Computational Electromagnetics and Applications
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Exercise 19
Method of Moment

We are now going to look into yet another interesting problem which is a problem of characteristic impedance of a transmission line.

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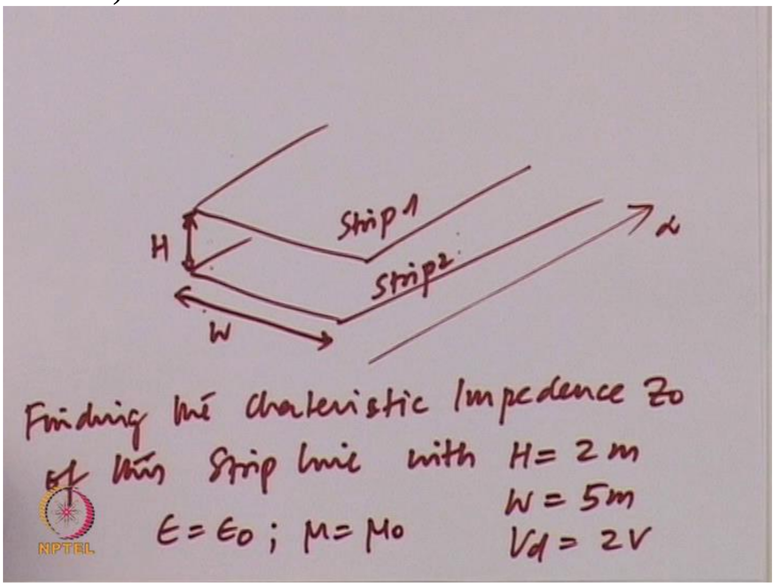

So in this case what we are going to have is a strip and we are going to approximate it using certain techniques and we are going to find out what is a characteristic impedance of this strip. So let us start looking into the problem itself and then we will discuss how we are going to solve it using the method of moments.

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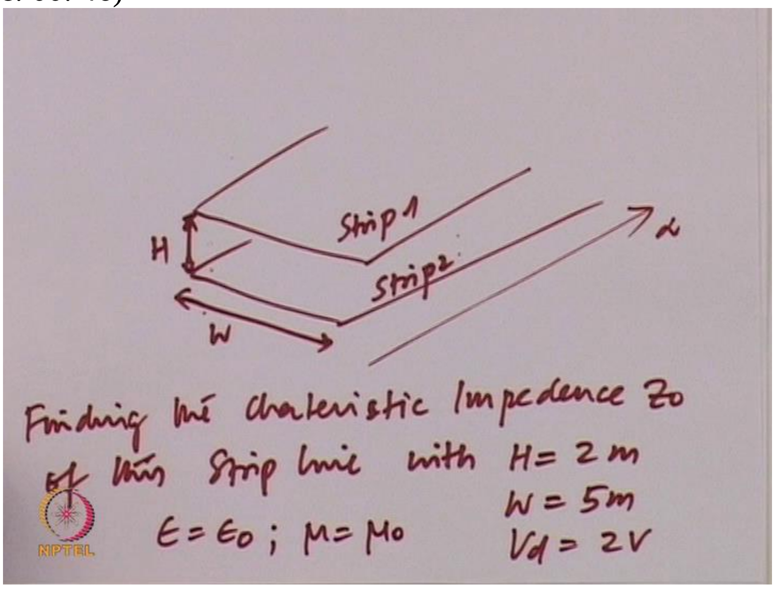

Let us look into the problem the problem is that of pair of strips that are lying in a lane and what we are assuming is the strip is infinitely long in one direction. So let us say this is strip 1 and this is strip 2 and it is infinitely long in this direction. And what we are interested is finding the characteristic impedance Z 0 of this strip line with certain approximation. Here the approximation is the distance between the strip is going to be H that is equal to 2 meter and we are talking about a width of the strip so this is the width this is the distance. The width of the strip is going to be equal to 5 meters. And we are assuming that the potential difference between them is going to be 2 volt. So potential difference between them is going to be 2 volt. And we are assuming there is a free space between them. Epsilon equal to Epsilon 0; and Mu is equal to Mu 0. So this is going to be our problem definition and what we are interested is in finding the characteristic impedance of this strip line.

So to solve this problem we are going to look at geometrical simplification of the problem, so what we are going to do is we are going to transform this problem into two dimension problem. And that is what we are going to do now.

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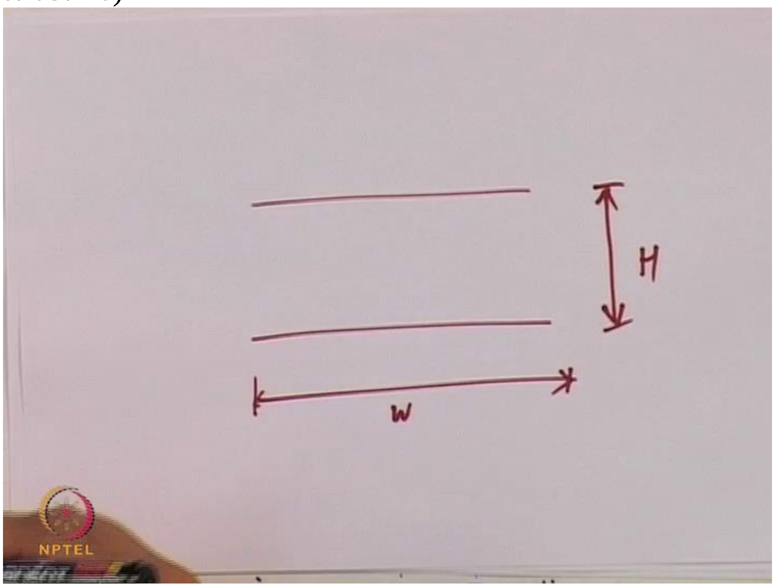

