in the slide this is the second order approximation and when you look at it from a 2 Dimensional point of view.

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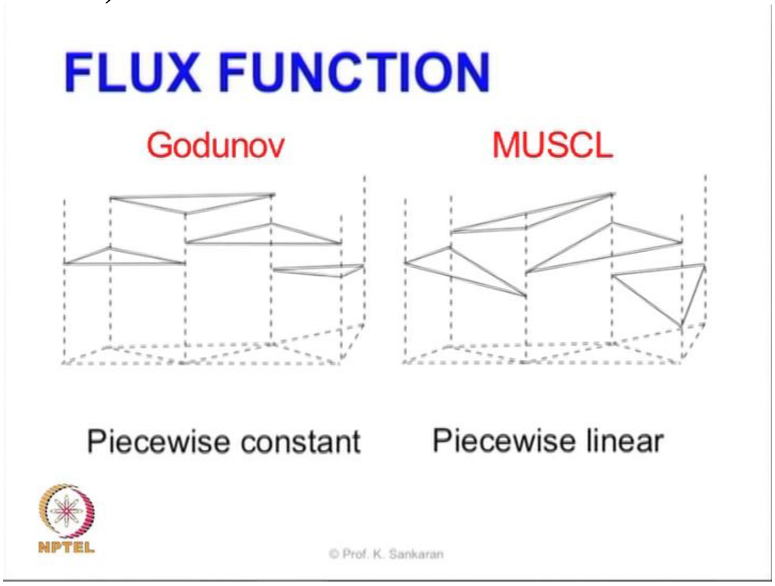

So what we will have is in the case of the Godunov scheme you will have a Piece wise constant so the triangles will have values which are equal across all the sides based on values at the centre. So based on that each of the neighbouring triangles will have different constant values. So a slight improvement to that will be the peice wise linear approximation where you will have values that will take into account the slopes. So you see that it is having a slope function within each of the domains and the each of the control volumes. And in 1D it will be simply a line instead of a plane.

So here we see that both Godunov scheme and also the MUSCL scheme they are quite good for computational fluid and a mix where they have quite a lot of instability. So in a way having a discipation what I mean by discipation you will come to know in the next slides. So in a way when the wave decase in its amplitude due to an inaccuracy that is coming from computation of the flux function it is good for them. And that works very fine when they are using computational fluid dynamics problems. But in our case the solution does not go into instability naturally. So numerical solutions become unstable but the physical solution itself is not like that so what happens is having any amount of dissipation or in accuracy that is coming from our way of computing the fluxes should be avoided. So numerical in accuracy or instability should be avoided.

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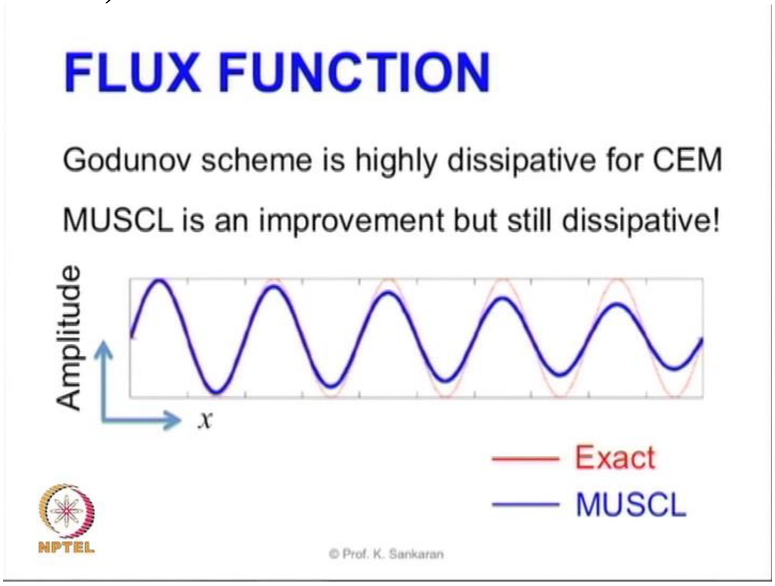

So as we said Godunov scheme is highly dissipative for CEM. We donot use that for any particular application. MUSCL scheme is an improvement definitely it has certain improvement but it is till dissipative. If you ask somebody who is working with fluid dynamics they will say that well we are happy with it but people with electromagnetics as I mentioned they are not happy with it. So we have to see how we can improve it.

