Computational Electromagnetics and Applications
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Indian Institute of Technology Bombay
Exercise No. 17
Finite Element Method –II

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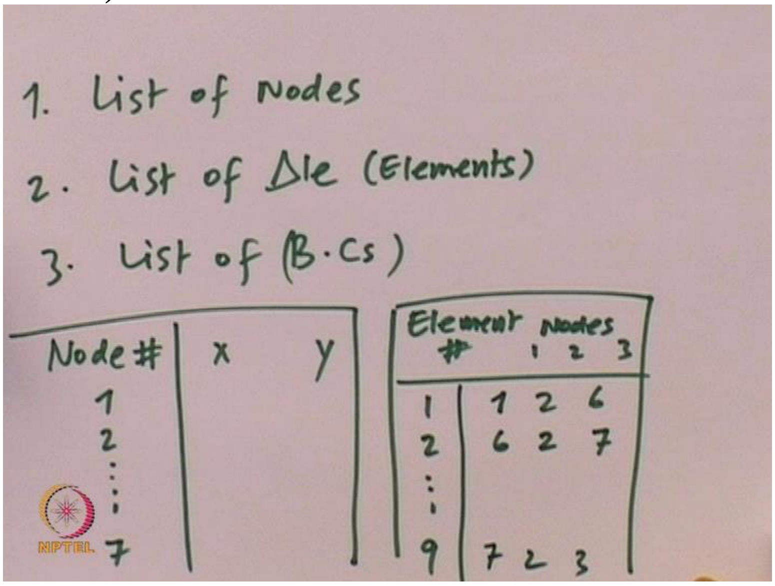

The finite element method gives us the flexibility of using unstructured grid. But it is also a pain to work with unstructured grid because we have to do intensive book keeping in terms of managing the number of nodes, number of elements what they are connected to, and how they are connected to so on and so forth. So when I am working on finite element method without telling you how this entire gamete of manipulating and using the unstructured grid in the numerical methods that we are going to use is a very important step.

And I do not want to leave without saying a word or two about how we are managing the unstructured grid itself. So we will start looking into the unstructured grid, the important parameters that we will be using like the list of nodes, the list of edges, the connectivity matrix so on and so forth.

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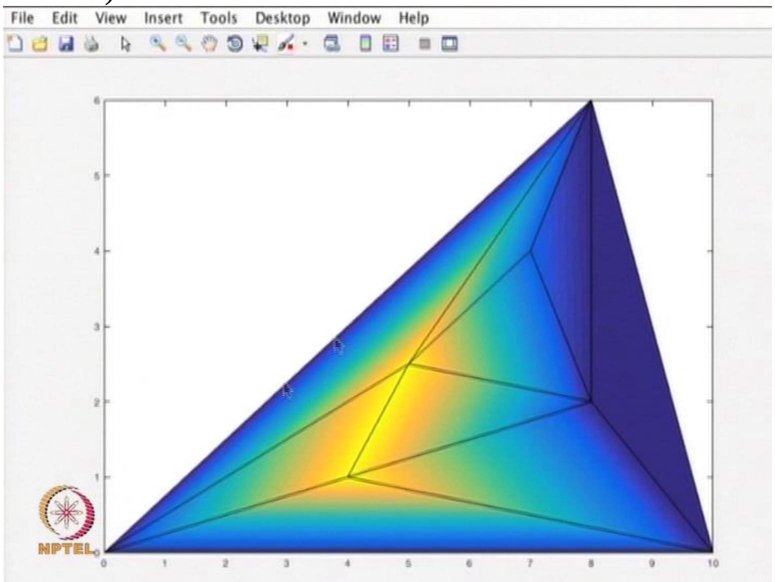

And also I am going to use simple Matlab based program to generate an unstructured grid for a triangular domain. So remember my domain itself is a triangle and I am triangulating it.

So I am using unstructured grid to triangulate the triangular domain, why did I take a triangular domain because if I take a circular domain or if I take a square domain it is easy to use rectangular grid. I can make the rectangular grid so small so as to confirmly map the domain.

Whereas when I am going to have a triangular domain I am going to have sharp corners. And I have to inevitably go and use triangles. And that is the power of Finite element method. So I am going to use that example to study a problem for modelling using finite element method. Before that it is important to pen sometime on unstructured grid, what are they made of? And how are they computed? How are they used in the finite element method? That is what we are going to see. So let us start with the problem we are going to look into.

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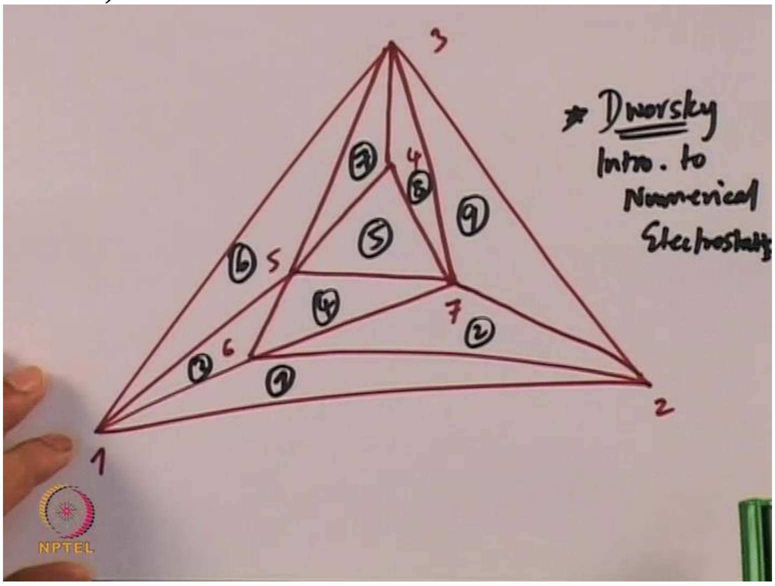

But before that, so let us look into the domain itself. So we start with the big triangular domain and triangular domain has various points. So this is the first point, this is the second node and this is the third node. And the fourth node is somewhere here. And the fifth node is somewhere here and the sixth node is somewhere, the seventh node is somewhere here. So these are the seven nodes. And the lines are edges are connected in this manner.

So let me number them before I forget, so this is a fourth node, this is the fifth node, this is the sixth node, this is the seventh node. So 4 and 5 are connected with a line, 5 and 6 are connected with a line, 1 and 5 are connected with a line, and 5 and 3 are connected with a line and 5 and 7 are connected with a line, 6 and 7 are connected with a line, 6 and 2 are connected with a line, and 7 and 2 are connected with a line, 4 and 7 are connected with a line, and 3 and 7 are connected with a line.

As you can see there are so many triangles. So let us number them one by one, this is my first element, this is my second element, this is my third element, this is my fourth element, this is my fifth element, this is my sixth element, this is my seventh element, this is my eighth element, this is my ninth element.

So I am taking this problem from a very classical text book, it is a introduction to numerical electrostatics by Lawrence Dworsky. So the source of this problem is Dworsky. And introduction to numerical electrostatics using Matlab. So I encourage you to try also other problems from this book. But as you see this domain has various complexities that we are going to talk about, even the simple domain of this sort which is a triangular domain which has been discretized by triangles. Using triangles and we have totally nine elements, so nine triangles and we have 7 nodes and we have so many edges. And we are going to talk about all these aspects step by step.

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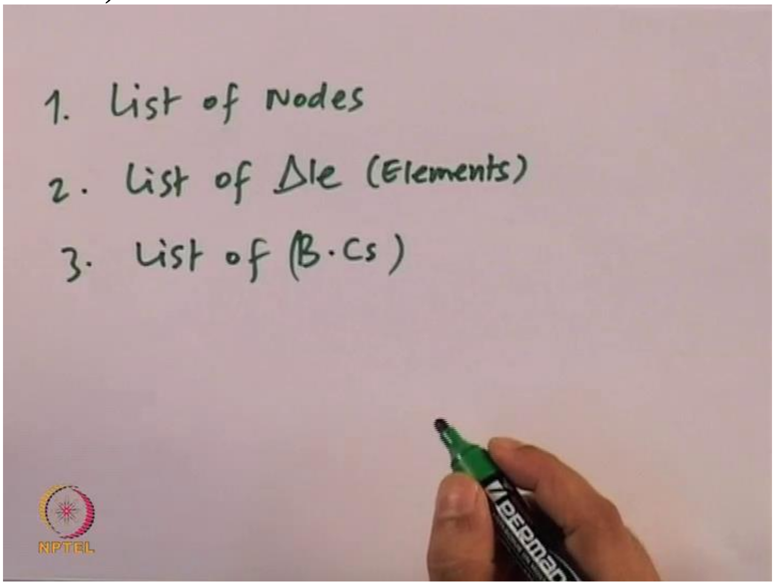

The first one we are going to talk about is the number of nodes itself. So the first one we are going to talk about in this problem is the list of nodes. So let us write down now, so this is the List of nodes, the second one will be the list of elements, so the list of triangles the elements and the third one we are going to talk about is the boundary conditions. So once we have that we are going to assign them numbers and we are going to use the Matlab structure of manipulating the matrices to take the problem.