So we take only the direction where we know that the plane is there and then we are able to write here the width and the height. And we are not considering the third dimension which is infinitely long. So we have reduced the 3D problem to a 2D transmission line problem of a line source in a plane.

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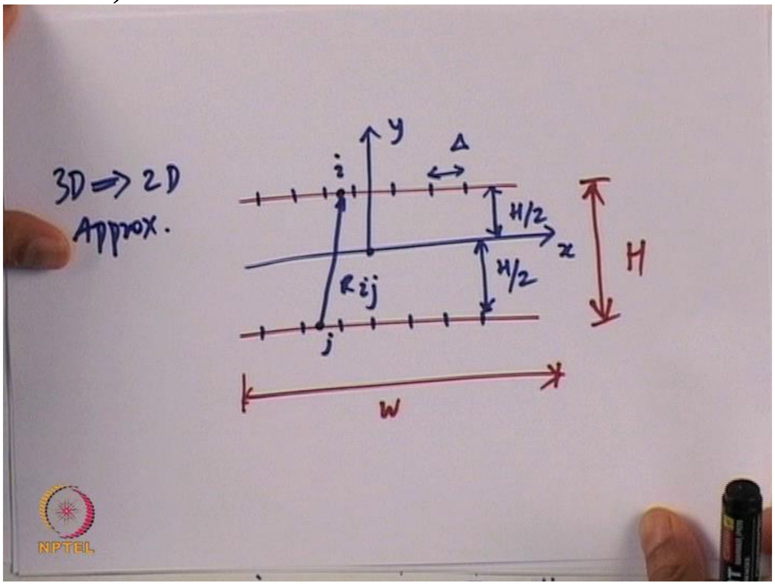

So let us assume that our x and y axis are located in this manner; this is my X this is my Y and I am finding a way to discritize this particular line into finite number of pieces. So 1,2,3,4,5,6,7 so on and so forth. And similarly I do that also for the low line. And I assume the distance or the grid size is going to be delta. And the origin is exactly at the center. So I have here h by 2 and here H by 2. And now I am interested in knowing the impact at a point i because of a point source at j. So assume that this distance is R i,j this is j this is i and obviously you can also do the reverse of that you can see the impact of the point i on j. So this is the way we simplify the 3D to 2D approximation. Once we have that we can go forward and write the surface charge density of the strip line as follows

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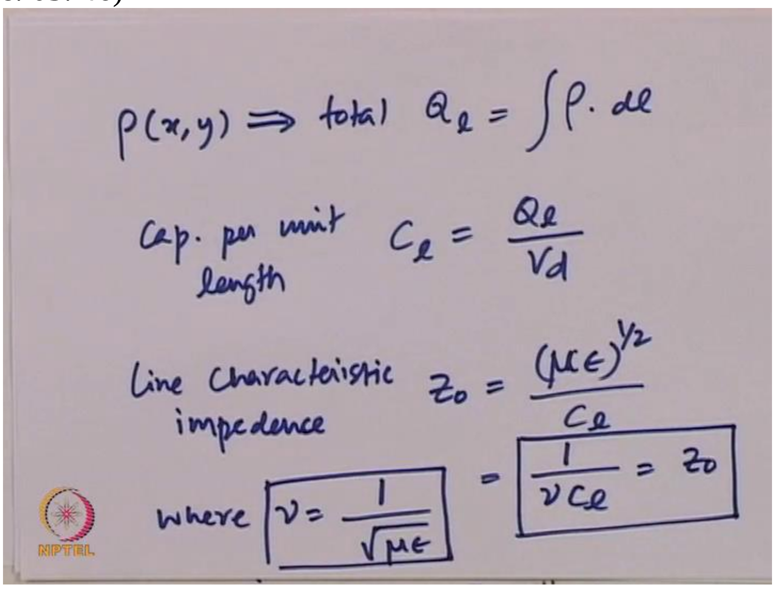

Ok so the surface charge density is going to be written as Rho of (x,y) is equal to total surface charge on the line. So the surface charge density Rho (x,y) is going to give the charge that is going to be on this line if we integrate it. So from the Rho(x,y) we can get the total Q l where l stands for the line as follows. So integral Rho dl. And the capacitance per unit length which is C l is going to be given by Q l divided by V d. We know V d we have to find Q l to get C l. And the lines characteristic impedance is going to be given by z 0 which is equal to Mu epsilon whole power 1 by 2 divided by C l which is nothing but 1 by V C l where v is the speed of the wave on the transmission line or the script line. So this is going to be equal to Z 0. So the wave we are going to compute our characteristic impedance is going to be along this line of thought.

