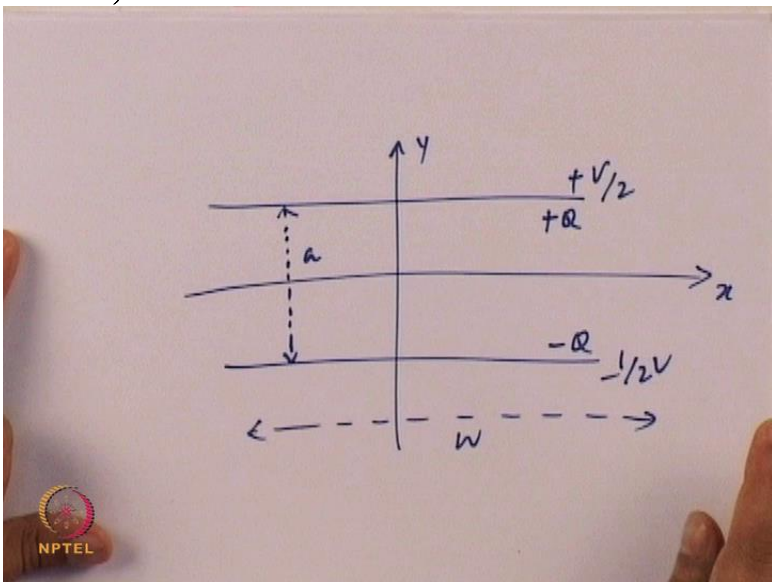
Computational Electromagnetics and Applications
Professor Krish Sankaran
Indian Institute of Technology Bombay
Exercise 18
Method of Moment

We went to look into simple capacitance problem in this module we are going to use Method of moments to solve this problem. So let us look into the problem geometry itself and I will explain you how we can later use method of moments to solve this problem.

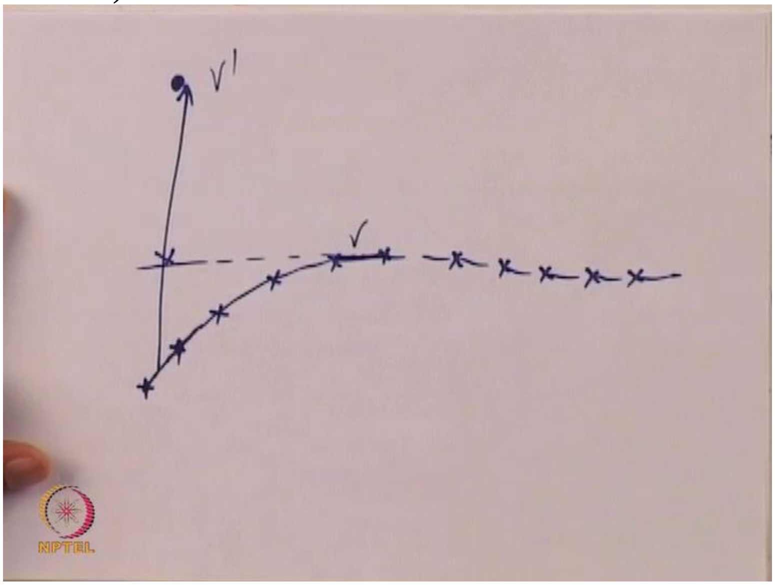
(Refer Slide Time: 00: 32)



Domain geometry is going to be a parallel plate capacitor and its having the x and y axis in such a manner so the top plate is going to be at v by 2 plus volt and the bottom plate is going to be minus 1 by 2 volt so the top plate is going to be plus half volt the lower plate is going to be at minus half volt. And the distance between the plates is going to be a. The top plate is going to have positive charges on it; the bottom is going to have the negative charges minus Q. And the width of the plate is going to be given by W. So this is going to be 2D plate capacitor problem that we are going to solve using Method of Moments.

As you know method of moments normally used when the surface phenomena become very important for example when these kind of problems it is not the bulk area which is in between the two parallel plates that is important whereas the surface phenomena that is the surface charges that are going to play an important role. So that is why method of moments will be a very very good tool to solve such problems. And not only that here we have got a parallel plate. But we do not need to have a parallel plate. We can have plate of orbitary shape. So we are going to take any shape and we are going to see release how such a model can lead to solution.

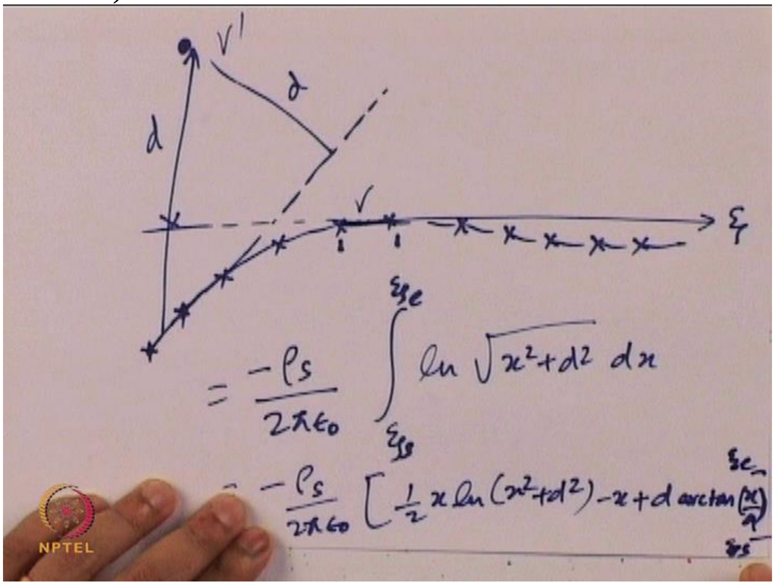
(Refer Slide Time: 02: 40)



So let us assume instead of parallel plate we are going to have a plate of orbitary shape. And this shape is going to have various elements. So the elements are going to be the ones that we are going to mark with x. So when you have such a arbitary shape what we are interested is we are going to find the value of the response at a distance from the source. For any charge it is going to be on the element you are trying to see the kind of respond that is going to happen for this element. So that is going to be given by the response function which is here going to be the Greens function.

So what we will do is we will start with a function D which is a minimal distance from a straight line extension to that element. So for example if you are talking about a particular element let us say this is the element you are interested in you are going to start talking about the distance that is going to be this . So for this particular element you are talking about this distance so the distance is the minimal distance from the straight line extending from that element. And the second thing is you are trying to find the contribution of this element to the potential at that observation point. The observation point is going to be r prime and the point where you are looking at will be r. So that is how we are going to see what is the impact of that particular charges that are at point r. And you are going to see the impact at the observation point at r prime.