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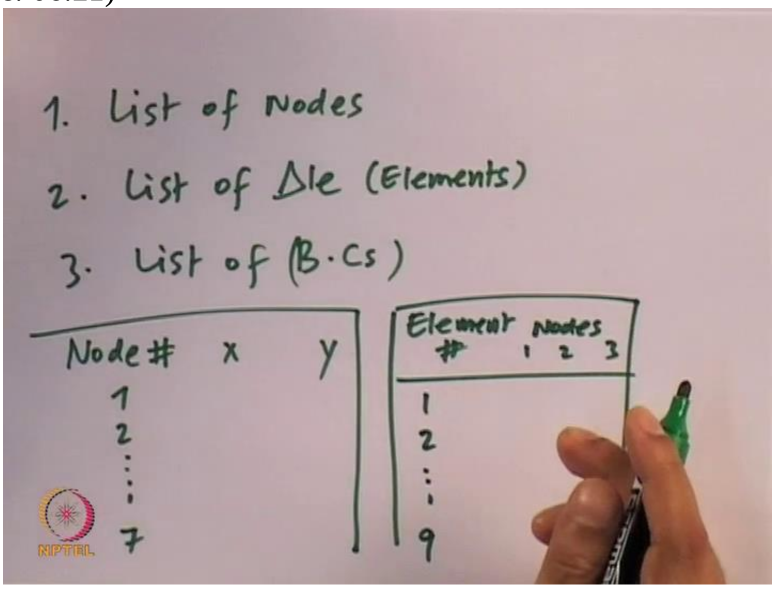

So for the list of nodes what we will have is node number and we will have the x and y coordinate, so in our case we will be having seven nodes. So it will go from 1 to 2 until 7, and we will have various coordinates x and y coordinates. So this will be the first one. Second one will be the list of elements. What we have is element number and the nodes. So we will have 1, 2 and 3 nodes per element. So we have 1, 2 until 9 elements and we will have the nodes.

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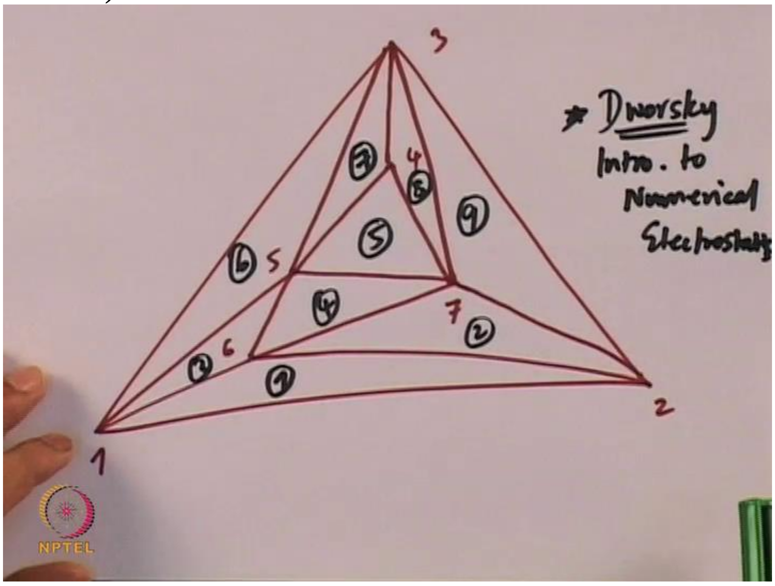

For example in this case when we have triangle number 1, we have 1 to 6 so if you see this particular triangle element 1, the first one will be 1, second one will be 2, and third one will be 6. They are going in the anti clock wise direction.

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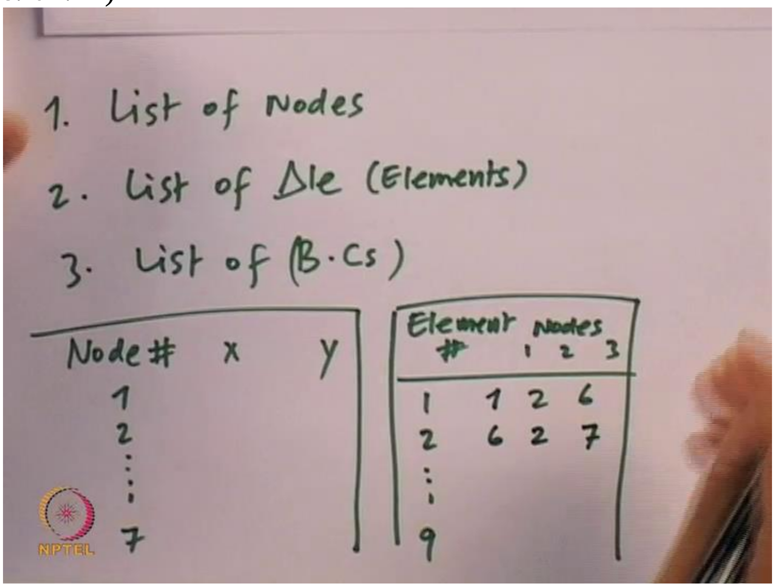

And we will stick to that throughout the example. So we will have 1 2 and 6. Similarly for the second one it will be given by 6, 2 and 7.

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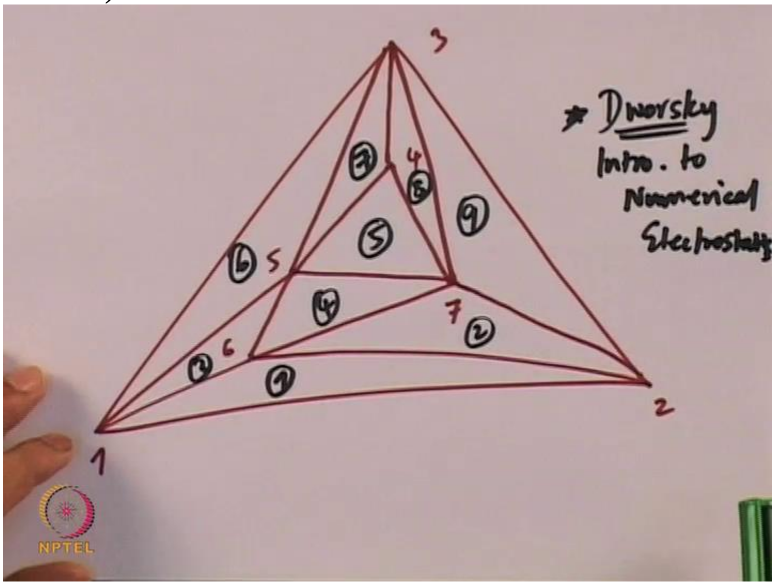

We will go and look at it here. For the second one 6, 2 and 7 so on and so forth. And the last one will be 7, 2 and 3. So 7, 2 and 3.

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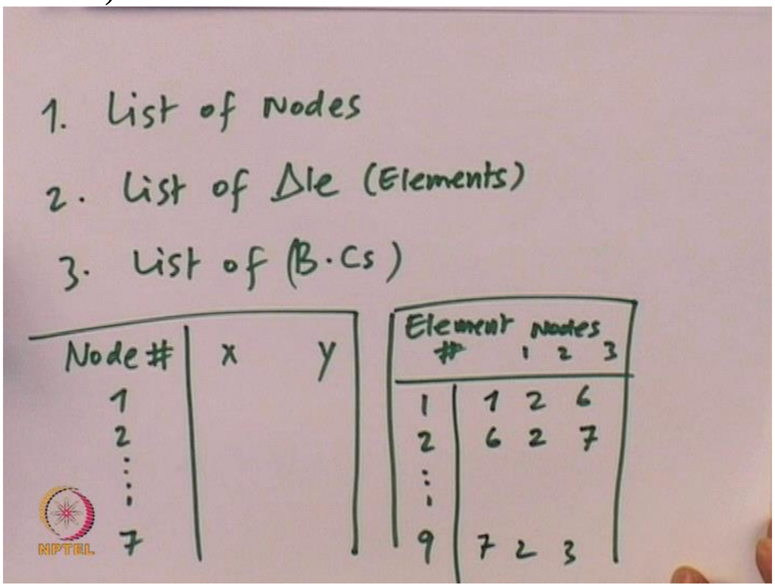

So you can write the last one as 7, 2 and 3. So this is the way we are going to use Matlab structure to register the list of nodes and list t of triangles. And the last point in the three aspects what we had is the list of boundary conditions, so let us also write it down.

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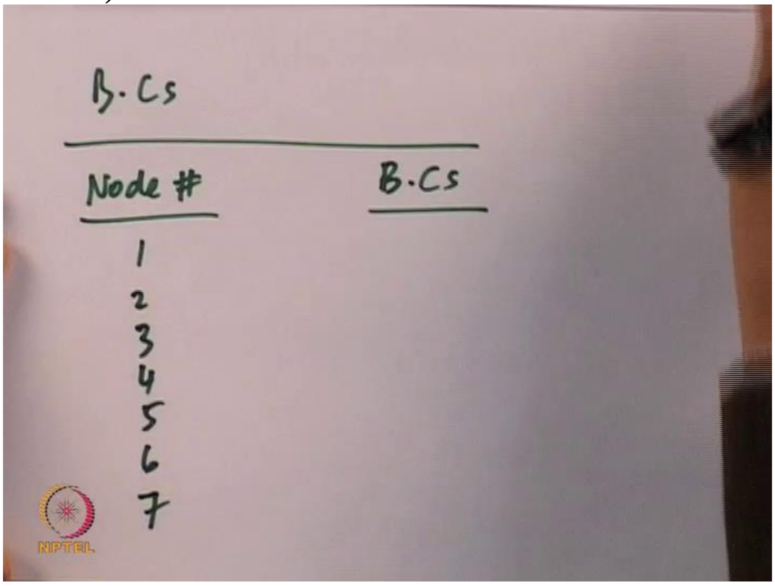

So the list of boundary condition is going to be given by so boundary conditions. So you have boundary nodes and then the node number and then boundary condition. So we have totally seven nodes. So we will give them the value so 1,2,3,4,5,6,7.