So now we have to find the value of Rho in order for us to get Q l and once you get Q l this is the pathway to get the characteristic impedance. So now let us focus on how do we compute Rho l.

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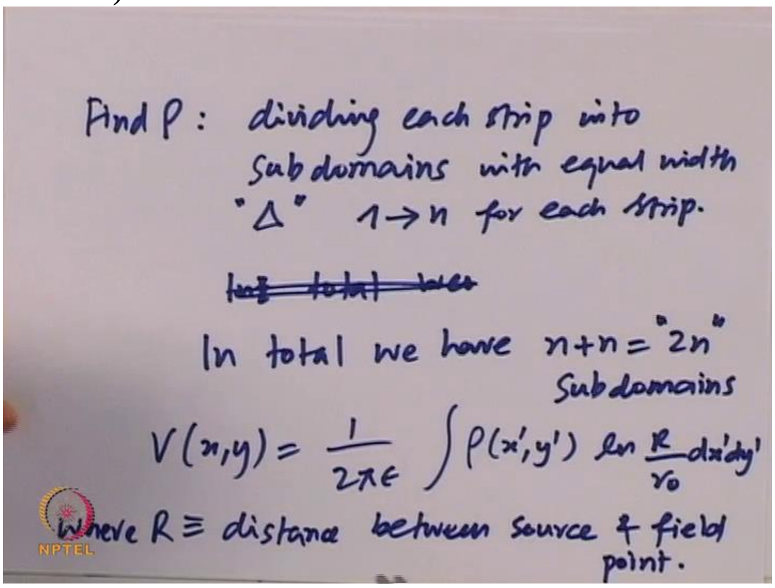

So let us start with the basic understanding what we have; so we have to find Rho. For that we are dividing each of the strip into sub domains with equal width delta. So this is the width of the each of the element. And it goes from 1 to n for each strip. So in total we have n plus n that is 2n sub domains. So we have V(x,y) 1 by 2 Pi epsilon integral Rho (x dash y dash) natural logarithm of R divided by r 0 dx dash dy dash. So for a source that is located at x dash y dashwe are computing the potential that is being calculated at (x,y). So this is a kind of a Green;s function algorithm for us to get the potential due to a source funtion which is the Rho(x dash y dash). And R is equal to the distance between source and the field point. So this is something that we know from the basic theory of method of moments that once we have this fromulation for the Green's function we can get the value of V using the method of moments as follows

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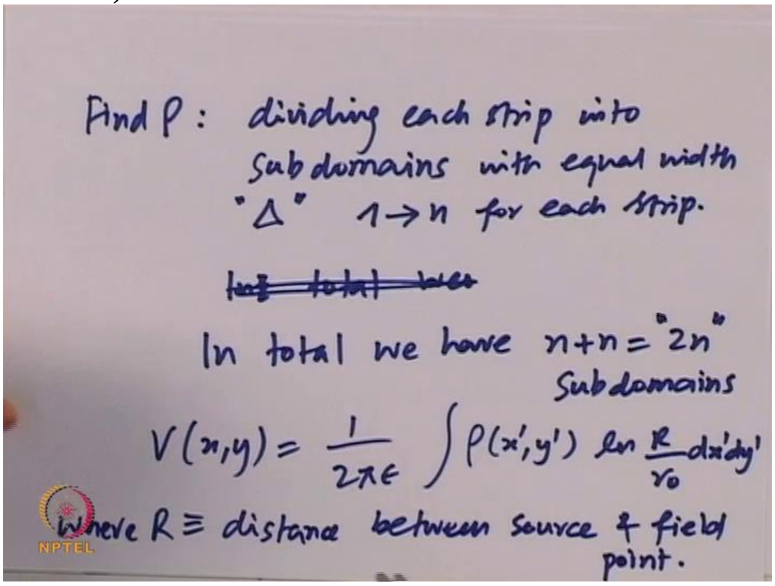

So here it is important that the R what we are calculating as the distance between the source and the field point can be directly derived from the knowledge of x and x dash and y and y dash.

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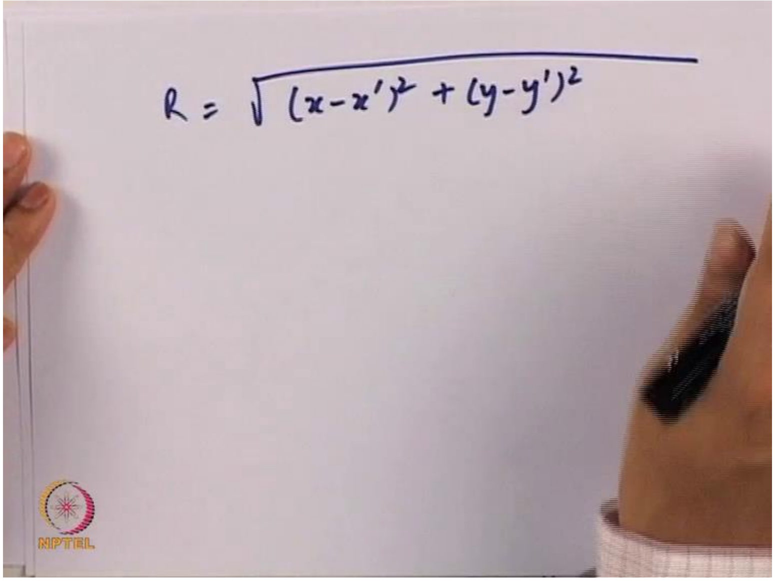

So how we do that is a very simple thing so R is given by the square root of (x minus x dash) square plus y minus y dash square and obviously in the third dimension you will also have a Z co ordinate.

