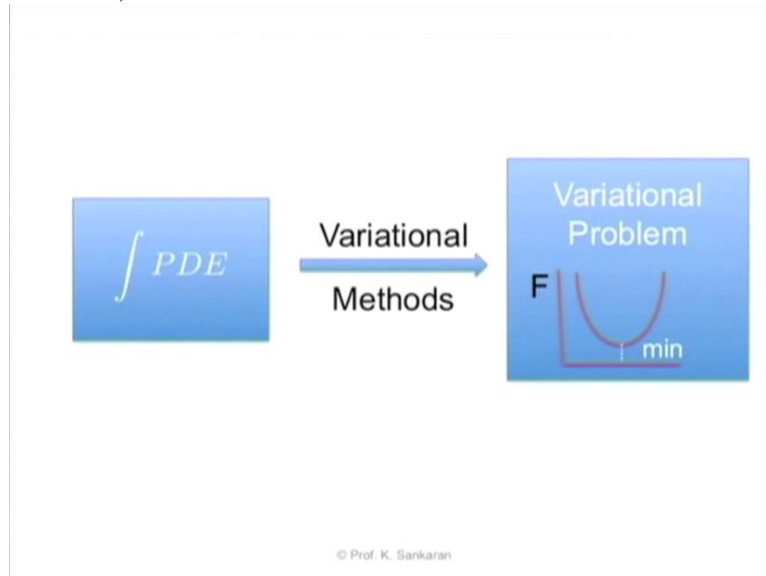


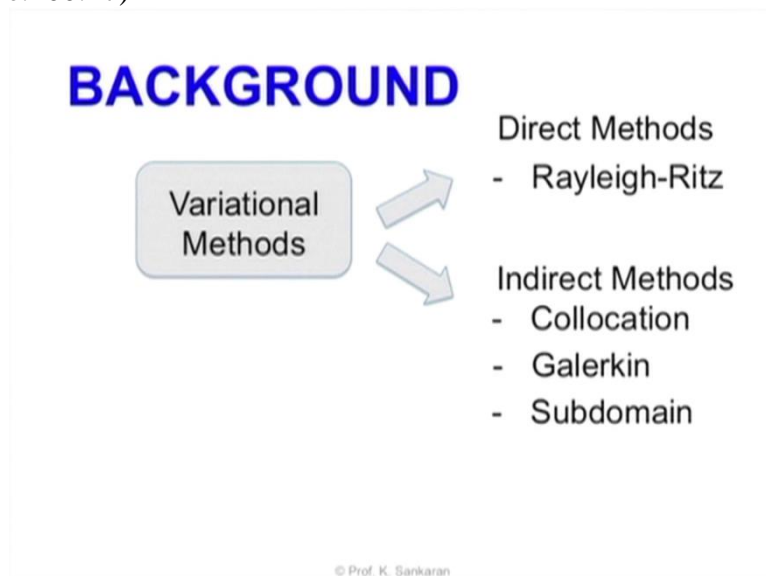
Computational Electromagnetics and Applications
Professor Krish Sankaran
Indian Institute of Technology Bombay
Summary of Week 5

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We introduced the basic theory behind variational methods starting from the concept of variational principle.

(Refer Slide Time: 00:27)



We discussed two broad categories of variational method namely the direct and indirect variational methods.

(Refer Slide Time: 00:33)

BACKGROUND

We define, **inner product** $\langle u, v \rangle$ of two functions u and v as,

$$\langle u, v \rangle = \int_{\Omega} uv^* d\Omega$$

* - complex conjugate

$$\langle u, v \rangle = \int_{\Omega} \mathbf{u} \cdot \mathbf{v}^* d\Omega$$

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We explained a physical and mathematical interpretations of inner product that we commonly used in variational methods and discussed some of its properties.