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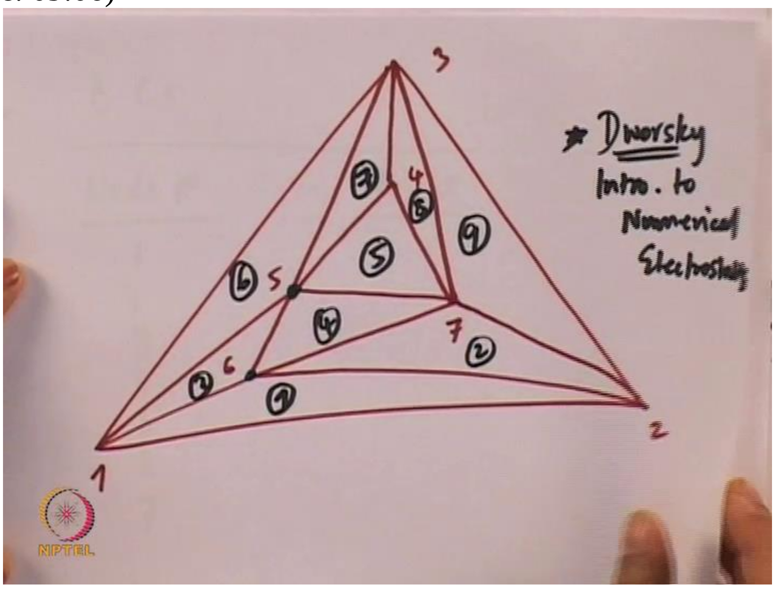

These are the nodes what we have here. So for the problem what we are going to do, we will assign the value of the potential on these two particular nodes to be 1 and 1. Whereas other nodes we will keep them as 0.

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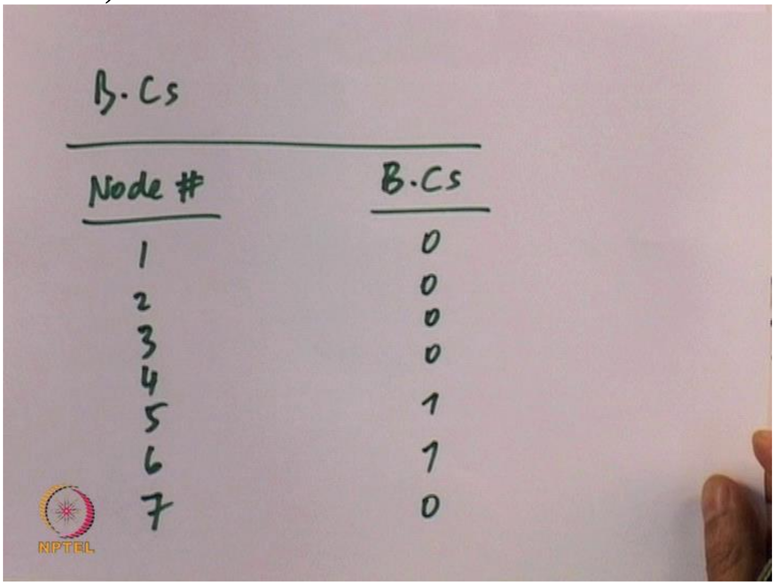

So we will put except for 5 and 6 we will put 1, whereas the ones we will put as 0.

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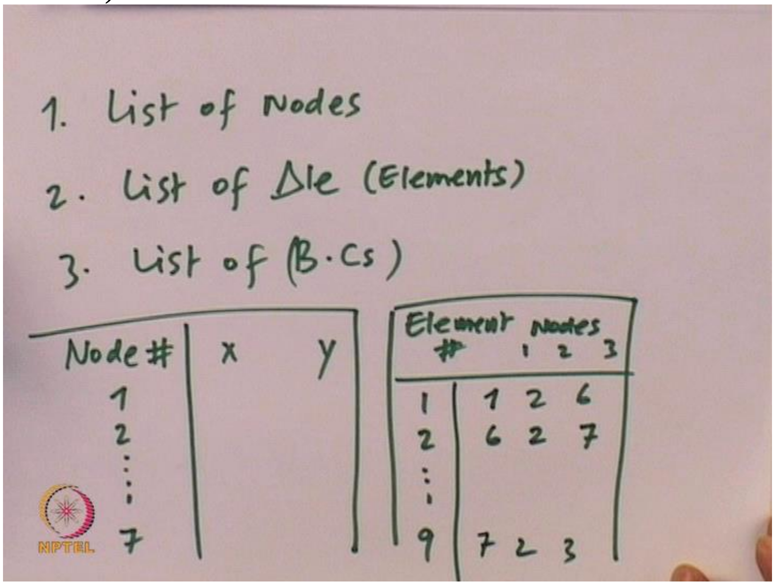

So this is the way we have going to start doing any problem, we have the list of nodes, we have the list of elements and the list of boundary conditions. And we have assigned them using the matrix.

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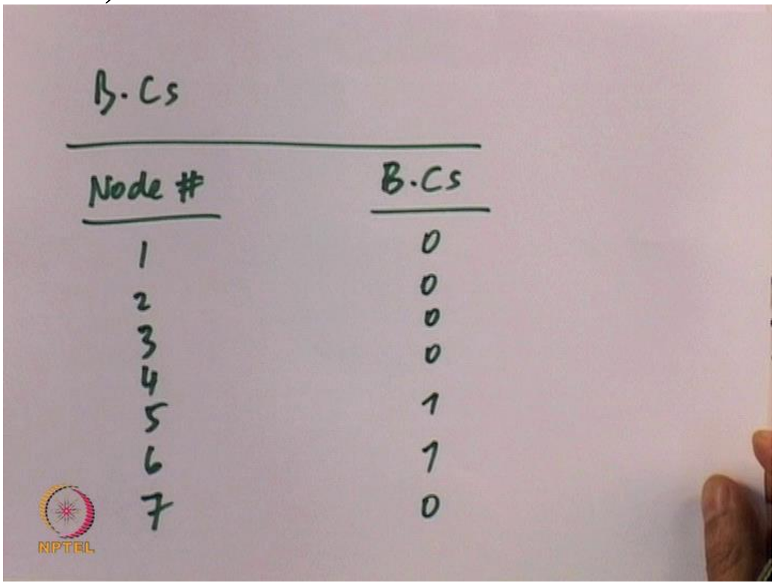

And we are also assigning the boundary conditions. Once we do that we are able to start with the problem itself. But before that look at the Geometry one more time.

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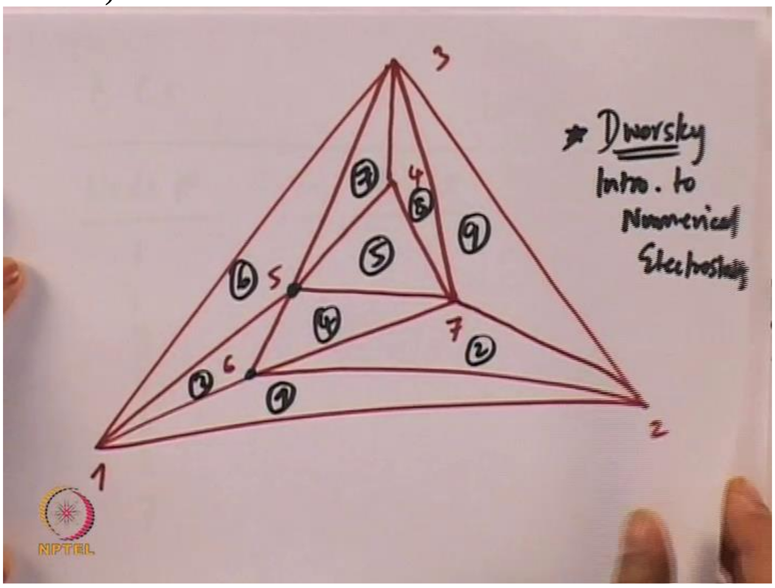

So look at the geometry of the problem that we are considering, there are several elements there are there, and these elements have different lengths for edges and different areas and they are going to impact the solution in different ways. Not all triangles have the same dimensions. So we have to somehow understand the properties of those triangles. One of the property or the geometrical property that we will be using is the area of the triangle itself.

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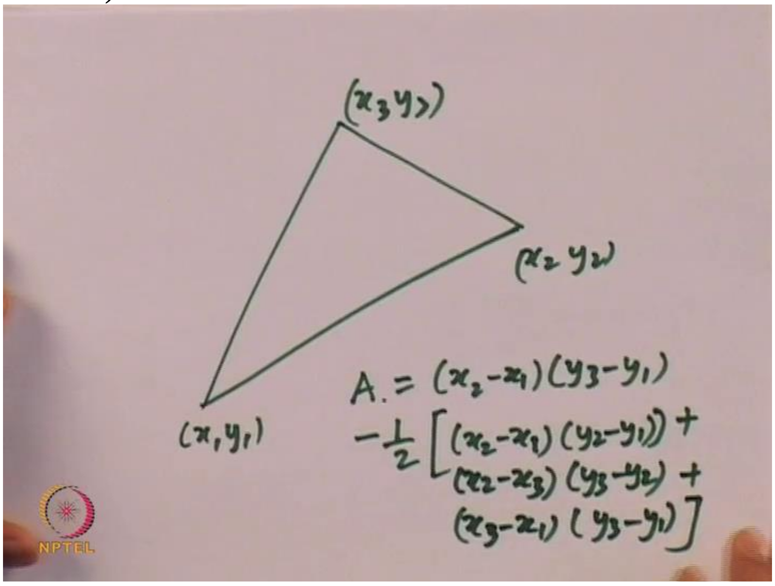

So if we have a triangle, let us say given by certain three coordinates. So we will take a triangle so you have (x1y1), then you have (x2y2), and you have (x3, y3). The way to compute the area of the triangle is going to be simply given by A of the element is equal to x2 minus x1) multiplied by (y3 minus y1)minus 1 by 2 [(x2 minus x1)(y2 minus y1) plus (x2minus x3) (y3 minus y2) plus (x3 minus x1) multiplied by (y3 minus y1)]. This is a very complicated big formula but we can actually simplify that by using the matrix manipulation.