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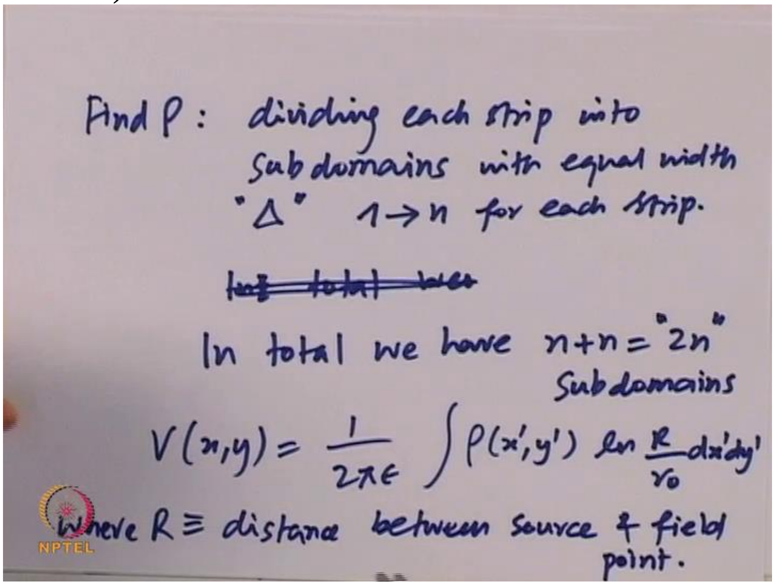

So the r 0 what we have in this particular equation is a scaling function which is set to unity for this problem

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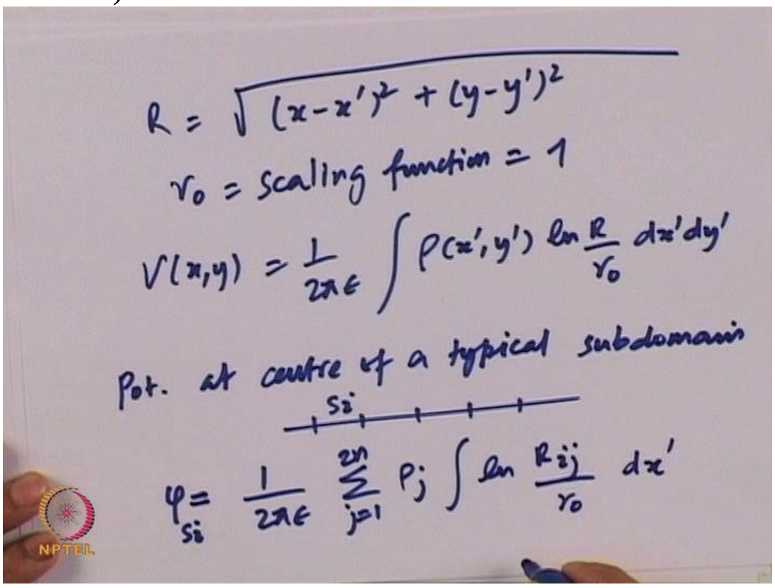

So r 0 is the scaling function set to unity. And now from above integral as in we get v of x,y is equal to 1 by 2 pi epsilon integral Rho (x prime y prime) Lu R by r 0 dx dash d y dash. from this integral the potential at the center of a typical subarea s1 s2 s3 so on and so forth can be calculated using the formula here. So what we can do is the potential at center of a typical sub areaor sub domain. And these sub domains are the domains what we have considered, so these are SIs. So the potential inside a sun area or sub domain SI is given by 1 by 2 Pi Epsilon summation of all the points so J goes from 1 to 2n. Remember we have two strip lines. So we have each strip line with n sub domain so we have two n sub domains in total. And Rho j integral Lu of R ij divided by R 0 of dx. The variation is only in one direction so we say dx prime. So this can be written in a simplified form.

