

Computational Electromagnetics and Applications
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Lecture No. 25
Method of Moment

So where are we now so we have got a big expression in front of us and we have to compute it, so we are going to now take the simple problem of thin wire antenna which we discussed in the previous modules on the Method of Moments.

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SET UP FOR GALERKIN METHOD

Poklington's integral equation is written as

$$\int_{-\frac{l}{2}}^{\frac{l}{2}} I_z(z') \left[k^2 + \frac{\partial^2}{\partial z'^2} \right] \frac{e^{-jkR}}{4\pi R} dz' = j\omega\epsilon E_z^{inc}(z)$$

This has a form $L[u] = g$

where $u = I(z)$ $g = E_z^{inc}(z)$

$$L[u] = \frac{1}{j\omega\epsilon} \int_{-\frac{l}{2}}^{\frac{l}{2}} I_z(z') \left[k^2 + \frac{\partial^2}{\partial z'^2} \right] \frac{e^{-jkR}}{4\pi R} dz'$$



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And we are going to use the Galerkin approach to compute this step by step and we will also give you a kind of an algorithm to do this on a Matlab based approach or you any of the computational softwares.

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SET UP FOR GALERKIN METHOD

Poklington's integral equation is written as

$$\int_{-\frac{l}{2}}^{\frac{l}{2}} I_z(z') \left[k^2 + \frac{\partial^2}{\partial z'^2} \right] \frac{e^{-jkR}}{4\pi R} dz' = j\omega\epsilon E_z^{inc}(z)$$


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So let us start with the Pocklington's integral equation which we introduced in the last module. So what we have get is an expression for the e incidence which is given by the term which is on the left hand side. And the e incidence field is on the right hand side. I am interested in finding the value of the induced current for certain incident fields I know so the incident field I know and I am interested in finding out what will be the value of the incident current on that object then this problem will be a problem of scattering problem. Whereas if I know the value of I z and then I am interested in finding out what will be the value of E z at certain point then it will be a problem of antennas.

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SET UP FOR GALERKIN METHOD


Pocklington's integral equation is written as

$$\int_{-\frac{l}{2}}^{\frac{l}{2}} I_z(z') \left[k^2 + \frac{\partial^2}{\partial z'^2} \right] \frac{e^{-jkR}}{4\pi R} dz' = j\omega\epsilon E_z^{inc}(z)$$

This has a form $L[u] = g$

where $u = I(z)$ $g = E_z^{inc}(z)$

$$L[u] \triangleq \frac{1}{j\omega\epsilon} \int_{-\frac{l}{2}}^{\frac{l}{2}} I_z(z') \left[k^2 + \frac{\partial^2}{\partial z'^2} \right] \frac{e^{-jkR}}{4\pi R} dz'$$




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So in this case we are starting with general Pocklington's integral equation which we introduced in the last module. We brought the differential operator inside the integration as you can see in this expression. And now we have this expression in the form of L(u) is equal to g. We know this term what we don't know is this term which we need to compute and this will be having some operator called L, where the u is the unknown value and here the unknown value is going to be the induced current and g is the known incident field value. And essentially the value of L(u) is going to be given by this term which is sitting here. So I am going to bring the jomega epsilon on to the other side and I will have an expression for L(u).

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RECALL GALERKIN METHOD

$$L[u] = g$$

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Recall that in the case of the Galerkin equation we started with $L(u)$ is equal to g and we used certain expansion functions to define the value of u . And expansion functions are going to be certain functions for which we know the behaviour. And what we are interested is finding out the expansion coefficients.


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RECALL GALERKIN METHOD

$$L[u] = g$$

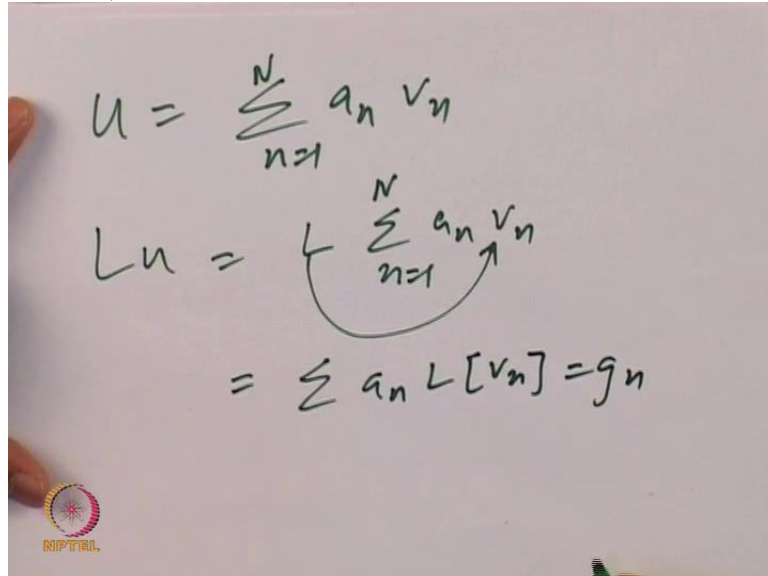
Step 1 – Expand unknown into set of basis functions

$$\sum_n a_n L[v_n] = g$$

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And that is what we see here step 1 is to start with set of basis functions which we call as v_n and I am going to bring in the L inside this term. So what I have done is I have started with u is equal to some basis function and goes to 1 to n $a_n v_n$.

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$$u = \sum_{n=1}^N a_n v_n$$
$$Lu = L \left(\sum_{n=1}^N a_n v_n \right)$$
$$= \sum_{n=1}^N a_n L[v_n] = g_n$$


And v_n is going to be the basis function and a_n are the coefficients that we are interested in. And Lu is equal to L of $\sum_{n=1}^N a_n v_n$ is equal to g_n . And I am going to bring this inside the equation and this is allowed in this operator manipulation. So sigma will be outside $a_n L[v_n]$ equal to g_n .

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RECALL GALERKIN METHOD

$$L[u] = g$$

Step 1 – Expand unknown into set of basis functions

$$\sum_n a_n L[v_n] = g$$


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So I am starting with this point.

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RECALL GALERKIN METHOD

$$L[u] = g$$

Step 1 – Expand unknown into set of basis functions

$$\sum_n a_n L[v_n] = g$$

Step 2 – Test both sides against basis functions

$$\left\langle v_m, \sum_n a_n L[v_n] \right\rangle \stackrel{!}{=} \langle v_m, g \rangle$$



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In the step 2 what we are essentially going to do is we are going to multiply on both sides using certain basis function, since we are starting with a Galerkin method the basis function will be the same or the weighting function will be the same as the basis function itself. So instead of w_m which we used in the case of weighted residual method, we are using the same term b which is a basis function itself. So we are multiplying this using certain basis function. So both sides we are multiplying it, this one will be the inner product term, this one will be also the inner product term.

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RECALL GALERKIN METHOD

Step 3 – Form matrix eqn $[Z_{mn}]\{a_n\} = \{g_m\}$

$$[Z_{mn}] = \begin{bmatrix} \langle v_1, L[v_1] \rangle & \langle v_1, L[v_2] \rangle & & \\ \langle v_2, L[v_1] \rangle & \langle v_2, L[v_2] \rangle & & \\ & & \ddots & \\ & & & \langle v_M, L[v_N] \rangle \end{bmatrix}$$
$$\{a_n\} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix} \quad \{g_m\} = \begin{bmatrix} \langle v_1, g \rangle \\ \langle v_2, g \rangle \\ \vdots \\ \langle v_M, g \rangle \end{bmatrix}$$


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And finally what we will get is a matrix expression which is $[Z_{mn}]$ multiplied by a vector $\{a_n\}$ which we don't know and then certain value g_m which we know. The way it will look individually will be given by this particular term and this we have already seen in our earlier

lectures on Method of Weighted residual and Galerkin approach which we are doing the same thing here.


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GALERKIN METHOD – STEP 1

Pocklington's integral equation

$$\int_{-\frac{L}{2}}^{\frac{L}{2}} I_z(z') \left[k^2 + \frac{\partial^2}{\partial z'^2} \right] \frac{e^{-jkR}}{4\pi R} dz' = j\omega\epsilon E_z^{inc}(z)$$

Expand current function into set of basis functions

$$I_z(z) = \sum_n a_n v_n(z)$$


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For the Pocklington integral equation what we have is I z will be expanded. This is the term that we do not know we are multiplying instead of I z using this term.

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GALERKIN METHOD – STEP 1

Pocklington's equation becomes

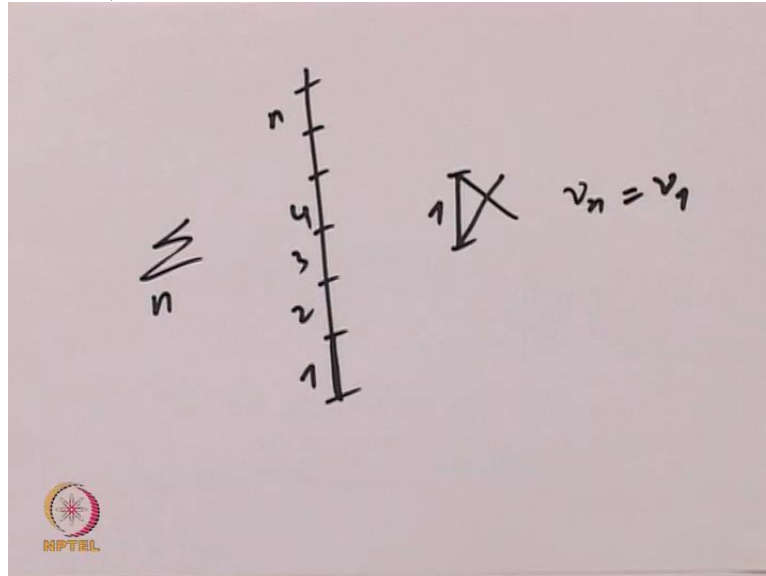
$$\int_{-\frac{L}{2}}^{\frac{L}{2}} \left[\sum_n a_n v_n(z') \right] \left[k^2 + \frac{\partial^2}{\partial z'^2} \right] \frac{e^{-jkR}}{4\pi R} dz' = j\omega\epsilon E_z^{inc}(z)$$

$$\sum_n a_n \int_{v_n} v_n(z') \left[k^2 + \frac{\partial^2}{\partial z'^2} \right] \frac{e^{-jkR}}{4\pi R} dz' = j\omega\epsilon E_z^{inc}(z)$$


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And we have the right hand side given by this equation. And what we are doing is we are bringing the a n out summation of a n out and we are keeping the rest of the terms inside. So what is happening here is inside each of the elements that is what we are interested in. What we have got here is instead of integration going minus 1 by 2 to 1 by 2. We are going to go into the individual elements. For individual elements the integration is going to be between the starting and ending point of the basis function itself. And that is why we have got v n here. And once we get this the summation here will take care of the entire thing.

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So once what we do is we have an antenna here and each of these elements are going to be the individual elements. And within this individual elements let us say the entire domain is going to be the summation of n that is going to be n number of elements so n number is going to be 1,2,3,4 until n . And once we do the summation we are going to finish off the entire line segment. But individually inside each of these thing it is going to be a summation between a starting point and an ending point. And you know that for this you will have two basis functions and those basis functions are defined by v_n for that element. The element number 1 v_n will be v_1 so on and so forth. Once we integrate it once we sum the entire thing we would have completed the entire line segment.

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GALERKIN METHOD – STEP 2

Test both sides with basis functions using inner product

$$\sum_n a_n \int_{v_n} v_n(z') \left[k^2 + \frac{\partial^2}{\partial z'^2} \right] \frac{e^{-jkR}}{4\pi R} dz' = j\omega\epsilon E_z^{inc}(z)$$
$$\left\langle v_m(z), \sum_n a_n \int_{v_n} v_n(z') \left[k^2 + \frac{\partial^2}{\partial z'^2} \right] \frac{e^{-jkR}}{4\pi R} dz' \right\rangle = \langle v_m(z), j\omega\epsilon E_z^{inc}(z) \rangle$$
$$\sum_n a_n \left\langle v_m(z), \int_{v_n} v_n(z') \left[k^2 + \frac{\partial^2}{\partial z'^2} \right] \frac{e^{-jkR}}{4\pi R} dz' \right\rangle = j\omega\epsilon \langle v_m(z), E_z^{inc}(z) \rangle$$



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That is what we are seeing in this particular expression here. This minus 1 by 2 to plus 1 by 2 is substituted by the summation of n. And individually we are doing it for those individual finite elements. And now we have the expression for the left hand side given by this term and the right hand side is given by the same thing that we have before. Now we are going to multiply this on both side with certain test functions. And this function is going to be given by the expression here.

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GALERKIN METHOD – STEP 2

Test both sides with basis functions using inner product

$$\sum_n a_n \int_{v_n} v_n(z') \left[k^2 + \frac{\partial^2}{\partial z'^2} \right] \frac{e^{-jkR}}{4\pi R} dz' = j\omega\epsilon E_z^{inc}(z)$$
$$\left\langle v_m(z), \sum_n a_n \int_{v_n} v_n(z') \left[k^2 + \frac{\partial^2}{\partial z'^2} \right] \frac{e^{-jkR}}{4\pi R} dz' \right\rangle = \langle v_m(z), j\omega\epsilon E_z^{inc}(z) \rangle$$
$$\sum_n a_n \left\langle v_m(z), \int_{v_n} v_n(z') \left[k^2 + \frac{\partial^2}{\partial z'^2} \right] \frac{e^{-jkR}}{4\pi R} dz' \right\rangle = j\omega\epsilon \langle v_m(z), E_z^{inc}(z) \rangle$$


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So let us start with the test function itself. As you can see in the slide we are going to multiply both sides with certain test functions which are going to be given by v m on both sides and I am going to manipulate this a little bit I am going to take the summation out and then I will have the integration on the left hand side with this manipulation and the right hand

side I am bringing the $j\omega\epsilon$ out and the remaining term will be inside the inner product.

