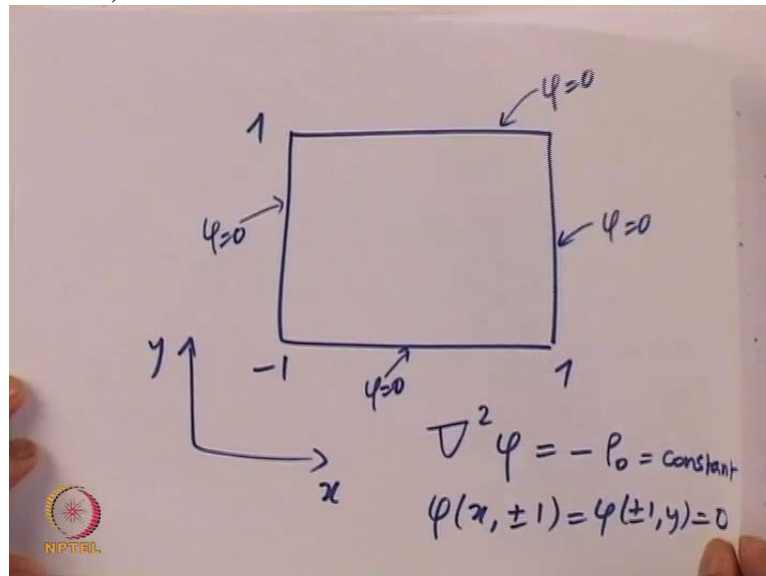


Computational Electromagnetics and Applications
Professor Krish Sankaran
Indian Institute of Technology Bombay
Exercise 10
Variational Methods

Today we are going to solve a very simple problem that we have been looking in various methods the Poisson Equation where we are going to use Rayleigh Ritz method a variational method to solve such a problem. So let us look into the problem geometry itself.

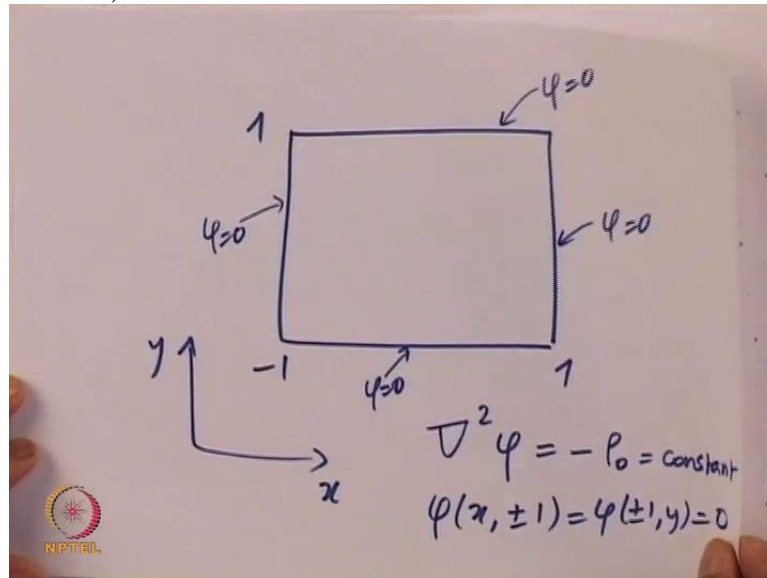
(Refer Slide Time: 00:33)



So we are going to have a square, so the square is going to be minus 1 to plus 1 in both direction in x and y direction. So if this is the x direction this is the y direction. So if this is going to be the x axis this is going to be the y axis so this is going to be our problem geometry. And we are going to solve the Poisson equation. So this is going to be $\nabla^2 \phi = -\rho_0 = \text{constant}$ and for the problem we are going to consider ρ_0 is equal to constant. So this is going to be constant across the entire domain and we are going to solve this problem subjected to some boundary conditions. The boundary condition is $\phi(x, \pm 1) = \phi(\pm 1, y) = 0$. So that means for all x but plus or minus 1 y we are going to have 0. So the potential is going to be 0 here, potential is going to be 0 here.

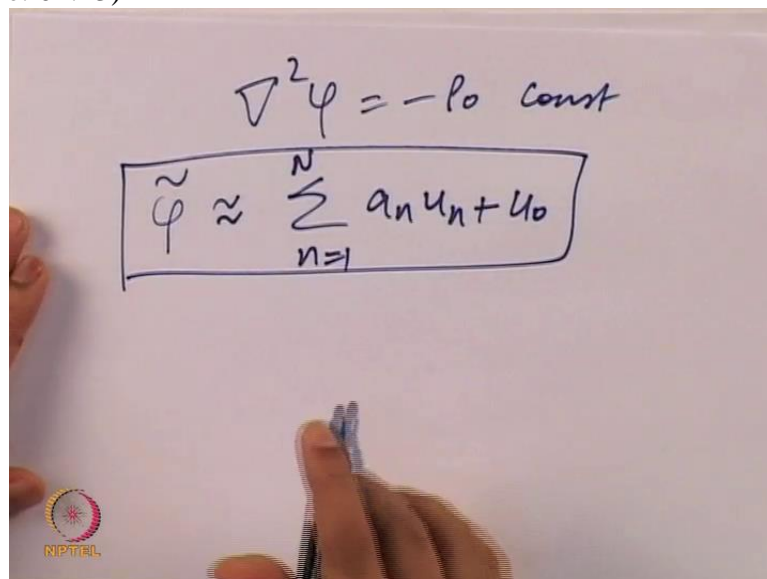
Similarly for all y x equal to minus 1 and x equal to plus 1 it is also going to be 0. So along the boundary the potential boundary is going to be 0. So $\phi = 0$, $\phi = 0$, $\phi = 0$ and $\phi = 0$. So this is the problem definition and let us go and look at how one can solve this problem using a simple technique of Rayleigh Ritz.

(Refer Slide Time: 02:26)



So we are going to take the problem from its symmetry. So if we look at this problem there is a kind of a symmetry so it is having x and y symmetry so we are going to use a basis function based on symmetry.

(Refer Slide Time: 02:43)



So the solution to this problem will be of the form. So we have the laplacian equation and we have set this is equal to constant the general solution to this thing will be of the form where we are searching for an approximate solution which is Phi tilde. It is going to be of the form sigma n equal to 1 to N $a_n u_n$ plus u_0 . So this is going to be the general form. And of course depending on the number of basis functions this is going to increase in terms.

(Refer Slide Time: 03:24)

$$\nabla^2 \phi = -\rho_0 \text{ const}$$

$$\tilde{\phi} \approx \sum_{n=1}^N a_n u_n + u_0$$

$$\begin{bmatrix} \langle L u_1, u_1 \rangle & \dots & \langle L u_1, u_N \rangle \\ \langle L u_N, u_1 \rangle & \dots & \langle L u_N, u_N \rangle \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_N \end{bmatrix} = \begin{bmatrix} \langle g_1, u_1 \rangle \\ \langle g_N, u_N \rangle \end{bmatrix}$$

$$\underline{A} \underline{X} = \underline{B}$$

So what we will have finally is of the form matrix equation which will have the inner product of $\langle L u_1, u_1 \rangle$ and the last term on the first row will be $\langle L u_1, u_n \rangle$. Similarly here first term on the last row will be $\langle L u_n, u_1 \rangle$ The last term here would be $\langle L u_n, u_n \rangle$. So this will be the first on the left hand side multiplied by the basis functions that multiplied by the coefficients which are going to be $[a_1 \dots a_n]$ and on the right hand side it is going to be a vector of $\langle g_1, u_1 \rangle$ in a product of g_1, u_1 . The last term is going to be $\langle g_n, u_n \rangle$. So this is nothing but in the matrix form AX equal to B . And we are interested in finding the value of X by taking the inverse. So this is going to be the way we are going to do the problem what we are interested is in finding these coefficients. So that is the way we are going to solve it and we will see what the coefficients are going to be in the following steps.

(Refer Slide Time: 05:04)

$$u_{mn} = (1-x^2)(1-y^2)(x^{2m}y^{2n} + x^{2n}y^{2m})$$

$$m, n = 0, 1, 2, \dots$$

So since the problem is symmetrical we can set the basis function we are going to use, it is going to be equal to u_{mn} is equal to $(1 - x^2)(1 - y^2)(x^{2m}y^{2n} + x^{2n}y^{2m})$ So for m, n equal to $0, 1, 2, \dots$ So on and so forth. So this is going to be the basis function we are going to use.

(Refer Slide Time: 05:55)

$$u_{mn} = (1-x^2)(1-y^2)(x^{2m}y^{2n} + x^{2n}y^{2m})$$

$$m, n = 0, 1, 2, \dots$$

$$\tilde{\Phi} = (1-x^2)(1-y^2) [a_1 + a_2(x^2+y^2) + a_3x^2y^2 + a_4(x^4+y^4) + \dots]$$

So let us say we have the problem using this basis function, so what we will get is we will get the solution itself as an approximation. So the approximate solution is going to be $\tilde{\Phi}$ is equal to $(1 - x^2)(1 - y^2)$ multiplied by $[a_1 + a_2(x^2 + y^2) + a_3x^2y^2 + a_4(x^4 + y^4) + \dots]$

(Refer Slide Time: 06:50)

$$u_{mn} = (1-x^2)(1-y^2)(x^{2m}y^{2n} + x^{2n}y^{2m})$$

$$m, n = 0, 1, 2, \dots$$

$$\tilde{\Phi} = (1-x^2)(1-y^2) [a_1 + a_2(x^2+y^2) + a_3x^2y^2 + a_4(x^4+y^4) + \dots]$$

Case 1 $\Rightarrow N=1$
 $m=n=0$

$$u_{00} = (1-x^2)(1-y^2)(1.1 + 1.1) = 2(1-x^2)(1-y^2)$$

So let us take a very very simple case where m is equal to n is equal to 0 the first case. So the case 1 is equal to the number of basis function is going to be 1 so that means m is equal to n

is equal to 0. So we have that let us club this into this equation what we get is $u(0,0)$ is equal to $(1 - x^2)$ multiplied by $(1 - y^2)$ and putting m equal to n equal to 0 what you will get is this value will become 1 and this value will become 1 plus this value will become 1 multiplied by this value will become 1. So the entire thing will become 2 times $1 - x^2$ multiplied by $1 - y^2$. So this will be our basis function. Once we do that we can multiply this into our equation in a clever way by taking the inner product.