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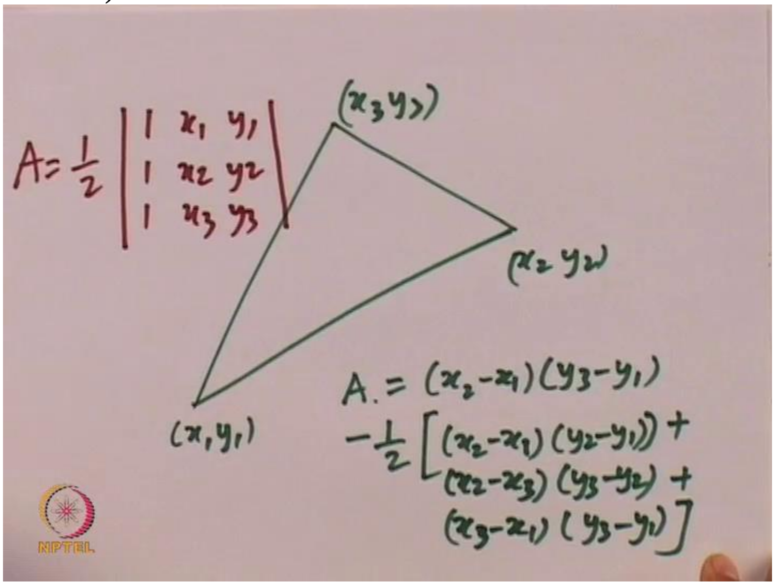

And when you do that what you will see is the area is given by i by 2 the value of the determinant. The determinant value is 1 x1 y1, 1 x2 y2, 1 x3 y3. So once you do the determinant and take half of it, this will be the value of the area of the triangle.

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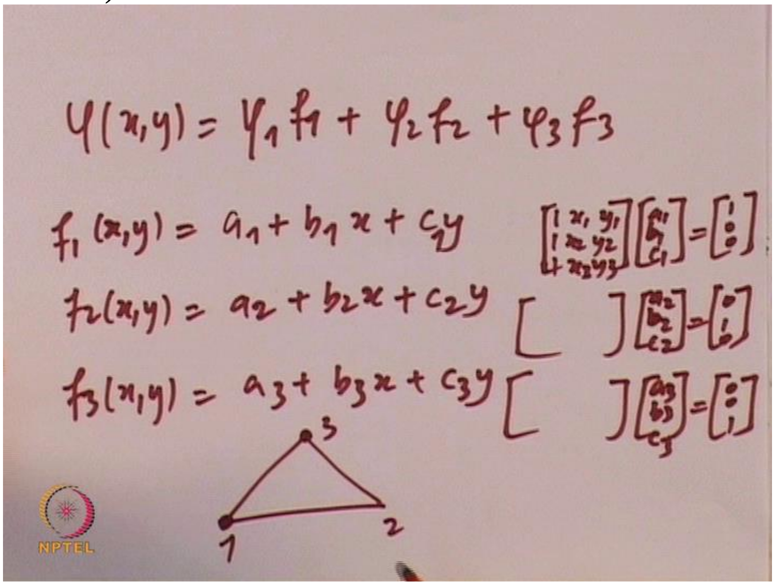

Similarly for the basis functions we are assigning let us say linear elements and we call them as, so let us say the potential inside the element (x,y) is going to be given by Phi 1 multiplied by f1 plus Phi 2 multiplied by f2 plus Phi3 multiplied by f3. Where f1 is a function of x, y is equal to a1 plus b1x plus c1y and f2 (x, y) will be given by a2 plus b2 xplus c2y and f3 (x, y) will be given by a3 plus b3x plus c3y. So this is the way you compute the values of the basis function and like before, what you can do is you will get a matrix equation of the form [] [] equal to something. So this would be matrix equation, for each of the basis function you will get three equations.

What you will see is right hand side of this equation when you rearrange the terms what you will get is for each of the elements, so what you will have is a1, b1and c1. Similarly you will have a2, b2, and c2 and a3, b3 and c3. So these are the elements that you need to compute the value for the geometry of them through 1 x1 y1, 1 x2 y2 and 1 x3 y3. Similarly you can do that for the other ones and you can see that there is a pattern here. So 1 0 0, 0 1 0, and 0 0 1.

What it means is the first basis function for a triangle like this; let us say this is the first node, second node and third node. The first basis function will have maximum value here and minimum value at these two points, so these are like slopes what you can see if i can draw it in three dimensions then you might be able to appreciate it. I will try my best to do that.

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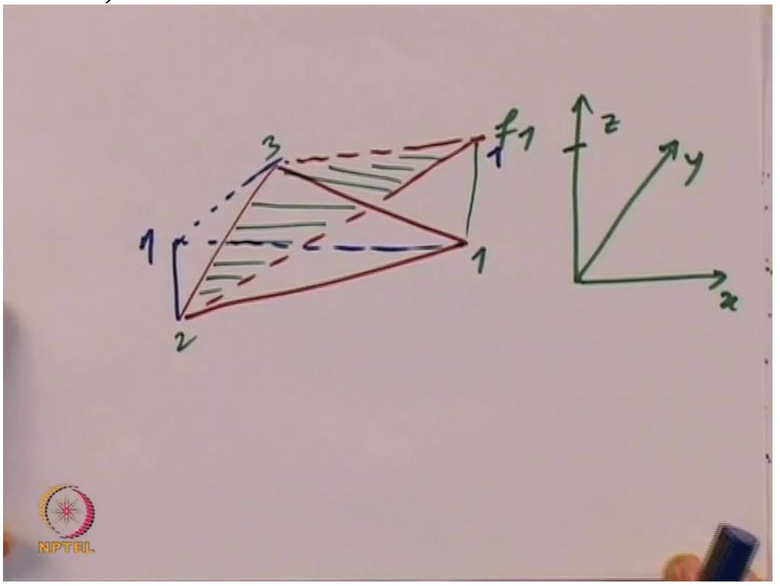

So what you will see is, so assuming that this is the so I am drawing the basis function in green stripes. So let us say this is node 1 and this is basis function f1. The f1 will have the maximum value so this is two dimensions. So this is the x direction this is the y direction. What you see is the value of the f1 is like a slope like this so you have a triangle and you have another triangle with the base on the same point and the maximum value it is going on the third dimension which is the z dimension.

And it has maximum value as 1 at the node which is 1, 2 whereas it has value 0, at the node 2 and 3, whereas the basis function f2 will have the value maximum here and goes to 0 here. If you are trying to replicate that in this graph, let me try it will be having a maximum value here will go to 0 at the other 2 points. So this is the value 1, so this is the value, this will be 1, so this will be 1. I hope you are able to see the three dimensional replication and so the basis functions are going to have different values.

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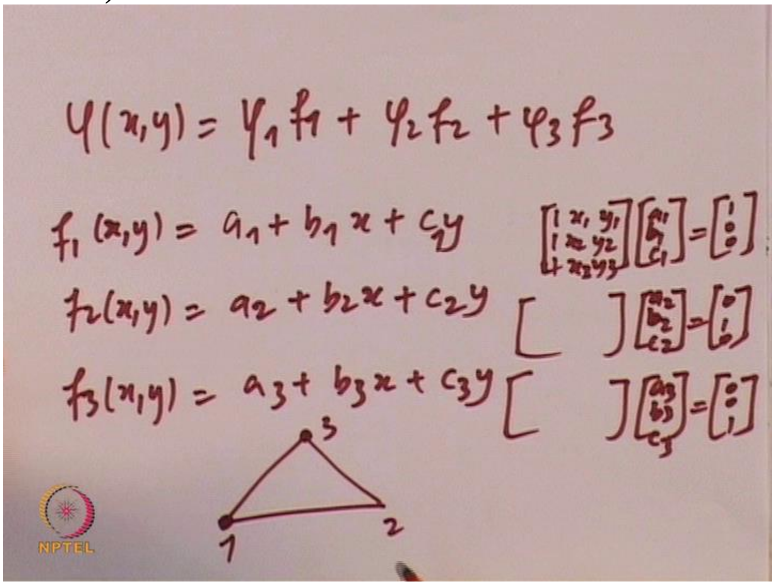

And that is what you are going to see for this particular case the maximum value is at node 2, maximum value is at node 3. The rest of the values are 0.

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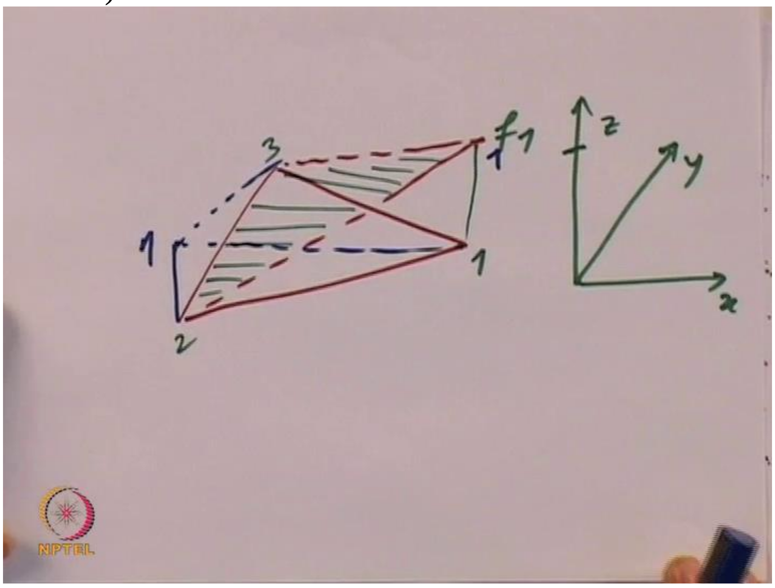

So once you get this thing clear first what we will do is first we will compute the electric field and from there we will compute the energy and once you have the energy we can use it to compute the capacitance.