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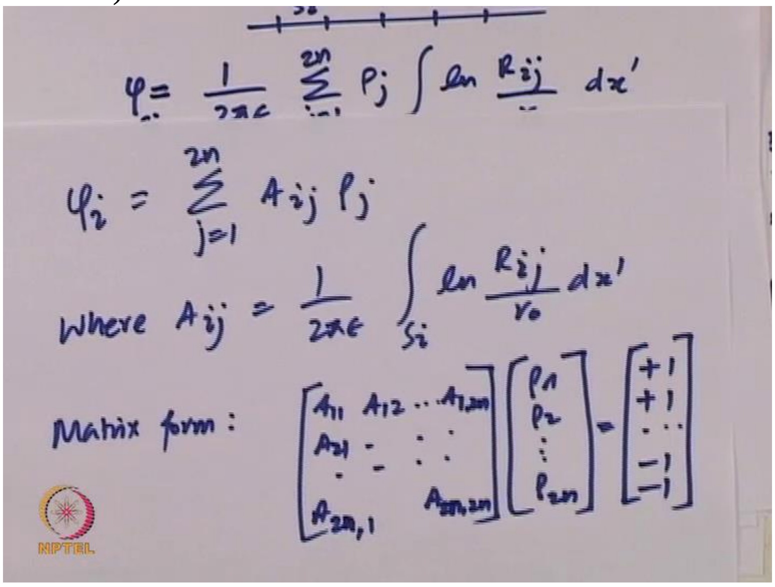

So we are going to simplify this even further writing Phi i equal to sigma j equal to 1 to 2n A ij Rho j. So this one says that a potential at point i due to the source at j. Where A ij is equal to 1 by 2 Pi epsilon Lu of the sub domain E ij divided by r 0 dx prime. So the matrix form of this equation is written as follows. Matrix form: So we have [A 11 A 12...... so on and so forth A 1,2n; A 21 so on and so forth A 2n,1 and A 2n,2n] multiplied by the column vector [Rho 1 Rho 2 ... so on and so forth Rho 2n] equal to the potential that we are given. So it is going to be plus 1plus 1 so on and so forth minus1 minus 1 at other points] . So in this case we assume that the sub domains are going to be in two different domains. One strip line is going to be in plus 1, the other strip line is going to be in minus 1. So the potentials here the right hand terms are going to be plus 1 or minus 1.

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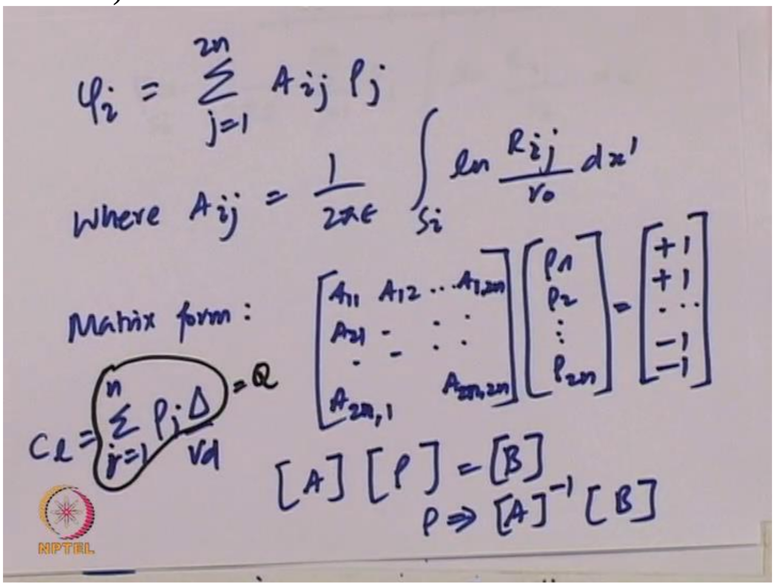

So this is nothing but [A] [Rho] equal to [B] . And we can compute Rho from this by taking the inverse of [A] and multiplying it with [B]. And this will give us the value of C l as sigma j goes from 1 to n Rho j Del divided by V d. So Rho j Del will be the value for charge that we are interested in. So this is going to be Q and Q divided by V dwill give you C l. So this is the way in which we are going to proceed doing this problem. So now let us go into the Matlab code and look at the way we have structured this code and how we are going to solve the problem and how the accuracy is going to change. And how the solution is converging.

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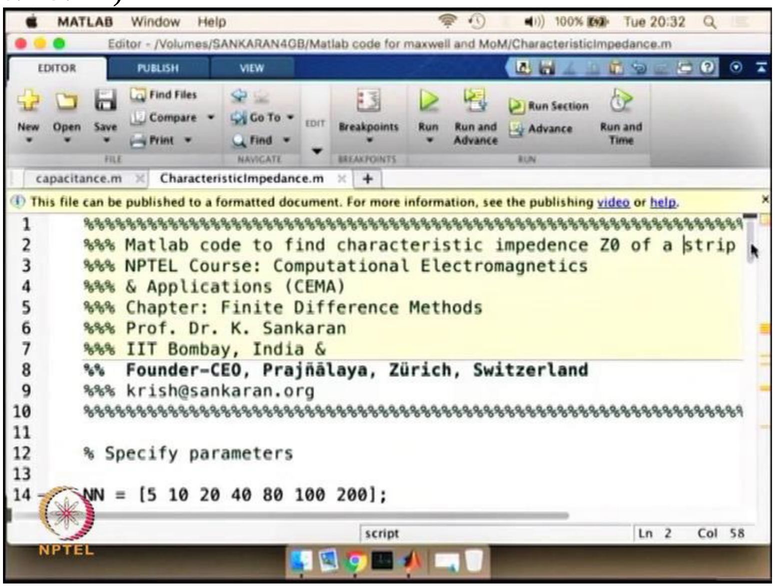

So let us look into the code to see how the numerical method is implemented to find out the characteristic impedance.