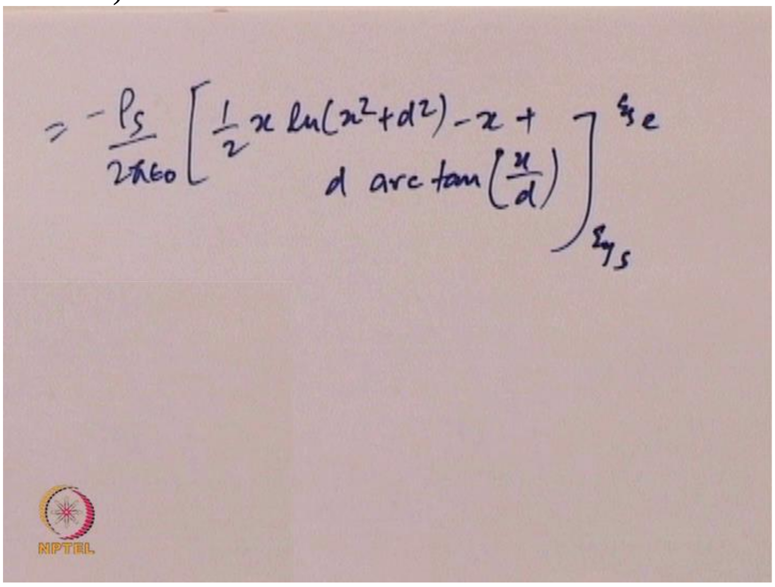
(Refer Slide Time: 04: 28)



So this is going to be the integral that we are talking about. So if you have this is as the zeta direction. And this is going to be the starting point and the ending point. So eta s is going to be the starting point and zeta e is going to be the ending point. So this is the starting and this is the ending point for this element. And Lu of square root of x square plus d square dx where d is the normal distance what we are talking about for this element.

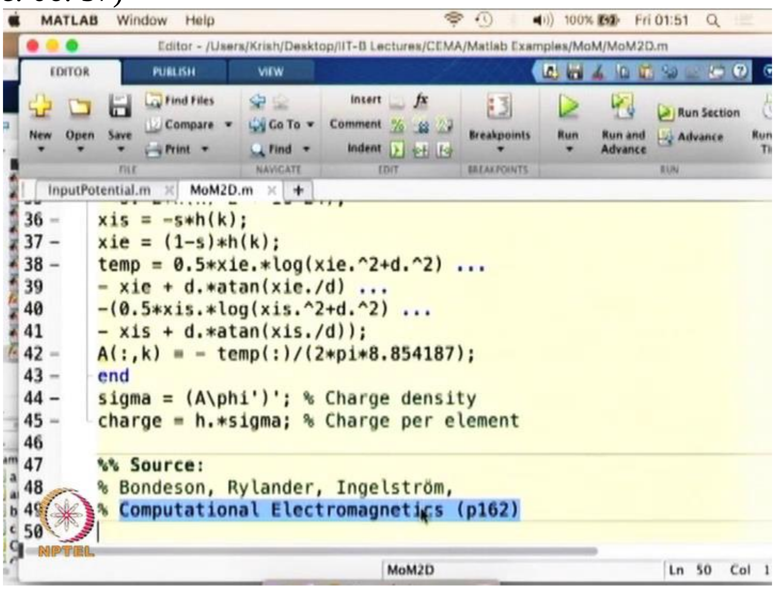
And when you are talking about this element you are going to extend this line and you are going to take the normal distance so this will become the d when you are talking about this element. It is going to change for each and every element but d is at normal distance and this is the way we compute the response function. And when you expand this what you get is minus Rho s divided by 2 Pi epsilon 0 [i by 2 x Lu (xsquare plus d square) minus x plus d arc tan x by d. This is going to be the value and its going to be integrated along the limit zeta s to zeta e. So this is going to be the starting point and the ending point of it.

(Refer Slide Time: 06: 05)



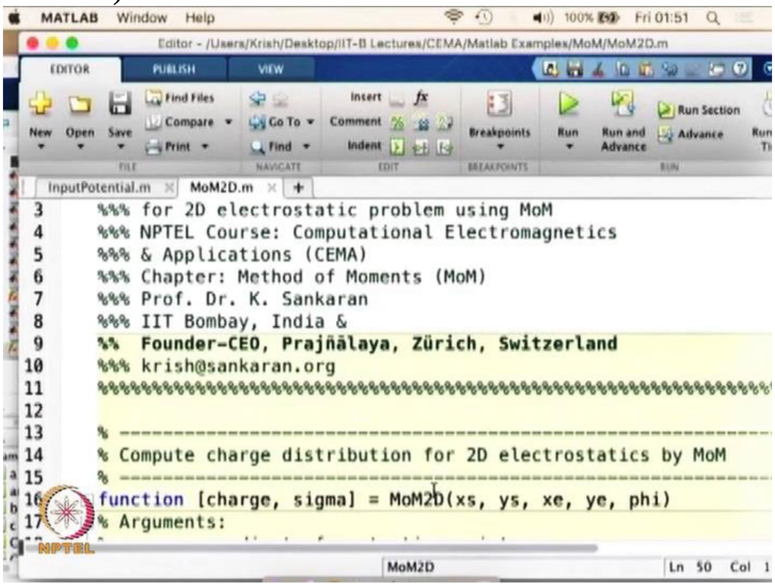
So let me write this expression in a much clearer form here. So what we have got is the response function is equal to minus Rho s divided by 2Pi Epsilon 0 [1 by 2 x Lu (x square plus d square) minus x plus d arc tan (x by d)] under the limit zeta s to zeta e. So this is going to be the way we are computing to each of that element. So this is for one element so you have to add up all the elements. So we are now going to do this problem using a Matlab environment.

(Refer Slide Time: 06: 57)



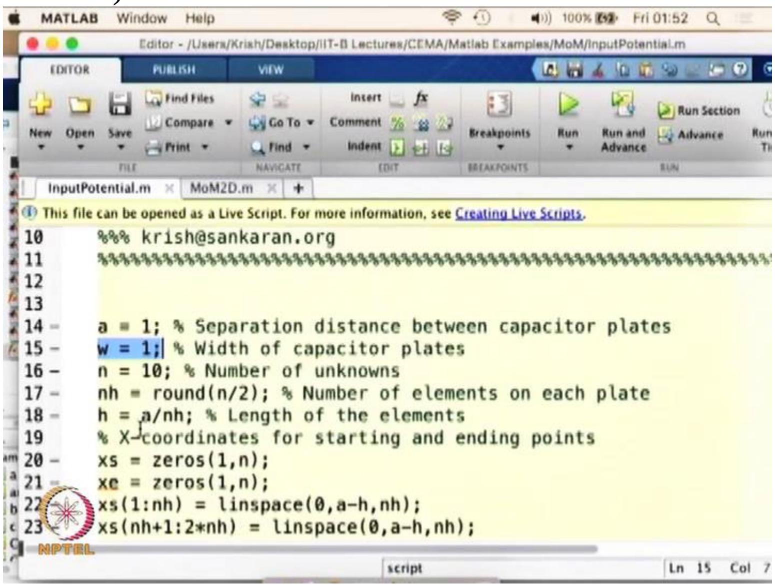
So we are going to use a Matlab program we have taken this code from the classical book of Rylander, Ingelstrom and Bodeson called Computational Electromagnetics and the page number is given here.