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GALERKIN METHOD – STEP 3

Construct matrix equation from inner products


$$\sum_n a_n \left\langle v_m(z), \int_{v_n} v_n(z') \left[k^2 + \frac{\partial^2}{\partial z^2} \right] \frac{e^{-jkR}}{4\pi R} dz' \right\rangle = j\omega\epsilon \langle v_m(z), E_z^{inc}(z) \rangle$$

↓

$$[\mathbf{Z}_{mn}] \{ \mathbf{a}_n \} = \{ \mathbf{g}_m \}$$

$$Z_{mn} = \int_{v_m} v_m(z) \int_{v_n} v_n(z') \left[k^2 + \frac{\partial^2}{\partial z^2} \right] \frac{e^{-jkR}}{4\pi R} dz' dz$$

$$g_m = j\omega\epsilon \int_{v_m} v_m(z) E_z^{inc}(z) dz$$



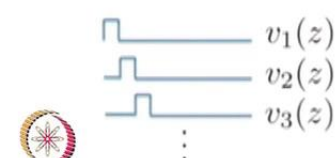
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And I am going to construct the matrix from the inner product, so what I will have is essentially a term for Z_{mn} multiplied by $\{a_n\}$ is equal to $\{g_m\}$. $\{a_n\}$ is the value that is sitting here z_m is the entire thing which is on the right hand side except for this particular term. And $\{g_m\}$ is entire the right hand side term. Obviously the right hand side term is a scaled value we will look at it in the next slides. And the individual expressions for the Z_{mn} , g_m etc are given by this term. We have just expanded this inner product. Remember the inner product is defined accordingly, so inner product between $v_m E_z^{inc}$ is given by inside individual elements, the integration of v_m and $E_z^{inc} dz$.

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PULSE BASIS FUNCTIONS

Let basis functions be pulse functions defined only on segments

$$v_m(z) = \begin{cases} 0 & z \text{ is outside } m^{\text{th}} \text{ segment} \\ 1 & z \text{ is inside } m^{\text{th}} \text{ segment} \end{cases}$$


This is called **point-matching**

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So we can choose various basis functions for simplicity we can choose a pulse basis function. Inside the z element which we are interested in the pulse basis function will give a value 1 outside it will be 0. In other words when you see for the first element v 1 will be equal to 1 only for the first element and 0 for other elements. And v 2 will be 1 for only the second element and 0 for other elements. And things follow accordingly until v n. This approach is called as the point matching approach.

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PULSE BASIS FUNCTIONS

Using these basis functions

$$Z_{mn} = \int_{v_m} v_m(z) \int_{v_n} v_n(z') \left[k^2 + \frac{\partial^2}{\partial z^2} \right] \frac{e^{-jkR}}{4\pi R} dz' dz$$

where $R = \sqrt{(z_m - z')^2 + a^2}$

$$Z_{mn} = k^2 \int_{z_m - \frac{\Delta z}{2}}^{z_m + \frac{\Delta z}{2}} \frac{e^{-jkR}}{4\pi R} dz' + \left[(z_m - z') \frac{1 + jkR}{R^3} e^{-jkR} \right] \Bigg|_{z' = z_m - \frac{\Delta z}{2}}^{z' = z_m + \frac{\Delta z}{2}}$$


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And we can substitute the value of E z accordingly for the value on the right hand side. And now for computing the matrix [Z mn] we can use this integration here and we can expand this integration inside the term and when the value of Z m is equal to Z n. There is something interesting going to happen that we will see in the next slide.

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PULSE BASIS FUNCTIONS

Using these basis functions

$$g_m = j\omega\epsilon \int_{v_m} v_m(z) E_z^{inc}(z) dz$$
$$g_m = j\omega\epsilon E_z^{inc}(z)$$


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So using the basis function we compute the value on the right hand side as g_m is equal to the value here. And we finally get the value for the expression accordingly because basically this value will be equal to 1 for that element and it will be 0 for other element. So essentially this integration will lead to this form.


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PULSE BASIS FUNCTIONS

When calculating impedance elements, get:

$$\int_{z_n - \frac{\Delta z}{2}}^{z_n + \frac{\Delta z}{2}} \frac{e^{-jkR}}{4\pi R} dz' \quad R = \sqrt{(z_m - z')^2 + a^2}$$

When $m = n$ use small argument approximation

$$\int_{z_m - \frac{\Delta z}{2}}^{z_m + \frac{\Delta z}{2}} \frac{e^{-jkR}}{4\pi R} dz' \approx \int_{-\frac{\Delta z}{2}}^{\frac{\Delta z}{2}} \frac{1 - jkR}{4\pi R} dz'$$


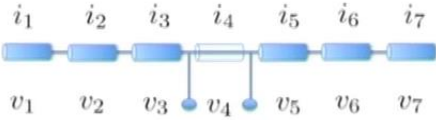
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And when calculating the impedance itself what we get is we can assume certain approximation. The approximation is when m is equal to n we use the small argument approximation. Where this term integration z_m plus Δz by 2 to z_m minus Δz by 2 will be approximately equal to the integration going from minus Δz by 2 to Δz by 2. So in that sense the integration has a close form expression which is given by this form. And we can get an analytical solution for this using this expression.


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PULSE BASIS FUNCTIONS

We can interpret $\{a\}$ as a column vector containing currents in each segment of antenna



The diagram shows a horizontal antenna divided into seven segments. Above each segment is a current label i_1 through i_7 . Below each segment is a voltage label v_1 through v_7 . The segments are connected in series, with a gap between v_3 and v_4 .

$$\{a\} = \{i\}$$
$$[Z_{mn}]\{a_n\} = \{g_m\}$$


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So we can interpret the value of $\{a\}$ which is the vector we are interested in finding. As the column vector containing the current in each of the line segment of that antenna. The current induced on those line segment. So the $\{Z_{mn}\}$ will be the impedance matrix. The $\{a_n\}$ will be the current induced in that individual elements and $\{g_m\}$ is the known incident field value.


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TRUE IMPEDENCE TRANSFORMATION

Matrix equation is $[Z_{mn}]\{a_n\} = \{g_m\}$

a_n coefficients are currents in each segment

g_m coefficients are scaled electric fields

$$[Z_{mn}]\{i_n\} = \{j\omega\epsilon E_z^{inc}(z_m)\}$$
$$E_z^{inc}(z_m) = \frac{V_m}{\Delta z}$$



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And obviously the value of $\{g_m\}$ is a scaled electric vector we can rescale it in order to get the right value by taking the true impedance value that we will do in the manner we have shown here. So the incident value is given by V_m divided by Δz . As you can see this is a gradient of the potential we are computing.

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TRUE IMPEDANCE TRANSFORMATION

$$\frac{j\Delta z}{\omega\epsilon} [Z_{mn}] \{i_n\} = \{V_m\}$$

$$\underbrace{\frac{j\Delta z\eta}{k} [Z_{mn}] \{i_n\}}_{\text{True } Z} = \{V_m\}$$


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And we can rescale the value of $\{v_m\}$ the electric field to compute the true impedance accordingly, so what we have done here is nothing but a manipulation that will allow us to compute the value of the true impedance. And this value of the true impedance is what is going to give the value of the individual currents that we are interested in.

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SINGULARITY IN POCKLINGTON'S EQN

Recall

$$Z_{mn} = \frac{1}{4\pi} \ln \left[\frac{\sqrt{1 + (\frac{2a}{\Delta z})^2} + 1}{\sqrt{1 + (\frac{2a}{\Delta z})^2} - 1} \right] - \frac{jk\Delta z}{4\pi} + \left[(z_m - z') \frac{1 + jkR}{R^3} e^{-jkr} \right] \Bigg|_{z'=z_n - \frac{\Delta z}{2}}^{z'=z_n + \frac{\Delta z}{2}}$$

Strong singularity!!

Slow convergence and poor accuracy

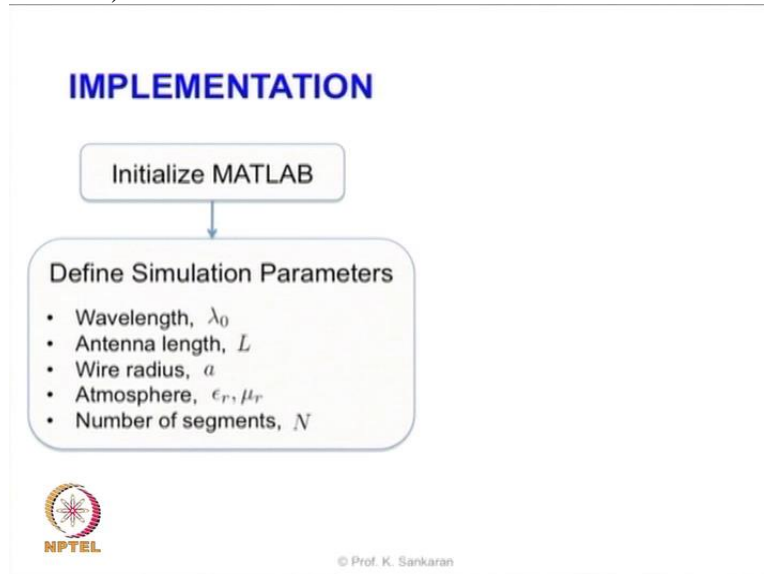


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And recall that we have got an approach from expression for Z_{mn} using the value that we have computed before. And based on that we can compute the value of the entire individual elements of Z_{mn} . But there is one problem, the problem is the term here R^3 , when R becomes small what happens is this term becomes very very small leads to a strong singularity. And this is something that also leads to poor accuracy and also slow convergence which we discussed in the earlier stage. But it is interesting for you to know where the singularity is coming from, and this is the point where the singularity comes from.

So now what we will do is we will take you through a step by step algorithm to compute whatever we have shown here, we have shown you a lot of equations. If you do not follow the expressions whatever we have derived you can go back in the lecture and see how we have derived each individual steps. But now we will look into the Matlab implementation itself. In the Matlab implementation we are going to given you a step by step algorithm to compute those individual parameters. The value of Z mn the value of different coefficients, and also how to use those values into the matrix form to get the solution in the Matlab program.

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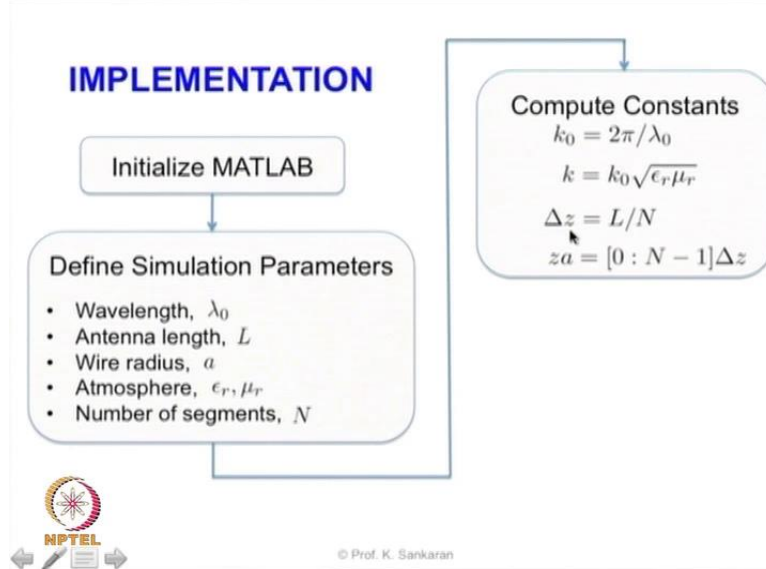


So let us start with the Matlab itself. Once we initialize the matlab what we are going to do is we are going to define certain basic simulation parameters. And those simulation parameters are nothing but the wavelength, the free space wavelength λ_0 ; the antenna length which is L ; and the Wire radius a , remember this approximation is valid when only a is very very small compared to L and the computational domain itself might have certain impedance and certain relative permeability and relative permittivity. So relative permittivity is given by the term here epsilon r and the relative permeability is given the μ_r and also the number of segments N .

Remember the convergence is fast in the case of the Hallen formulation, the Pocklington formulation it is slow. And when the number of elements goes high it goes very close to the required result which we are measuring or computing. And in the case of the method of moment as we already mentioned we will not do any volume discretization because we have transformed from a volume integration to a surface integration to a line integration. We are only interested in discretizing the surface and that too if the antenna is a very very thin

antenna it is going to be a line integration. So we are only going to talk about individual lines that are going to vary along z axis or any of the axis what you are interested in. Once you align the antenna in one axis in the case of our analysis we have aligned it along the z axis.

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Once we do that we are going to compute the values for the problem k_0 is going to be given by $2\pi/\lambda_0$ and k is going to be computed accordingly from the value of k_0 provided we know the value of ϵ_r and μ_r . And we are going to discretize the entire domain using n number of cells so if we have L is our dimension and N is the number of segments the individual components will have z as Δz and our value of z_a goes from 0 to $n - 1$. So each individual components of z_a will be given by the value of those individual terms that are going to vary from 0 to $N - 1$. And now we have to compute the impedance matrix itself.

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SINGULARITY IN POCKLINGTON'S EQN

Recall

$$Z_{mn} = \frac{1}{4\pi} \ln \left[\frac{\sqrt{1 + (\frac{2a}{\Delta z})^2 + 1}}{\sqrt{1 + (\frac{2a}{\Delta z})^2 - 1}} \right] - \frac{jk\Delta z}{4\pi} + \left[(z_m - z') \frac{1 + jkR}{R^3} e^{-jkr} \right] \Bigg|_{z'=z_n - \frac{\Delta z}{2}}^{z'=z_n + \frac{\Delta z}{2}}$$

Strong singularity!!

Slow convergence and poor accuracy



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So remember impedance matrix is basically this big ugly matrix. We need to compute this matrix.


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BUILDING IMPEDANCE MATRIX

Compute Diagonal Term

$$Z'_{mn} = \frac{1}{4\pi} \ln \left[\frac{\sqrt{1 + (\frac{2a}{\Delta z})^2 + 1}}{\sqrt{1 + (\frac{2a}{\Delta z})^2 - 1}} \right] - \frac{jk\Delta z}{4\pi}$$

Loop over all m and n



Calculation Step #1

$$Z'_{mn} = \begin{cases} Z'_{mn} & \text{for } m = n \\ \int_{z_n - \frac{\Delta z}{2}}^{z_n + \frac{\Delta z}{2}} \frac{e^{-jkr}}{4\pi R} dz' & \text{for } m \neq n \end{cases}$$


Calculation Step #2

$$r_1 = \sqrt{(Z_m - Z_n + \Delta z/2)^2 + a^2}$$

$$t_1 = (Z_m - Z_n + \Delta z/2) \frac{1 + jkr_1}{r_1^3} e^{-jkr_1}$$

$$r_2 = \sqrt{(Z_m - Z_n - \Delta z/2)^2 + a^2}$$

$$t_2 = (Z_m - Z_n - \Delta z/2) \frac{1 + jkr_2}{r_2^3} e^{-jkr_2}$$

$$Z_{mn} = k^2 Z'_{mn} + t_2 - t_1$$


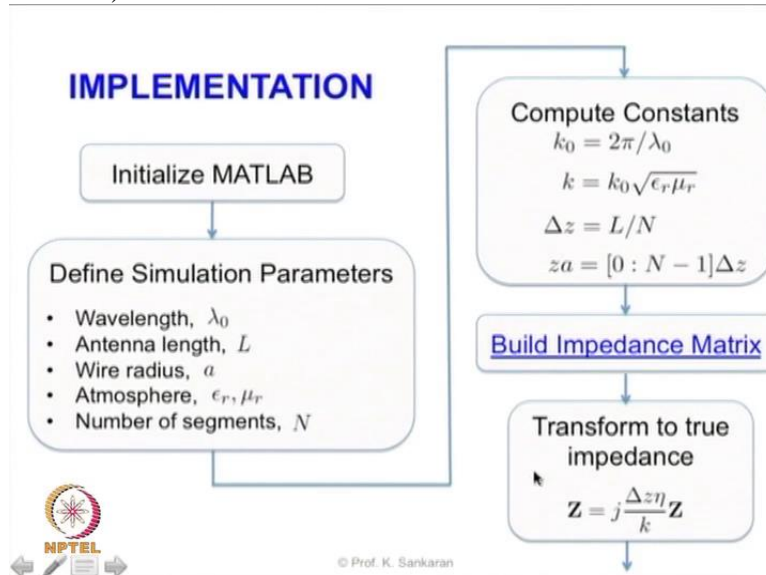
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And that is what we are going to do here in this step so to build the impedance matrix what we are going to do is we are going to compute this value of Z prime mn. Here we have done various approximations. The approximations are already discussed in the previous slides. So it is enough to start talking about this expression now. We are going to loop over all m and n to compute the value of this term which is going to be different for each of the m and n. And we are calculating z prime mn. And in the step 2 we are going to calculate the value of r 1, t 1, and the value of r 2, t 2 and Z mn is going to be basically the value that we have computed earlier Z prime mn from here we are multiplying it by k square plus t 2 minus t 1. Here what

we are taking into account is the time for the propagation from the point what we are interested is r_1 . And here the time t_2 is for r_2 .

