Computational Electromagnetics and Applications
Professor Krish Sankaran
Indian Institute of Technology Bombay
Exercise No.15
Finite Element Method-II

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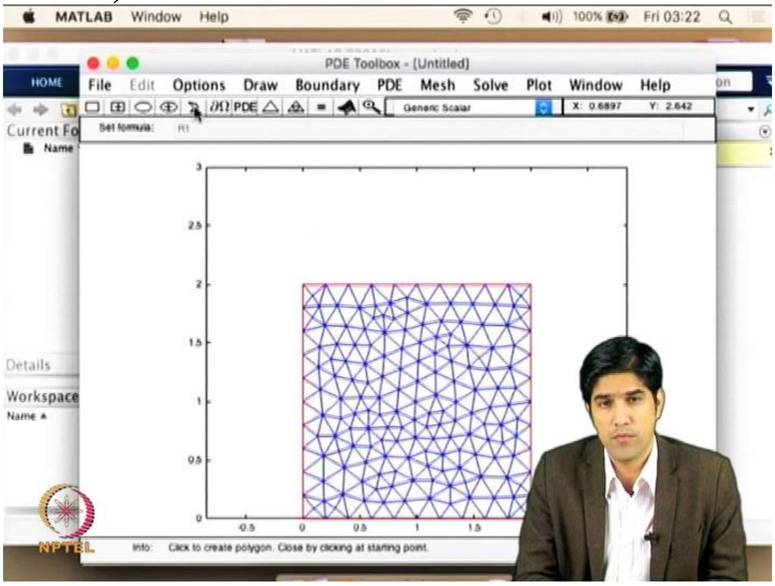

We are going to look into specific tool that comes in with Matlab which is called as PDE tool, which has several features that we have been using over the period of time either to mesh a problem domain or to use it as a entire tool to solve a problem inside the domain.

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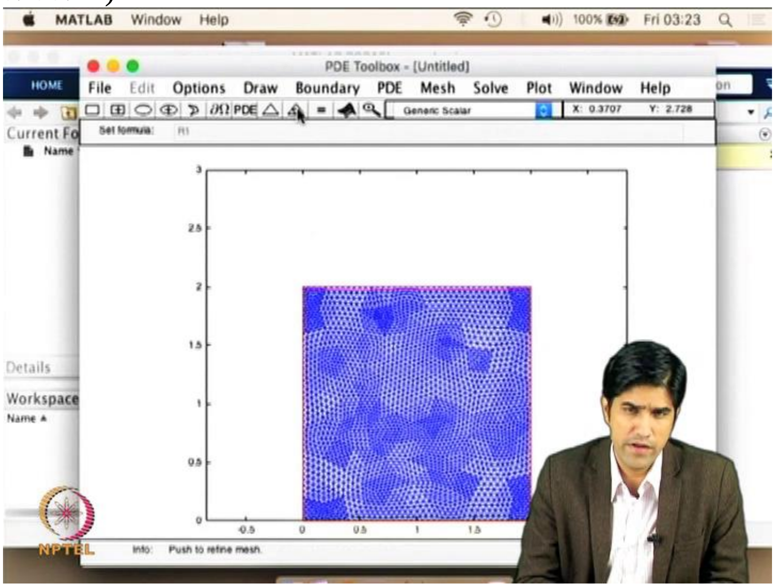

I went use PDE tool box using its entire capabilities but I have shown some of the examples while we looked at earlier aspects of modelling as when we tried it meshing a particular domain we have used PDE tool box. And also in some of the lab tours some of my students will be showing some examples using a PDE tool box.

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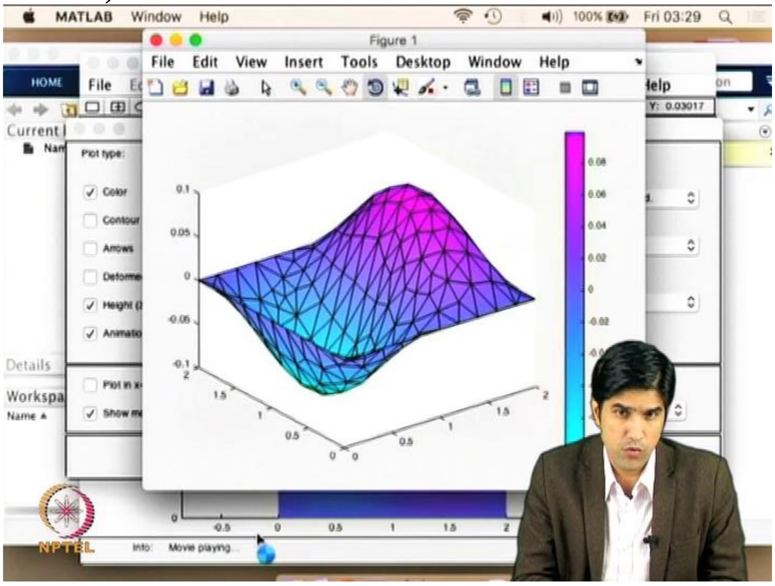

Right now what I am going to do is I am going to take a simple hyperbolic equation partial differential equation and I am going to solve the entire problem using PDE tool box. This is not characteristic of the numerical methods that I have been discussing whether it is finite difference finite element or in the future we will also be discussing finite volume algebraic topology this is going to be different this is going to be different because I wanted to show one of the examples where I am showing an entire problem modelling discretisation of the problem and also the entire solution process of the problem using PDE tool box. This is one of the problems I will be solving and I will also showcase how you will do that for other kinds of problems so that being case I have talked a lot about PDE tool now.So let me actally introduce the PDE tool. But before doing that let us start looking at the simple PDE that we are going to solve using this tool.

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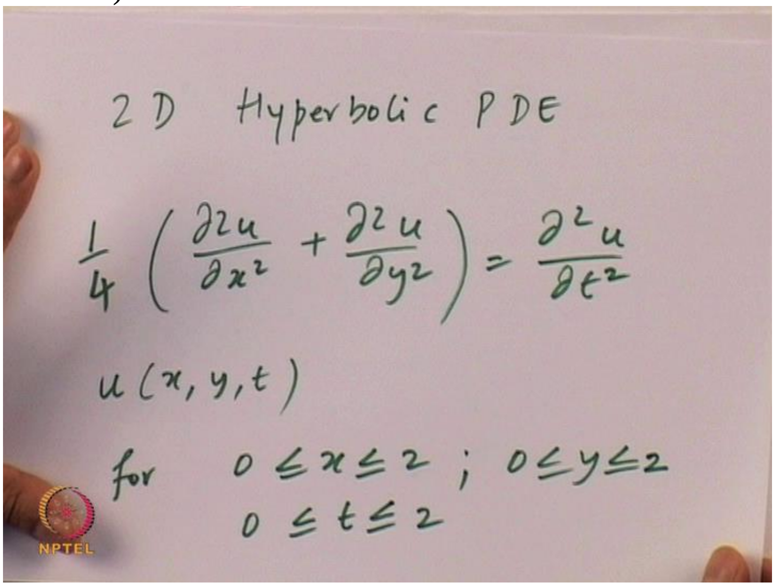

So we are going to use a hyperbolic PDE in 2 dimension. So it is going to be a 2D hyperbolic PDE. So the classical thing what we are going to do is wave equation in 2 Dimension and we are going to model this. So the wave equation we will take is let us say it is 1 by 4 (doe square u divided by doe x square plus doe square u by doe y square) equal to doe square u by doe t square. And of course we are setting that u is a function of (x,y,t) and we are going to solve this problem for the domain 0 less than or equal to x less than or equal to 2. Similarly 0 less than or equal to y less than or equal to 2. And for time steps 0 less than or equal to t less than or equal to 2. So it is going to be 0 to 2 in all these variables. And we are going to have certain boundary conditions. The boundary conditions are going to be given by these expressions.