(Refer Slide Time: 07: 14)



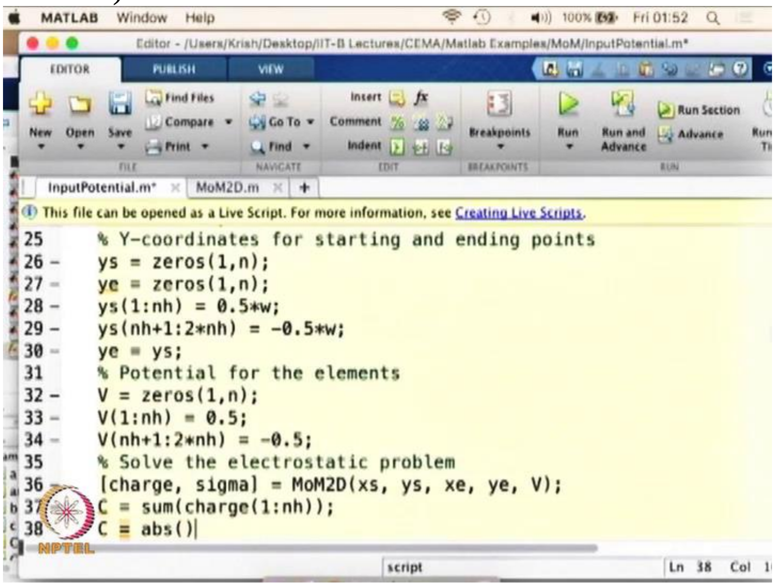
And the code is basically going to help us compute the value of the capacitance using the method of moments.

(Refer Slide Time: 07: 24)



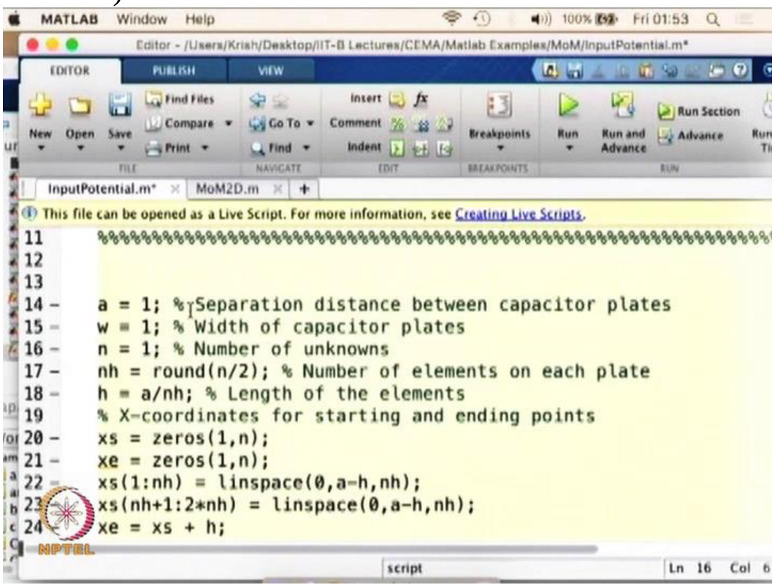
And we are going to solve for this problem by giving certain input parameters, the input parameters are going to be the separation between the plates the width of the plates and the number of elements we are going to choose and certain number of elements on each of the plates so on and so forth.

(Refer Slide Time: 07: 45)



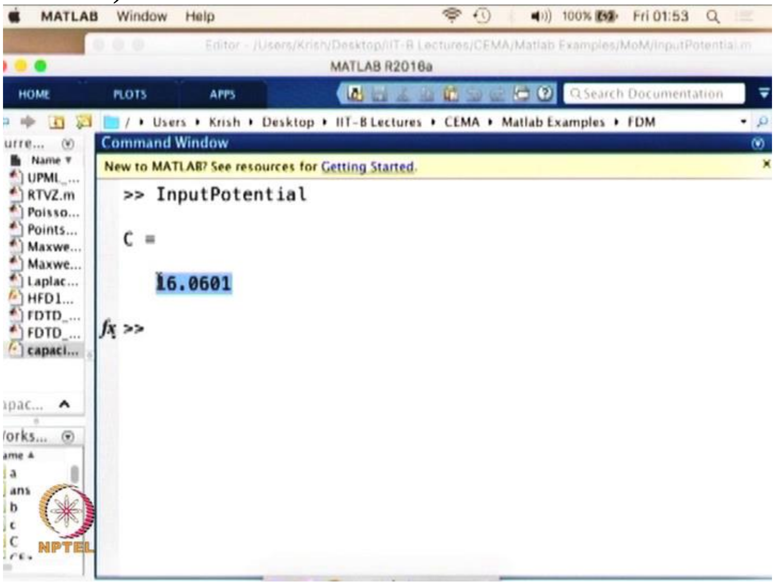
And we are going to compute the value of the capacitance, so in order for us to compute the capacitance we can write the value as the absolute value that we are interested in.

(Refer Slide Time: 08: 06)



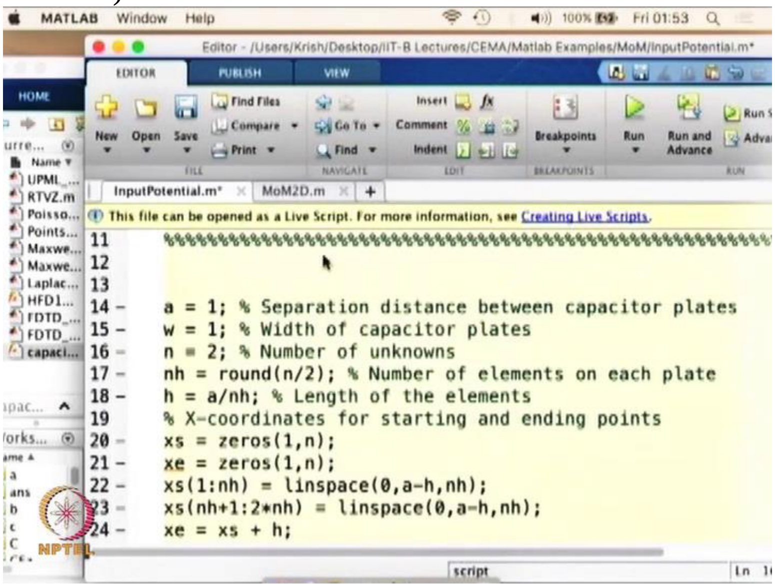
Let us say we have only 1 element. And if you are running the problem.