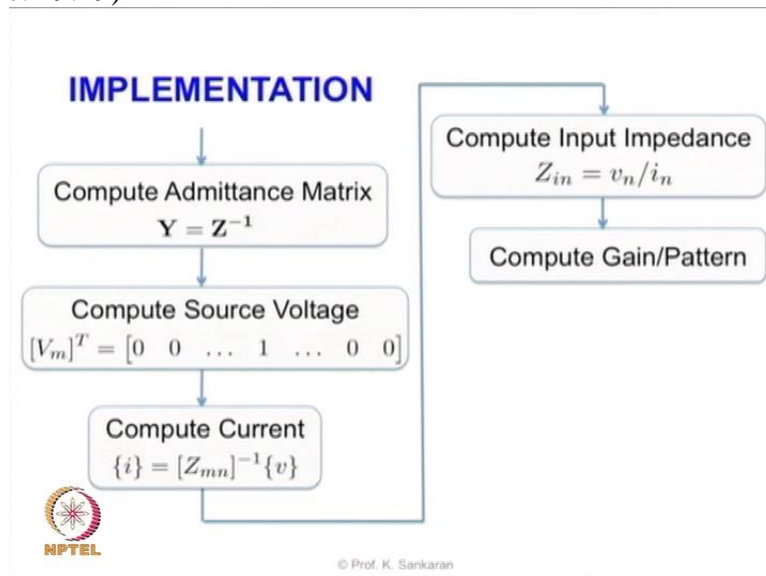
Once we have done that we have constructed the entire Z_{mn} impedance matrix. So once we have computed the impedance matrix based on that we can compute the value of the true impedance according to the expression given here.

(Refer Slide Time: 18:47)



So once we compute the Build Impedance Matrix procedure we can transfer that impedance value to the true impedance value using the value of eta which is given by the value of Epsilon Mu r. Because we can compute the value of impedance of the medium or the impedance of the atmosphere. Because we know Epsilon r and Mu r and eta can be calculated and we know delta z we know k so we can compute the value of true impedance accordingly.

(Refer Slide Time: 19:19)



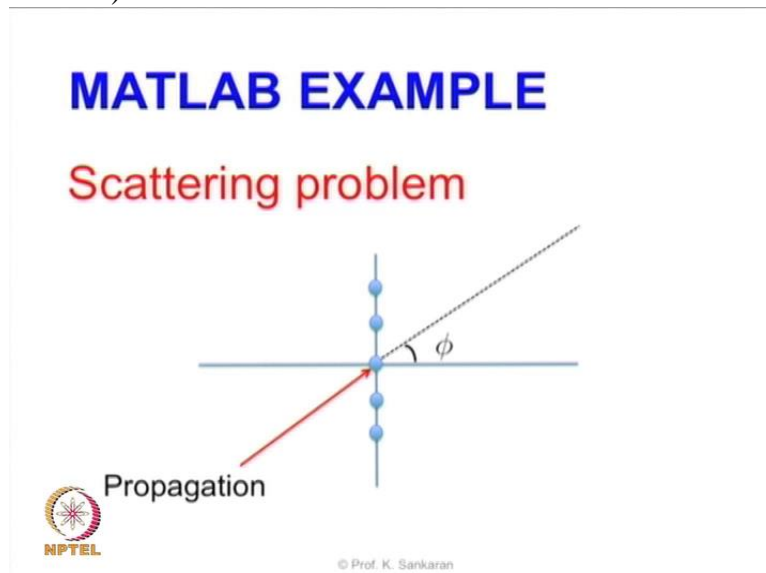
And once we have the true impedance value we can invert it to get the true Admittance value. And we have already known value of the source voltages which are v . So we can compute the values of current using the value of admittance which we computed and the source value which we know as v . And this will give us the value of z this will give us the value of i_n and we can compute the value of the input impedance by the ratio of v_n divided by i_n . Once we have that we can compute various parameters of the antennas like the gain; the antenna radiation pattern so on and so forth.

(Refer Slide Time: 20:00)



So now we will go into certain applications of this process what we have discussed.

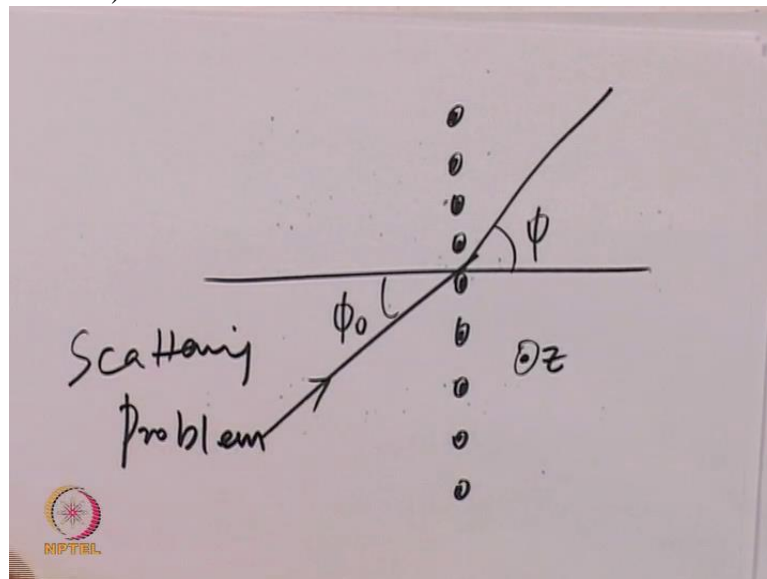
(Refer Slide Time: 20:10)



We will take a simple scattering problem as compared to the antenna problem what we have discussed now. So the scattering problem is going to have a geometry we have an input

propagation direction and the value of the input propagation direction is going to have certain angle of incidence.

(Refer Slide Time: 20:36)



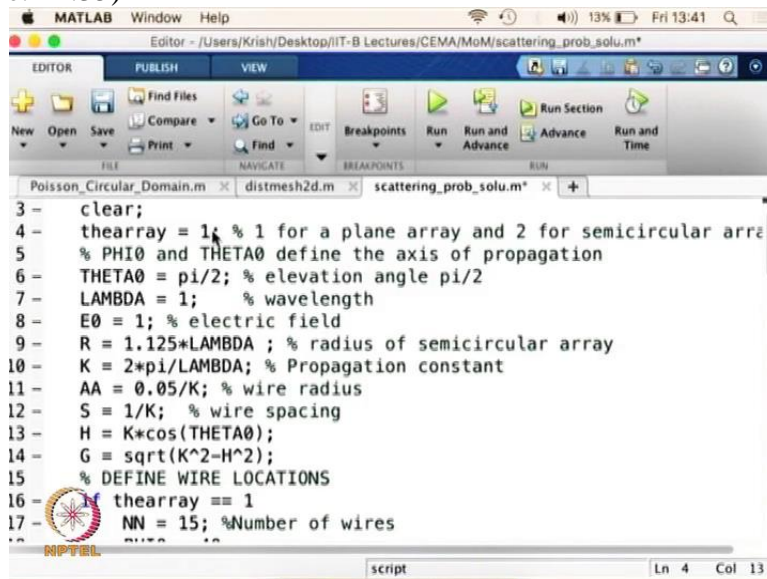
where the angle of incidence here is going to be given by and there are going to be some antennas that are going to sit here. And these are thin wire antennas and this value is the input value we call it as Φ_0 . And this angle is a scattering angle which we call it as Φ and this is the input direction and these are the individual wired antennas that are oriented in the direction z so assume that z is the direction which is coming out of this page. And we are going to talk about this problem which we call it as a scattering problem.

(Refer Slide Time: 21:29)

```
1 % THIS PROGRAM CALCULATES THE SCATTERING PATTREN OF AN ARRAY
2 % OF PARALLEL WIRES
3 clear;
4 thearray = 2; % 1 for a plane array and 2 for semicircular arra
5 % PHI0 and THETA0 define the axis of propagation
6 THETA0 = pi/2; % elevation angle pi/2
7 LAMBDA = 1; % wavelength
8 E0 = 1; % electric field
9 R = 1.125*LAMBDA; % radius of semicircular array
10 K = 2*pi/LAMBDA; % Propagation constant
11 AA = 0.05/K; % wire radius
12 S = 1/K; % wire spacing
13 H = K*cos(THETA0);
14 G = sqrt(K^2-H^2);
15 DEFINE WIRE LOCATIONS
```

So for this problem we are going to use the Matlab simulation

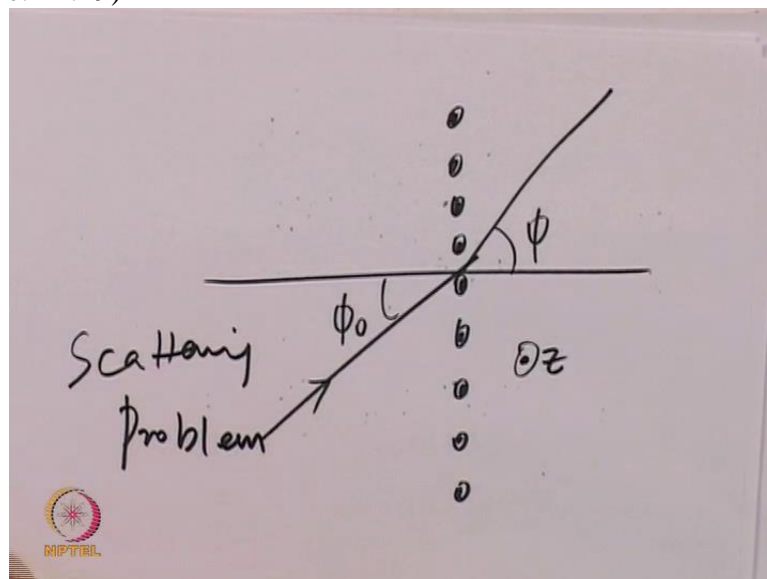
(Refer Slide Time: 21:35)



```
3- clear;
4- thearray = 1; % 1 for a plane array and 2 for semicircular arra
5- % PHI0 and THETA0 define the axis of propagation
6- THETA0 = pi/2; % elevation angle pi/2
7- LAMBDA = 1; % wavelength
8- E0 = 1; % electric field
9- R = 1.125*LAMBDA; % radius of semicircular array
10- K = 2*pi/LAMBDA; % Propagation constant
11- AA = 0.05/K; % wire radius
12- S = 1/K; % wire spacing
13- H = K*cos(THETA0);
14- G = sqrt(K^2-H^2);
15- % DEFINE WIRE LOCATIONS
16- if thearray == 1
17- NN = 15; %Number of wires
```

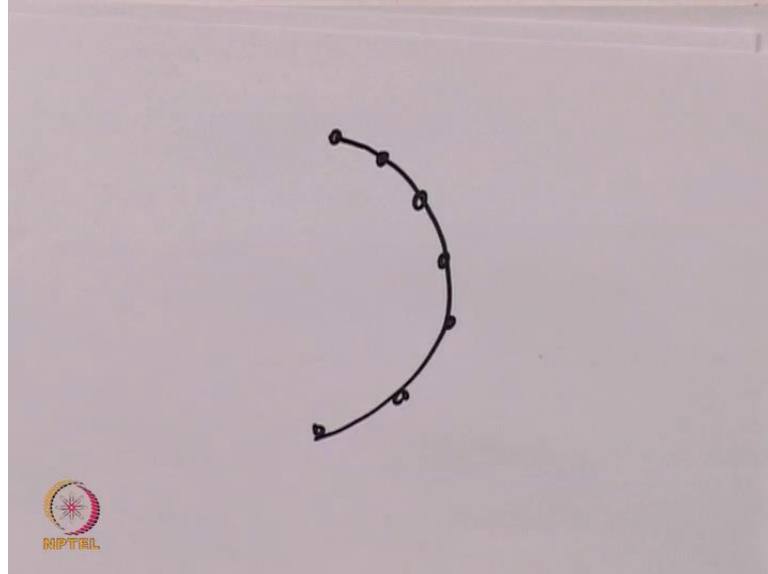
So basically we have defined the array so we can do two kinds of problem here. So in this example we will stick with a Linear array. What i mean by linear array is the antennas wires are sitting along the line;

(Refer Slide Time: 21:49)



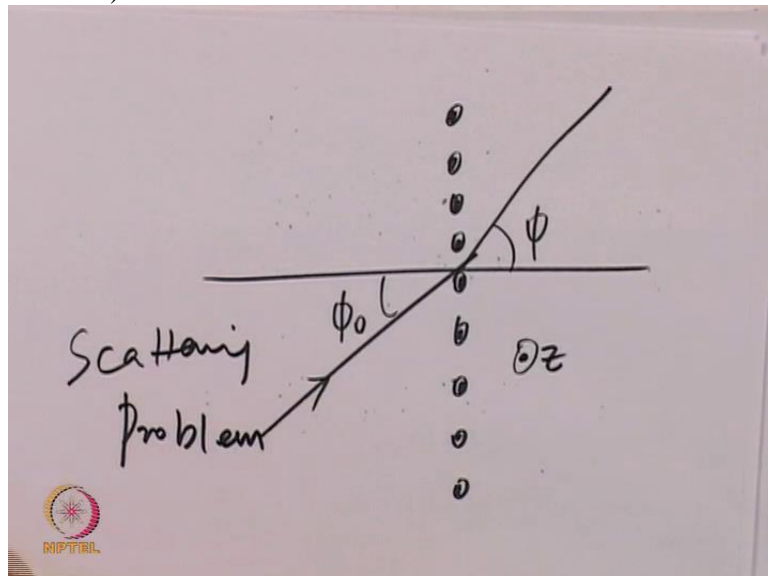
as I have shown in the slide here. So what you see here is they are linear.