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BACKGROUND

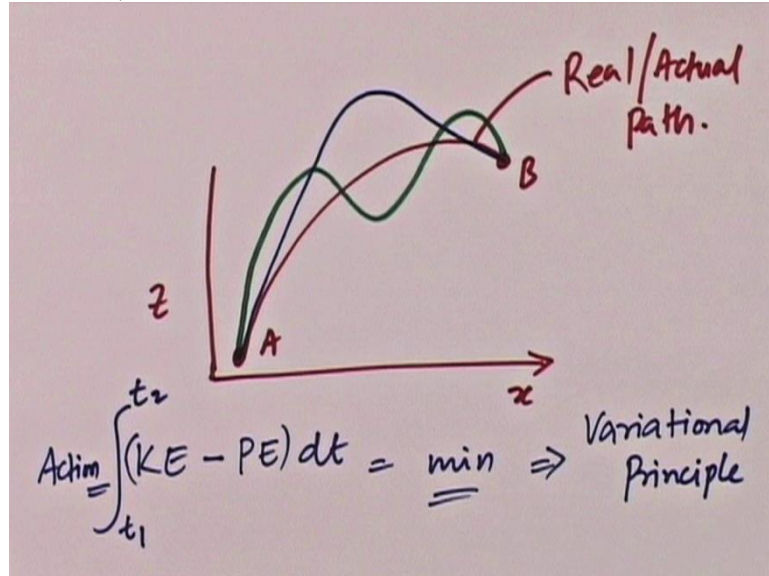
For each pair of u and v , we can get a number $\langle u, v \rangle$ such that

$$\langle u, v \rangle = \langle v, u \rangle^*$$
$$\langle \alpha u_1 + \beta u_2, v \rangle = \alpha \langle u_1, v \rangle + \beta \langle u_2, v \rangle$$
$$\langle u, u^* \rangle > 0 \quad \text{if } u \neq 0$$
$$\langle u, u^* \rangle = 0 \quad \text{if } u = 0$$

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We have also introduced the concept of action integral.

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We introduced the calculus of variations giving a simple example of throwing a stone in the air.

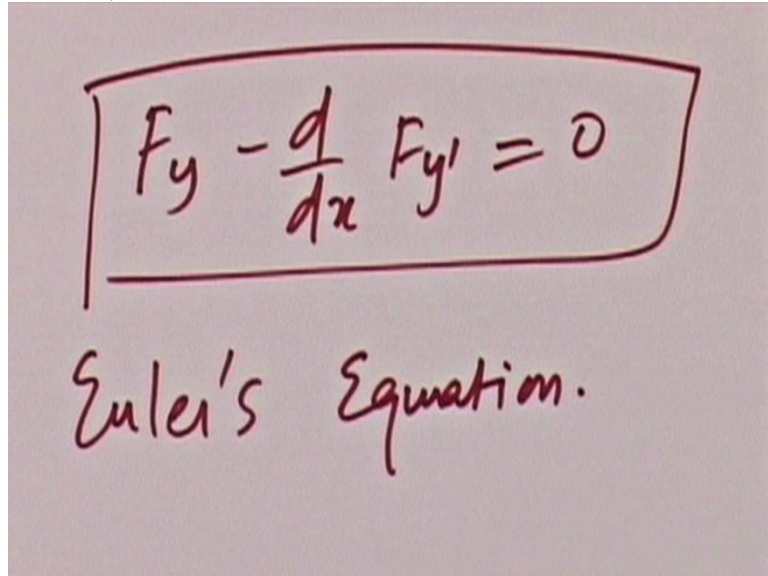
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$$\delta I = \int_a^b \left[\frac{\partial F}{\partial y} - \frac{d}{dx} \left(\frac{\partial F}{\partial y'} \right) \right] \delta y dx$$
$$\delta I = 0$$

$$\frac{\partial F}{\partial y} - \frac{d}{dx} \left(\frac{\partial F}{\partial y'} \right) = 0$$

We discussed the steps for deriving the partial differential equation for a given variational principle.

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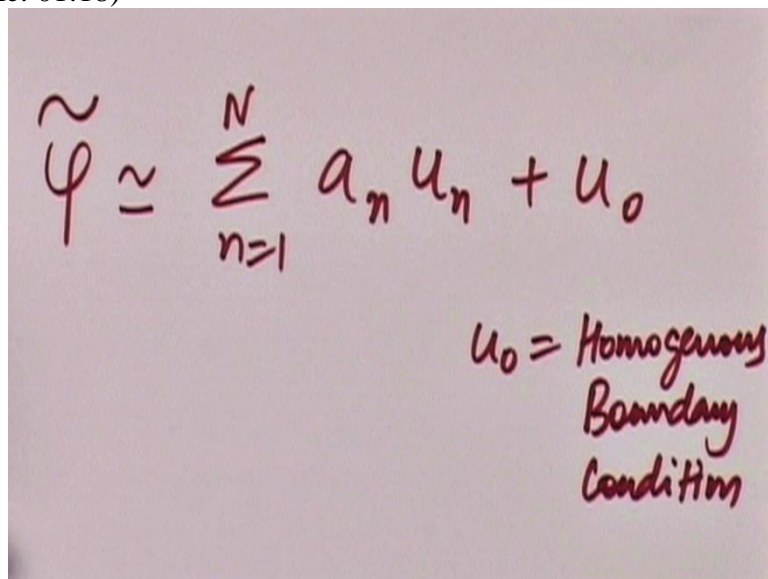
The image shows a handwritten equation in red ink on a light-colored background. The equation is enclosed in a hand-drawn rectangular box. Below the box, the text "Euler's Equation." is written in the same red ink.

$$F_y - \frac{d}{dx} F_{y'} = 0$$

Euler's Equation.