(Refer Slide Time: 08: 10)



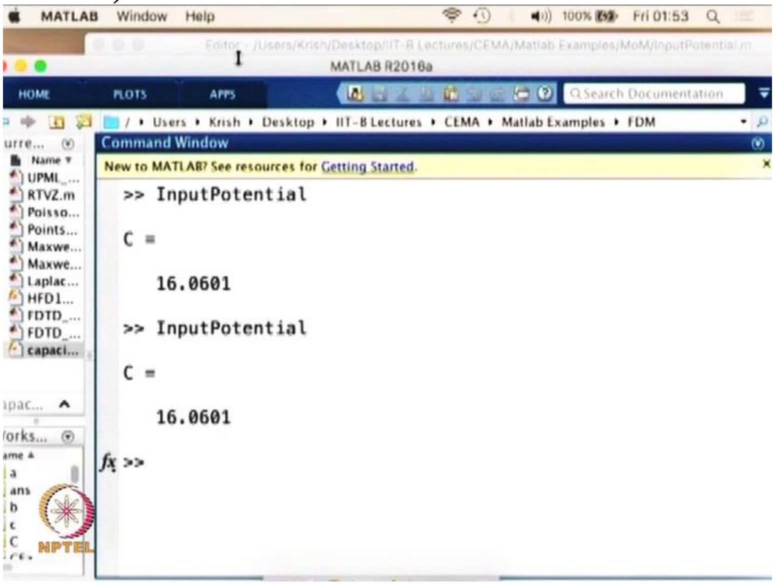
What we get is the value as 16 fared.

(Refer Slide Time: 08: 18)



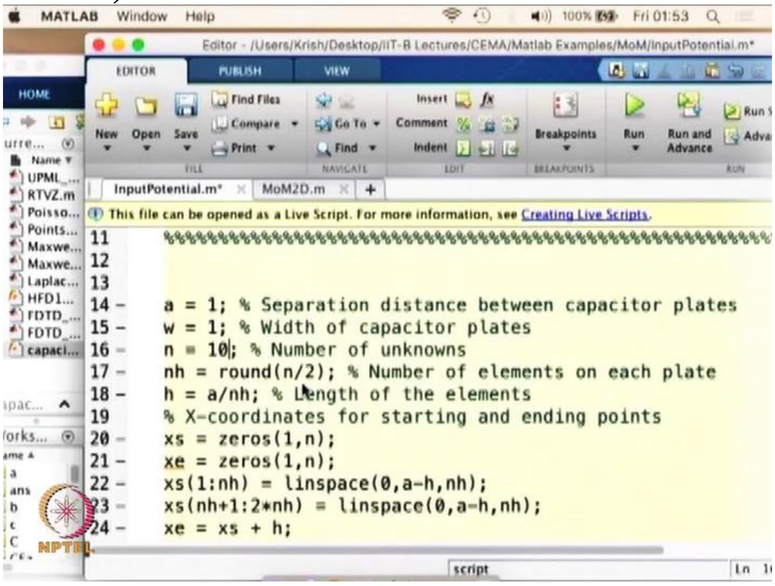
And now we are going to increase the number of elements to 2 we are doubling it.

(Refer Slide Time: 08: 22)



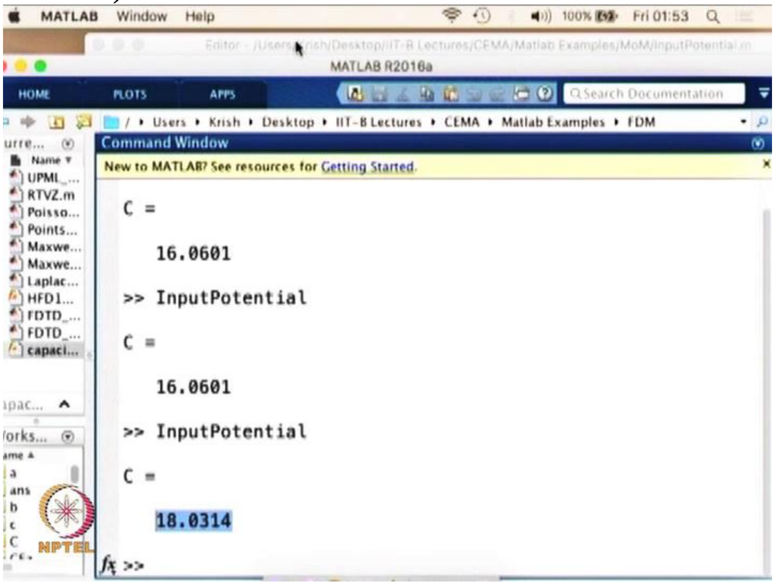
And we are seeing what is the value we are still getting 16

(Refer Slide Time: 08: 26)



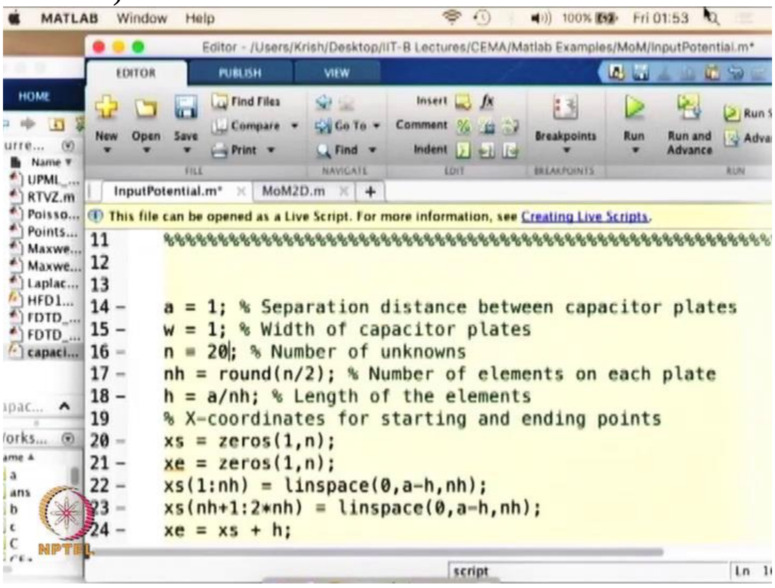
So we can keep increasing the value let us say we go to 10.

(Refer Slide Time: 08: 30)



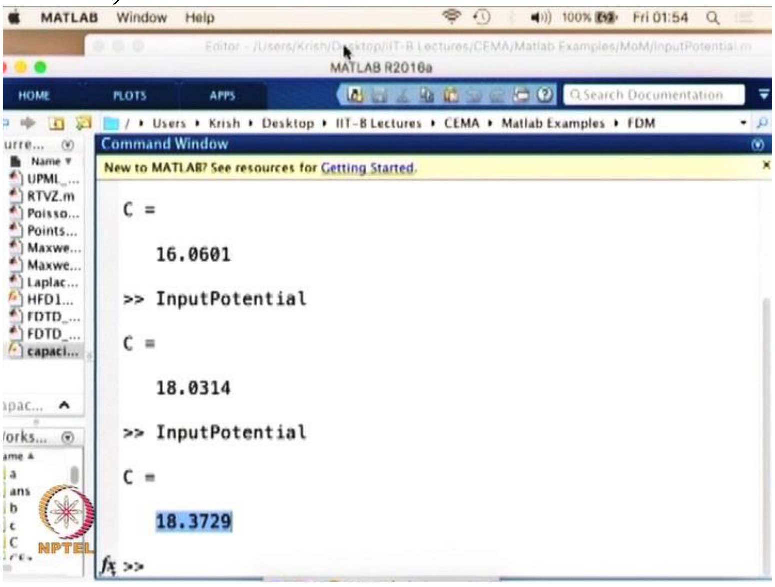
We see the value here it is 18 so it is still not converge the value is changing quite a bit.