(Refer Slide Time: 21:58)



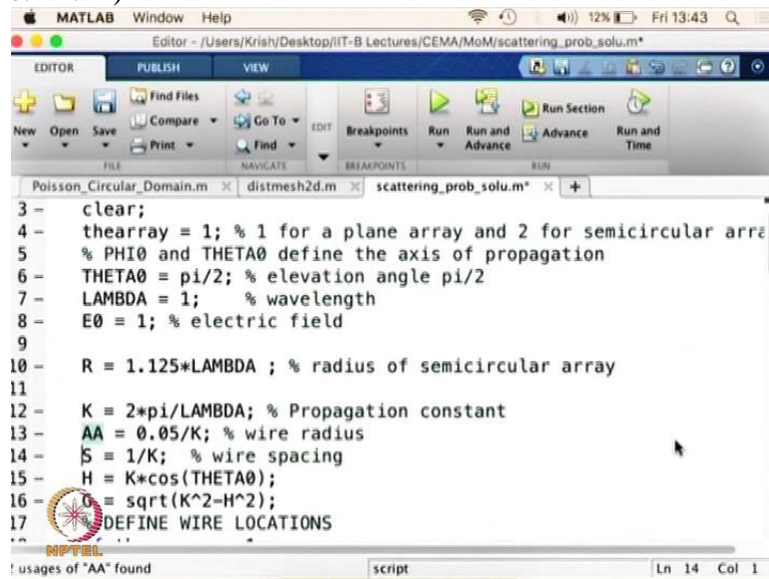
But you can also think of other geometry where the wires are in the semicircular manner in this manner. So we can do simulations for both.

(Refer Slide Time: 22:14)



But for the moment we will start with this problem where we are talking about linear array.

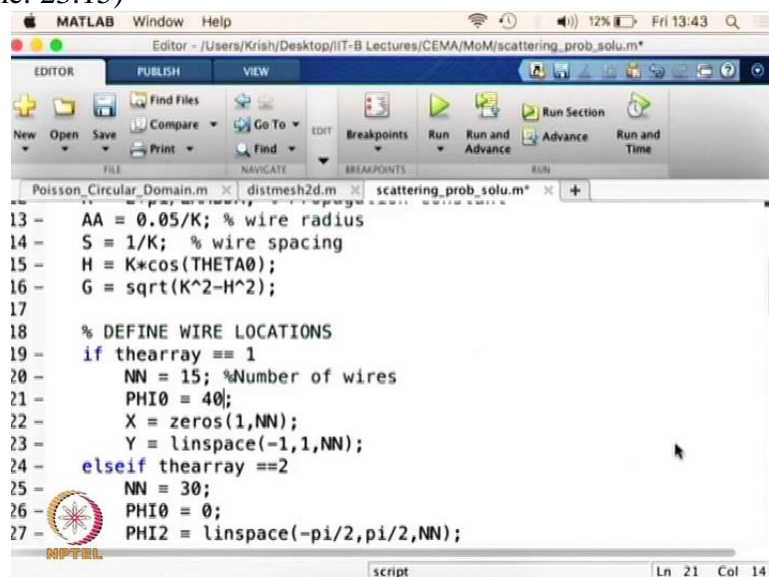
(Refer Slide Time: 22:22)



```
3- clear;
4- thearray = 1; % 1 for a plane array and 2 for semicircular arra
5- % PHI0 and THETA0 define the axis of propagation
6- THETA0 = pi/2; % elevation angle pi/2
7- LAMBDA = 1; % wavelength
8- E0 = 1; % electric field
9
10- R = 1.125*LAMBDA ; % radius of semicircular array
11
12- K = 2*pi/LAMBDA; % Propagation constant
13- AA = 0.05/K; % wire radius
14- S = 1/K; % wire spacing
15- H = K*cos(THETA0);
16- G = sqrt(K^2-H^2);
17- % DEFINE WIRE LOCATIONS
```

And if I want to choose a plain array I am going to choose the value 1. And the value of Phi 0 and Theta 0 are defined by the given angles of incident angles, so I am going to say that the Theta 0 is going to be Pi by 2. So the elevation angle is Pi by 2. And the Lambda is given by 1 Lambda. I have taken the value of Lambda as 1 meter. And the electric field E 0 is going to be given by 1 unit. And based on that I can compute. So we are not interested in the R 0 right now, because we are interested only in the plain array example. So depending on that we can compute the value of k which is a propagation constant and we can compute the value of wire radius and so on and so forth accordingly.

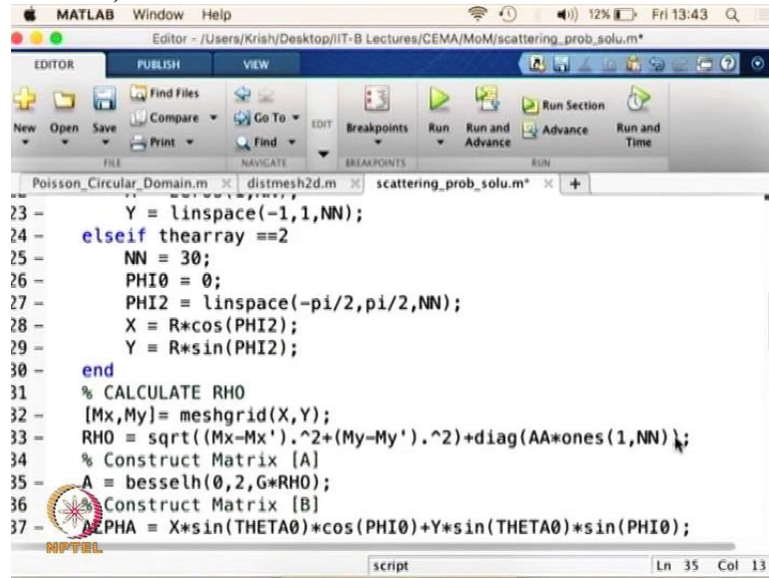
(Refer Slide Time: 23:15)



```
13- AA = 0.05/K; % wire radius
14- S = 1/K; % wire spacing
15- H = K*cos(THETA0);
16- G = sqrt(K^2-H^2);
17
18- % DEFINE WIRE LOCATIONS
19- if thearray == 1
20-     NN = 15; %Number of wires
21-     PHI0 = 40;
22-     X = zeros(1,NN);
23-     Y = linspace(-1,1,NN);
24- elseif thearray ==2
25-     NN = 30;
26-     PHI0 = 0;
27-     PHI2 = linspace(-pi/2,pi/2,NN);
```

And the value of the wire location. So initially we are considering 15 elements, 15 wires that are located as a scattering element. And the incidence angle is 40 degree.

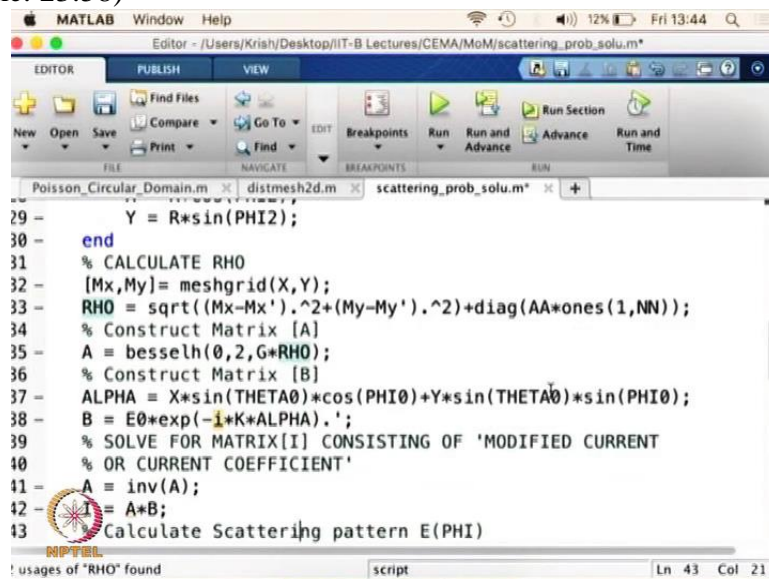
(Refer Slide Time: 23:30)



```
23 -     Y = linspace(-1,1,NN);
24 - elseif thearray ==2
25 -     NN = 30;
26 -     PHI0 = 0;
27 -     PHI2 = linspace(-pi/2,pi/2,NN);
28 -     X = R*cos(PHI2);
29 -     Y = R*sin(PHI2);
30 - end
31 % CALCULATE RHO
32 [Mx,My]= meshgrid(X,Y);
33 RHO = sqrt((Mx-Mx').^2+(My-My').^2)+diag(AA*ones(1,NN));
34 % Construct Matrix [A]
35 A = besselh(0,2,G*RHO);
36 % Construct Matrix [B]
37 ALPHA = X*sin(THETA0)*cos(PHI0)+Y*sin(THETA0)*sin(PHI0);
```

And I am going to use the Matlab function that is going to be given to me by Bessel functions. This is the Bessel H function and I am not going to go into the details of the Matlab code. But obviously it is easier once you know certain Matlab instructions so the Bessel H is going to give you the Bessel function and the inputs of those Bessel functions are 0, 2, and G and Rho 0.

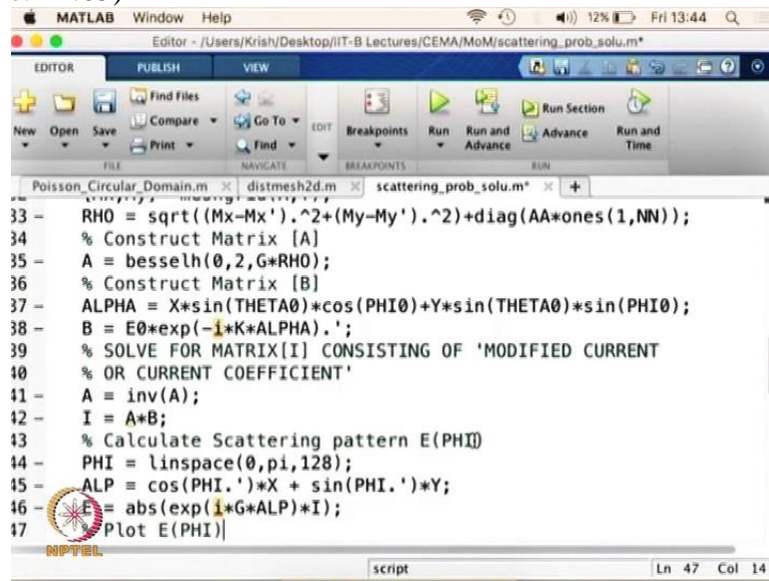
(Refer Slide Time: 23:58)



```
29 -     Y = R*sin(PHI2);
30 - end
31 % CALCULATE RHO
32 [Mx,My]= meshgrid(X,Y);
33 RHO = sqrt((Mx-Mx').^2+(My-My').^2)+diag(AA*ones(1,NN));
34 % Construct Matrix [A]
35 A = besselh(0,2,G*RHO);
36 % Construct Matrix [B]
37 ALPHA = X*sin(THETA0)*cos(PHI0)+Y*sin(THETA0)*sin(PHI0);
38 B = E0*exp(-i*K*ALPHA).';
39 % SOLVE FOR MATRIX [I] CONSISTING OF 'MODIFIED CURRENT
40 % OR CURRENT COEFFICIENT'
41 A = inv(A);
42 I = A*B;
43 Calculate Scattering pattern E(PHI)
```

And my value of Rho 0 is given by my input setting which I have given here.

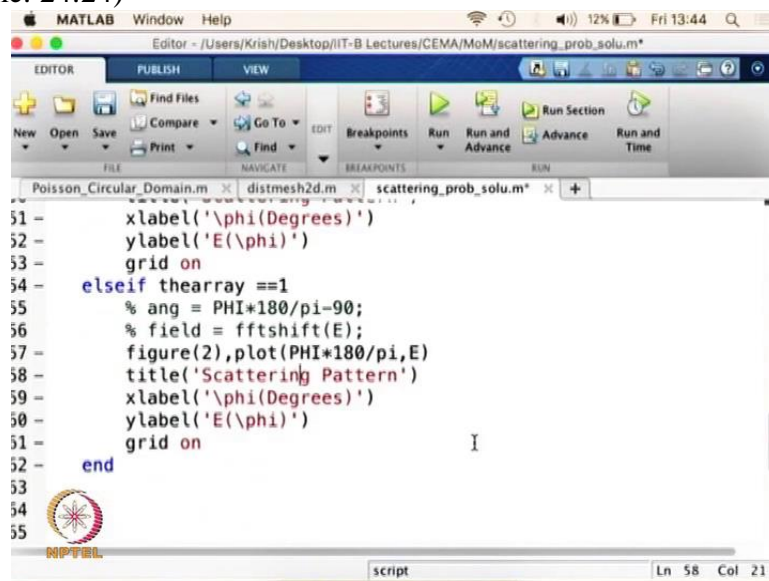
(Refer Slide Time: 24:05)



```
33 - RHO = sqrt((Mx-Mx').^2+(My-My').^2)+diag(AA*ones(1,NN));
34 % Construct Matrix [A]
35 - A = besselh(0,2,G*RHO);
36 % Construct Matrix [B]
37 - ALPHA = X*sin(THETA0)+cos(PHI0)+Y*sin(THETA0)*sin(PHI0);
38 - B = E0*exp(-i*K*ALPHA).';
39 % SOLVE FOR MATRIX[I] CONSISTING OF 'MODIFIED CURRENT
40 % OR CURRENT COEFFICIENT'
41 - A = inv(A);
42 - I = A*B;
43 % Calculate Scattering pattern E(PHI)
44 - PHI = linspace(0,pi,128);
45 - ALP = cos(PHI.)*X + sin(PHI.)*Y;
46 - E = abs(exp(i*G*ALP)*I);
47 Plot E(PHI)|
```

I am going to compute the value of the scattered E field as a function of PHI. So I am computing the value of the scattered E field as a function of PHI. I will give this code for you to test and try at your own time.