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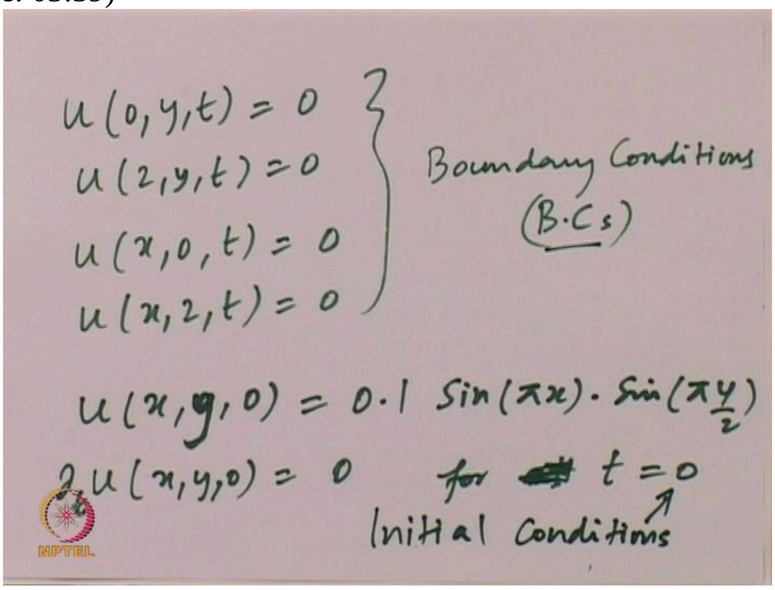

u (0,y,t) is equal to 0 u(2,y,t) is also equal to 0 u(x,0,t) is equal to 0 and u(x,2,t) is also equal to 0. So these are the boundary conditions. And we are going to also have some initial conditions and they are u(x,y,0) is equal to 0.1 sin(pi x) multiplied by sin (let us say pi by 2). And the other one is so since it is a second order equation in space and time we have to give two initial conditions so it is going to be d t u so the first derivative with respect to time of u (x,y,0) is equal to 0 for t equal to 0. So this is the basic problem definition so it has the basic equation that we are trying to solve and the domain boundaries that we are going to have for this governing equation and it has the boundary conditions and the initial conditions. It is called initial conditions because we are talking about for t equal to 0.So with that let us go and see how we can catch this equation in a PDE tool box.

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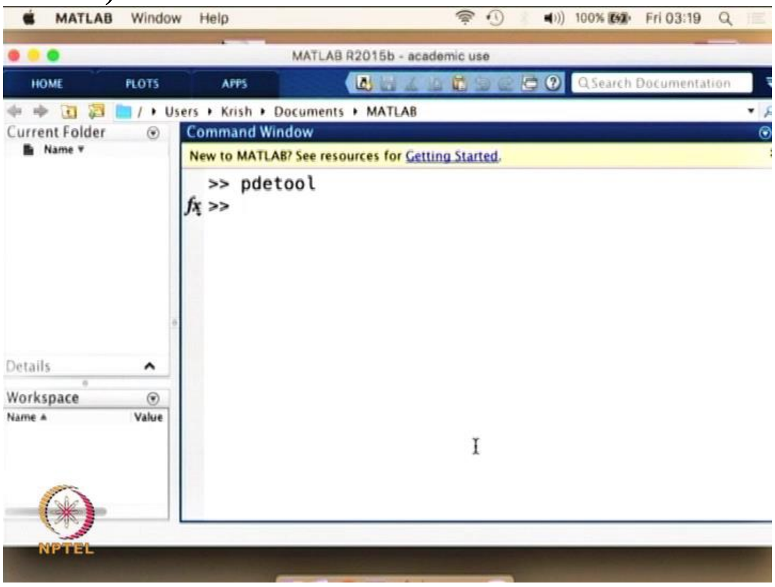

So what we are going to do is go into the Matlab and type PDE tool; type enter.

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What you get is a PDE tool window So we are going to open our PDE tool and this is the PDE tool.

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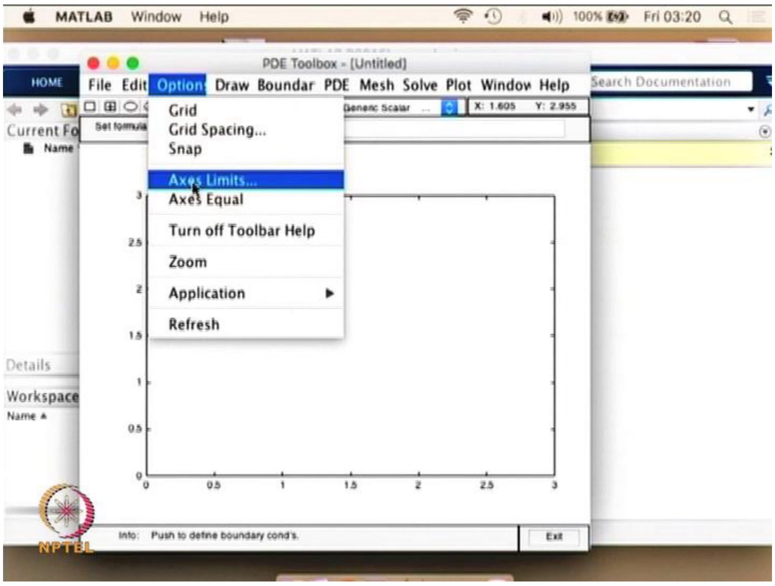

And in the PDE tool we are going to open the options ,and we are going to set the Axes limits.

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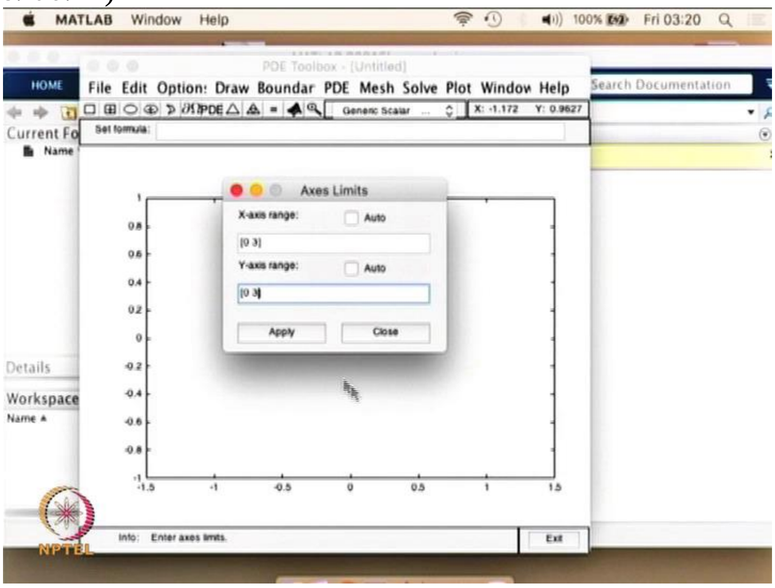

So we are going to start slightly bigger than our problem domain itself so let us say 0 to 3 for x and 0 to 3 for y. Apply and we close it.

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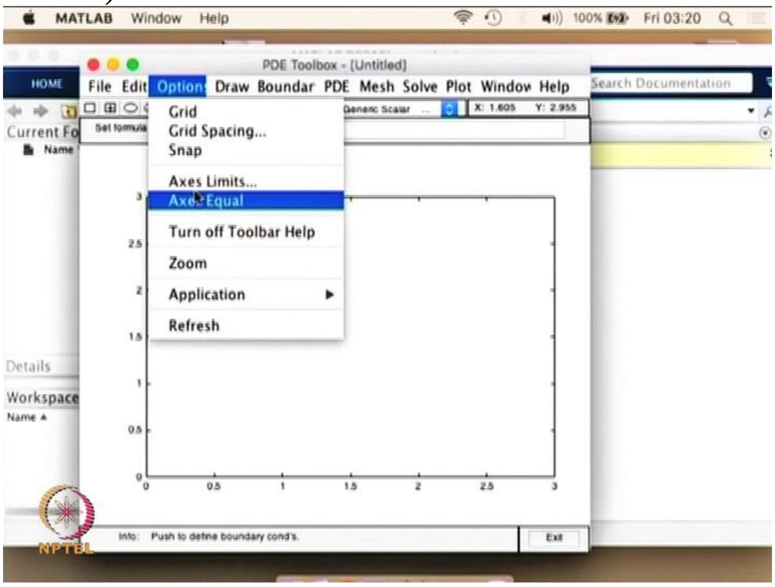

And then we go in option and say Axes equal. And now we are going to go and define the boundary.