(Refer Slide Time: 08: 39)



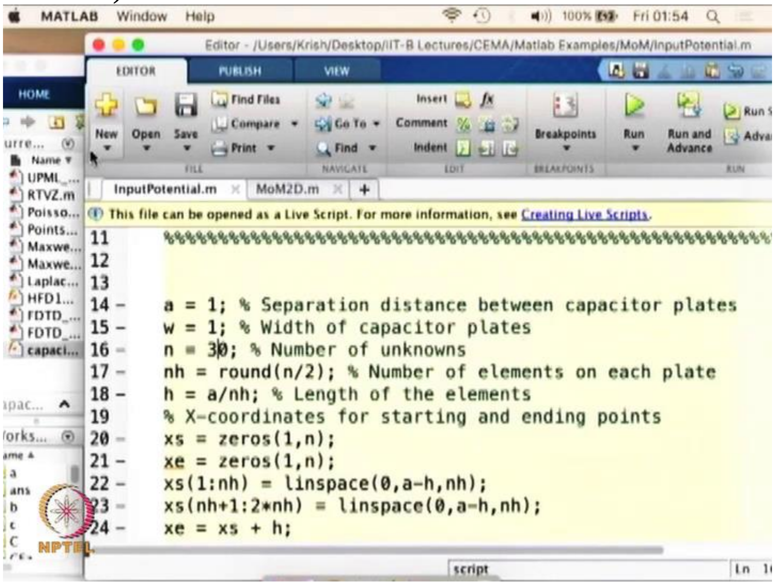
So we can increase the value even double it so we say 20 elements.

(Refer Slide Time: 08: 45)



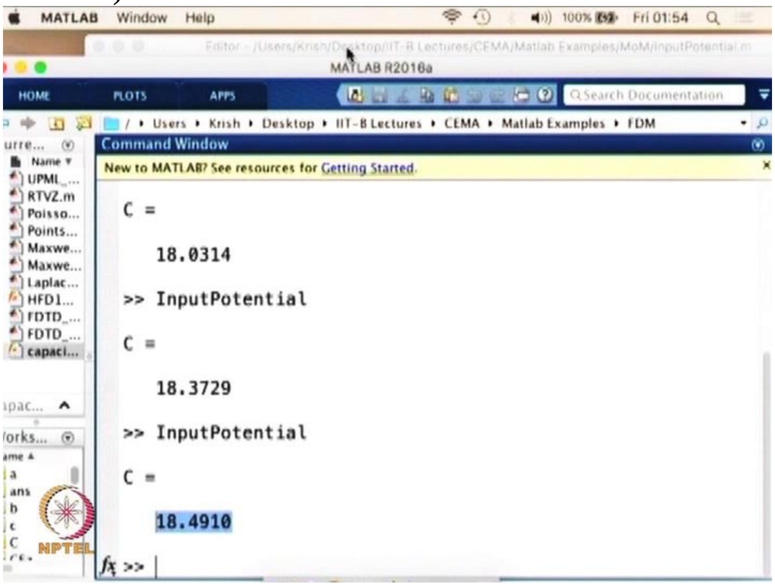
And we see the value is still within 18.something

(Refer Slide Time: 08: 52)



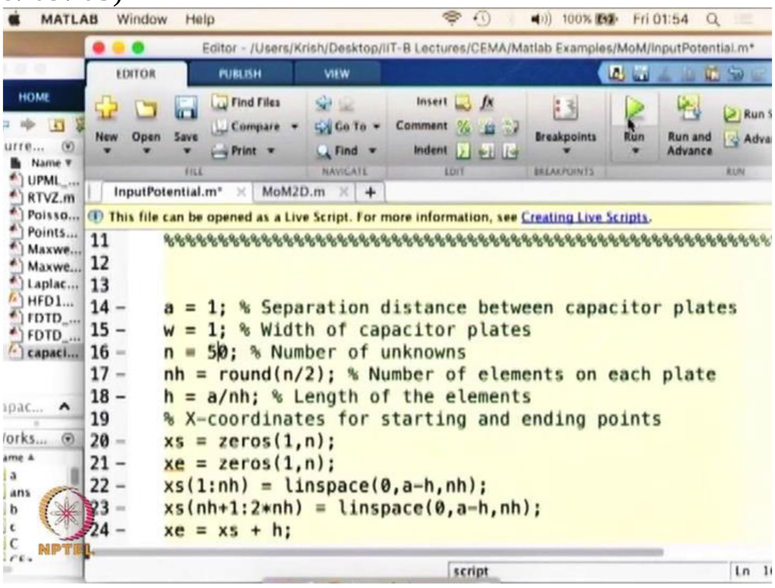
So we can increase it to 30, run the code;

(Refer Slide Time: 08: 56)



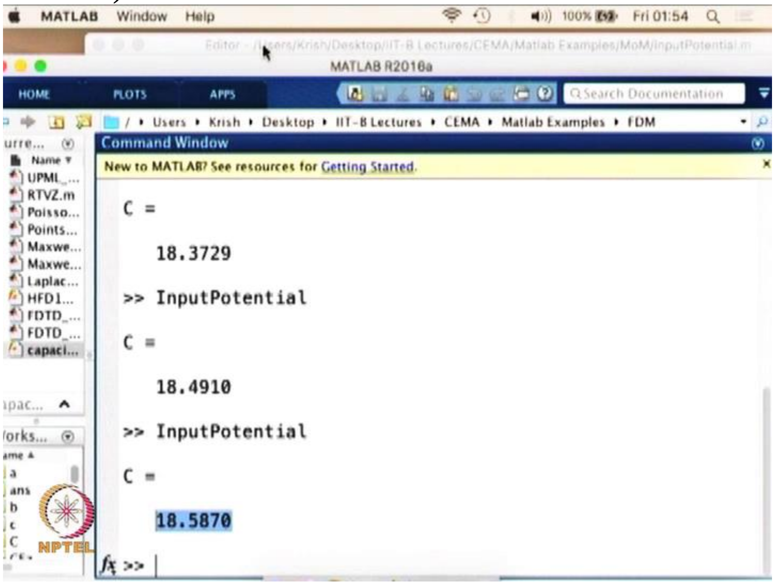
And see the value is increasing but not that much faster.