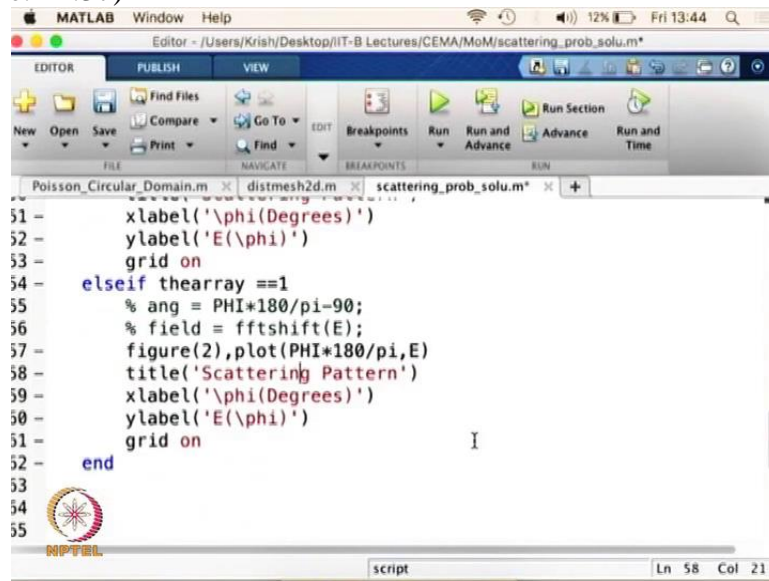
(Refer Slide Time: 24:24)



```
51 - xlabel('\phi(Degrees)')
52 - ylabel('E(\phi)')
53 - grid on
54 - elseif thearray ==1
55 % ang = PHI+180/pi-90;
56 % field = fftshift(E);
57 - figure(2),plot(PHI+180/pi,E)
58 - title('Scattering Pattern')
59 - xlabel('\phi(Degrees)')
60 - ylabel('E(\phi)')
61 - grid on
62 - end
```

But let us now take this example and compute the scattering pattern and plot the scattering pattern for this array antenna. And see how the solution looks like for various input parameters.

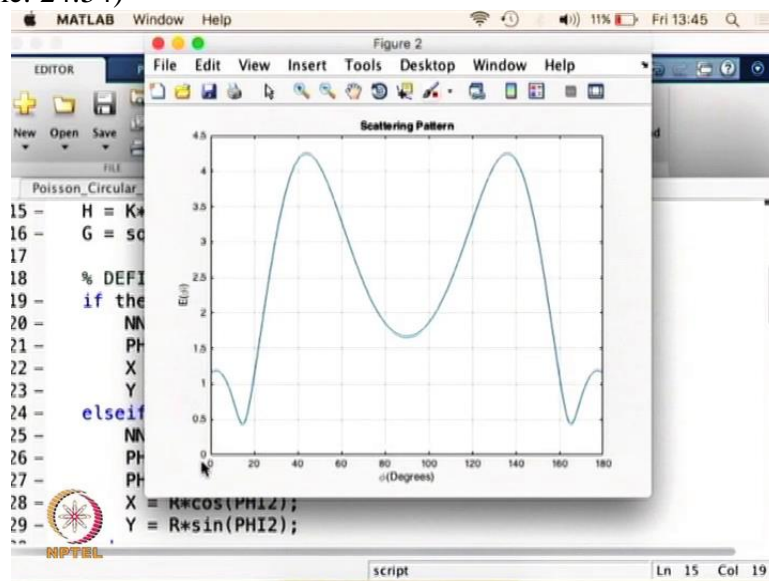
(Refer Slide Time: 24:37)



```
51 - xlabel('\phi(Degrees)')
52 - ylabel('E(\phi)')
53 - grid on
54 - elseif thearray ==1
55 - % ang = PHI*180/pi-90;
56 - % field = fftshift(E);
57 - figure(2),plot(PHI*180/pi,E)
58 - title('Scattering Pattern')
59 - xlabel('\phi(Degrees)')
60 - ylabel('E(\phi)')
61 - grid on
62 - end
63
64
65
```

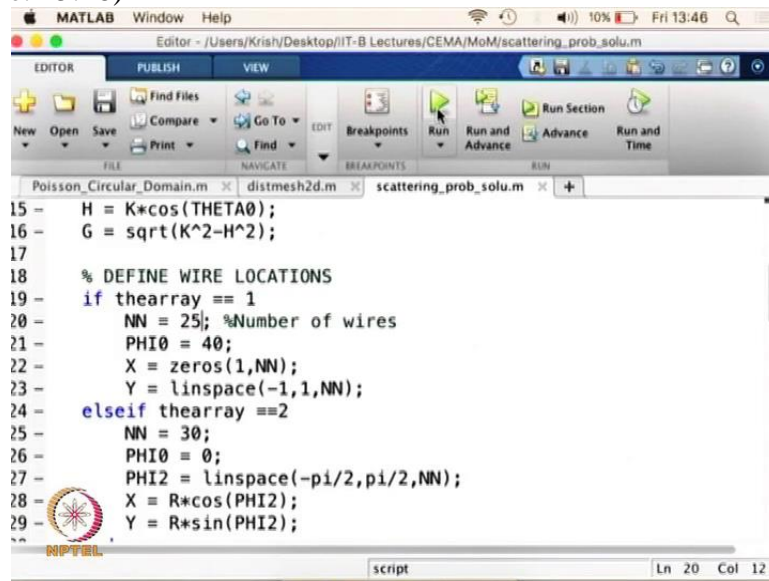
So I am going to change the number of wires and I am also going to change the number of Phi 0. So let us take for the moment the number of wires are going to 15. And we are going to have Lambda is equal to 1. And Phi 0 is equal to 40.

(Refer Slide Time: 24:54)



So once you do that what you have is? Scattering pattern as a function of Phi. So you see that E scattered will be the E as a function of Phi and then this is the Phi, Phi is going to vary from 0 to 90 and then it is going to repeat. So there is a repetition after 90. So 0 to 90 and then it starts reflect back and then there is a image of that reflection on the other side. So the image has to be symmetric around Phi equal to 90 degree.

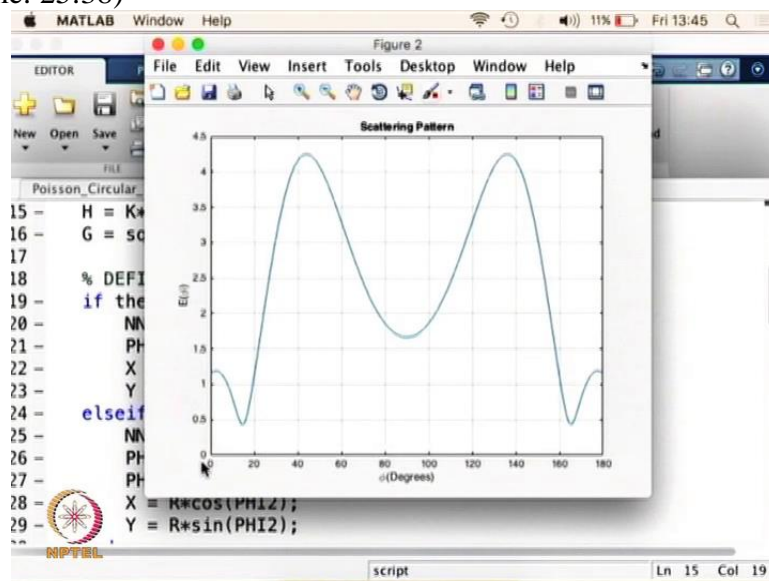
(Refer Slide Time: 25:28)



```
15 - H = K*cos(THETA0);
16 - G = sqrt(K^2-H^2);
17
18 % DEFINE WIRE LOCATIONS
19 - if thearray == 1
20 -     NN = 25; %Number of wires
21 -     PHI0 = 40;
22 -     X = zeros(1,NN);
23 -     Y = linspace(-1,1,NN);
24 - elseif thearray ==2
25 -     NN = 30;
26 -     PHI0 = 0;
27 -     PHI2 = linspace(-pi/2,pi/2,NN);
28 -     X = R*cos(PHI2);
29 -     Y = R*sin(PHI2);
```

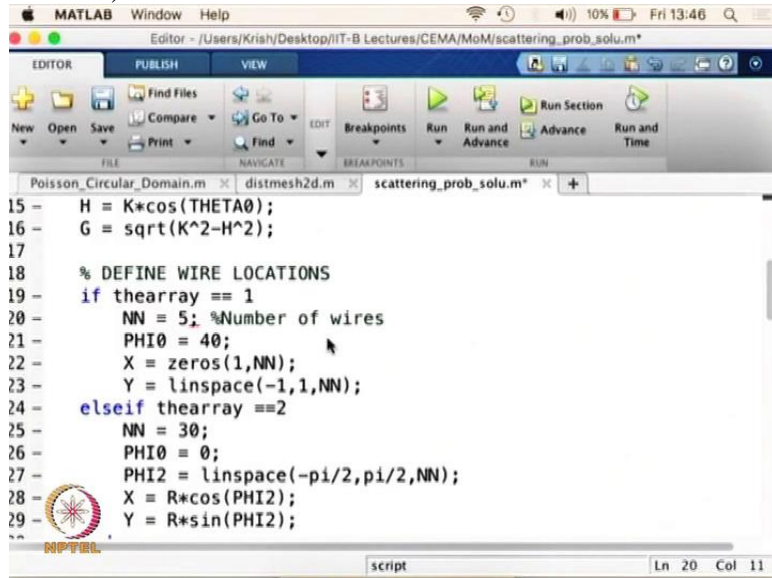
So now we can change certain things. We can change the number of elements, let us make the number of elements to let us say 25.

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So when you see that the number of elements increasing is actually not changing pretty much the scattering pattern.

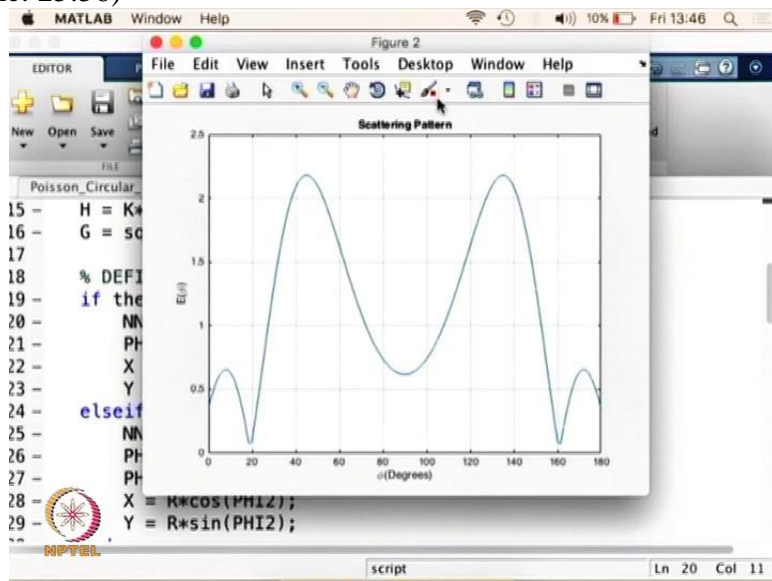
(Refer Slide Time: 25:48)



```
15 - H = K*cos(THETA0);
16 - G = sqrt(K^2-H^2);
17
18 % DEFINE WIRE LOCATIONS
19 - if thearray == 1
20 -     NN = 5; %Number of wires
21 -     PHI0 = 40;
22 -     X = zeros(1,NN);
23 -     Y = linspace(-1,1,NN);
24 - elseif thearray ==2
25 -     NN = 30;
26 -     PHI0 = 0;
27 -     PHI2 = linspace(-pi/2,pi/2,NN);
28 -     X = R*cos(PHI2);
29 -     Y = R*sin(PHI2);
```

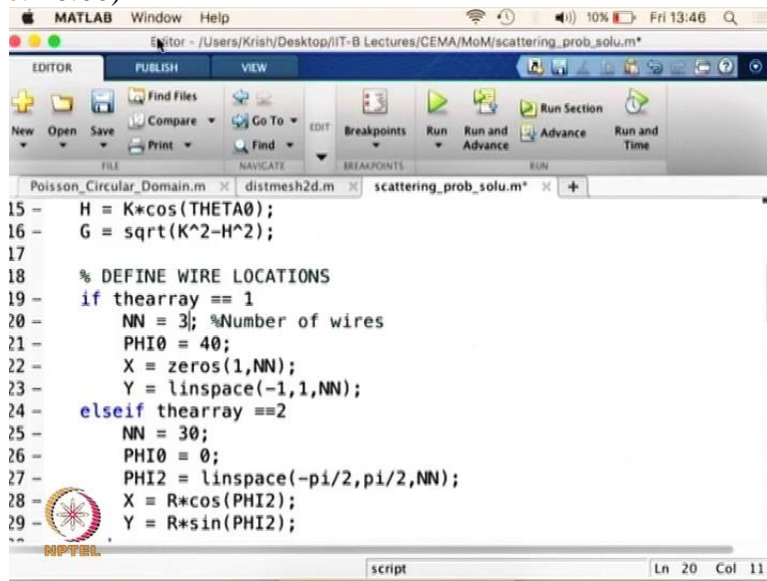
So if we change E further there is nothing happening but when we go down let us say when we have only 5 scatterers.