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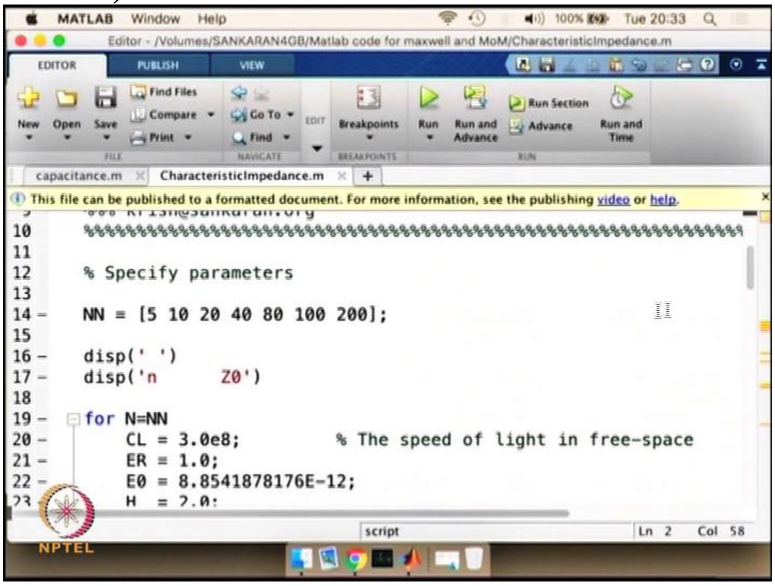

So here we have specified certain parameters n so this is going to be the number of points that I am going to have in the domain, if I have only 5 points, 10 points, 20 points in other words the sub domain are going to be 5,10,20,40,80 so on and so forth obviously when I have 4 more sub domains I will be having more resolution.

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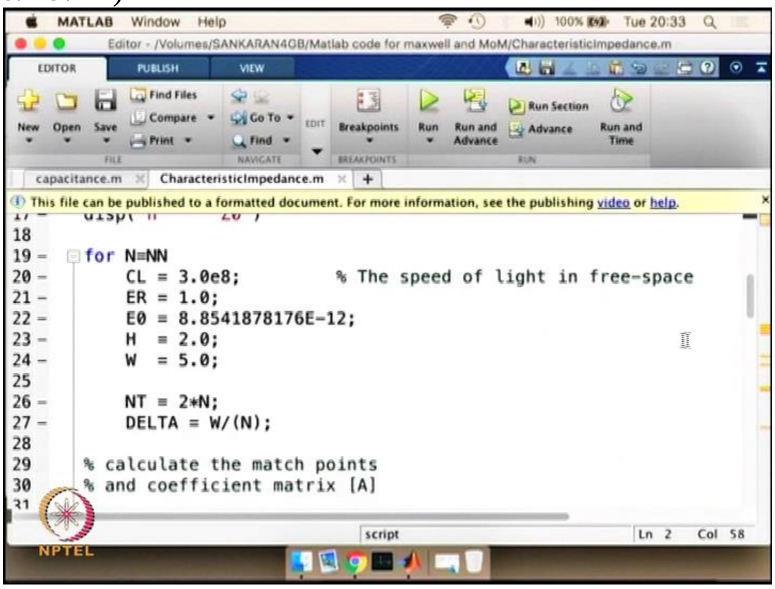

So I set certain parameters; so CL is going to be the speed of light and ER is going to be 1; Epsilon 0 is given here; H is given; W is given; and delta is going to be the Width divided by the number of points so that will give me the value of delta.

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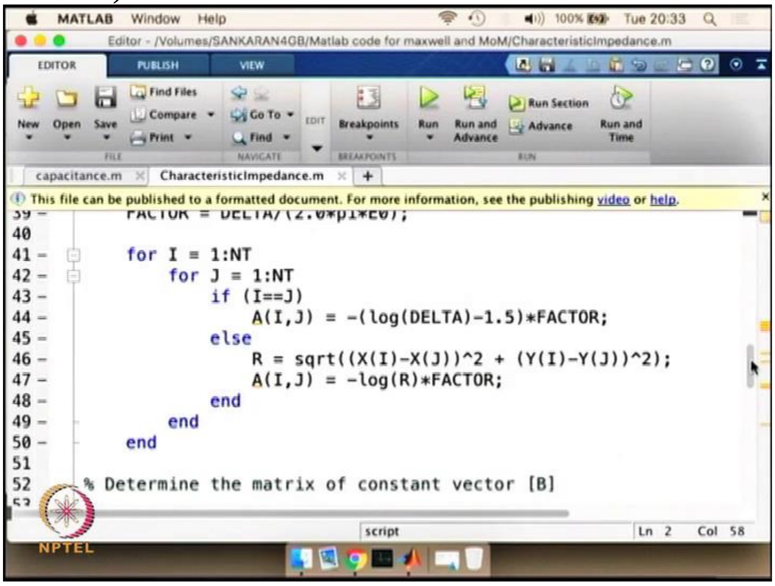

And I am going to compute it step by step using the Method of Moment algorithm what we have just discussed.