(Refer Slide Time: 09: 03)



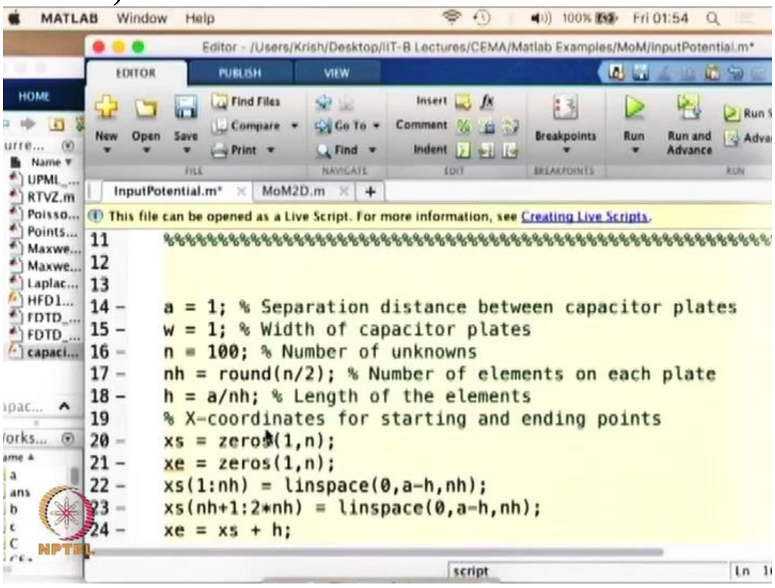
So if we put 50

(Refer Slide Time: 09: 08)



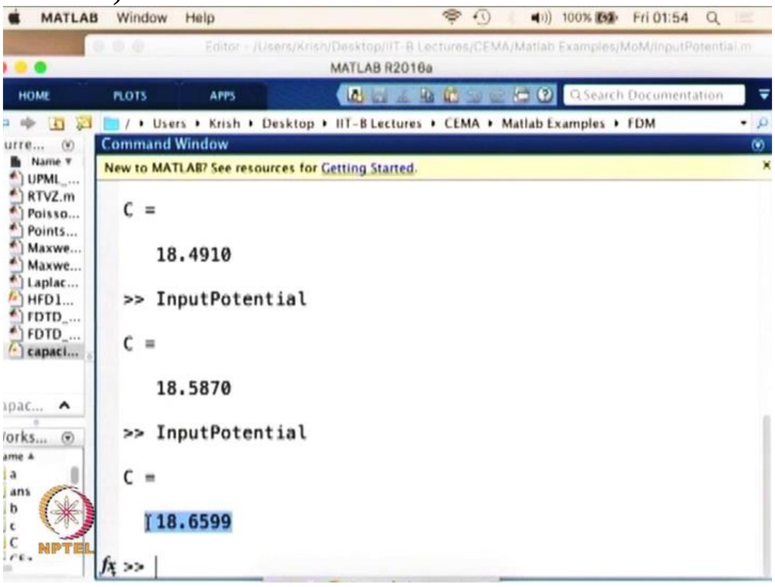
We will see that the value is almost converging very slowly.

(Refer Slide Time: 09: 14)



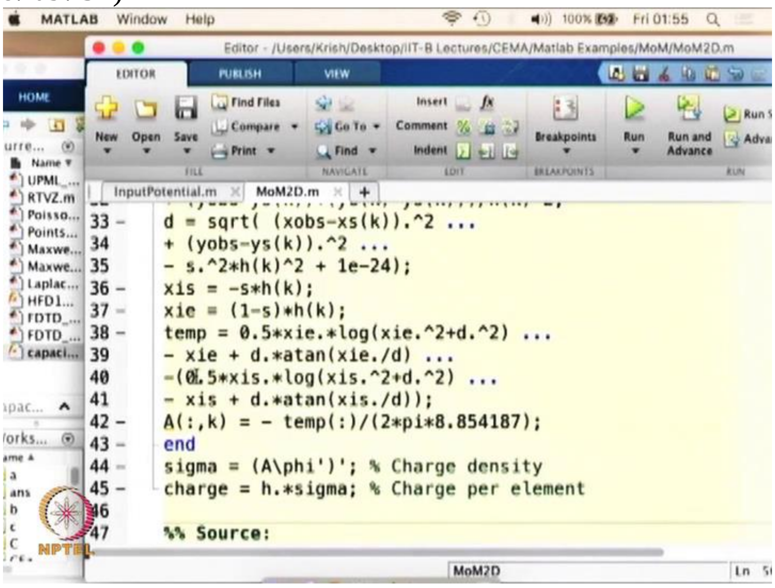
And if we increase it to 100. so this is going to be a very very high value

(Refer Slide Time: 09: 20)



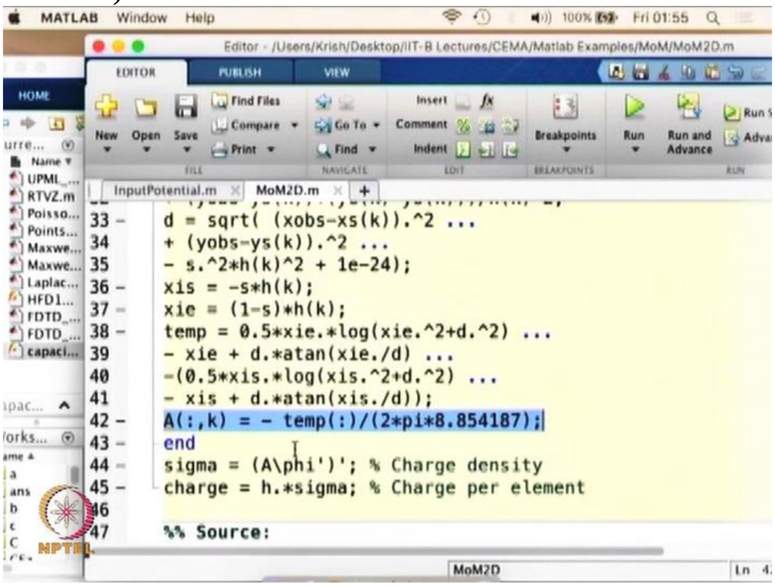
And you will see that the value is almost in the range of 18.5 to 18.6.