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You see that there is some variation

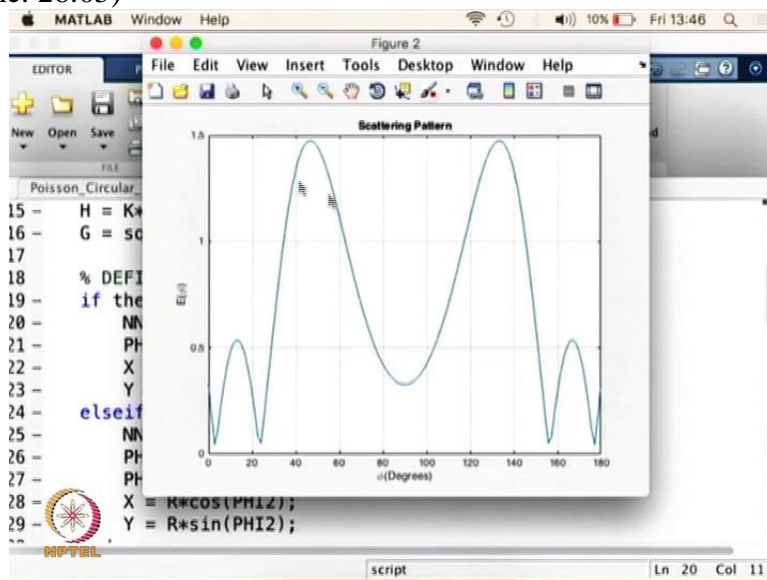
(Refer Slide Time: 26:00)



```
15 - H = K*cos(THETA0);
16 - G = sqrt(K^2-H^2);
17
18 % DEFINE WIRE LOCATIONS
19 - if thearray == 1
20 -     NN = 3; %Number of wires
21 -     PHI0 = 40;
22 -     X = zeros(1,NN);
23 -     Y = linspace(-1,1,NN);
24 - elseif thearray ==2
25 -     NN = 30;
26 -     PHI0 = 0;
27 -     PHI2 = linspace(-pi/2,pi/2,NN);
28 -     X = R*cos(PHI2);
29 -     Y = R*sin(PHI2);
```

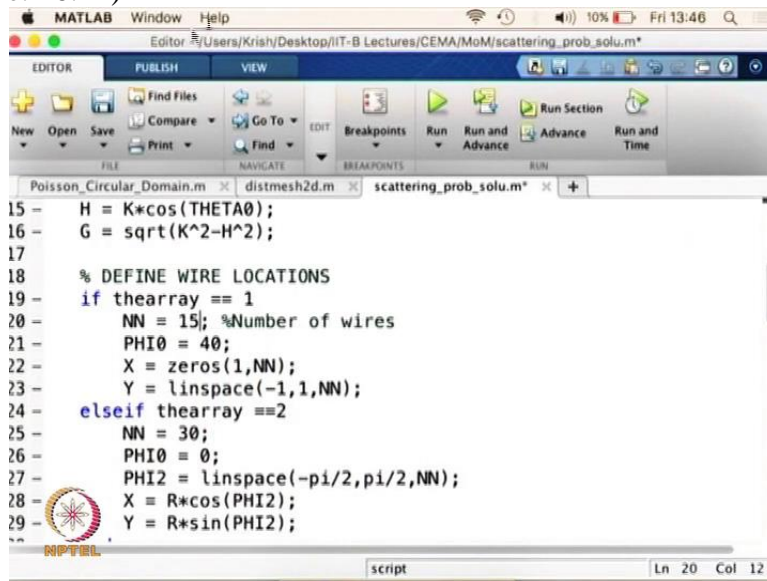
Let us say we have only 3 scatterers.

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You see that there is some variation happening the maximum amplitude is also changing.

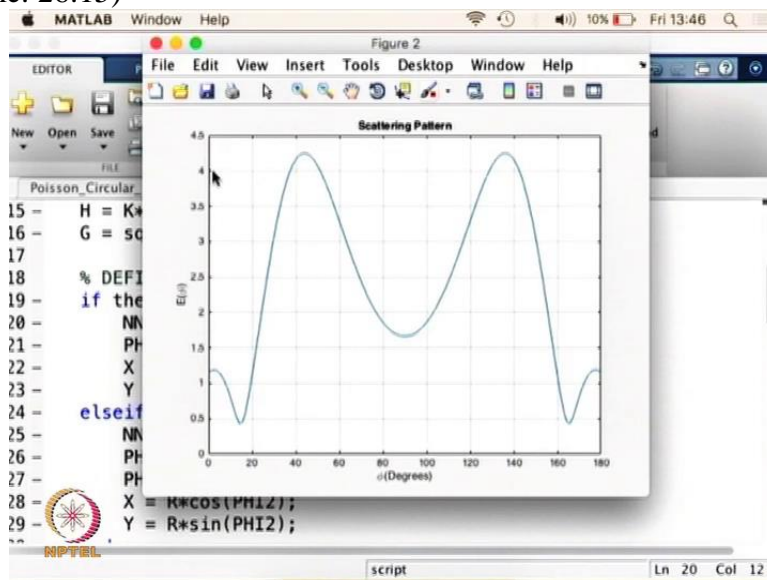
(Refer Slide Time: 26:14)



```
15 - H = K*cos(THETA0);
16 - G = sqrt(K^2-H^2);
17
18 % DEFINE WIRE LOCATIONS
19 - if thearray == 1
20 -     NN = 15; %Number of wires
21 -     PHI0 = 40;
22 -     X = zeros(1,NN);
23 -     Y = linspace(-1,1,NN);
24 - elseif thearray ==2
25 -     NN = 30;
26 -     PHI0 = 0;
27 -     PHI2 = linspace(-pi/2,pi/2,NN);
28 -     X = R*cos(PHI2);
29 -     Y = R*sin(PHI2);
```

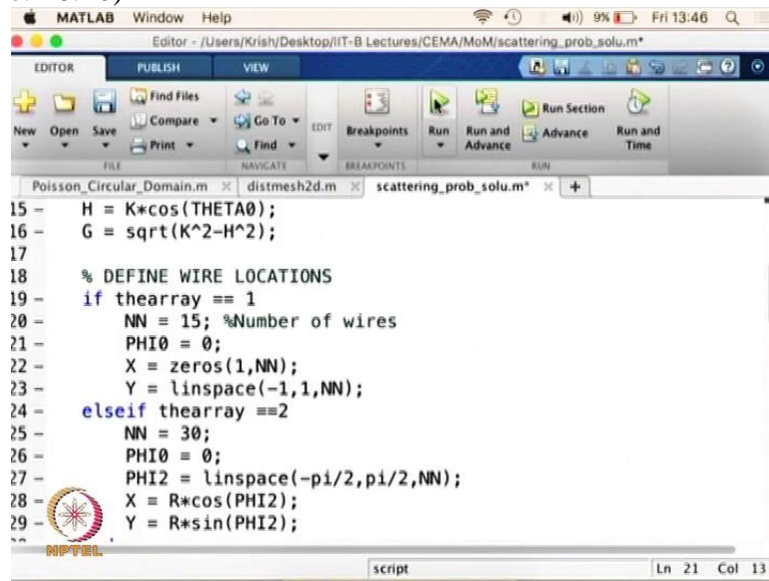
In the initial case when you had 15.

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the maximum scattering amplitude was in the range of 4.

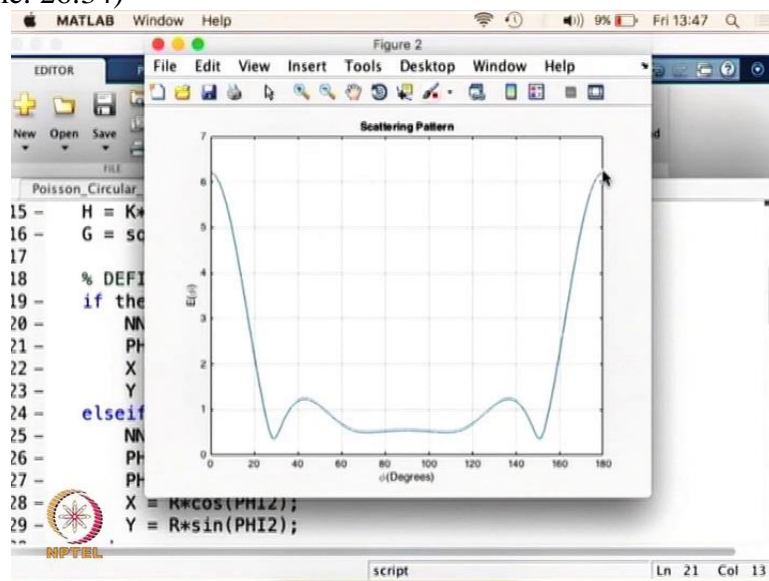
(Refer Slide Time: 26:20)



```
15 - H = K*cos(THETA0);
16 - G = sqrt(K^2-H^2);
17
18 % DEFINE WIRE LOCATIONS
19 - if thearray == 1
20 -     NN = 15; %Number of wires
21 -     PHI0 = 0;
22 -     X = zeros(1,NN);
23 -     Y = linspace(-1,1,NN);
24 - elseif thearray ==2
25 -     NN = 30;
26 -     PHI0 = 0;
27 -     PHI2 = linspace(-pi/2,pi/2,NN);
28 -     X = R*cos(PHI2);
29 -     Y = R*sin(PHI2);
```

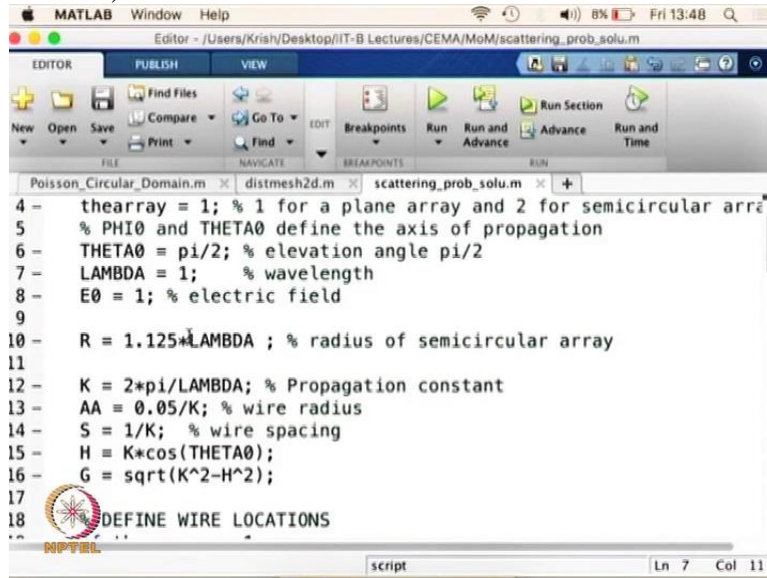
Where as in the case of less number of scatterers it goes to 1.5. That is one thing second thing is if I change the angle of incidence. If I go at 0 degree incidence.

(Refer Slide Time: 26:34)



You pretty much see at Phi equal to 0 you get a maximum scattering whereas at Phi equal to 90 you get a very very low scattering. So this is almost like a grating incidence here. And similarly at Phi is equal to 180 degree you get again maximum which is symmetric around 90 degree.

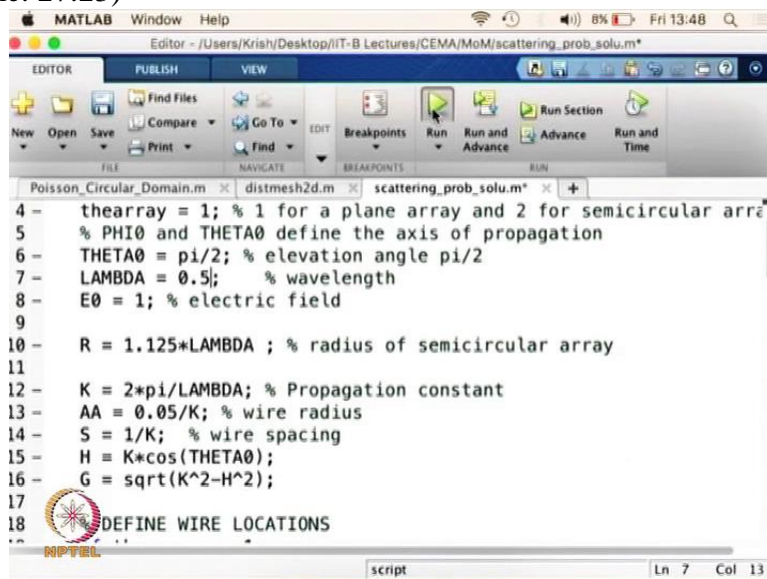
(Refer Slide Time: 26:56)



```
4- thearray = 1; % 1 for a plane array and 2 for semicircular arra
5- % PHI0 and THETA0 define the axis of propagation
6- THETA0 = pi/2; % elevation angle pi/2
7- LAMBDA = 1; % wavelength
8- E0 = 1; % electric field
9
10- R = 1.125*LAMBDA ; % radius of semicircular array
11
12- K = 2*pi/LAMBDA; % Propagation constant
13- AA = 0.05/K; % wire radius
14- S = 1/K; % wire spacing
15- H = K*cos(THETA0);
16- G = sqrt(K^2-H^2);
17
18- DEFINE WIRE LOCATIONS
```

What we can also see is once we change the Lambda value itself, when the Lambda value changes it also changes. And it also changes with respect to the length of the element itself. So remember the length of the element should be very big compared to the radius itself. So that also changes.

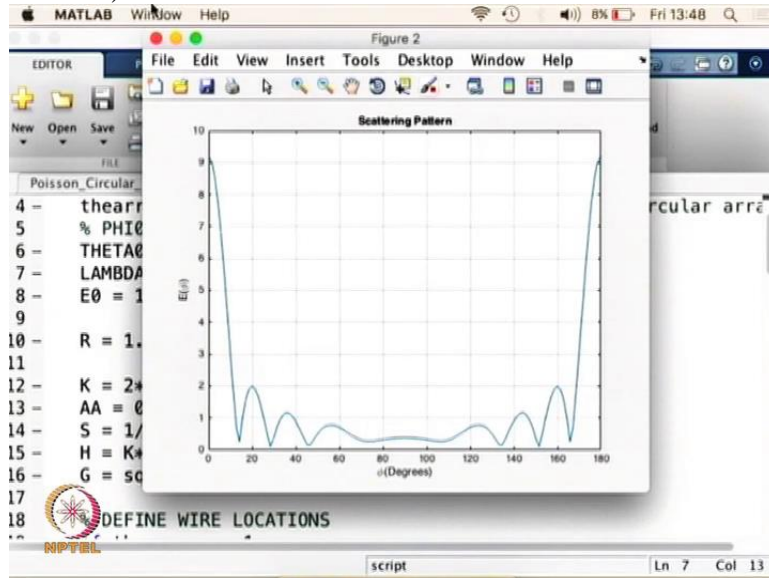
(Refer Slide Time: 27:23)



```
4- thearray = 1; % 1 for a plane array and 2 for semicircular arra
5- % PHI0 and THETA0 define the axis of propagation
6- THETA0 = pi/2; % elevation angle pi/2
7- LAMBDA = 0.5; % wavelength
8- E0 = 1; % electric field
9
10- R = 1.125*LAMBDA ; % radius of semicircular array
11
12- K = 2*pi/LAMBDA; % Propagation constant
13- AA = 0.05/K; % wire radius
14- S = 1/K; % wire spacing
15- H = K*cos(THETA0);
16- G = sqrt(K^2-H^2);
17
18- DEFINE WIRE LOCATIONS
```

So just for the sake of interest I am going to go lower in ; so instead of 1 lambda I am going on 0.5 Lambda.

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Once I do that you see that the pattern is changing quite a big.