(Refer Slide Time: 08:24)

$$A_{11} = \langle Lu_1, u_1 \rangle = \int_{-1}^1 \int_{-1}^1 \left(\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} \right) u_1 dx dy$$

$$\int u v dv = \langle u, v \rangle$$

$$L = \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

So we will have the first coefficient which is given by $\langle Lu_1, u_1 \rangle$. Remember the inner product is defined as $\int u v dv$, if we say this is the inner product of u, v . So in this case it is going to be a double integral for minus 1 to 1, minus 1 to 1 for x and y . And the L operator in our case is equal to the Laplacian operator. So we can say the Laplacian operator is going to be $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. So we are going to write the Laplacian operator here and we are going to multiply it with u_1 put this into a bracket and you have u_1 which is going to come. Remember this is the way we define inner product and multiply it by $dx dy$. So this is going to be the first step.

(Refer Slide Time: 09:21)

$$A_{11} = \langle Lu_1, u_1 \rangle = \int_{-1}^1 \int_{-1}^1 \left(\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} \right) u_1 dx dy$$
$$\int u v dv = \langle u, v \rangle$$
$$L = \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$
$$u_1 = 2(1-x^2)(1-y^2)$$

And the next step is to introduce u_1 and we have found the value of u_1 in our previous step which is going to be 2 multiplied by (1 minus x square) multiplied by (1 minus y square). And I am going to differentiate this twice with respect to x and add it with the result I get after differentiating twice with respect to y square and multiply it with u_1 . And this is going to be again multiplied by $dx dy$ and that is what I am going to do exactly now.

There is some level of symmetry in the problem and the symmetry is here very clear that it goes from minus 1 to 1, minus 1 to 1 on both sides. So I can say 0 to 1 0 to 1 and multiply it by 2 twice. So that is what I am going to do here.

(Refer Slide Time: 10:14)

$$A_{11} = -8 \int_0^1 \int_0^1 (2-x^2-y^2)(1-x^2)(1-y^2) dx dy$$

So I am going to write it in a simplified form A_{11} is equal to minus 8 integral 0 to 1 integral 0 to 1 I have taken half of the integral and I am going to multiply it with 2. And I am going to

do that also for x and y. So the outside multiplier will be minus 8 and the result of differentiating with respect to x twice and adding it with the differentiation with respect to y twice and multiplying with u1 will lead to this form. So (2 minus x square minus y square) multiplied by u1 which is (1 minus x square)(1 minus y square) dx dy. So this is going to be the first thing and we will see in the Matlab code how we can reduce it and get the value for A 11. So this is going to be the first one.

(Refer Slide Time: 11:10)

$$A_{11} = -8 \int_0^1 \int_0^1 (2 - x^2 - y^2) (1 - x^2)(1 - y^2) dx dy$$

$$[A][x] = [B]$$

$$[A_{11}][a_1] = [g, u_1]$$

The second one will be the right hand side. So what we will get on the right hand side will be. So this is the first coefficient that we are computing A 11 and we will also have the first term on the right hand side. So remember this is if you are using 1 basis function the matrix of [A][X] equal to B will have only A11 term. And the x term is going to be [a1] is equal to [g1, u1]. Since g is constant here g is nothing but the value of Rho itself Rho 0. So its a constant we are putting as g 0, g 1, g 2, g 3.


(Refer Slide Time: 11:57)

$$A_{11} = -8 \int_0^1 \int_0^1 (2 - x^2 - y^2) (1-x^2)(1-y^2) dx dy$$

$$[A][x] = [B]$$

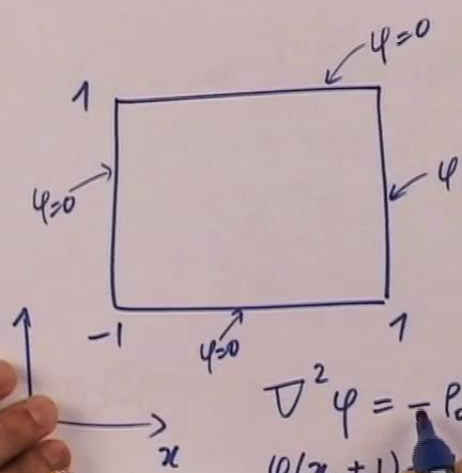
$$[A_{11}][a_1] = [g, u_1]$$

$$g = -\rho_0 \quad B_1 = \langle g, u_1 \rangle$$


$$= - \int_{-1}^1 \int_{-1}^1 (1-x^2)(1-y^2) \rho_0 dx dy$$


So here A_{11} is the first term and the only term that we have and we are interested in computing the right hand side term which is B term for B_1 we write it B_1 because we are in the first case with only one basis function N equal to 1 is equal to $\langle g, u_1 \rangle$. So this is going to be written as minus because g is minus ρ_0 in fact I forgot the minus sign we have to put a minus sign here. That is how we have defined the problem initially.

(Refer Slide Time: 12:48)



$$\nabla^2 \varphi = -\rho_0 = \text{constant}$$

$$\varphi(x, \pm 1) = \varphi(\pm 1, y) = 0$$


So the problem definition is the Poisson equation and the Poisson equation has a minus sign here.

(Refer Slide Time: 12:49)

$$A_{11} = -8 \int_0^1 \int_0^1 (2 - x^2 - y^2)(1-x^2)(1-y^2) dx dy$$

$$[A][x] = [B]$$

$$[A_{11}][a_1] = [g, u_1]$$

$$g = -\rho_0 \quad B_1 = \langle g, u_1 \rangle$$

$$= - \int_{-1}^1 \int_{-1}^1 (1-x^2)(1-y^2) \rho_0 dx dy$$

So that minus sign should be the value of g here so minus sign comes out. Minus 1 to 1 minus 1 to 1 (1 minus x square) (1 minus y square) Rho 0 dx dy. And when we simplify this what we get and we when we simplify this and apply the value of x and y what we get is B1 is equal to a value that we will compute in the program and we can also do that numerically by hand.

But we are going to do it in a Matlab code and I will show you how to do this.

So this is going to be for the first approximation. So now let us look at if I increase the value of n equal to 2 and n equal to 3 n equal to 4 so on and so forth.

(Refer Slide Time: 13:34)

$$\tilde{\varphi} = a_1 u_1 + a_2 u_2$$

$$u_{mn} = (1-x^2)(1-y^2)(x^{2m} y^{2n} + x^{2n} y^{2m})$$

$$m=n=0 \rightarrow u_1$$

$$m=n=1 \rightarrow u_2$$

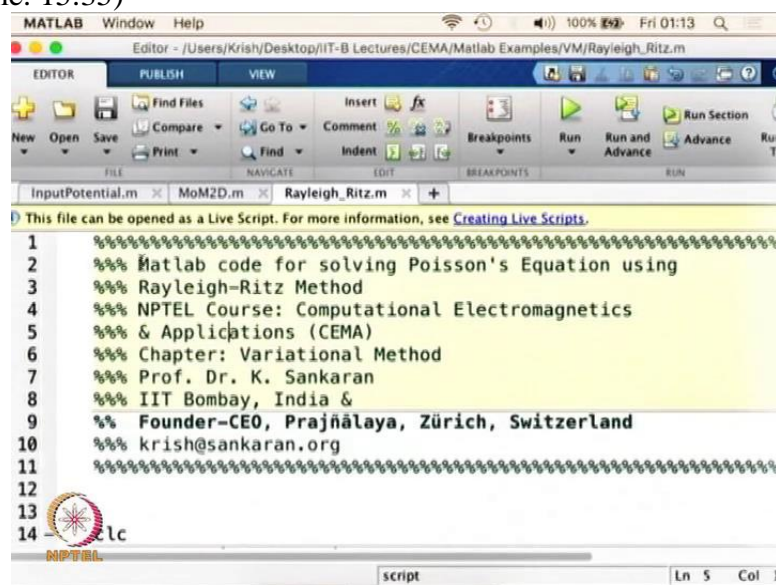
$$N=2 \begin{bmatrix} A_{11} & A_{12} & \dots \\ A_{21} & A_{22} & \dots \end{bmatrix} \quad A_{12} = A_{21}$$

So if n is equal to 2 what you will have is 2 basis functions. So the Phi tilde is equal to a1u1 plus a2u2. And you will compute the value of u2 from the general equation of u m,n which we described it as (1 minus x square) (1 minus y square) multiplied by x square m y square

m plus x square m y square m). So you are substituting the value for m equal to n equal to 1 and for the second case m equal to n equal to 0 for u1 and this is for u2.

So this is what we are going to do and we will get accordingly the different values for un and u2 and based on that we are going to compute not just 1 term but a series of term. The series of term will be [A 11, A 22, and here A 12 and A 21]. So this is the size of the matrix n equal to 2. The size of the matrix will increase for n equal to higher numbers having more and more terms on the matrix. And there is going to be some symmetry between the elements. So A 12 and A 21 should be same. So A 12 should be equal to A 21. Whereas the diagonal elements are going to be unique. And that is what we are going to see in the Matlab code.