(Refer Slide Time: 09: 32)



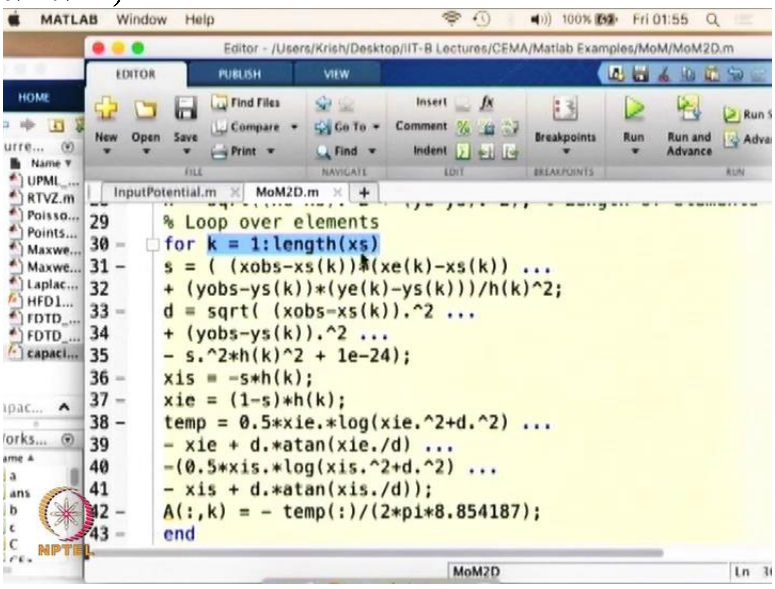
So this is the classical problem and the internal aspects of the problem are basically solving the greens function that we have described here and we are going to use the Greens function the way we have explained it in the expression we have shown.

(Refer Slide Time: 09: 51)



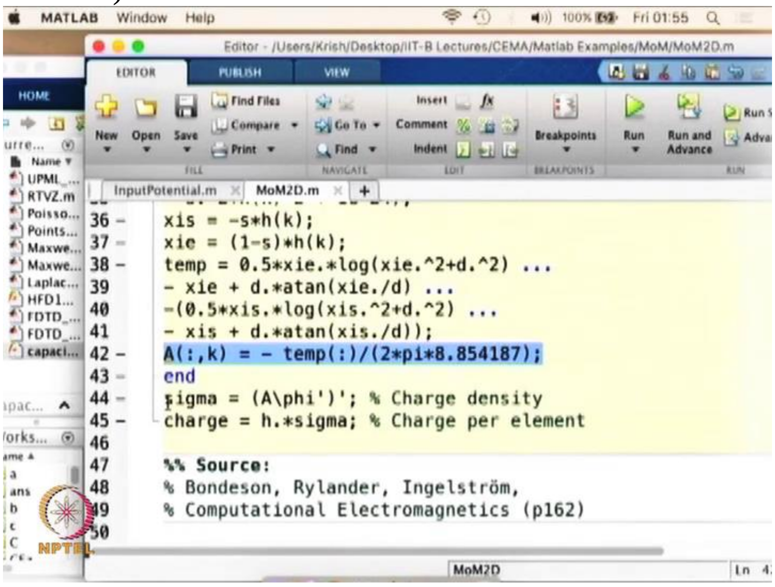
So if you see the value here is going to be given by this expression and we are taking the value ,and we are putting it in the A value which is the internal memory that we are using for computing the sigma later on once you go through each of the elements

(Refer Slide Time: 10: 11)



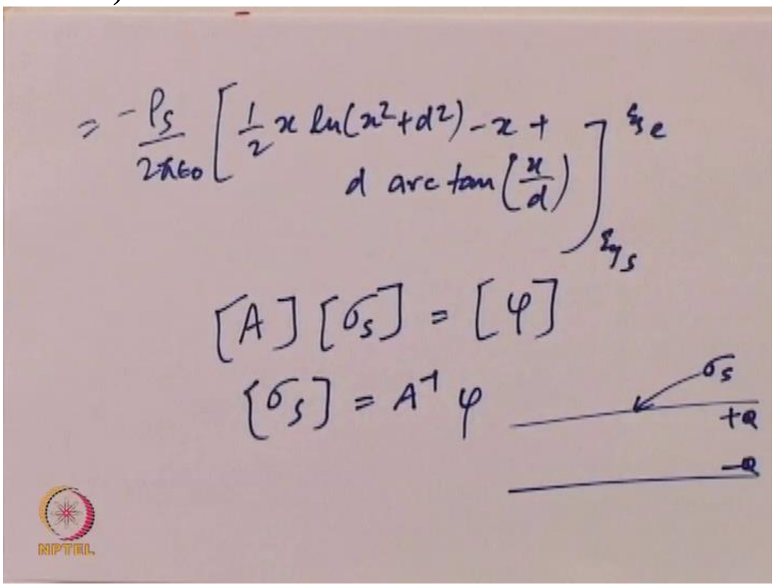
So this loop is basically going through k elements where k is from 1 to the length of (xs) itself.

(Refer Slide Time: 10: 18)



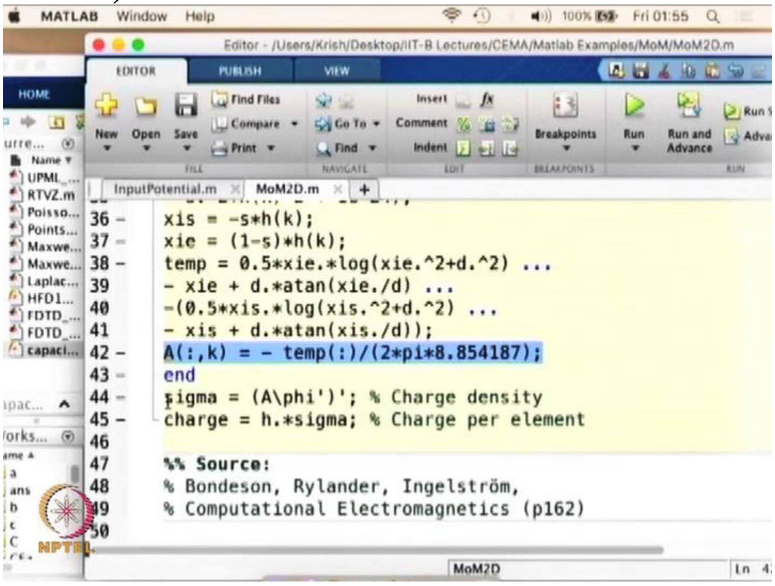
And once you compute all the A values so it is going to be the global matrix we are going to invert A, so initially what you get is A multiplied by sigma is equal to Phi.