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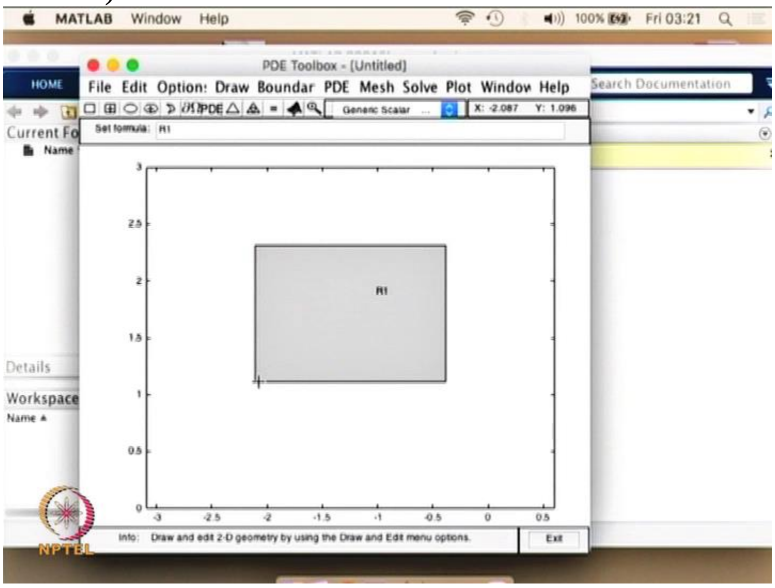

So we click the thing here and we click on this and we set the upper and lower thing.

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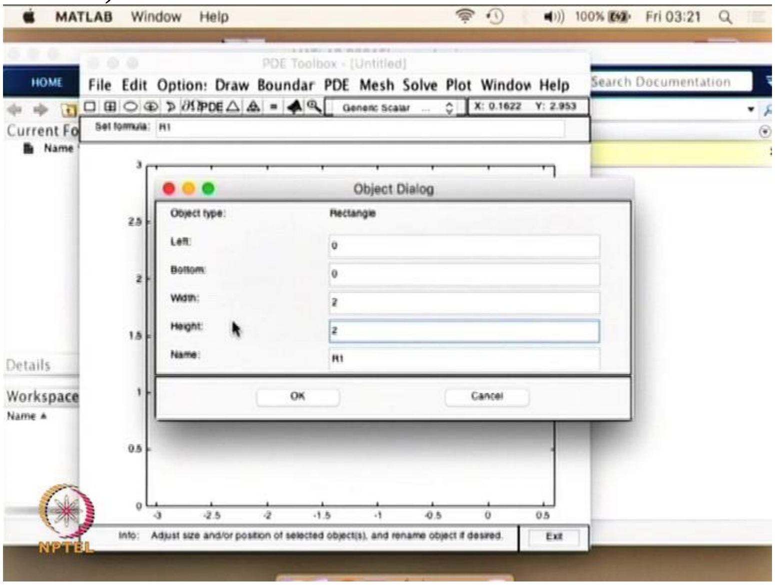

So we put left as 0 bottom as 0 width as 2 and height as 2. And we say ok

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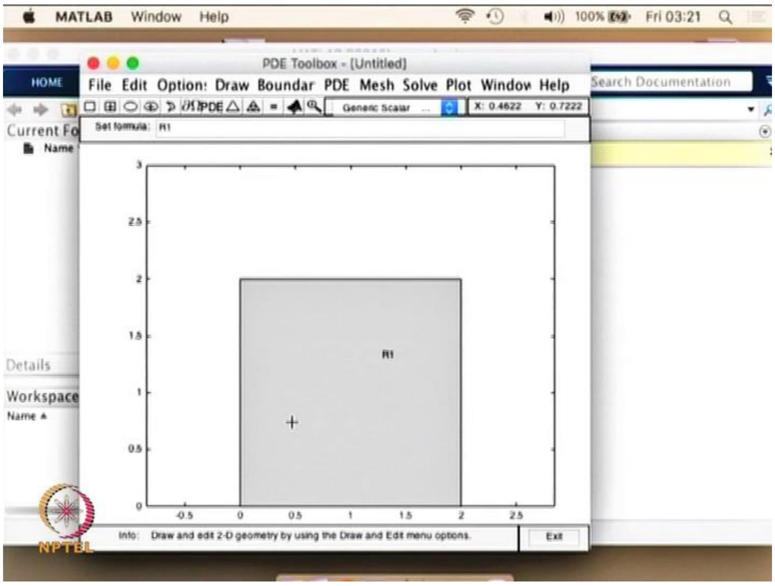

And we get this.

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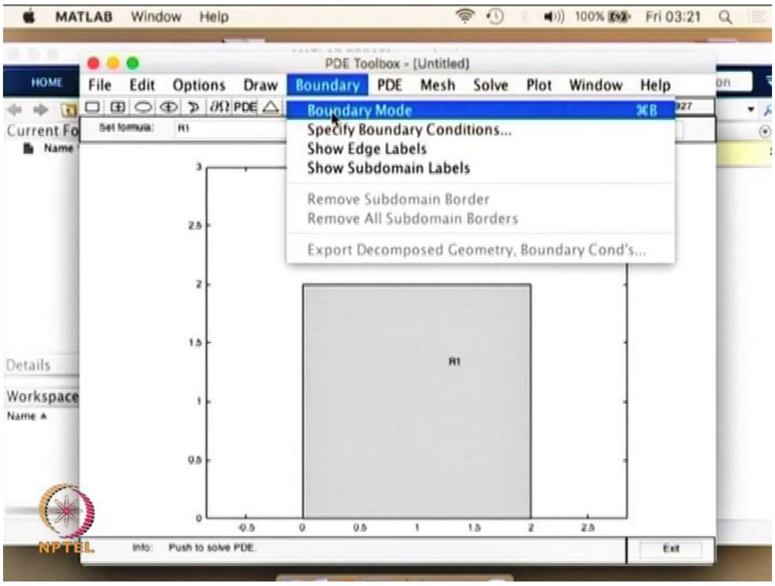

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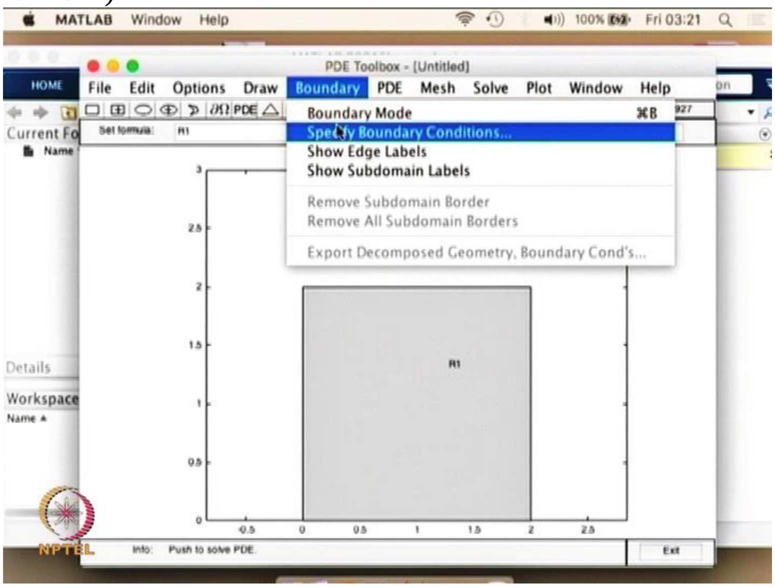

And we click on the boundary so we go and click on the boundary and boundary mode specify boundary conditions. So once we specify the boundary conditions.