So what I meant by dissipation is let us say you have an wave that is going in X direction. So the line which is in red is actually the exact value. The amplitude of the wave does not change should not change. But what happens is when you are trying to use numerical methods due to various reasons what we discussed just now the flux function computed is not accurate so it reads to slow DK in the amplitude. This is what is called as Dissipation, to be more specific we call it numerical dissipation.

So that being said we are now going to discuss what are the different ways in which we can improve this. We tried few techniques they work on a case by case basis. It is good to know them it is good to know where to apply when to apply them. In that sense we get an overview of various capabilities. Both the pros and cons of Finite Volume Time domain method.

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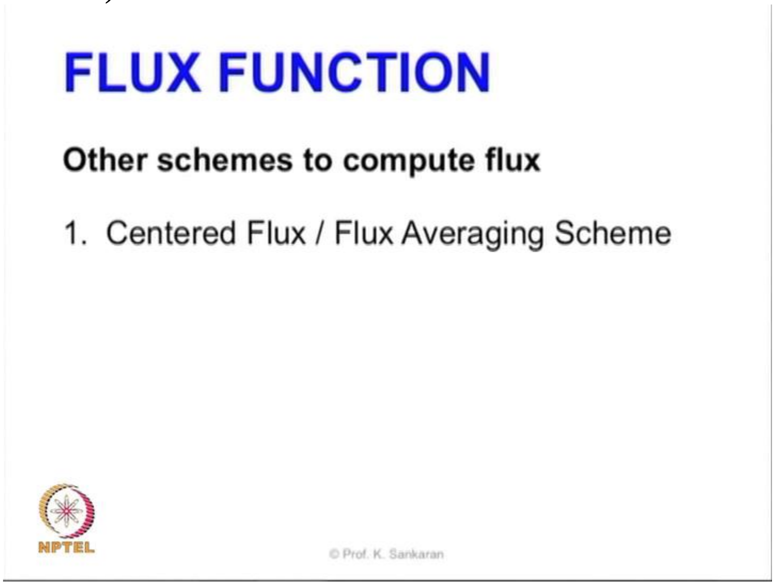

So one of the initial schemes I will look inti is the centered flux scheme. So its also called as Flux Averaging scheme. So the centered flux scheme or Flux Averaging scheme depending on who you are speaking to they mean the same thing and you will see how they are done in the case of electromagentic problem.

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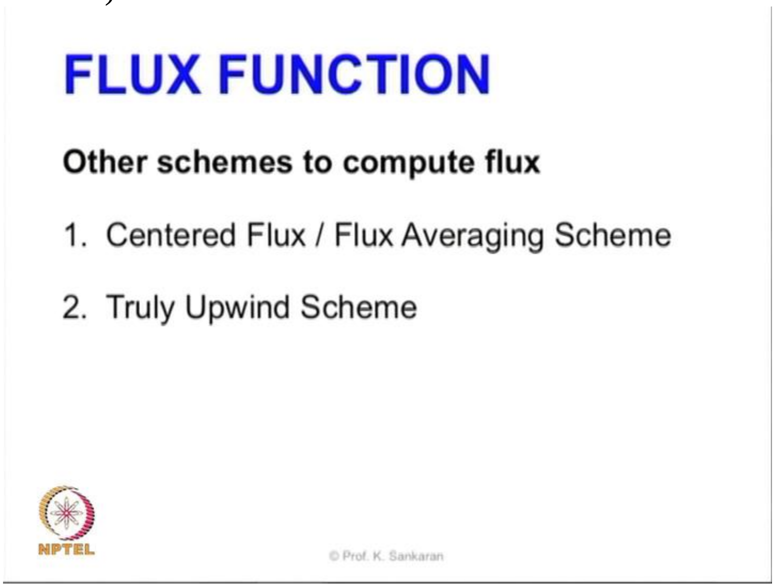

So the second one is truly Upwind scheme, the truly upwind scheme is basically mimicing more or less what MUSCL scheme the Monotonic Upwind scheme is doing but in much more structured way it looks into the geometrical parameters of a particular control volume and takes into account certain aspects.