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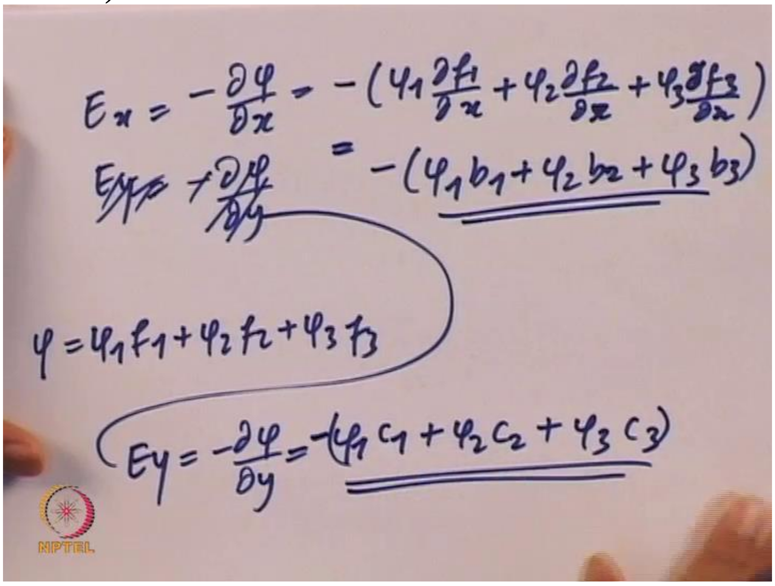

So we will start with the energy itself so the electric field components say has two components which are Ex and Ey. So we know that the gradient of the potential minus of the gradient of the potential, remember I am using the partial derivative here because there are two variables in the space x and y. So I have to use partial derivative is going to be given by, so I know the value of Phi from my earlier assumption, so Phi is going to be given by Phi1 f1 plus Phi2 f2 plus Phi 3 f3. Using this in this equation I am able to compute the value of Ex as minus (Phi1 Doe f1 divided by Doe x plus Phi2Doef2 by Doe x plus Phi3Doef3by Doe x). And knowing the value of doe f1, f2 and f3 I can write them in a simplified form as this is equal to so let me strike this one down and I will write this one later. Equal to minus (Phi 1 b 1 plus Phi 2 b 2 plus Phi 3 b 3).

So I can do the same thing for E y , So I will write E y here equal to minus Doe Phi by Doe y this is equal to Phi 1 c 1plus Phi 2 c2 plus Phi 3 c 3. So this is the value for the electric field so there will be a minus sign outside.

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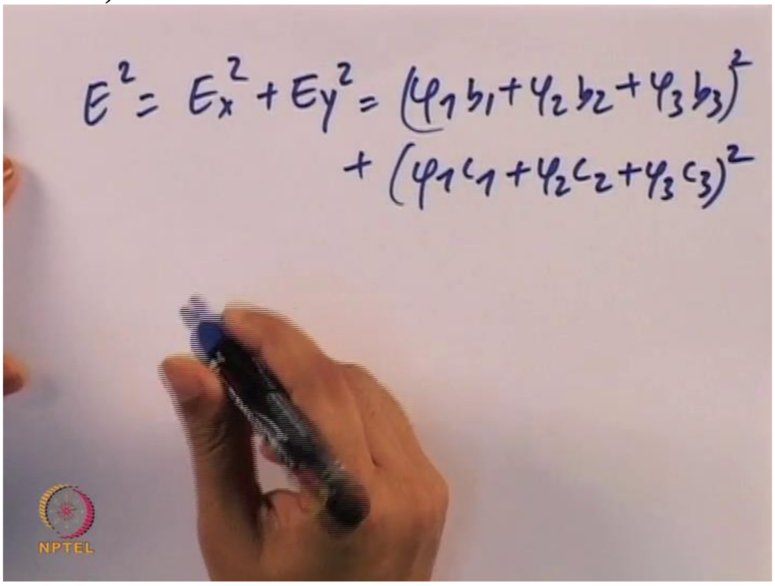

So once I know E x and E y I can compute the value of total electric field as E square is equal to E x square plus Ey square and that is going to be given by (Phi 1b1 plus Phi2b2 plus Phi3 b3)whole square plus (Phi 1c1 plus Phi2c2 plus Phi3c3)whole square.

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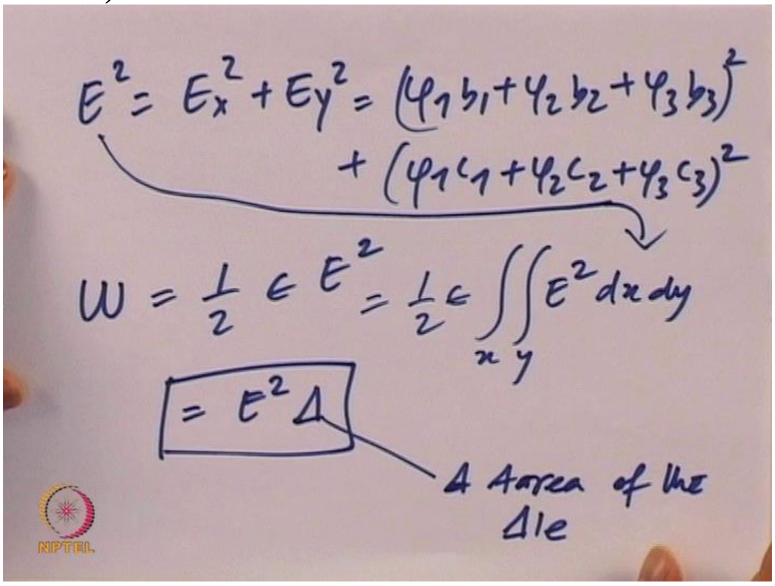

Since the energy what we are interested in it is not a function of x or y we can directly compute the energy using the formula w equal to 1 by 2 epsilon E square remember this we have computed this in the previous case it will be a double integral, so that is equal to 1 by2 epsilon double integral x and y E square dxdy so we can substitute this value into this equation to get the value for energy. And what we will get is essentially E square multiplied by the delta of the triangle that we are going to model. So this is the area delta is the area here; area of the triangle.

So what you see is I am taking out the E out of the equation, and I have this value that is given by the area itself. And now what we are going to do is now we are going to differentiate the value of the total electric field that we have computed with respect to each of the potential terms and that they have to go to 0. So the terms that must be added to the coefficients is what we are interested in. And we are going to do it step by step accordingly.

So what I have shown here is how the various basis functions are going to impact on the computation of the electric field, how we can see the impact of the geometry of the triangle impacting on the total energy that we are computing so on and so forth.

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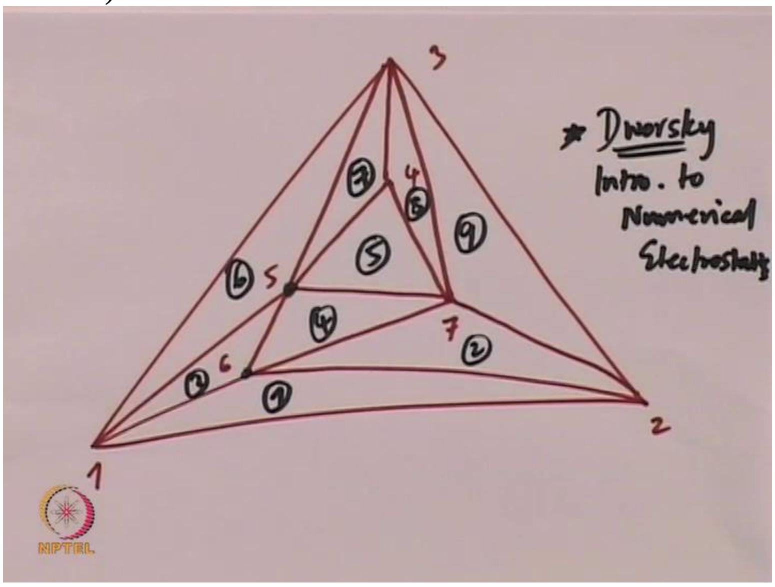

So let us look at the geometry one more time, so this is the geometry that we are going to simulate for this problem and we said we are going to assign for 5 and 6 the potential of 1.

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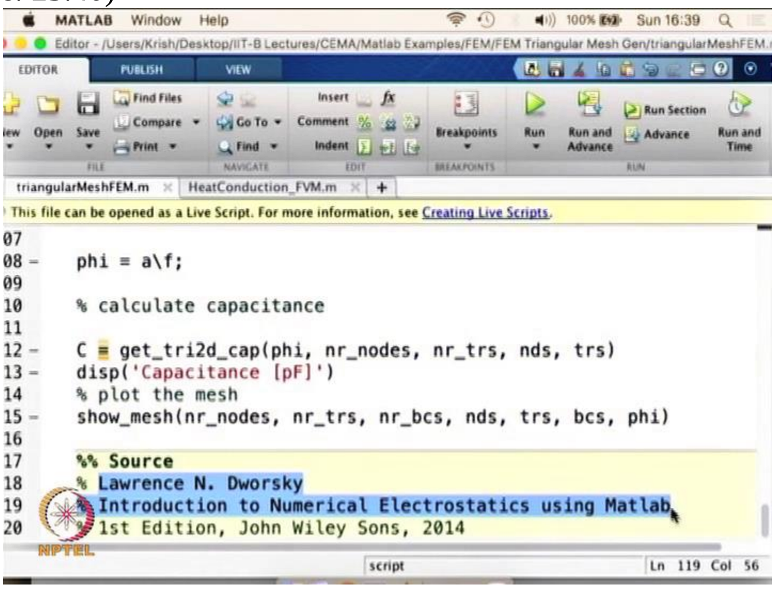

so let us go back here. So the source of the program what we are simulating is from this book. So let us go back to the starting of the program and see step by step what we are doing.