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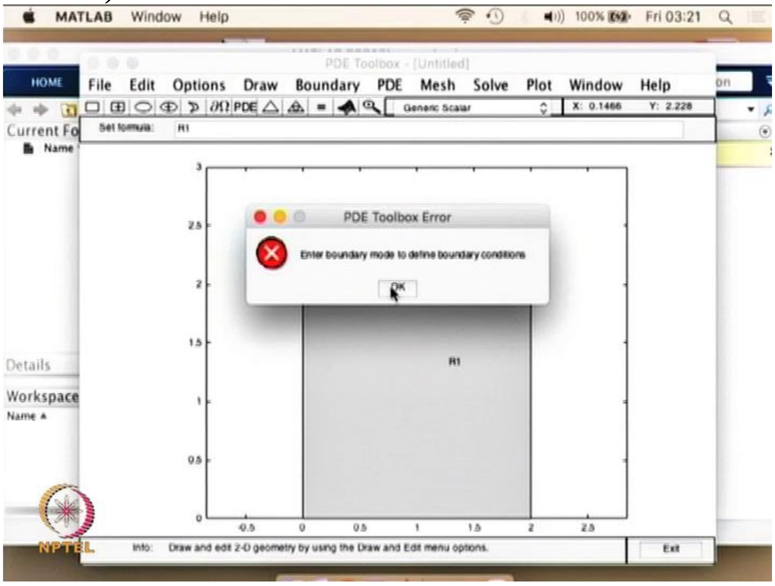

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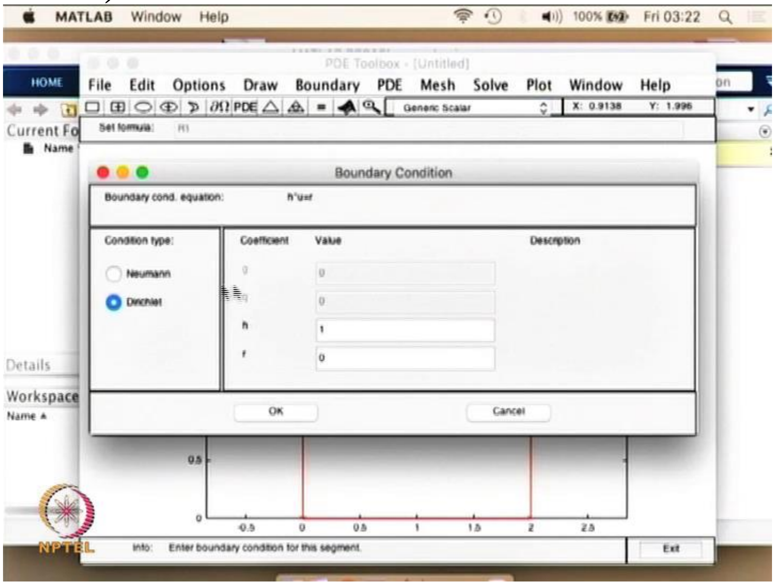

So now we are clicking the boundary here and when you click the boundary you can choose the boundary double click it you open and what you see is different sets of boundary condition that you can assign. If you are interested in assigning the value directly to the normal derivative of the function you click the Neumann boundary condition whereas if you are interested in forcing hard boundary conditions then you click on the dirichlet boundary condition. We stick to the Dirichlet boundary condition for this particular problem we click Ok . And we go into the meshing of the domain itself.

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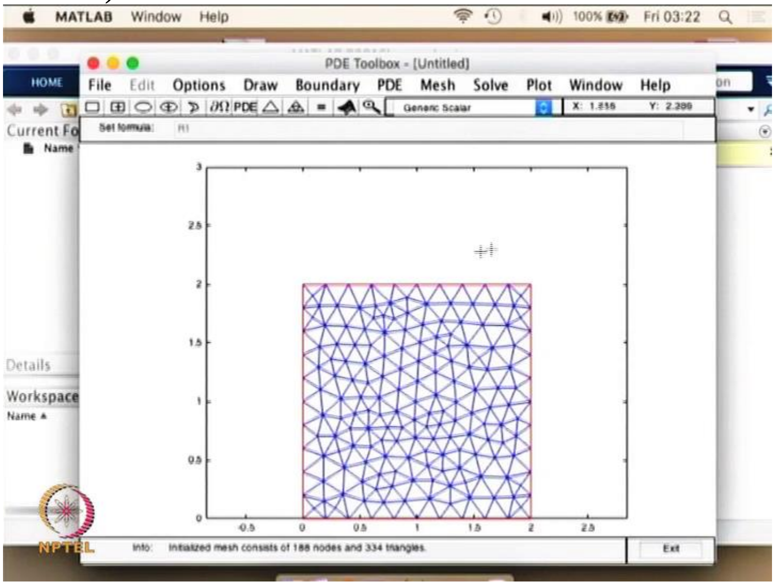

So we click the meshing and you see that the PDE tool box gives a very coarse mesh in this case.

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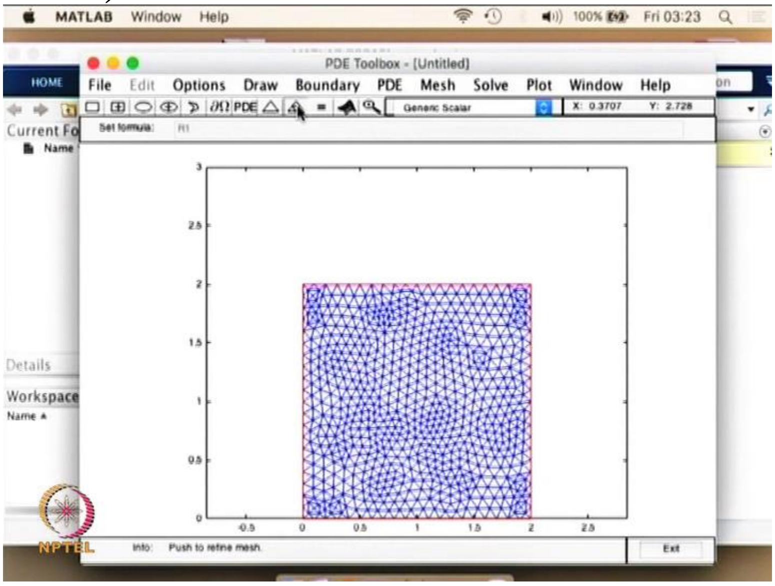

If you want to refine the mesh you can click on refining here.

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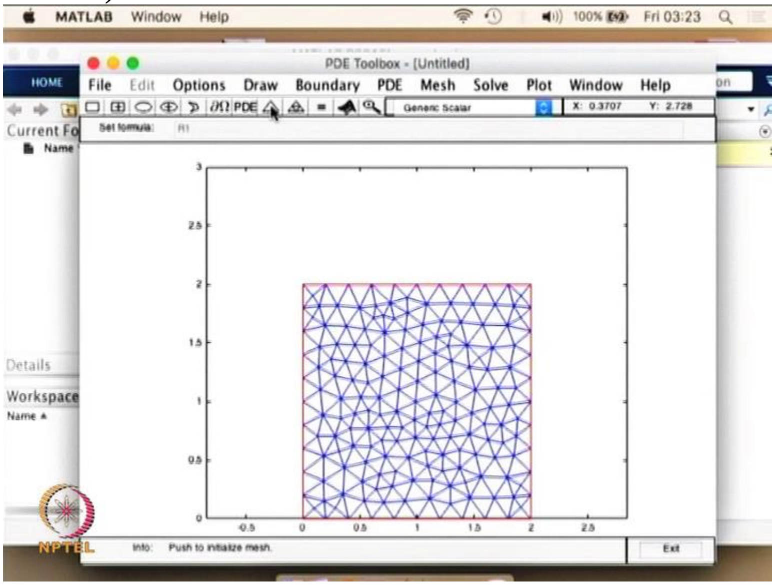

And if you wanted to have a coarse mesh you click on the basic one.