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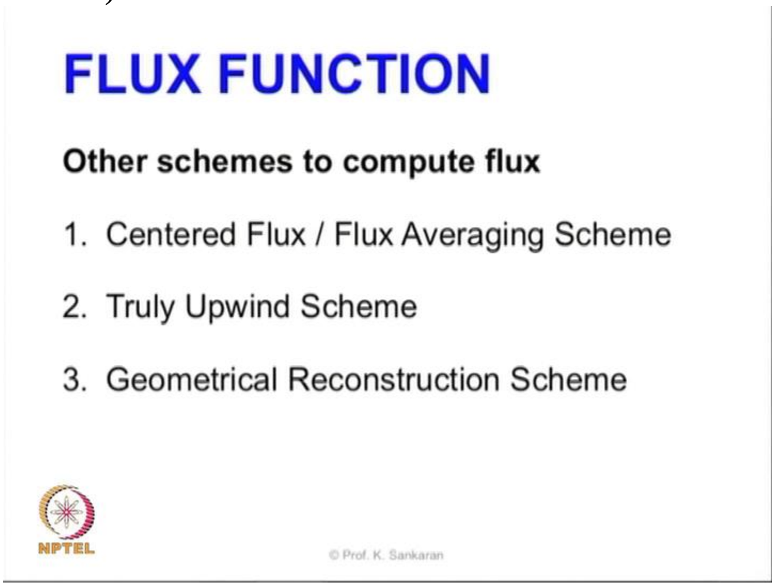

And the last one is the Geometrical reconstruction scheme which is a little bit more of an improvement compared to the Truly Upwind scheme.

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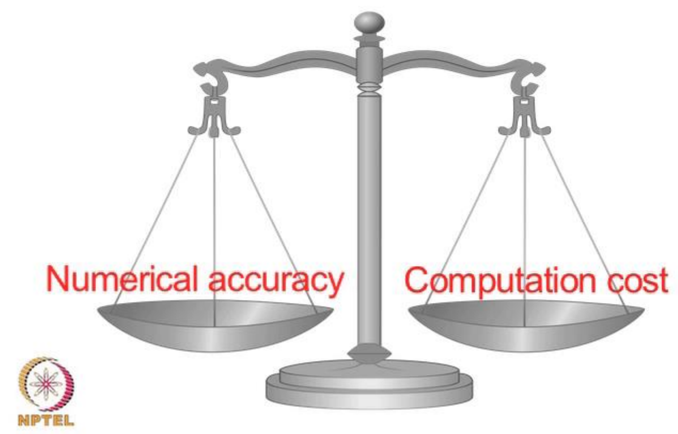

So a note I have to mention here is depending upon which ever scheme you are using the pros and cons are one is to look into the numerical accuracy that scheme brings but also we have to pay attention to the computational cost because we do not need to do too much computation in order to get a better approximation. So there is always a kind of a what should I say trade of compared to what kind of numerical method we use to compute the flux versus how much smaller we are go in terms of discretisation and also the better approximations we can get through those new functions.

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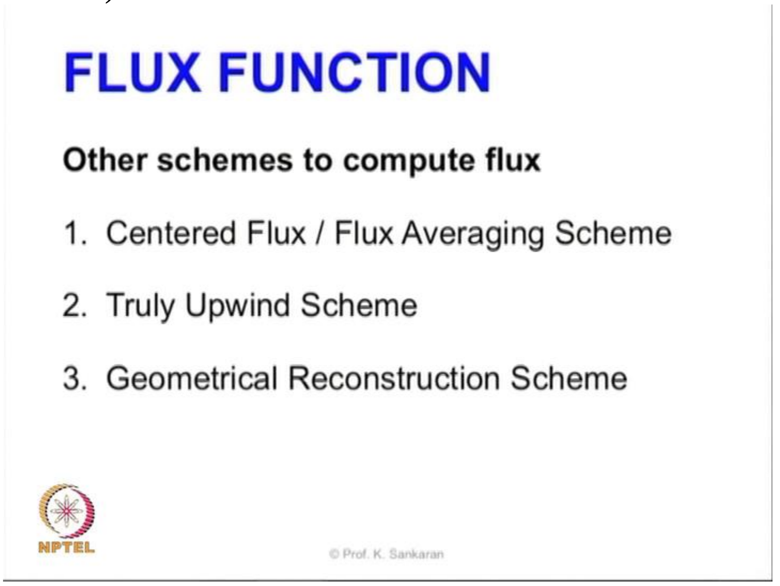

So with that being said in the next parts we will see into how we are going to compute the centered flux truly upwind scheme geometrical reconstruction scheme. So with that I will end this module.