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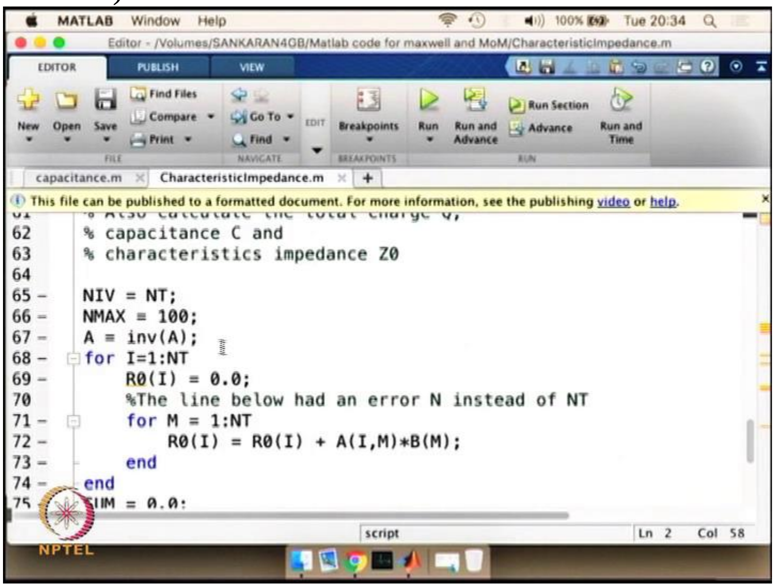

And when we set up the matrix form and we are going to invert the matrix form in the manner which is shown here. And this particular form is to invert the matrix, so I am using the internal Matlab function to invet matrix A.

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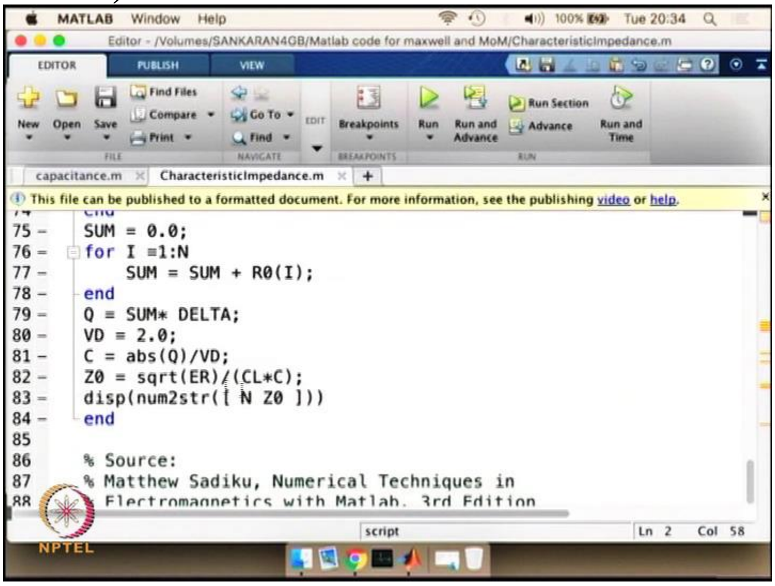

And finally I am displaying the result I have computed. So the code itself is a straight forward code we will give this code for you to try it out yourself. SO now we are going to run this code for various end. And ideally what we wanted to see is the convergence of characteristic impedance value that are computed with more number of points. So let us run it.

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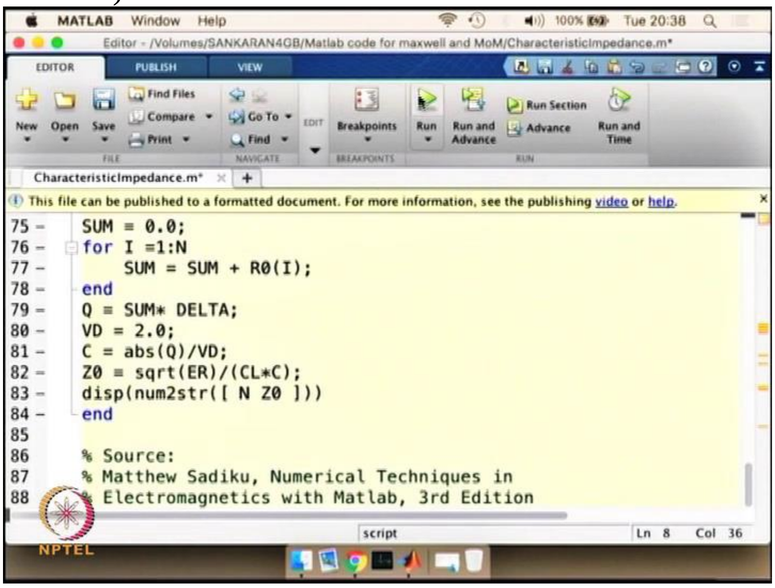

And what we are going to display at the end of the code is the value of the characteristic impedance for the various end. So let us run it