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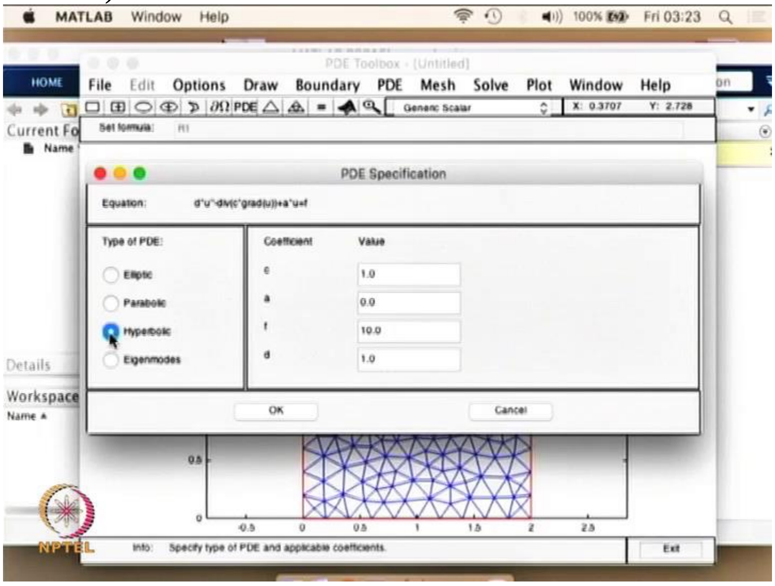

Once you do that you go and click the PDE specification you go into the PDE specification and there are different type of PDE s that you can model and we are interested in the hyperbolic PDE and in our case value of the c which is the value that we have as a quotient here.

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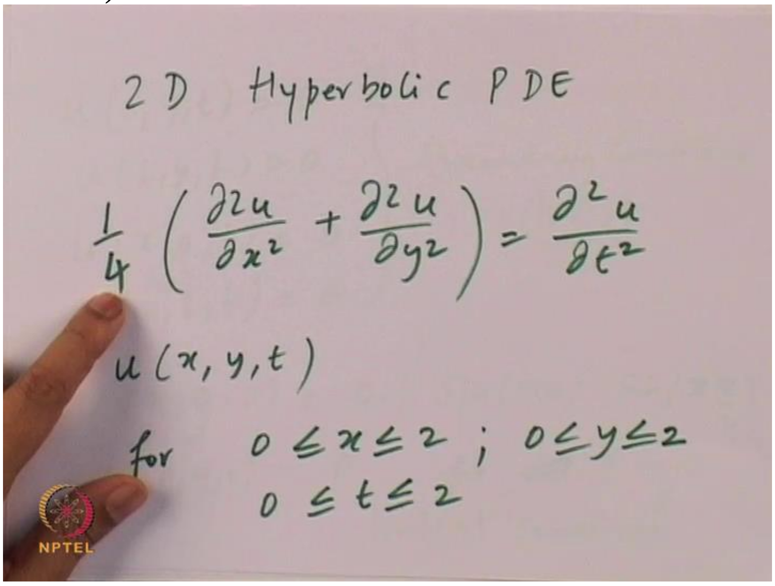

So here you have the value of the c value as 1 by 4.

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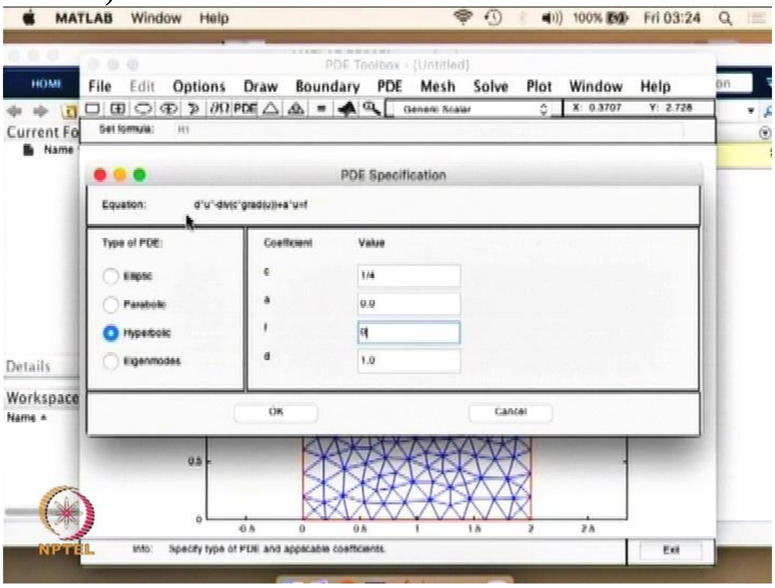

So we put here 1 by 4 and we put the value of a as 0 we put the value of f as 0 and we put the value of d as 1.

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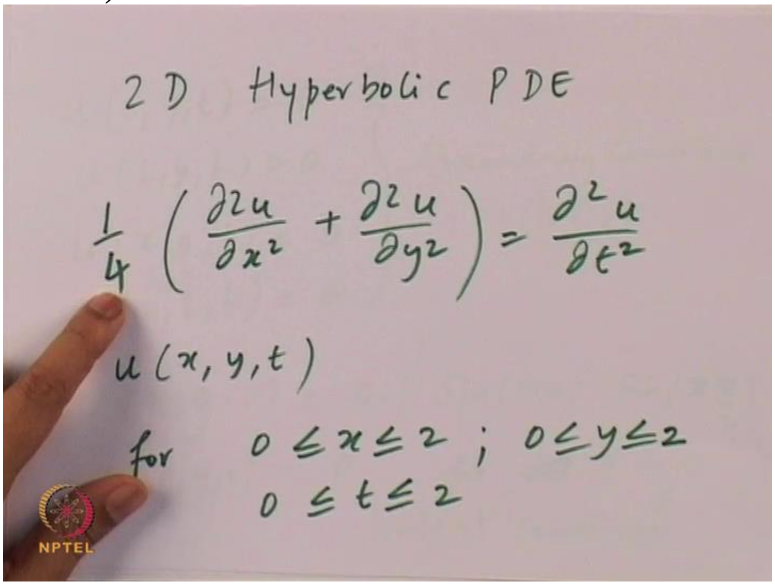

So this way you get the hyperbolic PDE that we are interested in and you click ok.

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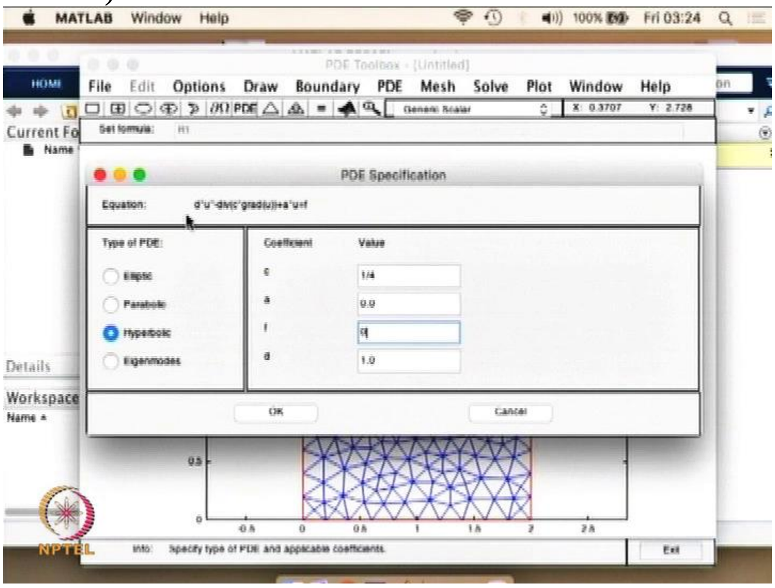

And one more thing I wanted to say is if you look at the PDE specification the equation that we are modelling will have a general form of this sort and of course when you put the values the way we have put here you will end up with the format that you are interested in. So if you see this equation you will see that putting a equal to 0 basically cuts this equation out of the form f equal to 0 will also make it the same. So this one will be u double prime minus divergence 1 by 4 gradient of u is equal to 0. So this is the basic equation what you have got and if you expand it you get the hyperbolic form that we have got in our paper. So we click ok.