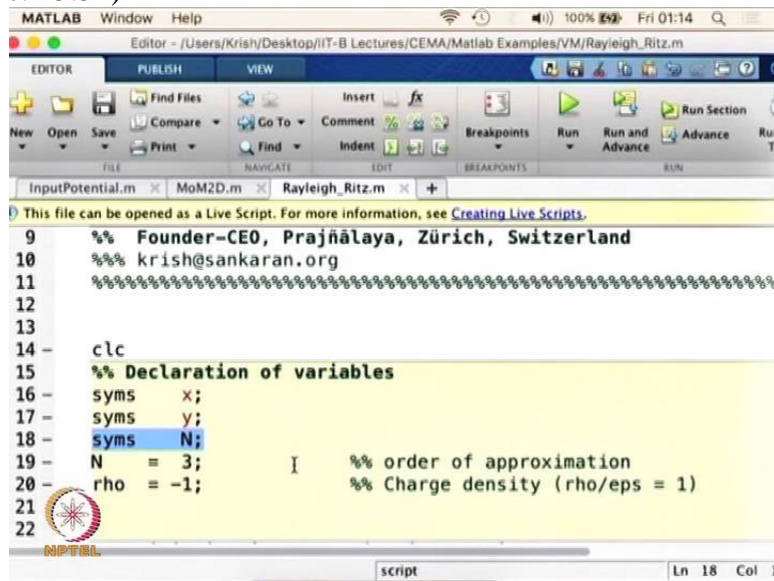
(Refer Slide Time: 15:35)



```
MATLAB Window Help
Editor - /Users/Krish/Desktop/IT-B Lectures/CEMA/Matlab Examples/VM/Rayleigh_Ritz.m
EDITOR PUBLISH VIEW
New Open Save Compare Go To Comment Breakpoints Run Run and Advance Run
Find Files Find Indent Breakpoints Run Run and Advance Run
FILE NAVIGATE EDIT BREAKPOINTS RUN
InputPotential.m MoM2D.m Rayleigh_Ritz.m
This file can be opened as a Live Script. For more information, see Creating Live Scripts.
1 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
2 %%% Matlab code for solving Poisson's Equation using
3 %%% Rayleigh-Ritz Method
4 %%% NPTEL Course: Computational Electromagnetics
5 %%% & Applications (CEMA)
6 %%% Chapter: Variational Method
7 %%% Prof. Dr. K. Sankaran
8 %%% IIT Bombay, India &
9 %%% Founder-CEO, Prajnālaya, Zürich, Switzerland
10 %%% krish@sankaran.org
11 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
12
13
14 NPTEL
script Ln 5 Col 1
```

So now let us go and see the Matlab code itself. So this is going to be the Matlab code that we are going to use we are going to use Matlab code for solving Poisson equation using Rayleigh Ritz method and we are going to use a specific tool box which is called as symbolic tool box.

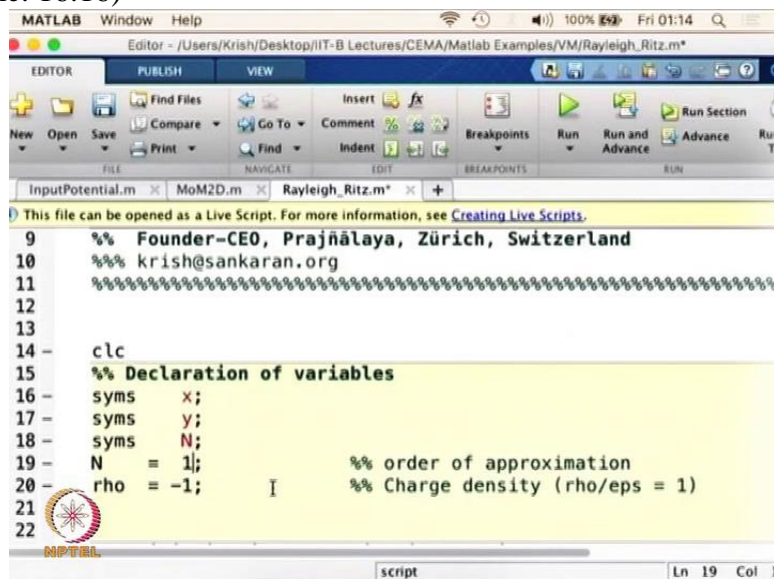
(Refer Slide Time: 15:54)



```
MATLAB Window Help
Editor - /Users/Krish/Desktop/IT-B Lectures/CEMA/Matlab Examples/VM/Rayleigh_Ritz.m
EDITOR PUBLISH VIEW
New Open Save Compare Go To Comment Insert fx Breakpoints Run Run and Advance Run Section
FILE NAVIGATE EDIT BREAKPOINTS RUN
InputPotential.m MoM2D.m Rayleigh_Ritz.m
This file can be opened as a Live Script. For more information, see Creating Live Scripts.
9 %% Founder-CEO, Prajñālaya, Zürich, Switzerland
10 %% krish@sankaran.org
11 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
12
13
14 - clc
15 %% Declaration of variables
16 - syms x;
17 - syms y;
18 - syms N;
19 - N = 3; I %% order of approximation
20 - rho = -1; %% Charge density (rho/eps = 1)
21
22
NPTEL
script Ln 18 Col 1
```

Some Matlab versions do not have the symbolic toolbox. If you do not have it you have to install it. It is very easy and elegant to do with symbolic toolbox that is why we are showing this. And for simple problems you can also do it differently but symbolic toolbox will be a very elegant thing because we wanted to see the expressions as well.

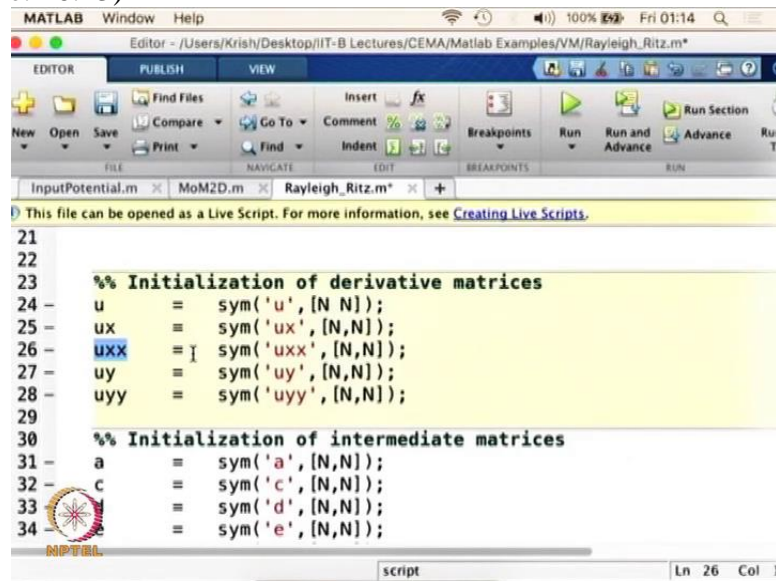
(Refer Slide Time: 16:16)



```
MATLAB Window Help
Editor - /Users/Krish/Desktop/IT-B Lectures/CEMA/Matlab Examples/VM/Rayleigh_Ritz.m*
EDITOR PUBLISH VIEW
New Open Save Compare Go To Comment Insert fx Breakpoints Run Run and Advance Run Section
FILE NAVIGATE EDIT BREAKPOINTS RUN
InputPotential.m MoM2D.m Rayleigh_Ritz.m*
This file can be opened as a Live Script. For more information, see Creating Live Scripts.
9 %% Founder-CEO, Prajñālaya, Zürich, Switzerland
10 %% krish@sankaran.org
11 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
12
13
14 - clc
15 %% Declaration of variables
16 - syms x;
17 - syms y;
18 - syms N;
19 - N = 1; %% order of approximation
20 - rho = -1; I %% Charge density (rho/eps = 1)
21
22
NPTEL
script Ln 19 Col 1
```

So for the first case we are going to put n equal to 1. IF we put n equal to 1.

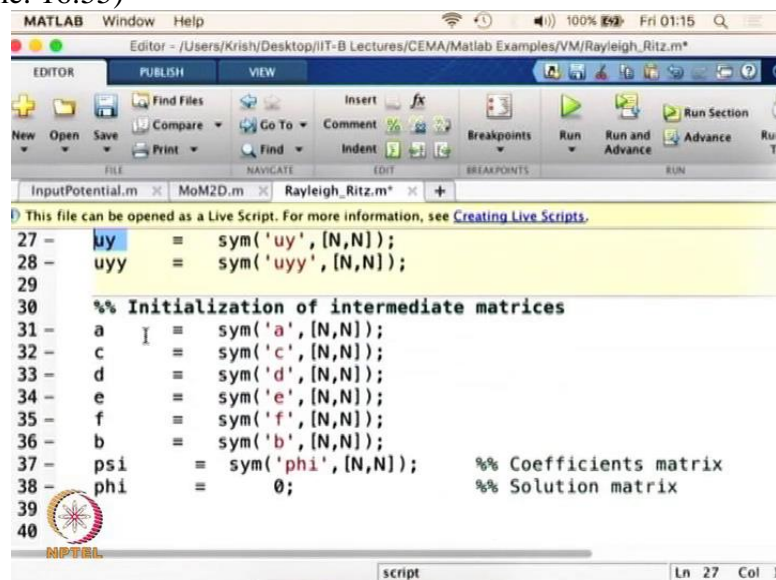
(Refer Slide Time: 16:23)



```
MATLAB Window Help
Editor - /Users/Krish/Desktop/IT-B Lectures/CEMA/Matlab Examples/VM/Rayleigh_Ritz.m*
EDITOR PUBLISH VIEW
New Open Save Compare Go To Comment Insert
Print Find Indent Breakpoints Run Run and Advance Run Section
FILE NAVIGATE EDIT BREAKPOINTS RUN
InputPotential.m MoM2D.m Rayleigh_Ritz.m*
This file can be opened as a Live Script. For more information, see Creating Live Scripts.
21
22
23 %% Initialization of derivative matrices
24 - u = sym('u', [N N]);
25 - ux = sym('ux', [N,N]);
26 - uxx = sym('uxx', [N,N]);
27 - uy = sym('uy', [N,N]);
28 - uyy = sym('uyy', [N,N]);
29
30 %% Initialization of intermediate matrices
31 - a = sym('a', [N,N]);
32 - c = sym('c', [N,N]);
33 - d = sym('d', [N,N]);
34 - e = sym('e', [N,N]);
script Ln 26 Col 1
```

And we are going to initialize the derivative matrix so the first derivative matrix the second derivative matrix because we need them in our definitions in our expressions. So if you see this part we are going to have both the derivatives. So we have to compute the second derivative. In order to compute the second derivative we have to compute the first derivative. And that is what we are doing here. So our goal is uxx and uyy. But to get uxx you have to derive ux and in order to derive uyy you have to get uy.