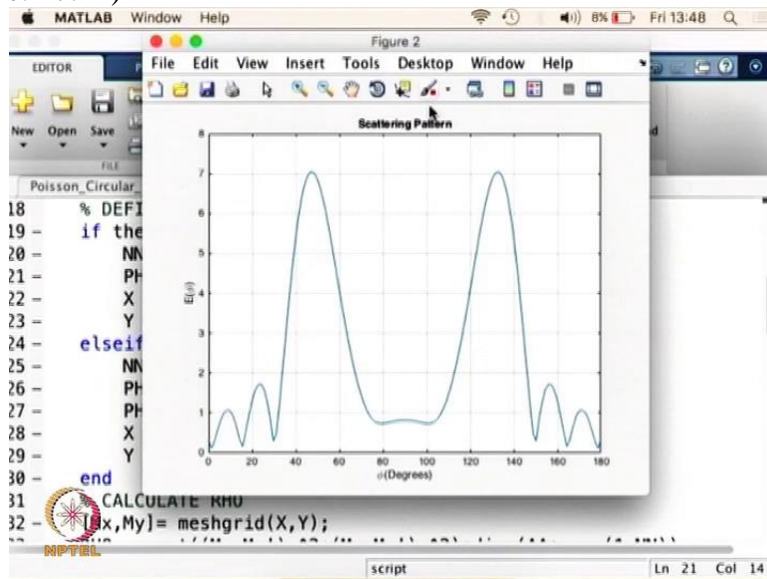
(Refer Slide Time: 27:36)

The figure shows a MATLAB script editor with the following code:

```
18 % DEFINE WIRE LOCATIONS  
19 if thearray == 1  
20     NN = 15; %Number of wires  
21     PHI0 = 40;  
22     X = zeros(1,NN);  
23     Y = linspace(-1,1,NN);  
24 elseif thearray ==2  
25     NN = 30;  
26     PHI0 = 0;  
27     PHI2 = linspace(-pi/2,pi/2,NN);  
28     X = R*cos(PHI2);  
29     Y = R*sin(PHI2);  
30 end  
31 % CALCULATE RHO  
32 [X,My] = meshgrid(X,Y);
```

So what we can do is we can change the value of Theta to be 40.

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And see that the value is changing quite a bit. The scattered field is going to be quite high

(Refer Slide Time: 27:50)

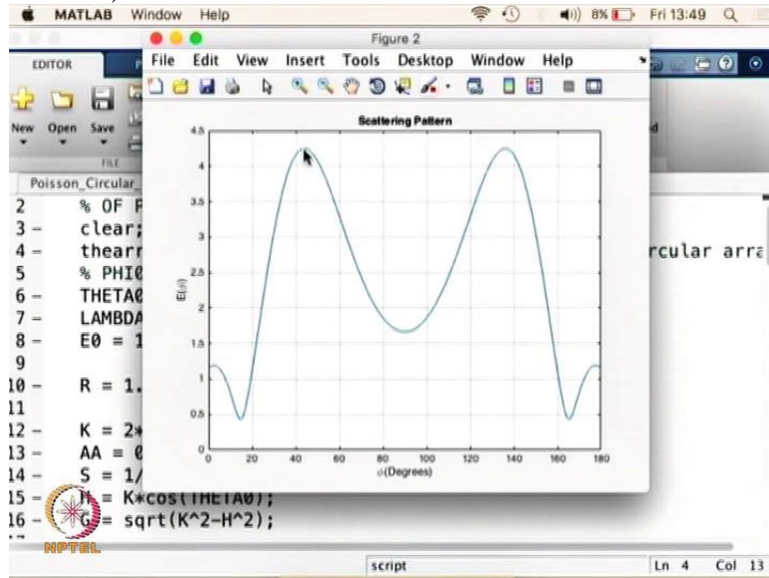
The figure shows a MATLAB window with the following code:

```
2 % OF PARALLEL WIRES
3 clear;
4 thearray = 1; % 1 for a plane array and 2 for semicircular array
5 % PHI0 and THETA0 define the axis of propagation
6 THETA0 = pi/2; % elevation angle pi/2
7 LAMBDA = 1; % wavelength
8 E0 = 1; % electric field
9
10 R = 1.125*LAMBDA; % radius of semicircular array
11
12 K = 2*pi/LAMBDA; % Propagation constant
13 AA = 0.05/K; % wire radius
14 S = 1/K; % wire spacing
15 H = K*cos(THETA0);
16 G = sqrt(K^2-H^2);
```

And the approximation is going to change also accordingly.

So just for the sake of interest I am also going to show you how the scattering pattern is going to change for a semi circular array

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So let us say we have simulated this for a plainer array. Remember the image is going to look like this. You have a maximum scattering E field around 4.5 unit compared to the minimum.

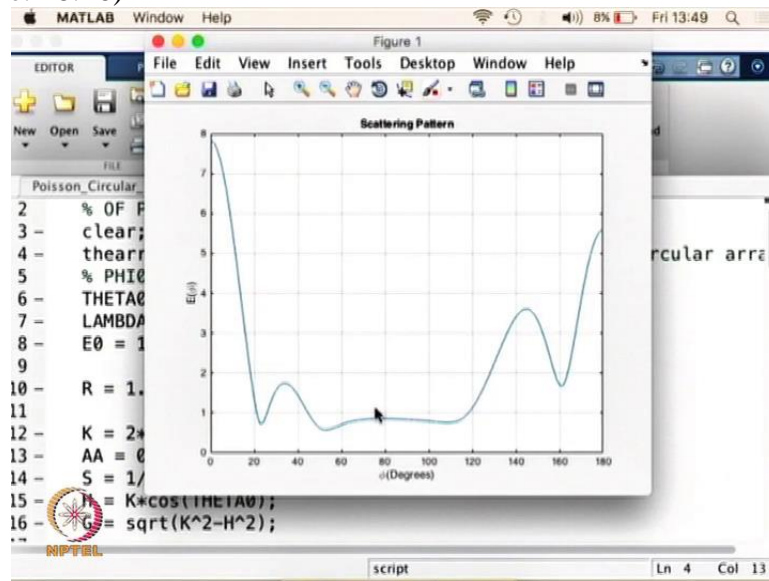
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The figure shows a MATLAB window with a script editor. The script contains the following code:

```
2 % OF PARALLEL WIRES  
3 clear;  
4 thearray = 2; % 1 for a plane array and 2 for semicircular arra  
5 % PHI0 and THETA0 define the axis of propagation  
6 THETA0 = pi/2; % elevation angle pi/2  
7 LAMBDA = 1; % wavelength  
8 E0 = 1; % electric field  
9  
10 R = 1.125*LAMBDA ; % radius of semicircular array  
11  
12 K = 2*pi/LAMBDA; % Propagation constant  
13 AA = 0.05/K; % wire radius  
14 S = 1/K; % wire spacing  
15 H = K*cos(THETA0);  
16 G = sqrt(K^2-H^2);
```

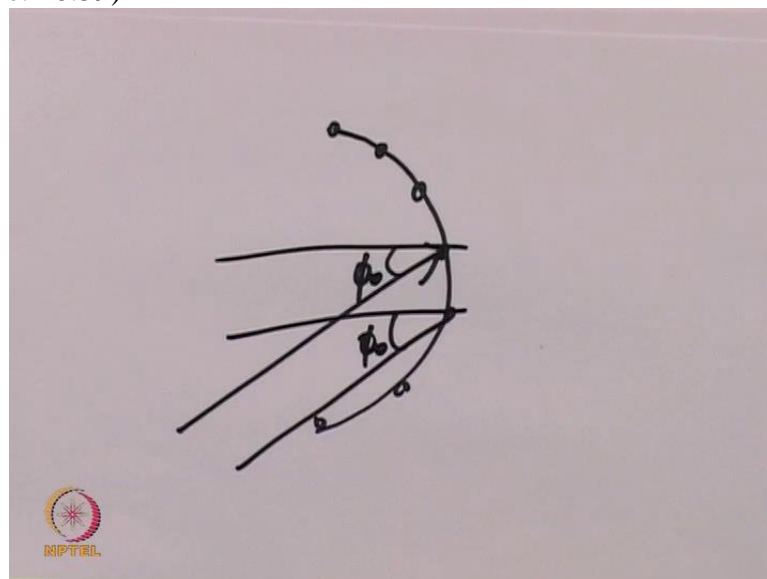
So let us see how this is going to change for the semi circular thing.

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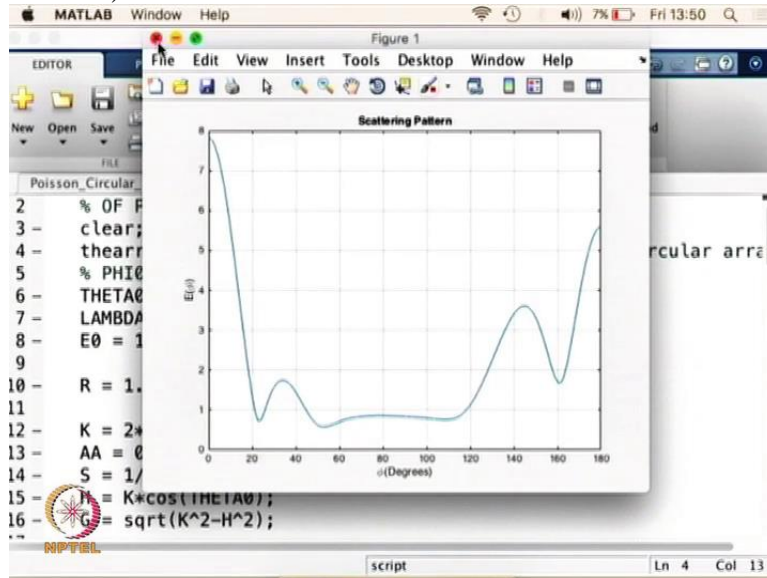
You can also try for the semi circular thing, you see that it is no longer symmetric. It is no longer symmetric because the angle in which you are coming in the incidence angle is not symmetric in both sides.

(Refer Slide Time: 28:39)



So for example when I come in this angle at this point my angle is going to be defined as this one. Whereas for this one for this element it is going to change accordingly. So that is why the scattering pattern is no longer similar.

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Whereas when you go at a angle equal to 0. You will see that it will have a symmetric pattern.

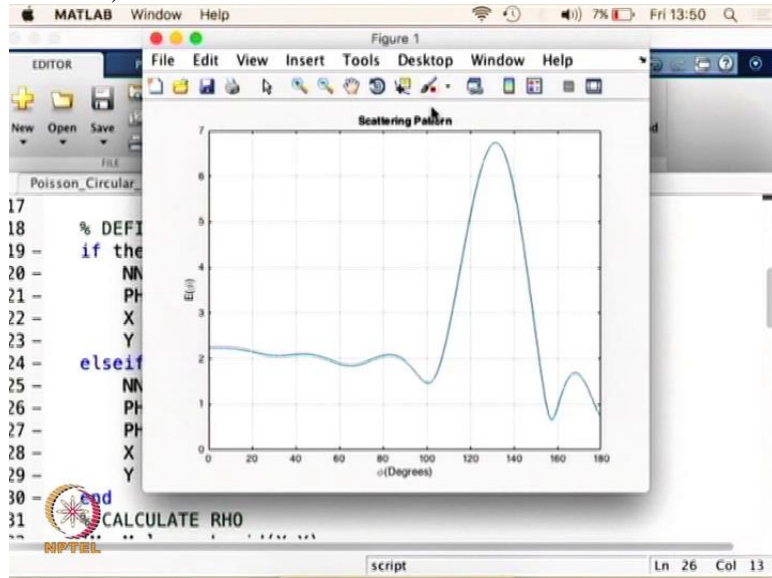
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The figure shows a MATLAB window with the following code:

```
17
18 % DEFINE WIRE LOCATIONS
19 if thearray == 1
20     NN = 15; %Number of wires
21     PHI0 = 40;
22     X = zeros(1,NN);
23     Y = linspace(-1,1,NN);
24 elseif thearray ==2
25     NN = 30;
26     PHI0 = 40;
27     PHI2 = linspace(-pi/2,pi/2,NN);
28     X = R*cos(PHI2);
29     Y = R*sin(PHI2);
30 end
31 CALCULATE RHO
```

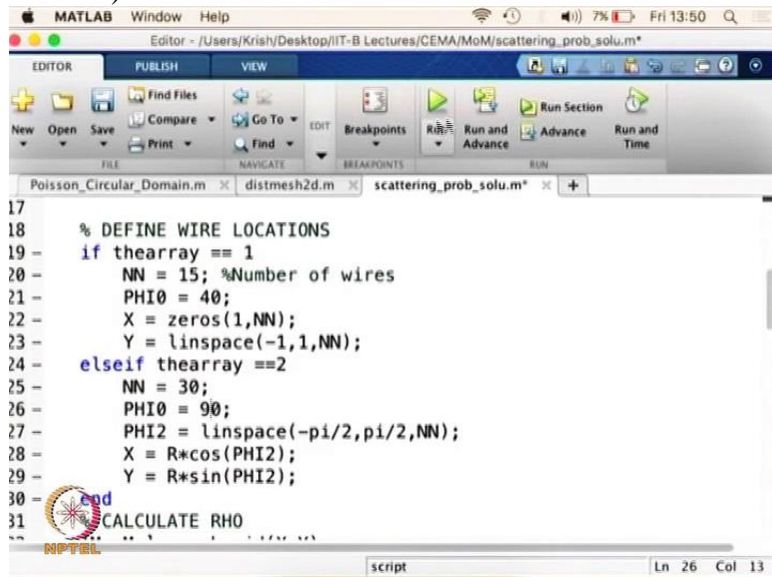
We can try this one when the incidence angle is going to be 0 here. So now I am going to change it to from 0 to 40.

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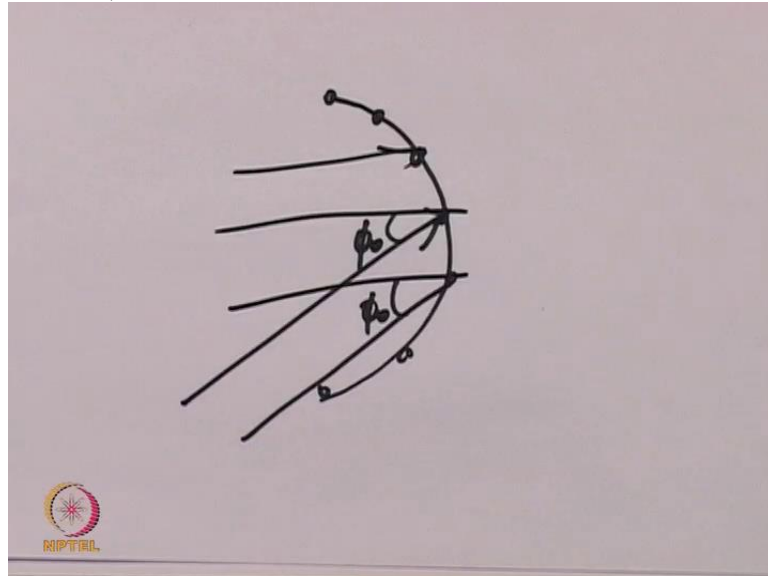
You see that it is still changing a bit.