We have also discussed how one can derive the Euler equation for a given partial differential equation.

(Refer Slide Time: 01:18)



The image shows a handwritten equation in red ink on a light-colored background. The equation is $\tilde{\varphi} \approx \sum_{n=1}^N a_n u_n + u_0$. Below the equation, the text " $u_0 =$ Homogeneous Boundary Condition" is written in the same red ink.

$$\tilde{\varphi} \approx \sum_{n=1}^N a_n u_n + u_0$$

$u_0 =$ Homogeneous
Boundary
Condition

Later we explained how we can approximate any unknown function using a set of known basis functions.

(Refer Slide Time: 01:27)

RAYLEIGH-RITZ METHOD

In matrix form

$$\begin{bmatrix} \langle Lu_1, u_1 \rangle & \langle Lu_2, u_2 \rangle & \dots & \langle Lu_1, u_N \rangle \\ \vdots & & \vdots & \\ \langle Lu_N, u_1 \rangle & \langle Lu_N, u_2 \rangle & \dots & \langle Lu_N, u_N \rangle \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_N \end{bmatrix} = \begin{bmatrix} \langle g, u_1 \rangle \\ \vdots \\ \langle g, u_N \rangle \end{bmatrix}$$
$$[A]\{X\} = \{B\}$$

where $A_{mn} = \langle Lu_m, u_n \rangle, B_m = \langle g, u_m \rangle, X_n = a_n$

Above equation is **Rayleigh-Ritz** system

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Under direct variational method we discussed the mathematical formulation of the famous Rayleigh Ritz method.

(Refer Slide Time: 01:38)

METHOD OF WEIGHTED RESIDUALS

This equation can be written in matrix form as,

$$\sum_n a_n \langle w_m(x), L[v_n(x)] \rangle = \langle w_m(x), g(x) \rangle$$



$$[Z_{mn}]\{a_n\} = \{g_m\}$$

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Under the indirect method we introduced the Weighted Residuals

(Refer Slide Time: 01:40)

GALERKIN METHOD

The weighting functions are made to be the same as the basis functions

$$w_m(x) = v_m(x)$$

The matrix equation becomes,

$$\sum_n a_n \langle w_m(x), L[v_n(x)] \rangle = \langle w_m(x), g(x) \rangle$$



$$[\mathbf{Z}_{mn}] \{\mathbf{a}_n\} = \{\mathbf{g}_m\}$$

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And Galerkin Method. These techniques form the basis for two of the most important computational techniques namely the finite element method and the method of moments.

(Refer Slide Time: 01:55)

FUNCTIONALS FROM PDE

According to **Mikhlin**, if L is real, self-adjoint and positive definite

$$I(\varphi) = \langle L\varphi, \varphi \rangle - 2\langle \varphi, g \rangle$$

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We also discussed how one can directly derive the functional for a given PDE using Mikhlin approach.

(Refer Slide Time: 02:05)

FUNCTIONALS FROM PDE

Consider **Poisson's equation** $\nabla^2\varphi = -g(x, y)$
where $\nabla^2 = L$

Take S1

$$\begin{aligned}\delta I &= \int \int [-\nabla^2\varphi - g]\delta\varphi dx dy = 0 \\ &= - \int \int \nabla^2\varphi\delta\varphi dx dy - \int \int g\delta\varphi dx dy\end{aligned}$$

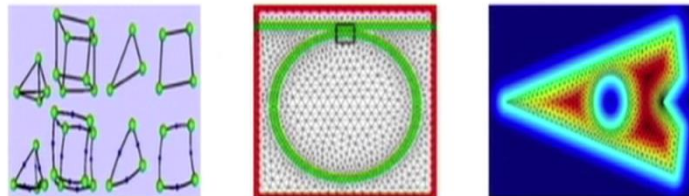
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We used these techniques for solving two dimensional Poisson equation mastering

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FINITE ELEMENT METHOD (FEM) Part – I

Prof. Krish Sankaran



The basics of variational methods are going to be very helpful when we are introducing the finite element method in the next modules.

So please practice diligently the simulations and examples that we have discussed in this week and post your questions on the forum. We will see you next week till then Good Bye!