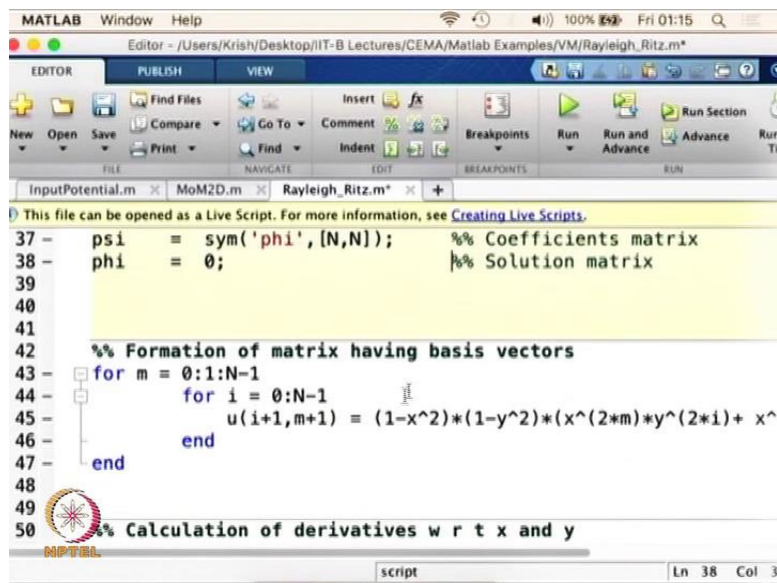
(Refer Slide Time: 16:55)



```
MATLAB Window Help
Editor - /Users/Krish/Desktop/IT-B Lectures/CEMA/Matlab Examples/VM/Rayleigh_Ritz.m*
EDITOR PUBLISH VIEW
New Open Save Compare Go To Comment Insert
Print Find Indent Breakpoints Run Run and Advance Run Section
FILE NAVIGATE EDIT BREAKPOINTS RUN
InputPotential.m MoM2D.m Rayleigh_Ritz.m*
This file can be opened as a Live Script. For more information, see Creating Live Scripts.
27 - uy = sym('uy', [N,N]);
28 - uyy = sym('uyy', [N,N]);
29
30 %% Initialization of intermediate matrices
31 - a = sym('a', [N,N]);
32 - c = sym('c', [N,N]);
33 - d = sym('d', [N,N]);
34 - e = sym('e', [N,N]);
35 - f = sym('f', [N,N]);
36 - b = sym('b', [N,N]);
37 - psi = sym('phi', [N,N]); %% Coefficients matrix
38 - phi = 0; %% Solution matrix
39
40
script Ln 27 Col 1
```

And you are initializing some of the intermediate matrices and the coefficient matrix is going to be c and the solution matrix is going to be Phi. Initially you set solution to be 0. So this is the way we start doing the problem and while we simulate the problem and get the solution. The solution will be different so initially we set it to 0.

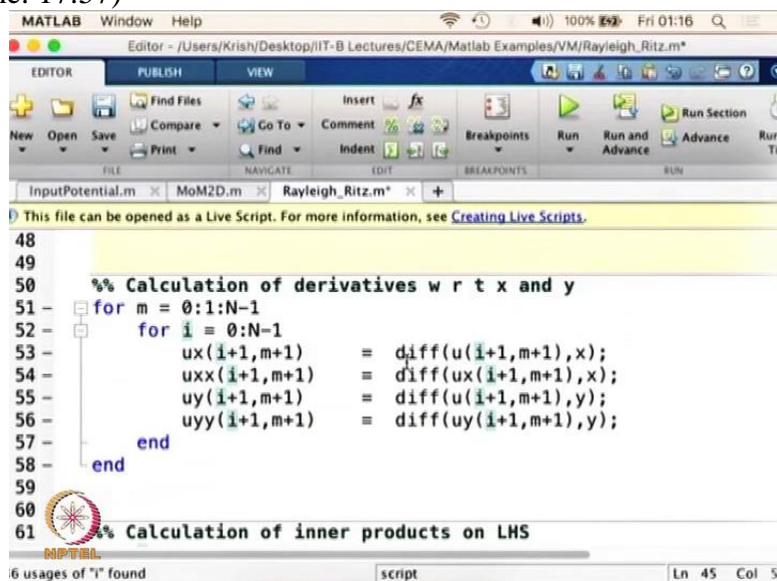
(Refer Slide Time: 17:23)



```
MATLAB Window Help
Editor - /Users/Krish/Desktop/IIT-B Lectures/CEMA/Matlab Examples/VM/Rayleigh_Ritz.m*
EDITOR PUBLISH VIEW
New Open Save Compare Go To Comment Breakpoints Run Run and Advance Run
FILE NAVIGATE EDIT BREAKPOINTS RUN
InputPotential.m MoM2D.m Rayleigh_Ritz.m*
This file can be opened as a Live Script. For more information, see Creating Live Scripts.
37 - psi = sym('phi',[N,N]); %% Coefficients matrix
38 - phi = 0; %% Solution matrix
39
40
41
42 %% Formation of matrix having basis vectors
43 - for m = 0:1:N-1
44 -     for i = 0:N-1
45 -         u(i+1,m+1) = (1-x^2)*(1-y^2)*(x^(2*m))*y^(2*i)+ x^
46 -     end
47 - end
48
49
50 %% Calculation of derivatives w r t x and y
NPTEL
script Ln 38 Col 3
```

So we are forming the matrices and the matrices are formed based on the equation we have (1 minus x square) (1 minus y square)(x power 2m) (y power 2i) and we are using i instead of n here because we are using n for a different variable so we are using i instead of n, so wherever there is i we have to substitute it in the derivation which we showed before as n.

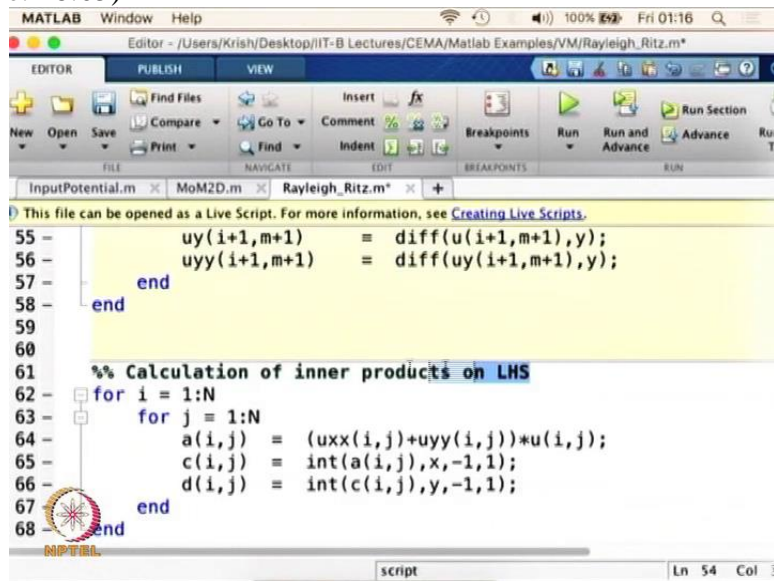
(Refer Slide Time: 17:57)



```
MATLAB Window Help
Editor - /Users/Krish/Desktop/IIT-B Lectures/CEMA/Matlab Examples/VM/Rayleigh_Ritz.m*
EDITOR PUBLISH VIEW
New Open Save Compare Go To Comment Breakpoints Run Run and Advance Run
FILE NAVIGATE EDIT BREAKPOINTS RUN
InputPotential.m MoM2D.m Rayleigh_Ritz.m*
This file can be opened as a Live Script. For more information, see Creating Live Scripts.
48
49
50 %% Calculation of derivatives w r t x and y
51 - for m = 0:1:N-1
52 -     for i = 0:N-1
53 -         ux(i+1,m+1) = diff(u(i+1,m+1),x);
54 -         uxx(i+1,m+1) = diff(ux(i+1,m+1),x);
55 -         uy(i+1,m+1) = diff(u(i+1,m+1),y);
56 -         uyy(i+1,m+1) = diff(uy(i+1,m+1),y);
57 -     end
58 - end
59
60
61 %% Calculation of inner products on LHS
NPTEL
6 usages of "i" found
script Ln 45 Col 5
```

And once you do that you do the derivation of it using inbuilt function called diff. So this is a Matlab function which allows you to do that.

(Refer Slide Time: 18:05)



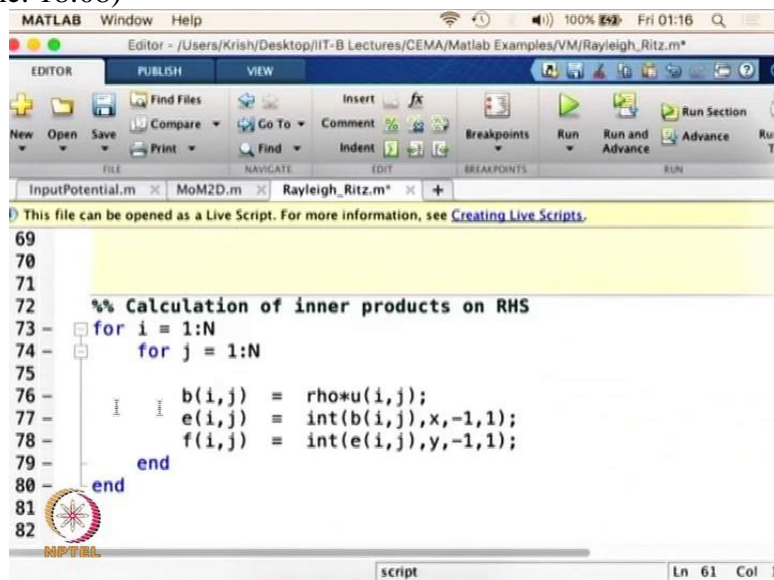
The image shows a MATLAB editor window with the following code:

```
55 -         uy(i+1,m+1) = diff(u(i+1,m+1),y);
56 -         uyy(i+1,m+1) = diff(uy(i+1,m+1),y);
57 -     end
58 - end
59
60
61 %% Calculation of inner products on LHS
62 for i = 1:N
63     for j = 1:N
64         a(i,j) = (uxx(i,j)+uyy(i,j))*u(i,j);
65         c(i,j) = int(a(i,j),x,-1,1);
66         d(i,j) = int(c(i,j),y,-1,1);
67     end
68 end
```

The status bar at the bottom indicates "Ln 54 Col 3".