(Refer Slide Time: 10: 29)



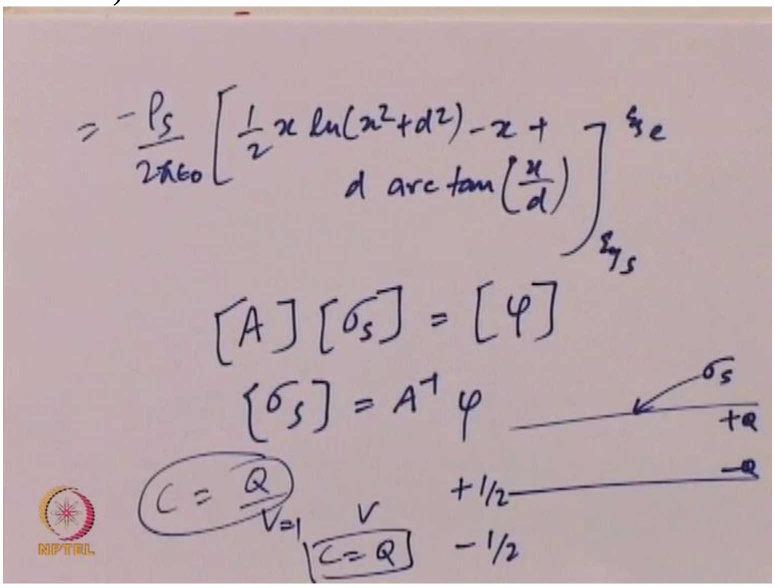
Since it is a simple Poisson equation what we get is a matrix equation which is [A] [sigma] is equal to the potential which is the Phi that we are trying to solve, and once you solve this what you get is the value for sigma and once you get the charged density which is we write it as surface charged density because we are interested in the charges that are on the surface of the parallel plate capacitor. If one side is Rho s the other side will be minus Rho s because we have plus Q and minus Q. And once we know that we compute the value of the charge by taking the A inverse on multiplying it with Phi. And once we know that we multiply the value of the charge density multiplied by the H which is the length of the element itself.

(Refer Slide Time: 11: 37)



And that will give us the value for the charge.

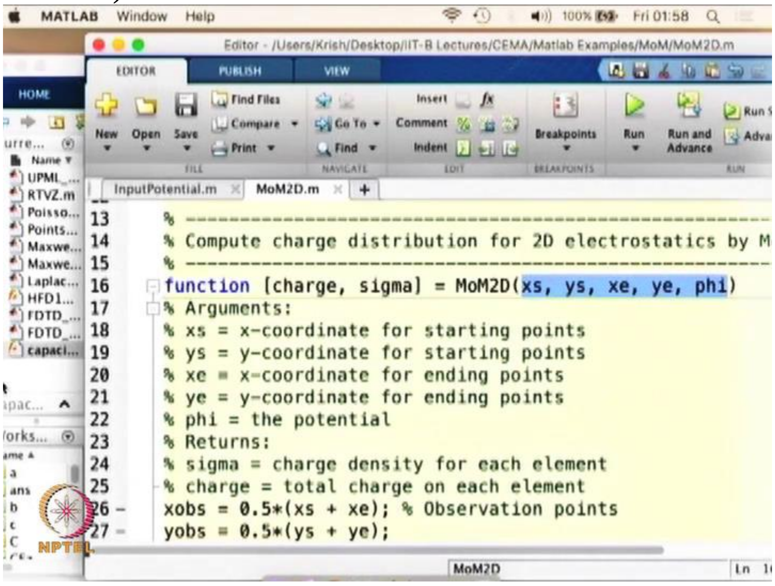
(Refer Slide Time: 11: 40)



Once we know the charge we can go and compute the value of the capacitance by using the formula c equal to Q by V.

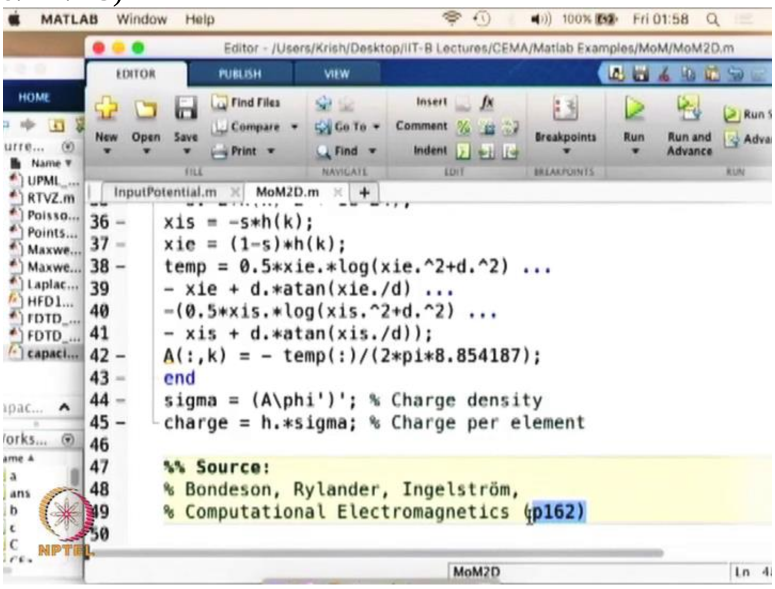
And we know the v is the potential difference between the two plates so its going to be plus 1 by 2 and minus 1 by 2 so the difference is going to be 1. And we can compute the value of c by making the value of v equal to 1, so it is going to be C equal to Q itself for this problem.

(Refer Slide Time: 12: 17)



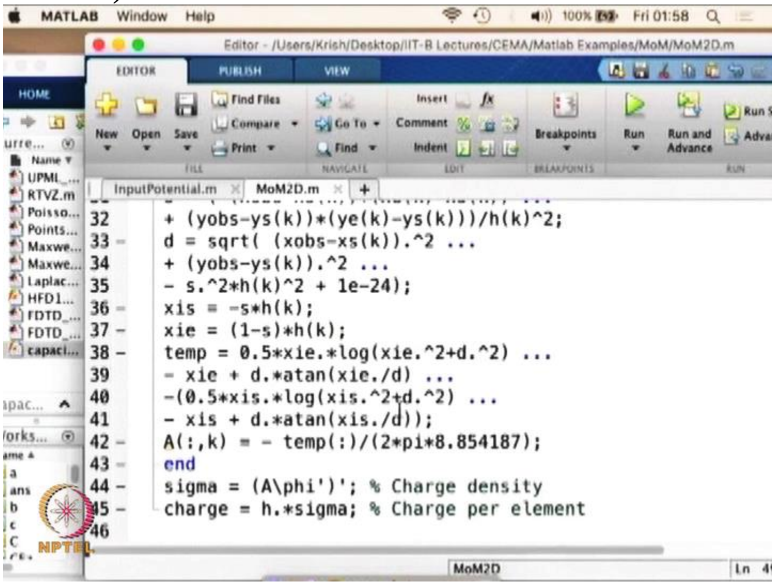
So that is what we are doing using this code and the code itself is self explanatory it has two modules one is the input potential function itself where you give the domain and its domain definition and once the domain definition is given it calls for a subroutine which is basically a function which gives the value of charge and sigma for a given inputs which are here.

(Refer Slide Time: 12: 45)



So I encourage you to take the code and test it for yourself. This is an excellent example from the classical book of Rylender, Ingelstrom and Bondeson is given at the bottom of the code itself and the page references are given here

(Refer Slide Time: 13: 06)



so I urge you to simulate such problems from classical text books. So that you can understand very much the basics of simulation on one hand and the other hand you can also understand how one can model such physical problems in a computational environment like Matlab Thank you!