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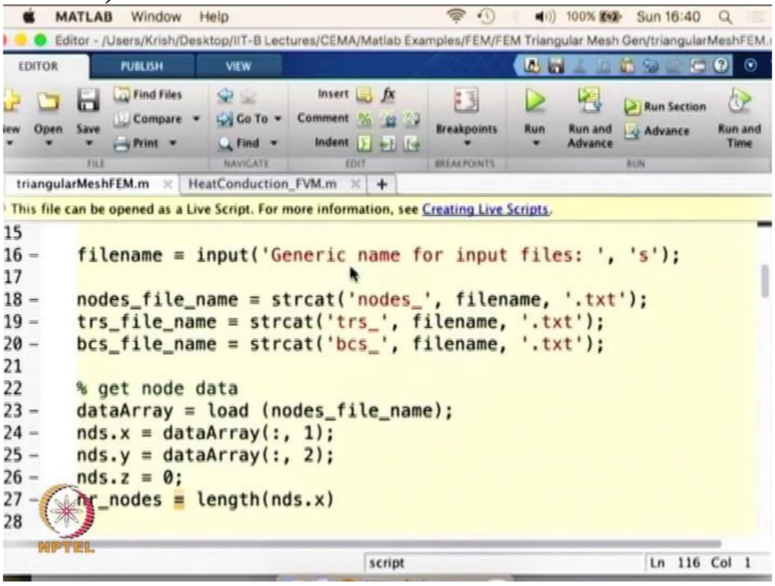

So the program asks for a input file, so the input file is basically the file which contains the value for the various parameters that we have discussed here. So these are the parameters, so let us look in to the parameter one more time.

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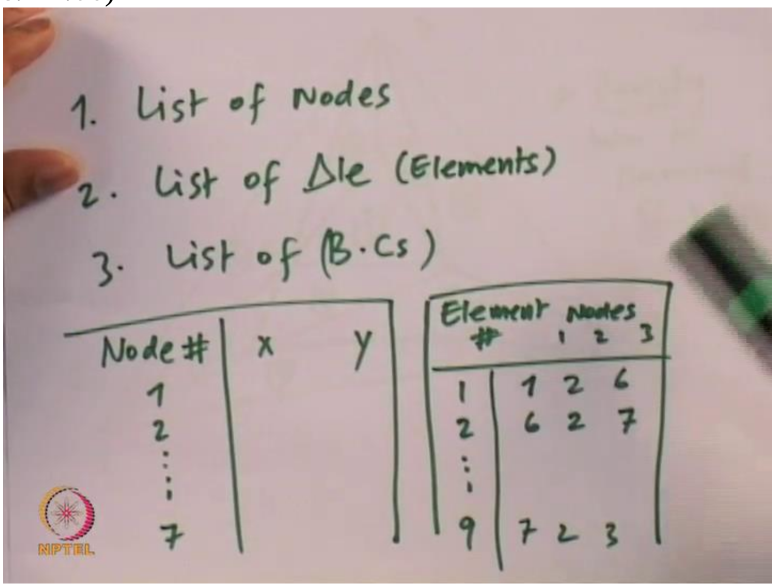

So these are the parameters, so these are the input files inputs that we are going to give using the input files.

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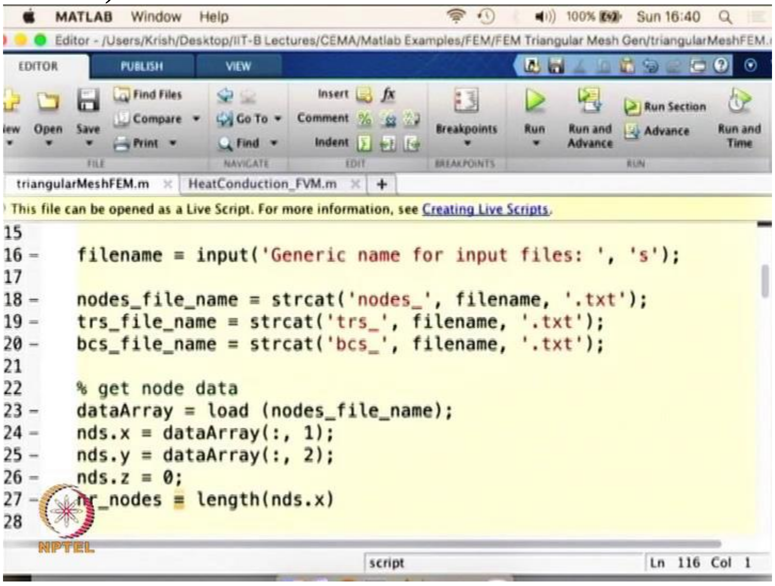

So the file name we have to give a generic name so we will call it as input and then various parameters will get loaded. So the node data will be loaded into the node file name, and then the triangle data will be stored in the triangle name. So these are the elements and the boundary conditions will come from the boundary conditions file.

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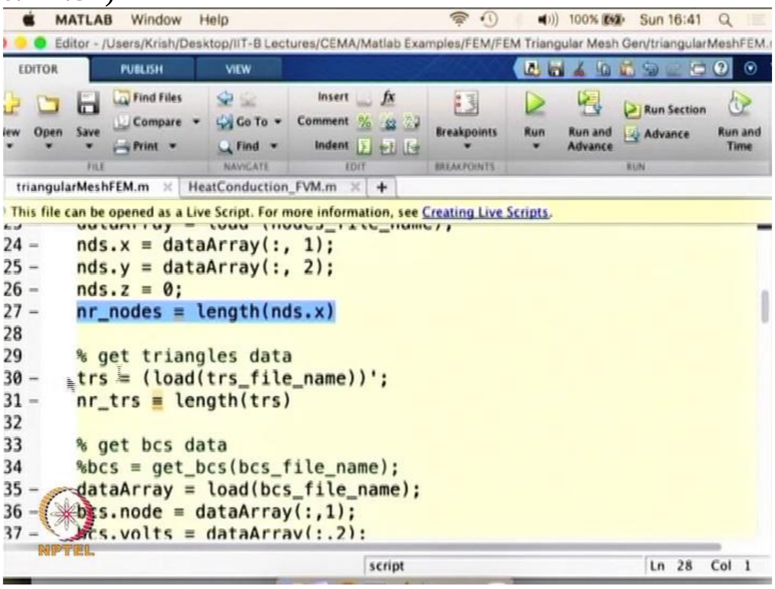

So once we get that we are going to load those values into each of the parameters and we are going to compute the number of nodes using the length function, the number of triangles using also the length function of these individual parameters.

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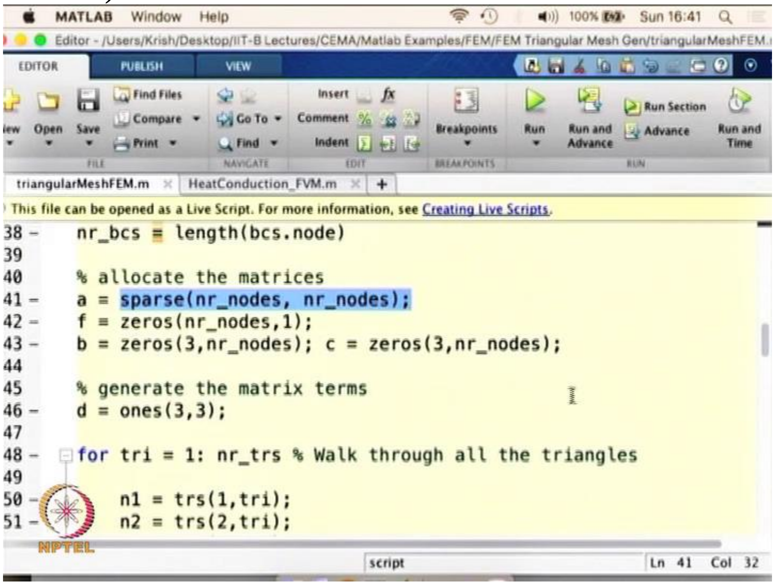

And once we do that we are able to assign certain parameters for preallocating the varaibles that we are going to use into the code, so a,f and b; a is declared as sparse matrix; the reason for doing that is it actually quickens the code. If you are interested in writing the codes that are running fast. You have to use certain inbuilt functionalities of Matlab that will allow you to declare certain matrices as sparse. So the calculation can be done quicker.