And you are computing the inner products on the left hand side;

(Refer Slide Time: 18:08)



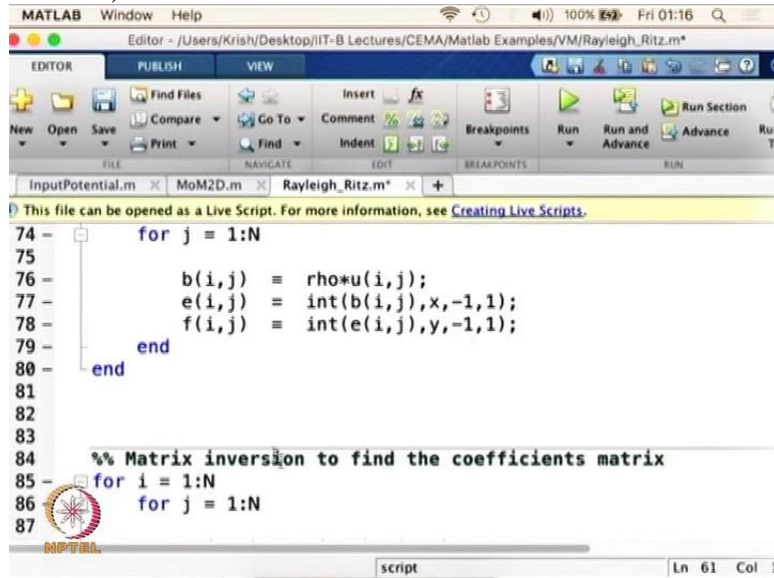
The image shows a MATLAB editor window with the following code:

```
69
70
71
72 %% Calculation of inner products on RHS
73 for i = 1:N
74     for j = 1:N
75
76         b(i,j) = rho*u(i,j);
77         e(i,j) = int(b(i,j),x,-1,1);
78         f(i,j) = int(e(i,j),y,-1,1);
79     end
80 end
81
82
```

The status bar at the bottom indicates "Ln 61 Col 1".

And you are computing the inner products on the right hand side as follows.

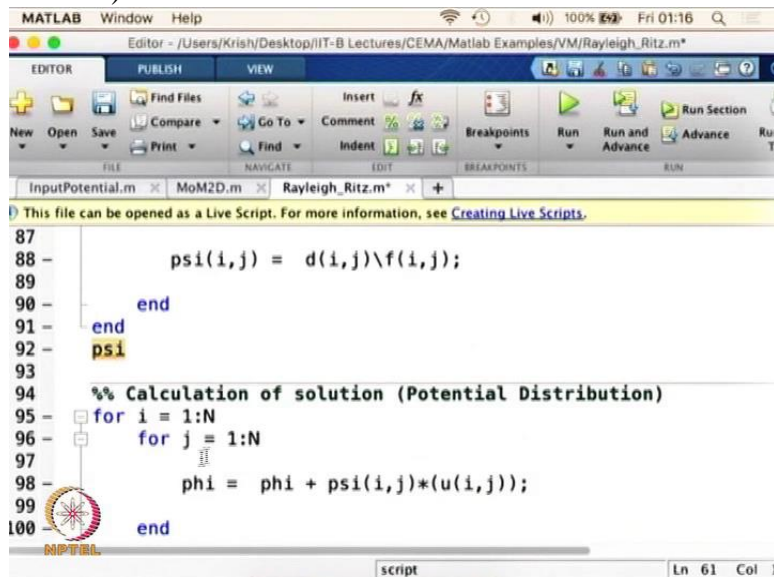
(Refer Slide Time: 18:10)



The image shows a MATLAB editor window with the following code:

```
74 - for j = 1:N
75 -     b(i,j) = rho*u(i,j);
76 -     e(i,j) = int(b(i,j),x,-1,1);
77 -     f(i,j) = int(e(i,j),y,-1,1);
78 - end
79 - end
80 - end
81
82
83
84 %% Matrix inversion to find the coefficients matrix
85 - for i = 1:N
86 -     for j = 1:N
87
```

(Refer Slide Time: 18:11)



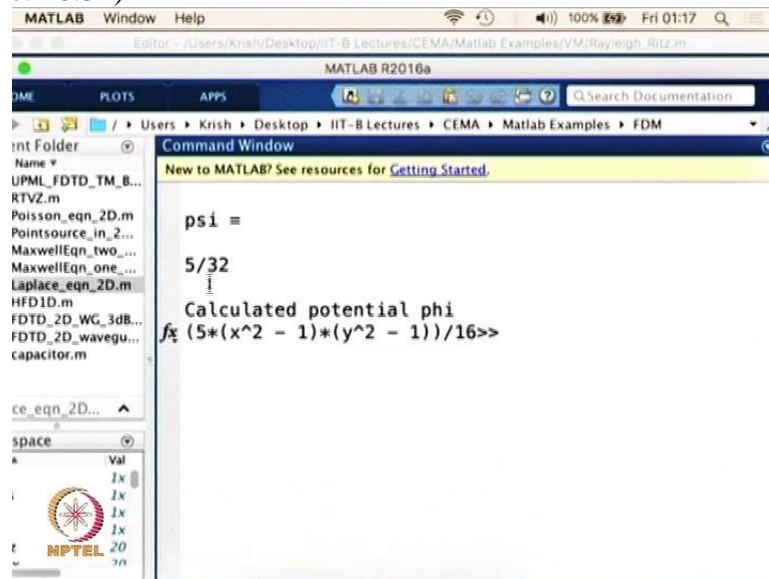
The image shows a MATLAB editor window with the following code:

```
87
88 -     psi(i,j) = d(i,j)\f(i,j);
89 - end
90 - end
91 - end
92 - psi
93
94 %% Calculation of solution (Potential Distribution)
95 - for i = 1:N
96 -     for j = 1:N
97 -         phi = phi + psi(i,j)*(u(i,j));
98 -     end
99 - end
100
```

And once you do that you do the matrix inversion to compute the value of psi and you compute the solution for the domain.

So this is a very straight forward and we are going to print the solution in the final step. So now we are going to do this for n is equal to 1.

(Refer Slide Time: 18:34)



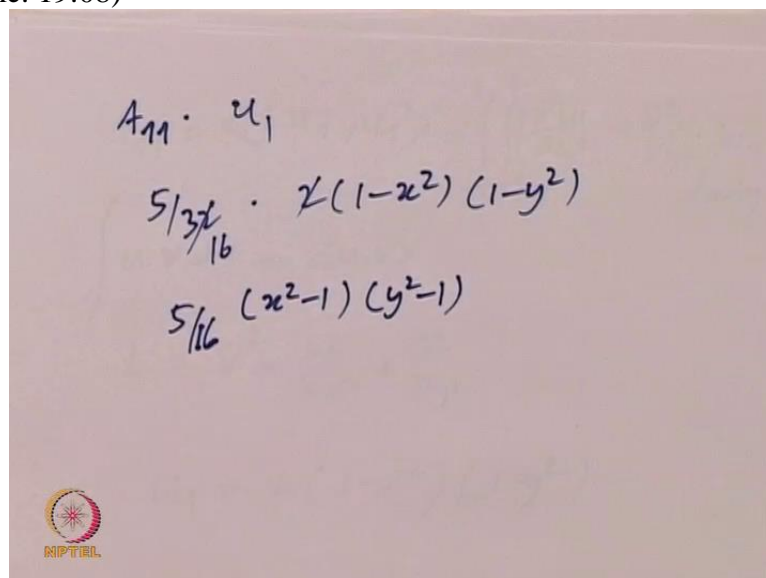
The image shows a MATLAB R2016a Command Window. The Command Window displays the following text:

```
psi =  
5/32  
Calculated potential phi  
fx (5*(x^2 - 1)*(y^2 - 1))/16>>
```

So let us run it and let us go back to the code itself here. What you see is you get 5 by 32 as a value and the potential itself is going to have this function.

So what it means is the Phi value is going to depend on the value that we are going to compute here 5 by 32 is a coefficient that we have which is coming here as 5 by 16 because do not forget the value which we have chosen for u_1 is going to be.

(Refer Slide Time: 19:08)



The image shows a handwritten derivation of the potential function. The equations are:

$$A_{11} \cdot u_1$$
$$\frac{5}{32} \cdot 2(1-x^2)(1-y^2)$$
$$\frac{5}{16} (x^2-1)(y^2-1)$$

So let me explain this. So the coefficient is A_{11} multiplied by the value of u_1 that we are having. So here the value of A_{11} is going to be 5 by 32 and u_1 is going to be the value that we have which is 2 multiplied by (1 minus x square) (1 minus y square) 2 and 32 it goes away it becomes 5 by 16 . And if we swap it 2 x square minus 1 there will be a minus sign and this minus sign goes here and then we swap it one more time and gets cancelled. So we will get $\frac{5}{16} (x^2 - 1)(y^2 - 1)$.

So that is what a solution what you get on the Matlab program here. $5 \text{ by } 16 (x^2 \text{ square minus } 1) (y^2 \text{ square minus } 1)$.