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So if I go to 90

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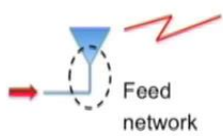
You see that it is no longer symmetric in either cases. Because for the simple reason which we talked about in the case of the semi circular array each of the array elements is going to see the angle of incidence going to be different. And that is why you don't get the symmetric pattern as we saw in the case of the planar one

(Refer Slide Time: 29:50)

WHAT IS EXCITATION?

Antenna parameters are easily calculated when it is treated as a transmitting device

Excitation is manner in which energy is fed so as to get radiated



Antenna properties depend on how and where energy is applied

NPTEL

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The slide contains text explaining the concept of excitation in antennas. It states that antenna parameters are easily calculated when the antenna is treated as a transmitting device. It defines excitation as the manner in which energy is fed so as to get radiated. A diagram shows a blue antenna element connected to a dashed circle labeled 'Feed network'. A red arrow points into the feed network, and a red lightning bolt symbol indicates radiation from the antenna. The NPTEL logo is in the bottom left, and the copyright notice '© Prof. K. Sankaran' is at the bottom center.

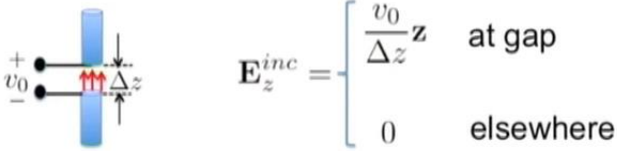
The antenna parameters are easily calculated once you understand how we are treating the antenna and the antenna is treated as a transmitting device it is easy to extract the various parameters of the antenna but for us to treat it as a transmitting device the way in which we are going to feed it how we are feeding it and where we are feeding it is going to greatly impact. So in that sense it is important to know various techniques that normally come into play. So the excitation manner in which the antenna is getting fed is going to radiate that is

one point the second point is so the properties depend on how and where we are going to feed the antenna.


(Refer Slide Time: 30:30)

DELTA-GAP SOURCE

It models the feed as if incident field exist only in small gap at terminals


$$\mathbf{E}_z^{inc} = \begin{cases} \frac{v_0}{\Delta z} \mathbf{z} & \text{at gap} \\ 0 & \text{elsewhere} \end{cases}$$

Simplest source to implement, but less accurate for impedance calculations




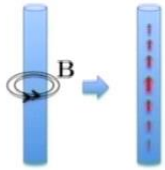
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One way of looking at it is very simple straight forward very rudimentary approach is to assume that the incident field exists only in the small gap between the terminals. So this will approximate the incidence field as the voltage difference divided by the distance between the terminal at the gap and elsewhere the antenna incident field is 0. So that is a very very simple but it is less accurate. In my opinion it is not useful for practical impedance calculation.

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MAGNETIC FRILL SOURCE

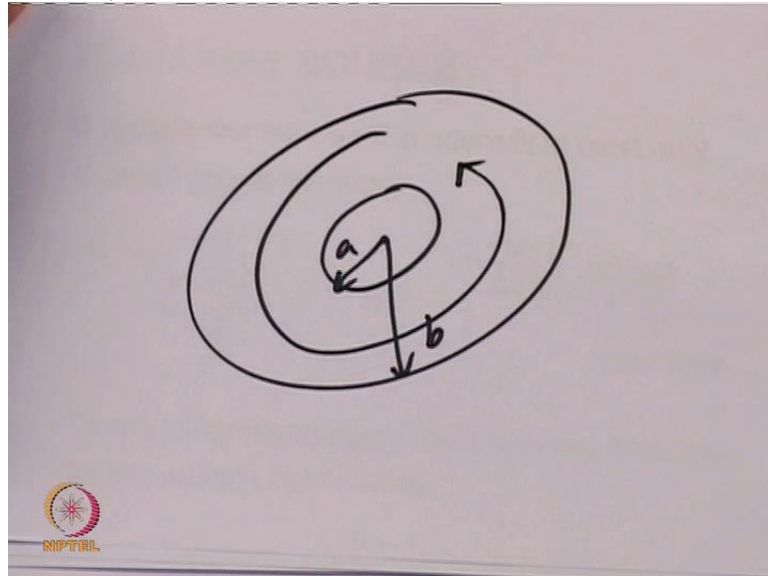
It models the feed magnetic field circulating around thin wire at feed



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The other approach is to take the magnetic field approach assume that you have a coaxial input plane where you are interested.

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So there is a coaxial plane the inner radius is a and the outer radius is b. so once you have that you can talk about the value of certain magnetic field that are circulating around this thing. So this is the approach of the magnetic field.

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MAGNETIC FRILL SOURCE

It models the feed magnetic field circulating around thin wire at feed

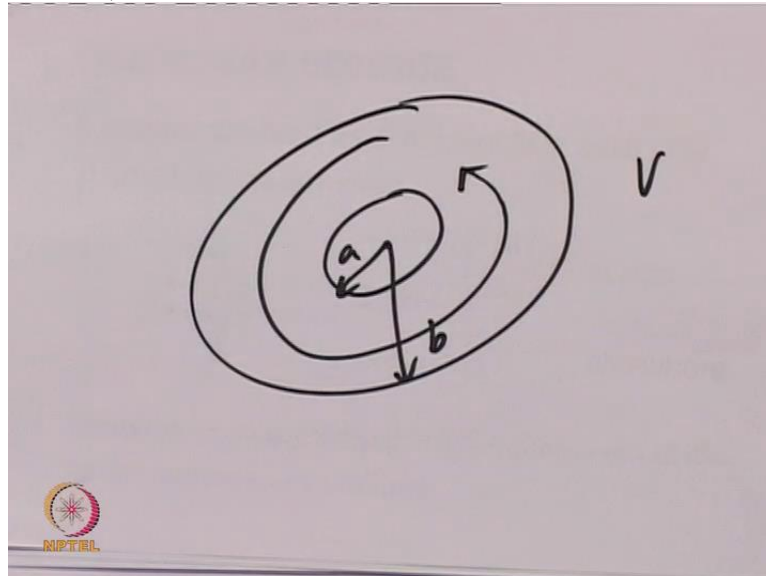
$$\mathbf{E}_z^{inc}(z) = \frac{1}{2 \ln(b/a)} \left(\frac{e^{-jkr_a}}{r_a} - \frac{e^{-jkr_b}}{r_b} \right)$$

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The slide contains a title 'MAGNETIC FRILL SOURCE' in blue. Below it is a descriptive sentence: 'It models the feed magnetic field circulating around thin wire at feed'. To the left of the equation is a diagram showing a vertical blue wire with a circular arrow labeled 'B' around it, and a blue vertical bar with red dashed lines representing the frill. The equation is in the center. At the bottom left is the NPTEL logo and navigation icons. At the bottom right is the copyright notice '© Prof. K. Sankaran'.

As we have shown in this slide. So once you do that you basically can approximate the value of the e incident as logarithmic value of the b by n.

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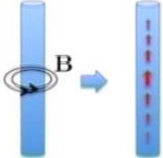


Because it is a coaxial aspect so you can compute the voltage that exists between the two coaxial planes and then we can compute the value of V once we know the v we can divide it by the distance.

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MAGNETIC FRILL SOURCE

It models the feed magnetic field circulating around thin wire at feed



$$\mathbf{E}_z^{inc}(z) = \frac{1}{2 \ln(b/a)} \left(\frac{e^{-jkr_a}}{r_a} - \frac{e^{-jkr_b}}{r_b} \right)$$

$$r_a = \sqrt{z^2 + a^2} \quad r_b = \sqrt{z^2 + b^2} \quad b \approx 3a$$

Difficult to implement and more calculations,
but more accurate

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And that is what we have done here. this expression is much more accurate compared to the other one where inner radius of the magnetic field is given by a the outer radius is given by b and the incidence field is given by the value that is the difference of the two values that we have computed. So it is e^{-jkr_a} is the component that is coming from the inner radius and e^{-jkr_b} divided by r_b is the one that is coming from the outer radius term.

This way we can basically approximate the value much more elegantly and the value of r_a and r_b are given by the term and for this approximation to work you need an outer radius that is at least thrice the inner radius. If b is not in the range of 3 times a this approximation is not

that accurate but mostly when you choose the value of b such that it is thrice the value of the inner radius the value is much more accurate. As I said before it is little bit to compute because you have this term which is bit complicated compared to the simple delta gap approach which we showed before. But it gives us more accurate result.

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IMPEDANCE LOADING

Pocklington's
Integral Equation

$$\frac{j}{\omega \epsilon \Delta z} \int_L I(z') \left(\frac{\partial}{\partial z} + k^2 \right) \frac{e^{-jkR}}{4\pi R} dz' = V(z)$$

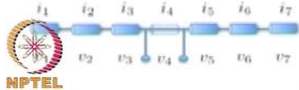
MOM

→


Matrix
Equation

$$[Z]\{I\} = \{V\}$$

Perfectly Conducting
Dipole



Impedance Loaded
Dipole



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In the case of the Impedance loading what we have is a Pocklington equation with the term of the derivatives inside and what you can do is you can do method of moment approximation to get the value of the entire term in the matrix form. And you can approximate it using the perfect conducting Dipole or the impedance loaded Dipole.

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IMPEDANCE LOADING

Perfectly Conducting Dipole

$$\begin{bmatrix} z_{11} & z_{12} & z_{13} & z_{14} & z_{15} & z_{16} & z_{17} \\ z_{21} & z_{22} & z_{23} & z_{24} & z_{25} & z_{26} & z_{27} \\ z_{31} & z_{32} & z_{33} & z_{34} & z_{35} & z_{36} & z_{37} \\ z_{41} & z_{42} & z_{43} & z_{44} & z_{45} & z_{46} & z_{47} \\ z_{51} & z_{52} & z_{53} & z_{54} & z_{55} & z_{56} & z_{57} \\ z_{61} & z_{62} & z_{63} & z_{64} & z_{65} & z_{66} & z_{67} \\ z_{71} & z_{72} & z_{73} & z_{74} & z_{75} & z_{76} & z_{77} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{bmatrix}$$

Impedance Loaded Dipole

$$\begin{bmatrix} z_{11} + Z_1 & z_{12} & z_{13} & z_{14} & z_{15} & z_{16} & z_{17} \\ z_{21} & z_{22} + Z_2 & z_{23} & z_{24} & z_{25} & z_{26} & z_{27} \\ z_{31} & z_{32} & z_{33} + Z_3 & z_{34} & z_{35} & z_{36} & z_{37} \\ z_{41} & z_{42} & z_{43} & z_{44} + Z_4 & z_{45} & z_{46} & z_{47} \\ z_{51} & z_{52} & z_{53} & z_{54} & z_{55} + Z_5 & z_{56} & z_{57} \\ z_{61} & z_{62} & z_{63} & z_{64} & z_{65} & z_{66} + Z_6 & z_{67} \\ z_{71} & z_{72} & z_{73} & z_{74} & z_{75} & z_{76} & z_{77} + Z_7 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{bmatrix}$$

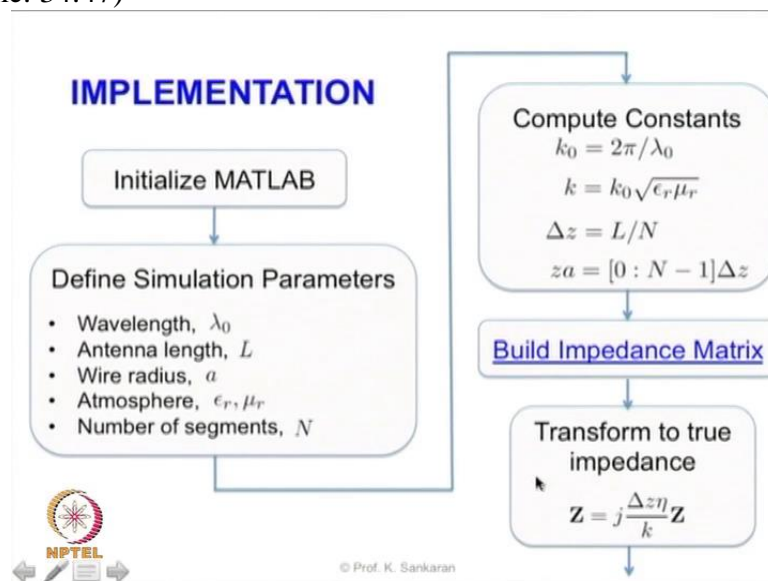
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And based on whatever expressions you are using the perfectly conducting Dipole will give you a standard impedance matrix multiplied by the unknown current terms that is going to be equal to the known voltage terms on the right hand side. And when you use the Impedance

loaded Dipole and the individual values of the impedances in the terms will given by z_1, z_2, z_3 . And they will nicely sit along the diagonal elements. And like before you have the unknown current terms and you have the right hand side known voltage terms.

So what we have done so far is we have given you a pretty elaborate approach on various elements of Method of Moments. The method itself is quite largely used for various radiational problems scattering problems and Antenna problems. We have showed you some examples from each of those.

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And we have also given you an elaborate Matlab procedure to compute the impedance matrix using the Pocklington's formulation. We also discussed alongside certain drawbacks of Pocklington formulation.

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SINGULARITY IN POCKLINGTON'S EQN

Recall

$$Z_{mn} = \frac{1}{4\pi} \ln \left[\frac{\sqrt{1 + (\frac{2a}{\Delta z})^2} + 1}{\sqrt{1 + (\frac{2a}{\Delta z})^2} - 1} \right] - \frac{jk\Delta z}{4\pi} + \left[(z_m - z') \frac{1 + jkR}{R^3} e^{-jkr} \right] \Bigg|_{z'=z_n + \frac{\Delta z}{2}}^{z'=z_n - \frac{\Delta z}{2}}$$

Strong singularity!!

Slow convergence and poor accuracy



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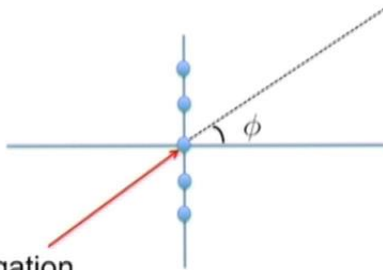
we have the singularity term that is going to affect the accuracy of the method. And also as a number of elements are going to increase a convergence of the solution that is computed is going to be equal to the experimental value or accurate value that we are using from various other methods.

So that brings us pretty much to the end of this module on Method of moments. We have some examples Matlab codes that are available for you to test.

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MATLAB EXAMPLE

Scattering problem



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And try for various scattering problems whether its a planar case or a semicircular case that will enable you to really get certain sense of how to compute various parameters for practical applications.

With that being said thanks for following this module and I hope you have learned quite a bit on the method of moments and its applications With that being said thanks for being with us thank you!