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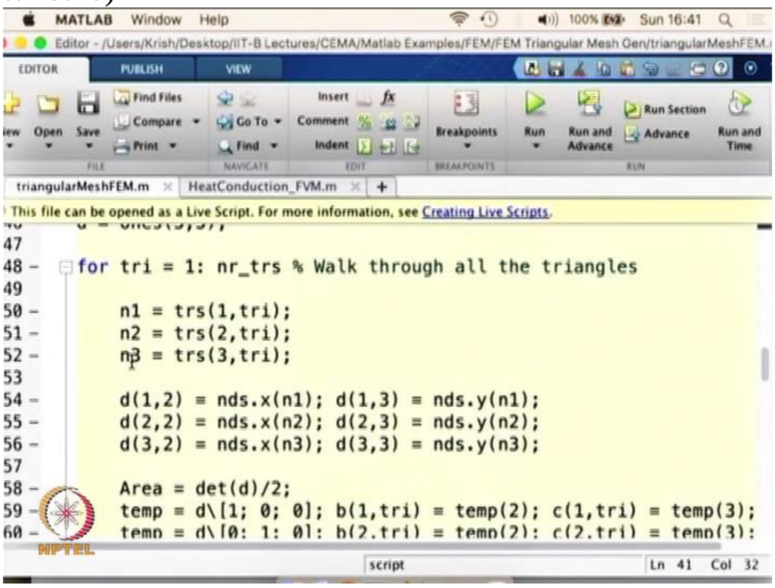

So once we do that we are able to go through all the triangles and collect the data that we are interested in.

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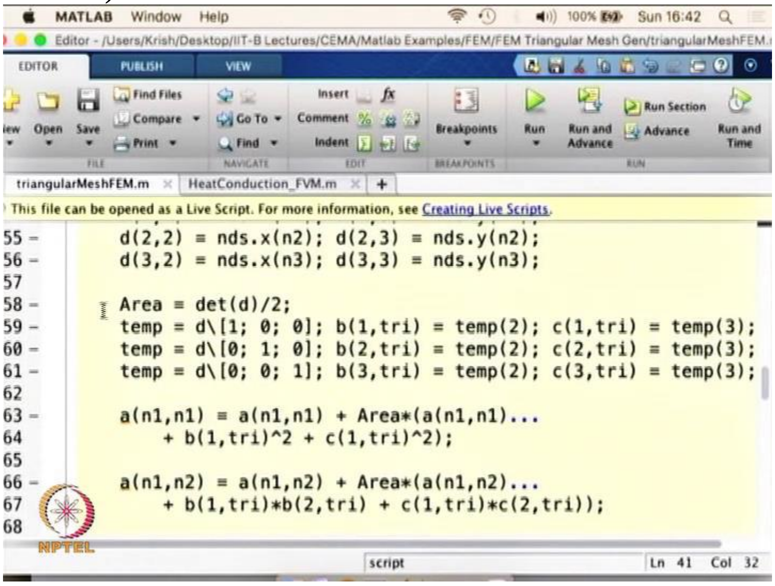

for example the area information that I have described so it is the half of the determinant which we are calculating.

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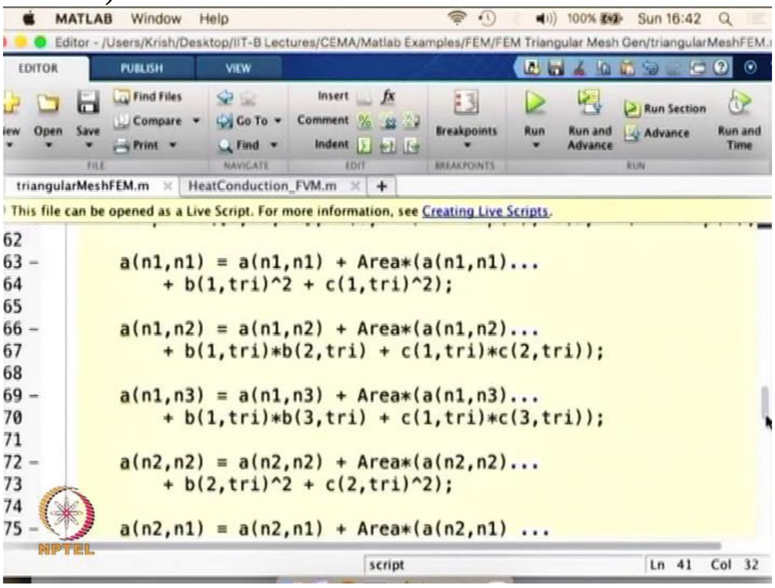

and these are the parameters that we have described in the coefficients a1, a2and a3; b1, b2 and b3; c1, c2 and c3.

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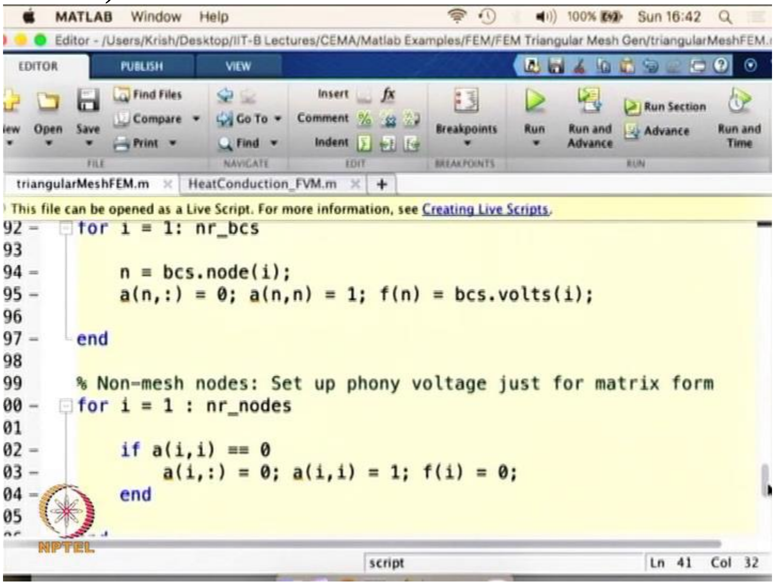

Once we have that we are able to calculate the value of the potential at various points in the triangular domain;

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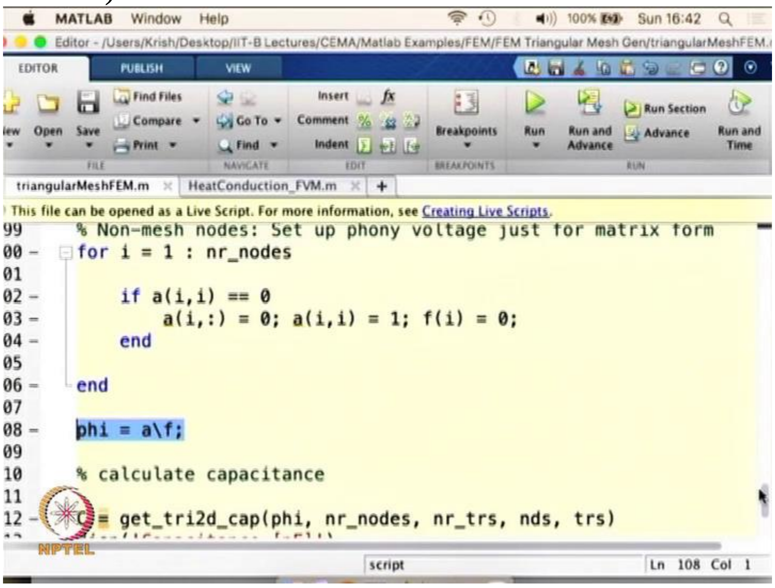

using a simple Matlab calculation where Phi is given by inverse of a multiplied by f and where f is the boundary condition. As you can see I am setting the value of the boundary to be 1 for certain nodes whereas I am setting the boundary to be 0 for other nodes.

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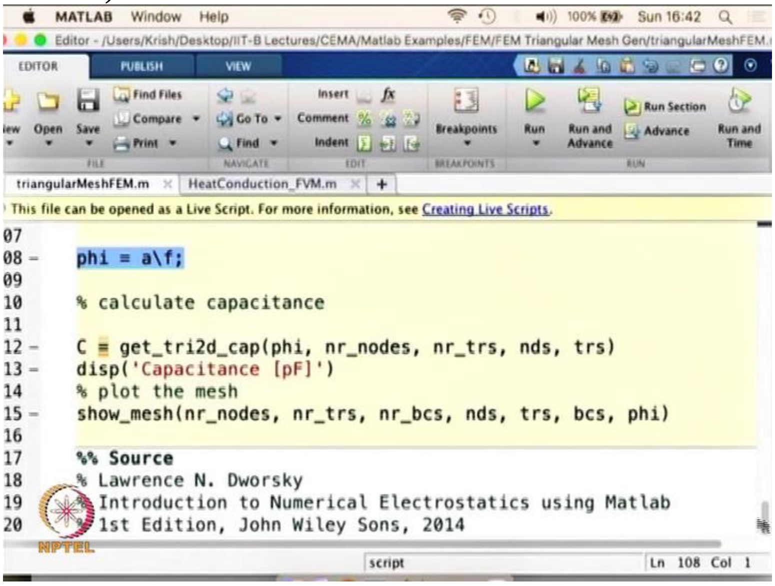

So 5and 6 will be in this example set to 1 whereas the other ones will be set to 0.

And once we have that we are able to generate the mesh and we are able to also calculate the capacitance as a function of various points that we are interested, so it will be a function of various points.