(Refer Slide Time: 20:08)

```

18 - syms N;
19 - N = 2;           %% order of approximation
20 - rho = -1;       %% Charge density (rho/eps = 1)
21
22
23 %% Initialization of derivative matrices
24 - u = sym('u', [N N]);
25 - ux = sym('ux', [N,N]);
26 - uxx = sym('uxx', [N,N]);
27 - uy = sym('uy', [N,N]);
28 - uyy = sym('uyy', [N,N]);
29
30 %% Initialization of intermediate matrices
31 - a = sym('a', [N,N]);
  
```

Now we will do it for n equal to 2. So for n equal to 2 you will have 4 values in the Matrix of A so when we run it.

(Refer Slide Time: 20:22)

```

psi =

[ 5/32, 105/352]
[ 105/352, 147/352]

Calculated potential phi
fx (5*(x^2 - 1)*(y^2 - 1))/16 + (105*(x^2 + y^2)*(x^2
  
```

So we will see that there are four values the first value is the same value that we had before $5 \text{ by } 32$ which is good and we are getting another value which is $147 \text{ divided by } 352$. And as I explained before A_{12} which is $105 \text{ divided by } 352$ is the same as A_{21} which is also $105 \text{ divided by } 352$.

(Refer Slide Time: 20:51)

$A_{11} = \frac{5}{32}$
 $A_{22} = \frac{147}{352}$
 $A_{12} = A_{21} = \frac{105}{352}$

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

In other words I write it again A 11 A 22 A 12 and A 21. This is going to be the matrix and we are getting A 11 in the first case and also in the second case as 5 by 32. We are getting A 22 as 147 divided by 352 and we are getting A 12 is equal to A 21 is equal to 105 divided by 352. And this is what we are seeing in the matrix here.

(Refer Slide Time: 21:26)

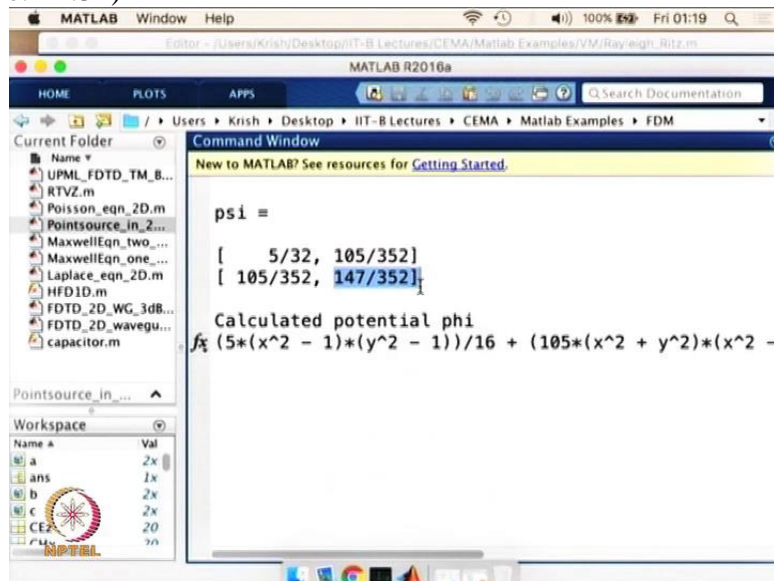
```

psi =
[ 5/32, 105/352]
[ 105/352, 147/352]

Calculated potential phi
phi = (5*(x^2 - 1)*(y^2 - 1))/16 + (105*(x^2 + y^2))*(x^2 - 1)
    
```

In the C matrix in the equation and as we had before we are going to have one term because of this element

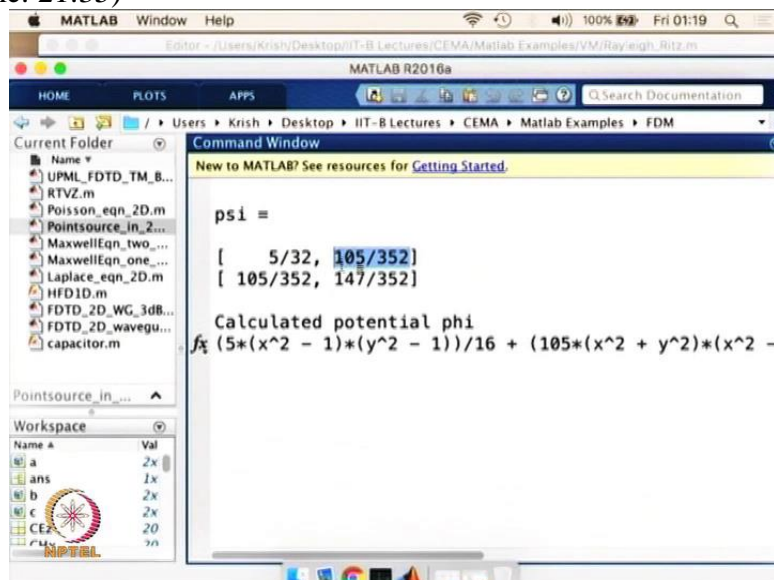
(Refer Slide Time: 21:32)



```
psi =  
[ 5/32, 105/352]  
[ 105/352, 147/352]  
  
Calculated potential phi  
∫ (5*(x^2 - 1)*(y^2 - 1))/16 + (105*(x^2 + y^2))*(x^2 -
```

One term because of this element

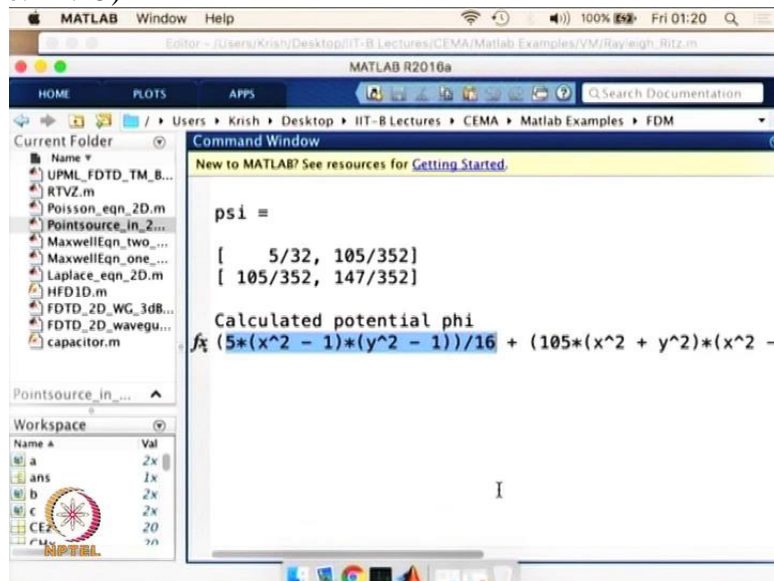
(Refer Slide Time: 21:35)



```
psi =  
[ 5/32, 105/352]  
[ 105/352, 147/352]  
  
Calculated potential phi  
∫ (5*(x^2 - 1)*(y^2 - 1))/16 + (105*(x^2 + y^2))*(x^2 -
```

And one term because of this element because it is getting repeated so it will get added up. So we will have totally three.

(Refer Slide Time: 21:45)

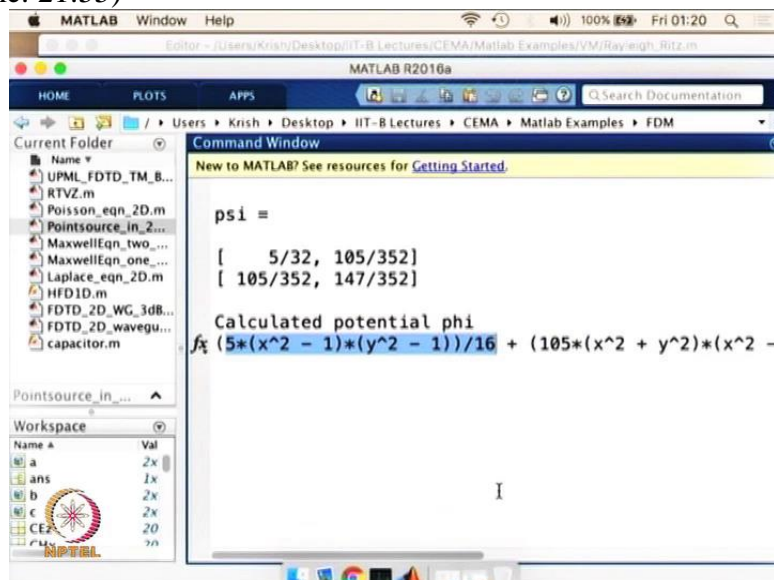


The screenshot shows the MATLAB R2016a Command Window. The workspace contains variables 'a' (2x), 'ans' (1x), 'b' (2x), 'c' (2x), 'CE' (20), and 'r' (?). The Command Window displays the following output:

```
psi =  
[ 5/32, 105/352]  
[ 105/352, 147/352]  
  
Calculated potential phi  
 $\int_0^x (5*(x^2 - 1)*(y^2 - 1))/16 + (105*(x^2 + y^2))*(x^2 -$ 
```

So this is the first term and this is the second term so until here is a second term

(Refer Slide Time: 21:55)