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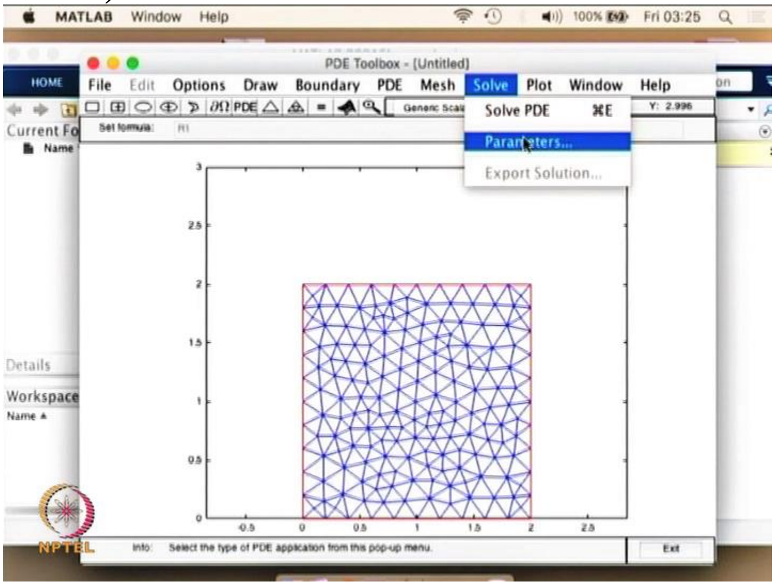

And now we go into the solve. So the PDE solve itself

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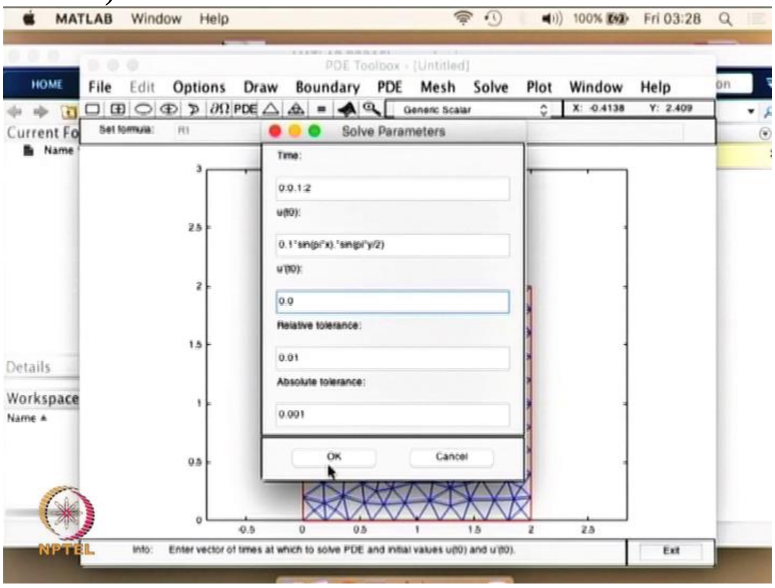

So you set the parameters So you set the time as 0 to 2 but with a stepping of 0.1 as the delta t. So you can set the value maximum value as 2 and it goes from 0 to 2 with the time step of 0.1. You can set the initial value the way we have got here.

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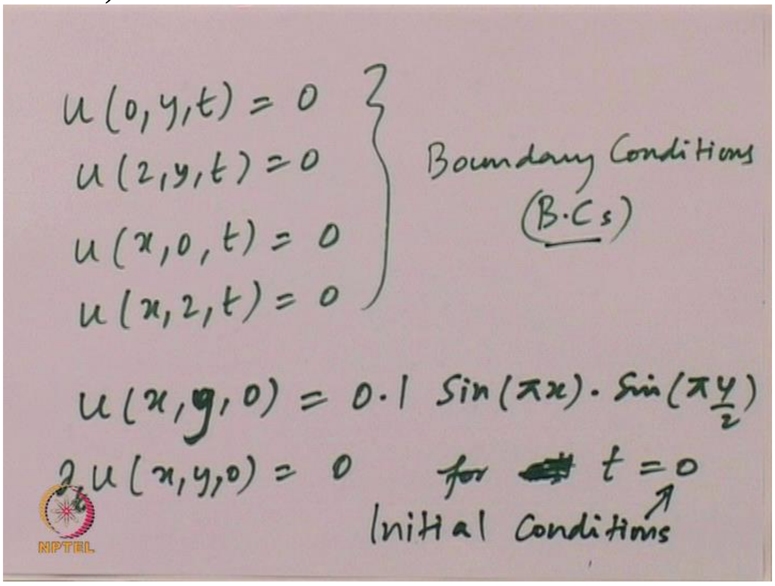

So we go and see the equation here the initial value is given here. So for u (t) equal to 0 we have to set the initial value so the initial value is going to be 0.1 multiplied by sin (pi x) multiplied by sin(pi y by 2).

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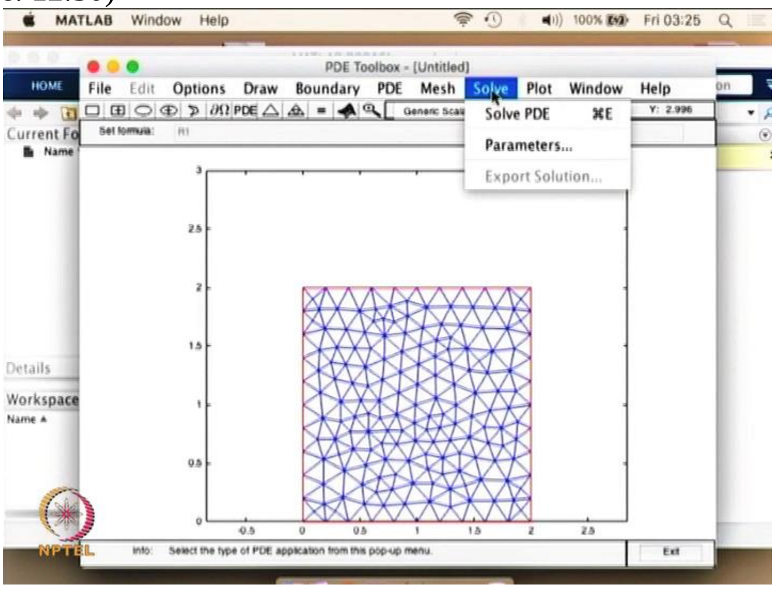

So that is what I am going to type here so 0.1 multiplied by sin(pi multiplied by x) and multiplied by, so since it is the matrix multiplication. So we are putting dot star sin(pi multiplied by y by 2). So this is the way we set the initial condition. So 0.1 multiplied by sin (pi x) dot star sin(pi y by 2).

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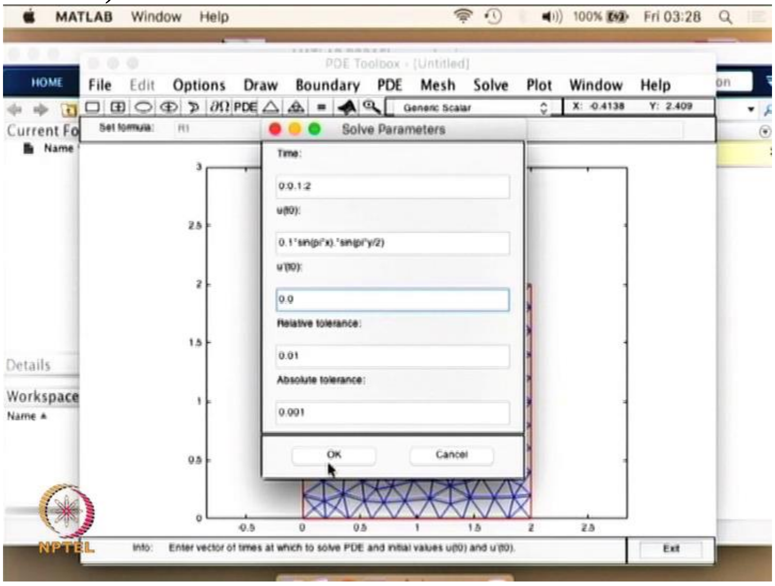

And the second initial condition is d t u is equal to 0. So we put here the value as 0. So the tolerance and absolute tolerance value we are not changing anything and we are setting ok.