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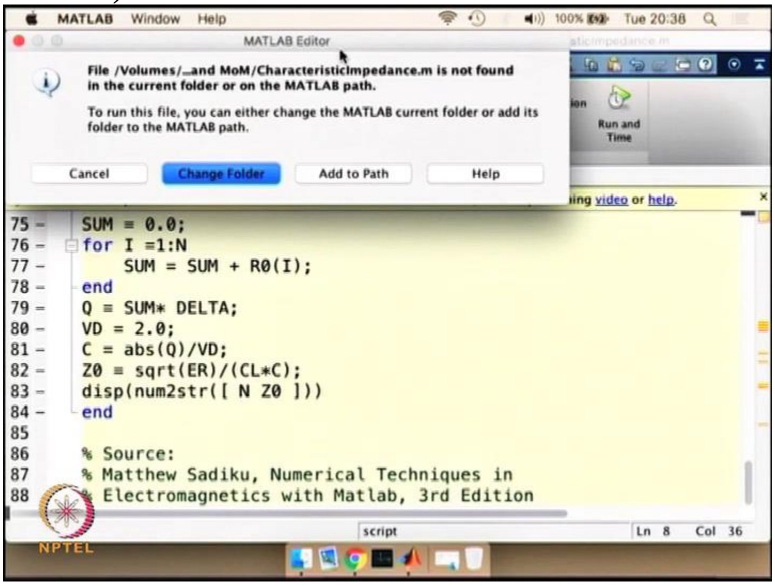

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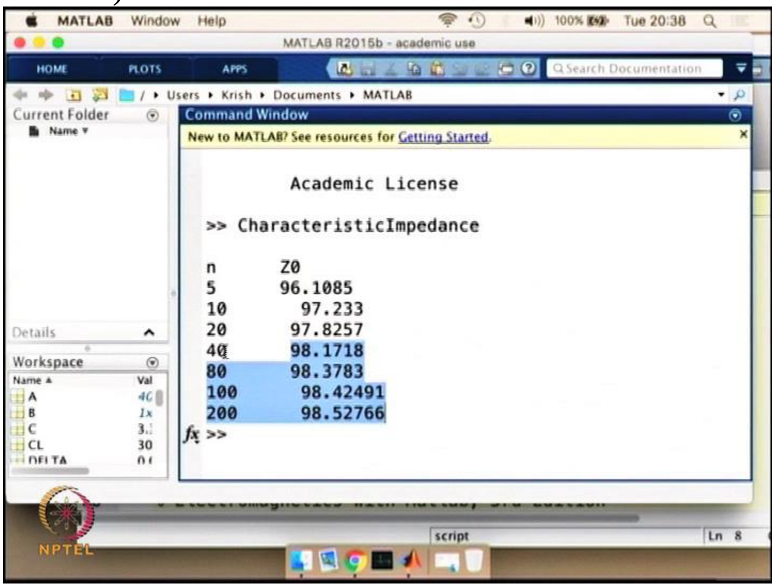

And we can go and see here the result and the result is showing certain convergence. And the value of n was 5 it was 96 the characteristic impedance. And when it goes to 10 it goes slightly higher value, and 20 even slightly higher value and 40 onwards it is staying in the 98 zone. S we can see that even while doubling tripling and making ten times the value compared to 20 the value is still in the 98 area. So that show certain convergence.

So what I would like to request you is to take this code and practice it for yourself. How the entire problem is structured and how we can reduce a 3 Dimensional problem into a 2 Dimensional problem. And how we can easily solve it using the Matlab program like this.

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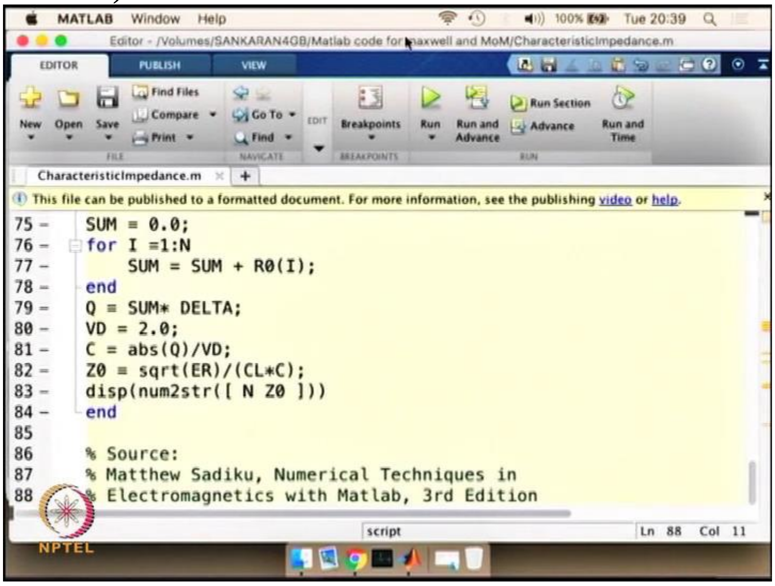

And the source of the code is also given below for you to try further problems and learn for yourself a techniques that are being used for solving similar problems using method of moments. And we have covered the part of the physical problem definition and we have also showed how the Matlab implementation is done. So with this we will stop the exercise on the method of moments . And I request you to practice it for yourself so as to get certain confidence incoding your own problems and using method of moments Thank You!