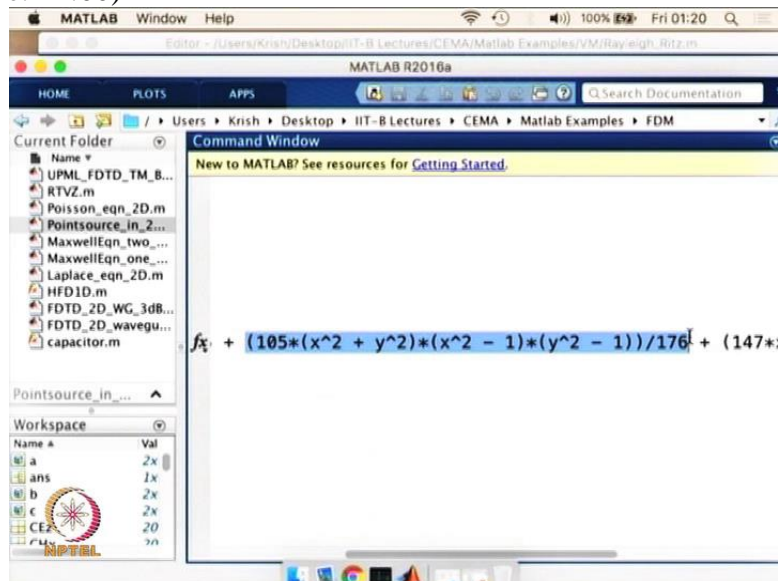


The screenshot shows the MATLAB R2016a Command Window. The workspace contains variables 'a' (2x), 'ans' (1x), 'b' (2x), 'c' (2x), 'CE' (20), and 'r' (?). The Command Window displays the following output:

```
psi =  
[ 5/32, 105/352]  
[ 105/352, 147/352]  
  
Calculated potential phi  
 $\int_0^x (5*(x^2 - 1)*(y^2 - 1))/16 + (105*(x^2 + y^2))*(x^2 -$ 
```

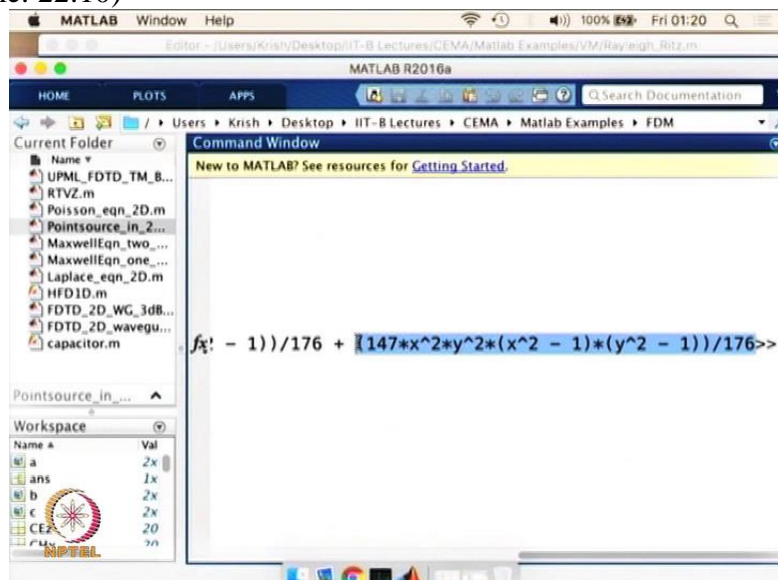
This is the first term

(Refer Slide Time: 22:00)



This is the second term

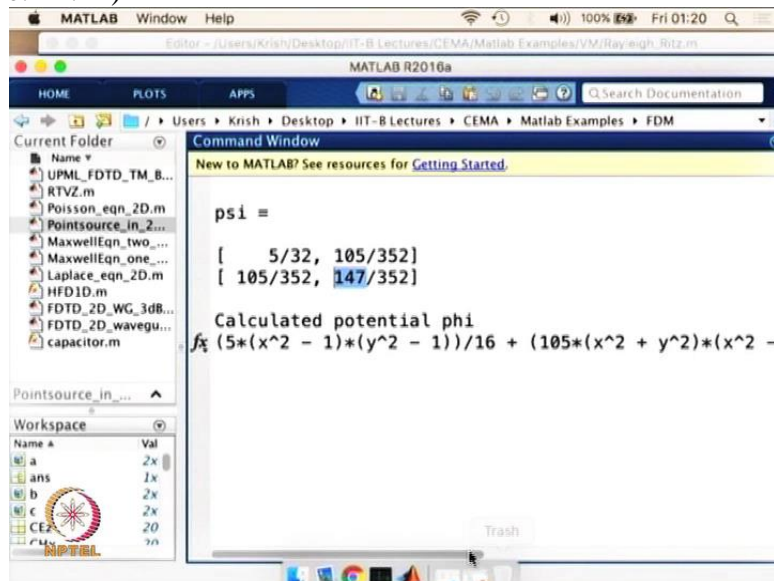
(Refer Slide Time: 22:10)



And you have the third term which is going to be from here until the end of the equation.

So this is the third term

(Refer Slide Time: 22:22)



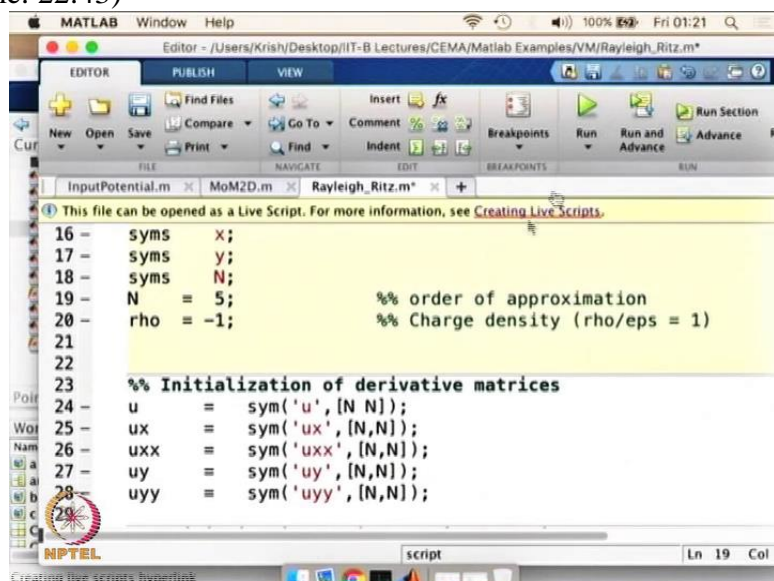
```
psi =  
[ 5/32, 105/352]  
[ 105/352, 147/352]  
  
Calculated potential phi  

$$\int_{\mathbb{R}^2} (5*(x^2 - 1)*(y^2 - 1))/16 + (105*(x^2 + y^2))*(x^2 -$$

```

And the numerators have to be same as here and the denominators will be divided by 2 because there is a numerator 2 from u n definition itself. So anyway so this is the way we have computed for n equal to 2. And you can go in higher orders. The beauty of doing it with symbolic tool box is it allows you to have different orders.

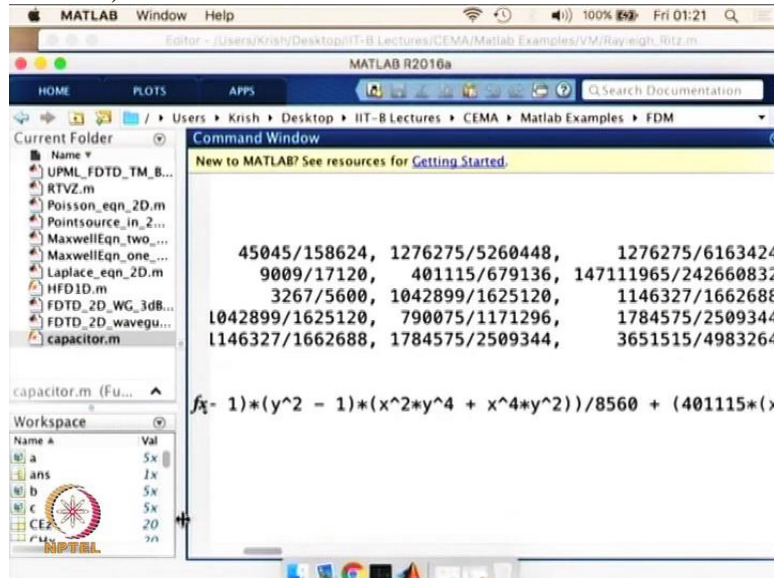
(Refer Slide Time: 22:43)



```
16 - syms x;  
17 - syms y;  
18 - syms N;  
19 - N = 5;           %% order of approximation  
20 - rho = -1;       %% Charge density (rho/eps = 1)  
21  
22  
23 %% Initialization of derivative matrices  
24 - u = sym('u', [N,N]);  
25 - ux = sym('ux', [N,N]);  
26 - uxx = sym('uxx', [N,N]);  
27 - uy = sym('uy', [N,N]);  
28 - uyy = sym('uyy', [N,N]);
```

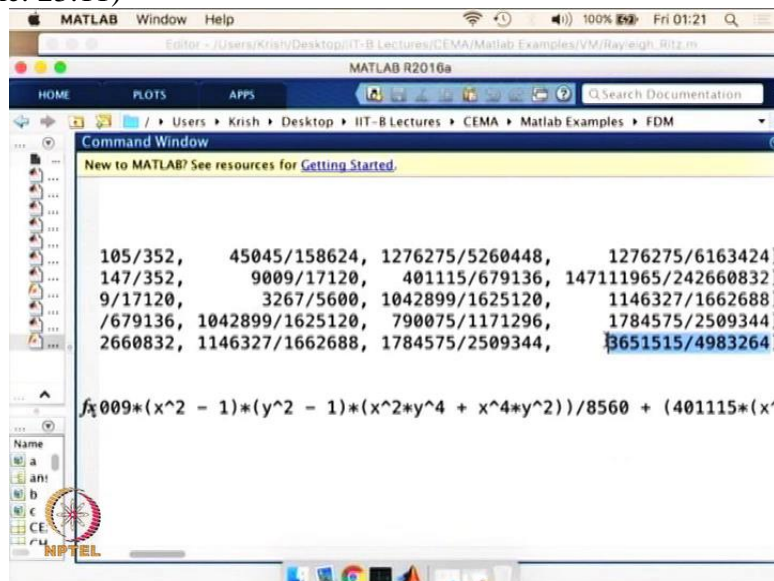
So for example if you have n equal to 5 you simulate the program what you will get is 5 terms on the leading axis for c.