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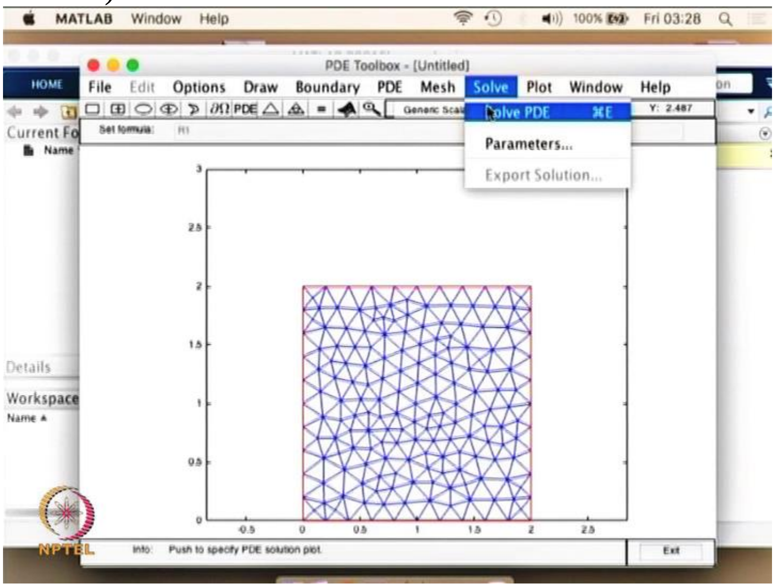

And once we do that we can go into the solve PDE button.

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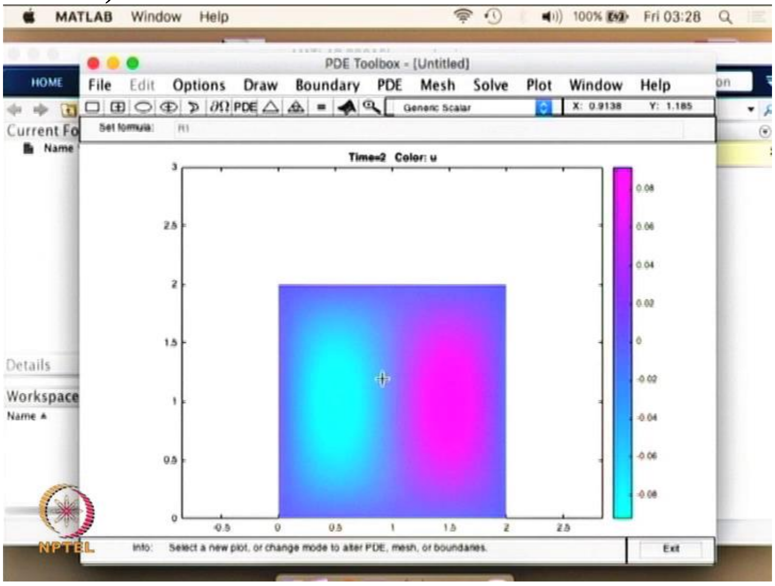

We get the value that we are looking for the time period 0 to 2 like this. Of course we wanted to see the entire thing in motion so what we can do is we can click the equal button. So of course what we are seeing is a final result.

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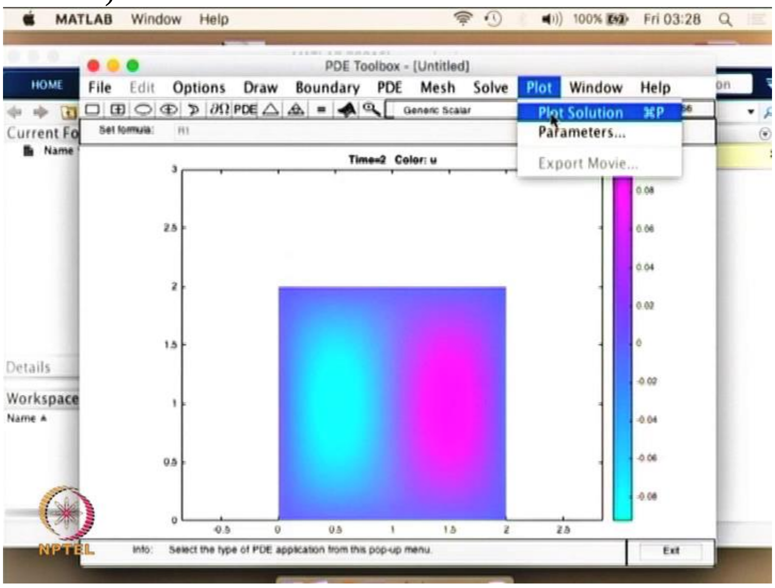

What we need to see is the entire plot solution so we can click the plot solution button.

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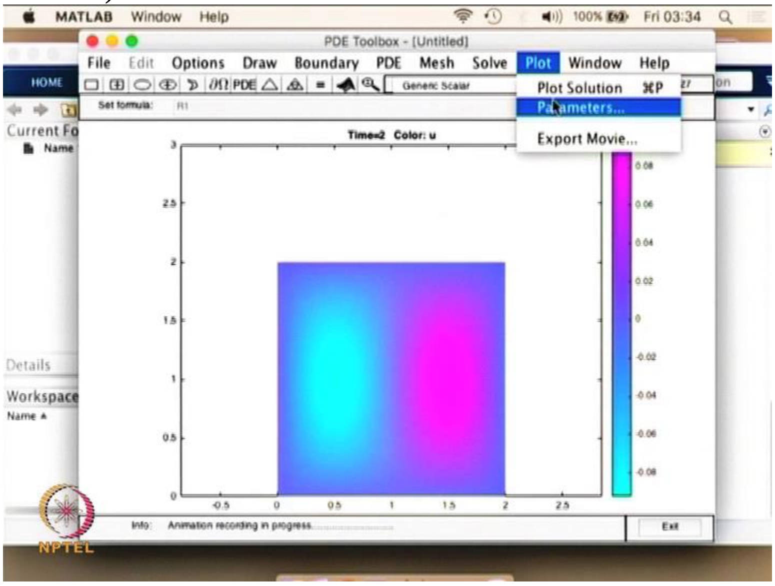

Once you click that you can click the parameters that you are trying to plot.

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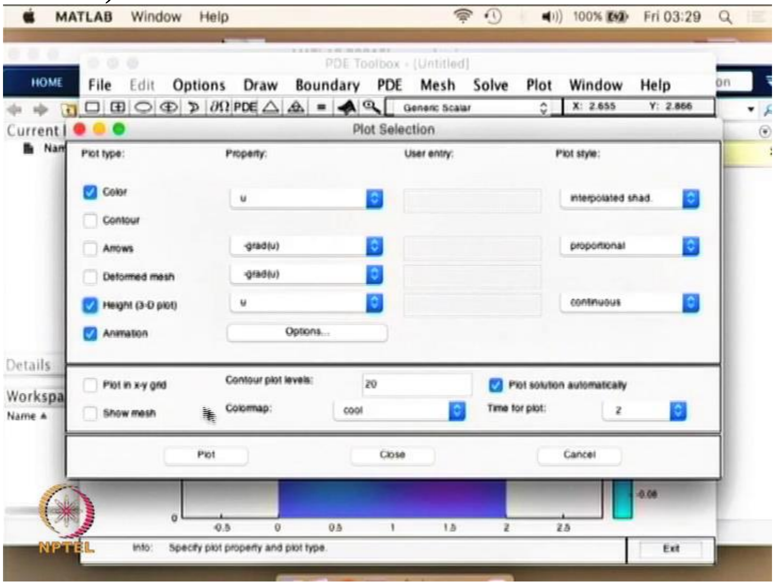

So you wanted to see the entire thing is an animation then you can click animation if you want to have a 3D plot you can click the 3D plot and if you want to see the mesh you can click the mesh. So then you can say ok.