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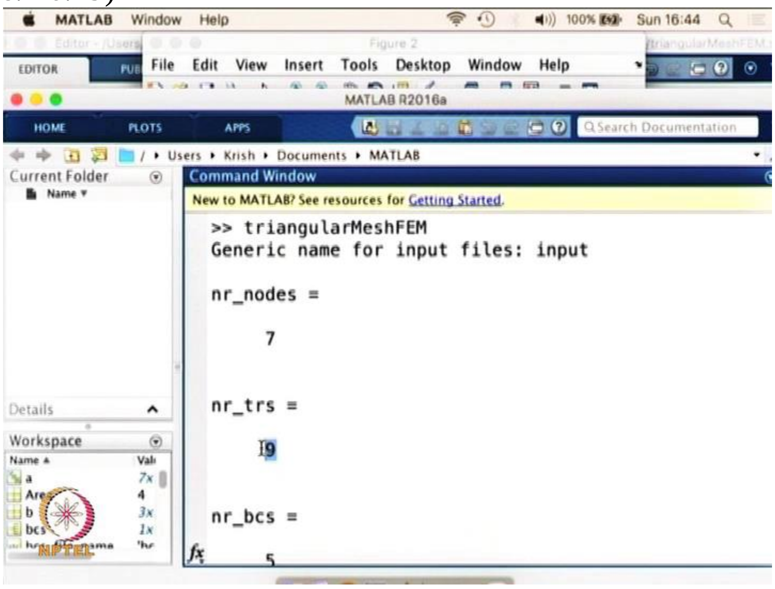

So let us run this code, once I run this code it is going to ask me for certain inputs, I set the input name will be input and what it does is it initially calculating various things. In our case the total number of nodes what we have is going to be 7, and the total number of triangles that we are going to have is going to be 9.

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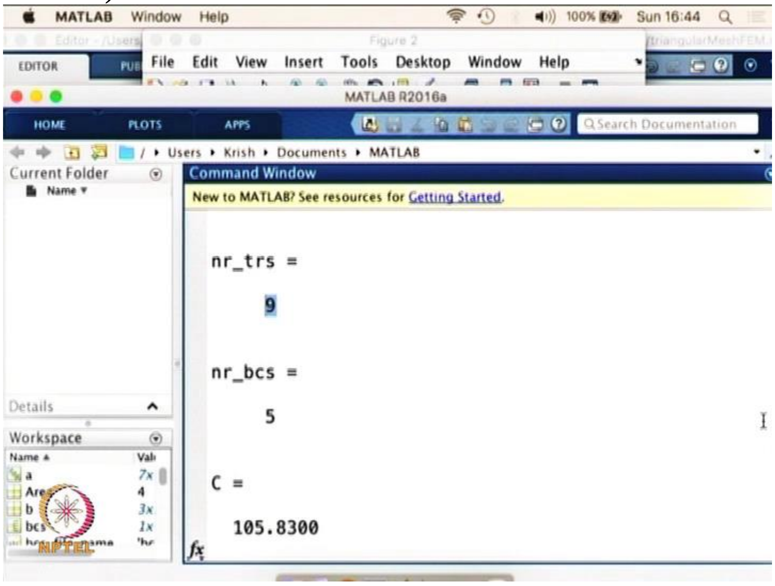

And the total number of boundary conditions that we are going to assign is going to be 5, so why 5 although there are only 3 here.

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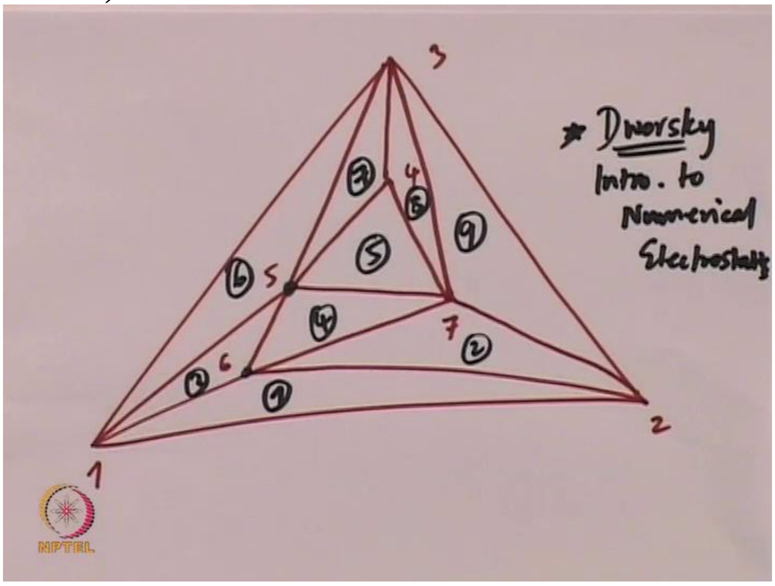

If we look at the case here there are only 1, 2 and 3. We are going to also force certain boundary condition for these two n odes. So that is why it is going to be 3 plus 2. And once we do that we are going to get the value of the capacitance. So let us look at the field plot itself.

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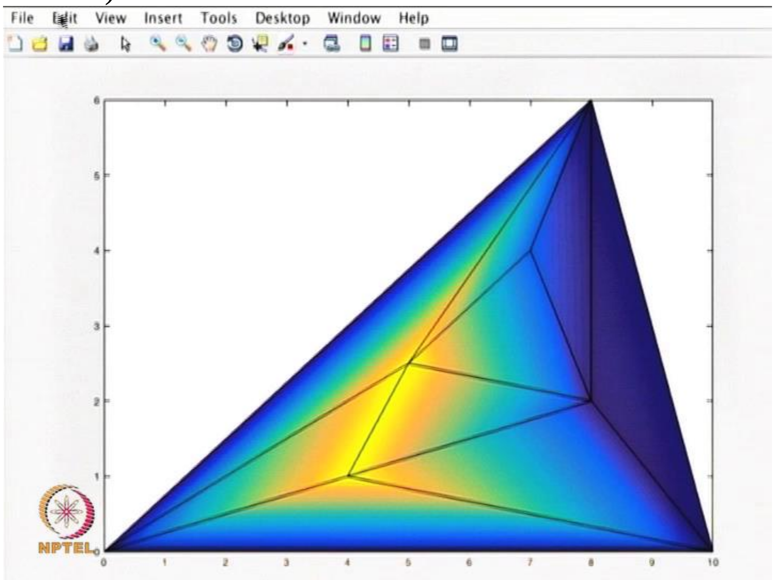

So this is the potential plot what you are seeing is the one the 5 and 6 nodes are given 1 whereas the other nodes are given 0. And the field plot looks like this for this problem. And the capacitance that we are going to compute for this problem is given by the value that we have given that we have printed in the Matlab file.

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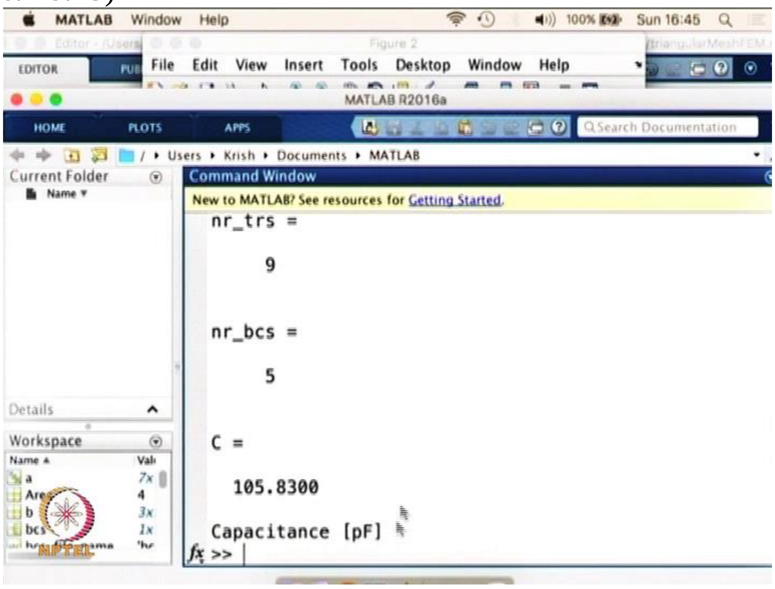

So what we are getting is 105.83 Pico Fared.

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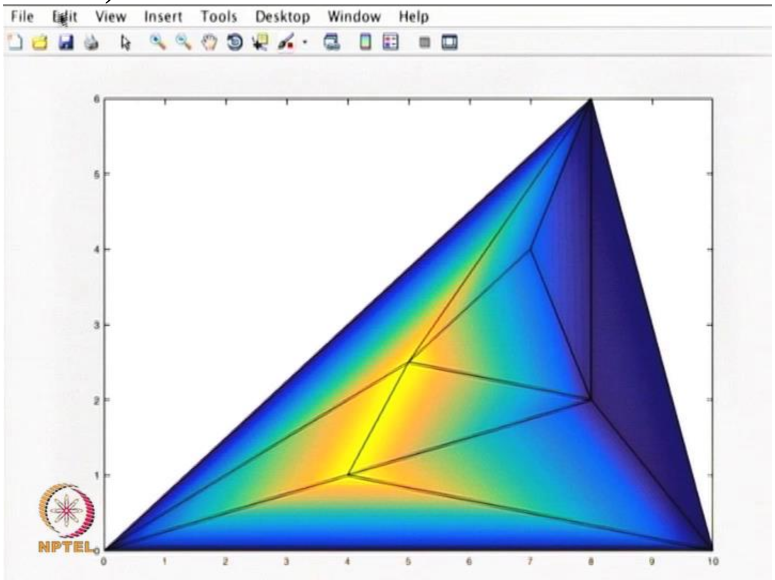

So this is an excellent program for you to learn two things; one thing is how unstructured grids are used in finite element method, so we have talked about the advantage of using a unstructured grid, advantage of going to finite element method. But we have not really covered the integrities of manipulating unstructured grid, managing the data and using those data in computation. So what I have tried to show is a very basic aspect of unstructured grid. So you get to know what are the important parameters in the mesh that we are generating, the list of nodes, and the list of elements the connectivity of those element the boundary conditions so on and so forth.

I have also showcased some simple calculations that you have to do to calculate the parameters, like the area the basis function so on and so forth. And once we have that we are able to assign boundary conditions to the problem and you are able to simulate and see how the finite element method works for that problem.

So we encourage you to practice this code at your free time and get a sense of how the finite element works. Thank You!