(Refer Slide Time: 23:05)



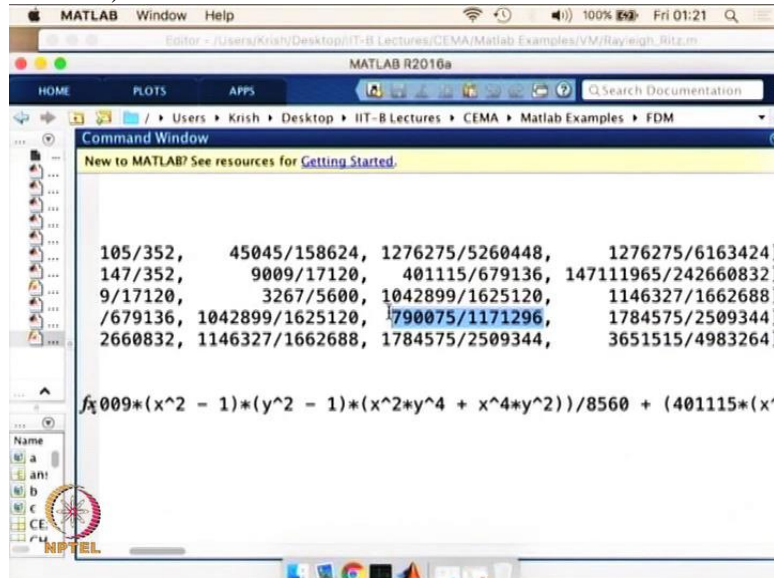
So one term here one term here one term here one term here and one term here. So these are the five terms.

(Refer Slide Time: 23:11)



First this is the last term

(Refer Slide Time: 23:14)



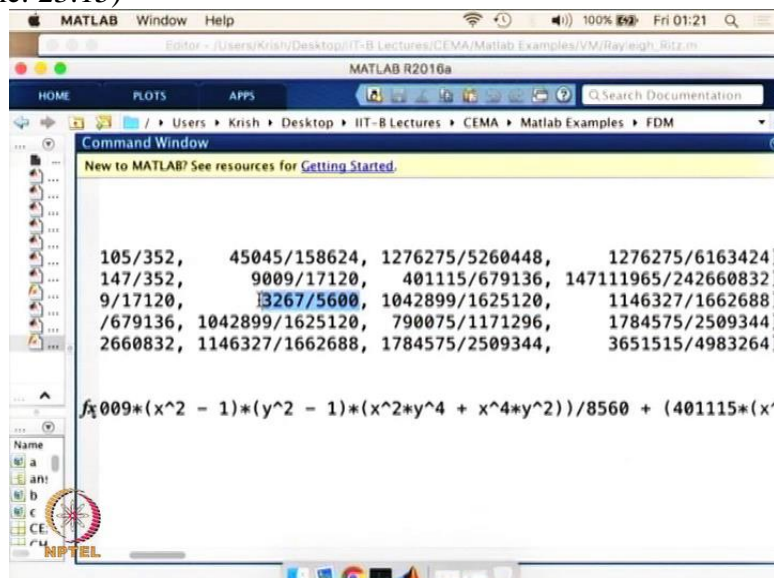
The screenshot shows the MATLAB R2016a Command Window. The window title is 'MATLAB R2016a'. The path is '/Users/Krish/Desktop/IIT-B Lectures/CEMA/Matlab Examples/VM/Rayleigh_Ritz.m'. The Command Window displays a 4x4 matrix of rational numbers:

105/352,	45045/158624,	1276275/5260448,	1276275/6163424]
147/352,	9009/17120,	401115/679136,	147111965/242660832]
9/17120,	3267/5600,	1042899/1625120,	1146327/1662688]
/679136,	1042899/1625120,	790075/1171296,	1784575/2509344]
2660832,	1146327/1662688,	1784575/2509344,	3651515/4983264]

Below the matrix, the start of an equation is visible: $f_{009}(x^2 - 1)(y^2 - 1)(x^2*y^4 + x^4*y^2)/8560 + (401115*(x^2$

And this one term above,

(Refer Slide Time: 23:15)



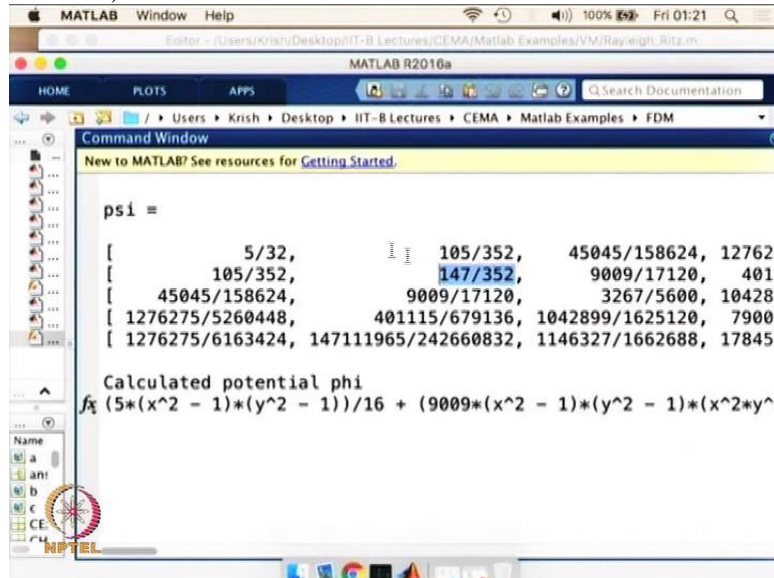
The screenshot shows the MATLAB R2016a Command Window. The window title is 'MATLAB R2016a'. The path is '/Users/Krish/Desktop/IIT-B Lectures/CEMA/Matlab Examples/VM/Rayleigh_Ritz.m'. The Command Window displays a 4x4 matrix of rational numbers:

105/352,	45045/158624,	1276275/5260448,	1276275/6163424]
147/352,	9009/17120,	401115/679136,	147111965/242660832]
9/17120,	3267/5600,	1042899/1625120,	1146327/1662688]
/679136,	1042899/1625120,	790075/1171296,	1784575/2509344]
2660832,	1146327/1662688,	1784575/2509344,	3651515/4983264]

Below the matrix, the start of an equation is visible: $f_{009}(x^2 - 1)(y^2 - 1)(x^2*y^4 + x^4*y^2)/8560 + (401115*(x^2$

This is the third term

(Refer Slide Time: 23:19)



```
MATLAB R2016a
Command Window
New to MATLAB? See resources for Getting Started.

psi =

[      5/32,      105/352,  45045/158624, 1276275/5260448,
 [      105/352,      147/352,      9009/17120,  401115/679136,
 [  45045/158624,      9009/17120,      3267/5600, 1042899/1625120,
 [ 1276275/5260448,  401115/679136, 1042899/1625120,  79009/1662688,
 [ 1276275/6163424, 147111965/242660832, 1146327/1662688, 17845/1662688]

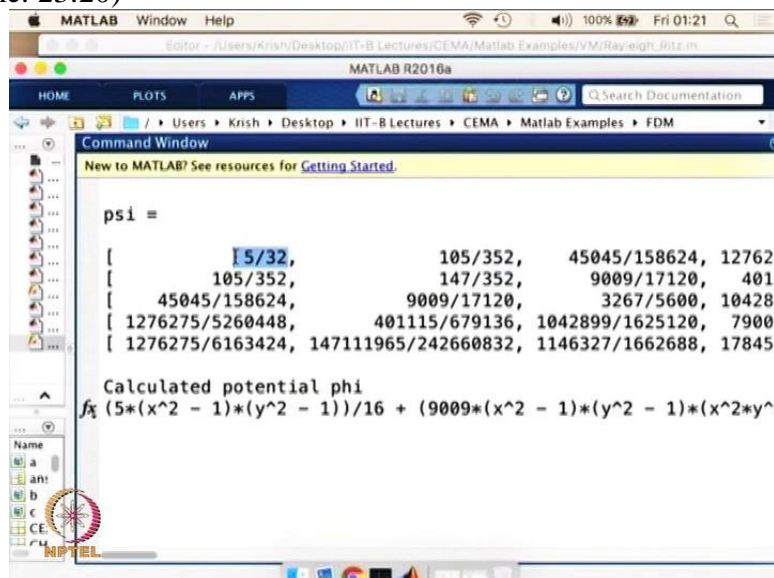
Calculated potential phi

$$\phi = \frac{5(x^2 - 1)(y^2 - 1)}{16} + \frac{9009(x^2 - 1)(y^2 - 1)(x^2 y^2)}{16}$$

```

And this is the fourth term

(Refer Slide Time: 23:20)



```
MATLAB R2016a
Command Window
New to MATLAB? See resources for Getting Started.

psi =

[      5/32,      105/352,  45045/158624, 1276275/5260448,
 [      105/352,      147/352,      9009/17120,  401115/679136,
 [  45045/158624,      9009/17120,      3267/5600, 1042899/1625120,
 [ 1276275/5260448,  401115/679136, 1042899/1625120,  79009/1662688,
 [ 1276275/6163424, 147111965/242660832, 1146327/1662688, 17845/1662688]

Calculated potential phi

$$\phi = \frac{5(x^2 - 1)(y^2 - 1)}{16} + \frac{9009(x^2 - 1)(y^2 - 1)(x^2 y^2)}{16}$$

```

And this is the fifth term.

So these are the leading diagonal terms and you can see 5 by 32, 105 by 32, 105 by 352, and those terms are repeating and that is how it should repeat because we are going higher in number of basis function whereas the lower basis function values will get repeated.

So what we have done in this problem is we have simulated standard Poisson Equation using a very useful toolbox called as symbolic tool box from Matlab and we have simulated it for very many basis functions. The reason for going for symbolic toolbox is to allow you to create a general program and you can do it for different number of basis function.

And we have showed for this case please take the code and try it for yourself and as I said before you need a specific toolbox which is a symbolic toolbox which is something you can

download for Matlab. And try it out for yourself how one can use the in built functionalities cleverly to solve such simple problems. Thank you!!