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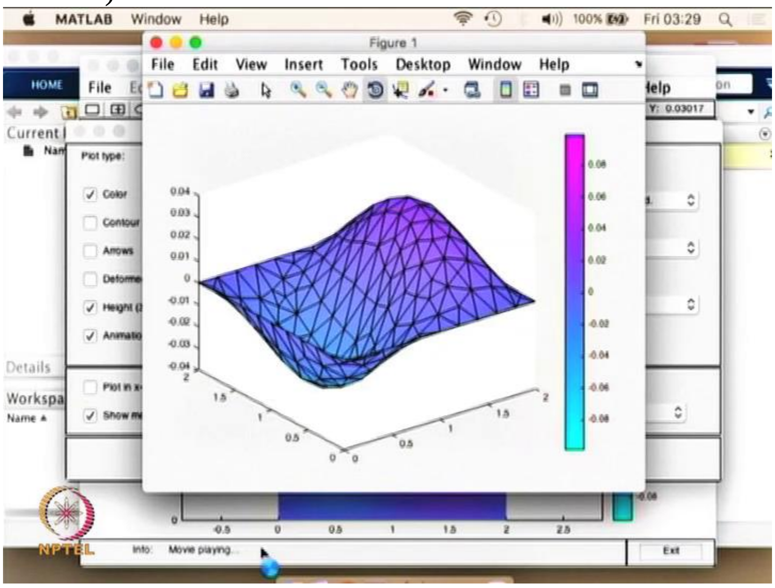

And what you will see is a variation as a time simulation so this is a simple example. Of course this is an example that is mostly used in the mechanics rather than electromagneitcs but still it is a very simple equation that one can look and model and if you wanted to see the variation also in 2D and the variation in the simulation as well so you can keep both of them side by side and see the variation happening. So you can plot once more.

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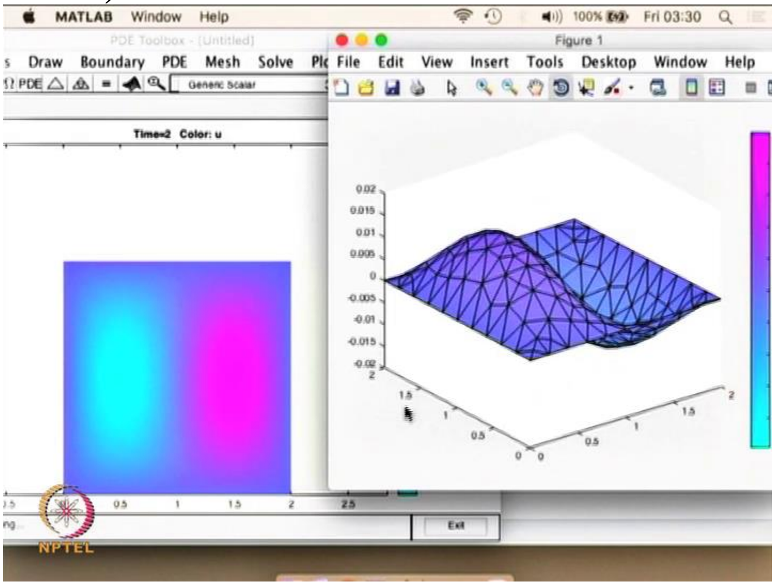

So you can see the variation as a function of space x and y and time. So what we can also do is in order to make this simulation go slow because what we have given here as initial conditions. So if you look at the parameters we have set. We have set 0 to 2 is the time limit and we are going 0.1 so basically we have got 20 time steps so what you can do is you can increase the time steps delta t by making it even finer and you can see the simulation will be much more smoother. So you can basically solve the PDE and it takes a little bit time because the animation is recording in the progress as you can see down here. And it takes a long time because there are going to be more number of steps so we are going to have roughly 200 steps. So it is taking a little long time.

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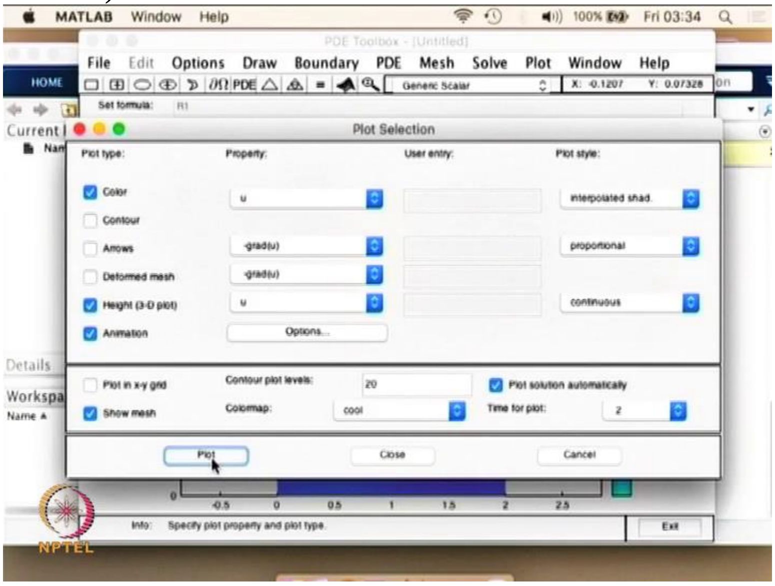

So now the simulation is finished so we can go and see the simulation by clicking on the parameters we are interested in the 3D plot we are looking at the animation we wanted to see the mesh and we are going to plot it.

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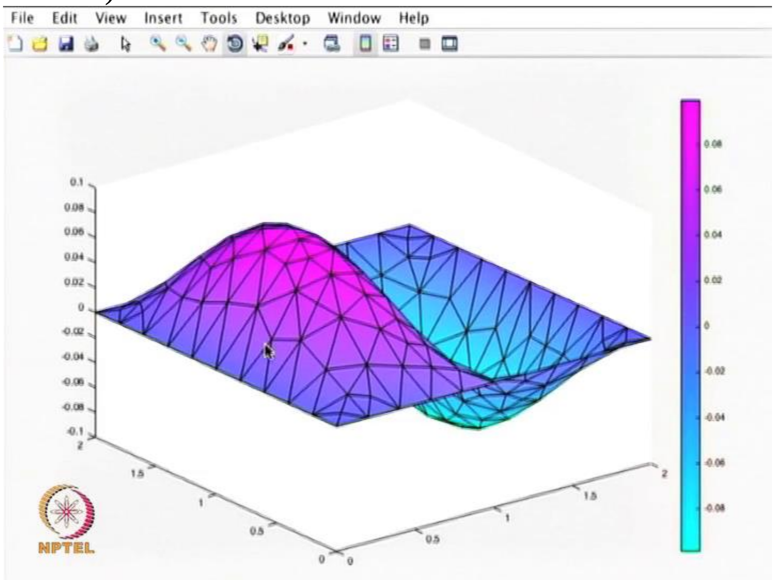

Once you are plotting it what you see is the variation much more finer and it is showing the way the variation happens as a vibration so since it is a wave equation that is representing the vibration so you are seeing the changes in a much more smoother manner. And you also see that the discretisation itself shows some kind of averaging. So there is some kind of averaging along the space of the element. So since it is a finite element method so what it does it takes the entire domain of the triangle to have the same value so that is the reason we see sharp points at some of the things which are not very physical.

So what we have done in this example is a way to show how one can model PDE using PDE tool box and ofcourse we have not shown any example from Maxwell equation or Electromagnetics because we want to keep it very general you can use this kind of PDE tool which are inbuilt functionalities of Matlab to model your own problems and of course you can model an eleptical problem hyperbolic problem or a parabolic problem so in this case we have done a hyperbolic problem. Often times you come across elliptical problems and parabolic problems as well.

So one of the easiest problem that we do is the hyperbolic problem in Maxwell equation but other than that we have done Laplace equation and Poisson equation so which are a kind of electric PDE to model. So if you are doing a laplace equation or a Poisson equation then you end up in a elliptical PDE setup. So in order of us to go away from a standard problems because we have solved Poisson equation or Laplace equation using various methods but we have not looked into the hyperbolic equation itself. When we go and model the Maxwell equation for finite volume method later on during the course of this lecture we will be using the hyperbolic formulation. There I will describe more in detail what is an hyperbolic PDE.

For now it is enough to say that hyperbolic PDE has a form like the parabolic PDE or the elliptic PDE. So for now this is the problem that I wanted to solve using the PDE tool box and we have shown you how to solve it using the PDE tool.

We encourage you to try it to use PDE tool from your own problems and also learn how meshing can be done how one can define the boundary condition and how one can solve problems by changing the mesh discretisation or the time discretisation like the way I have shown now.

So with that we come to the end of this module I encourage you to try this particular tool and practice it for your problems. Thank You!