Computational Electromagnetics and Applications
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Lecture No 13
Boundary Conditions

So today we are going to cover one of the most important aspect of numerical simulation domain truncation in every simulation they play an important role the accuracy of the results that one can get using any numerical method is really dependent on domain truncation techniques in that sense they normally play a major role in restricting the computational domain to finite space but also greatly affect the accuracy of the results so one has to really pay attention to what kind of dormant truncation techniques that we are using and also how one can improve the accuracy of those dumb and truncation techniques.

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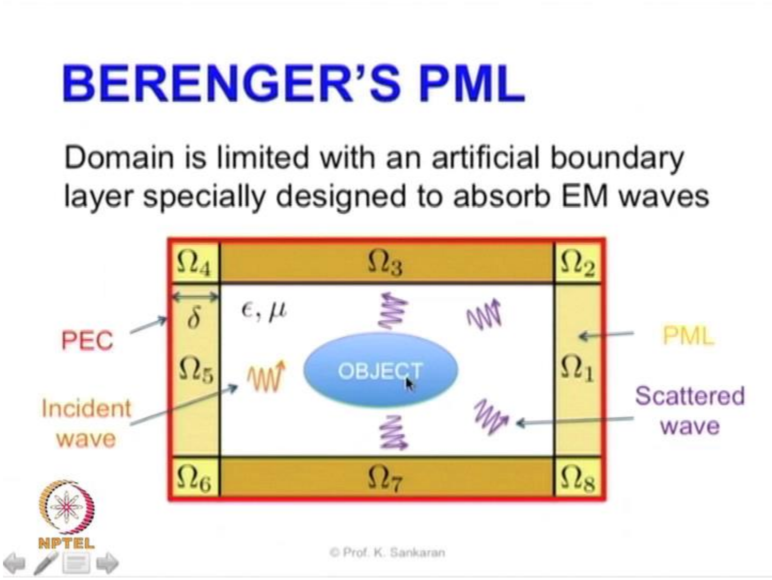

When I introduce finite difference method I said in 1994 it was the greatest innovation in Domain truncation technique that cleared the biggest bottleneck that the finite difference method was facing in that sense the perfectly matched layer otherwise called as PML is one of the most important innovation that happened in numerical techniques in general particularly in a finite difference method we have talked about PML in larger aspects using simulation so on and so forth but we haven't looked into them using a particular example so I am going to take the next out of to discuss formulation and also to present a simulation that one can Re test and try for himself and one can see one can implement the PML so the kind of PML that I am going to look into in today's module is called as uniaxial PML.

Normally when one talks about PML one talks about the Benergen PML the classical split field formulation of PML that was brought into the finite difference technique by the French engineer called Benergen however the method has developed over a period of time and due to several of those techniques been brought into the numerical methods the methodology of the PML itself is a vast area so we cannot discuss each and every PML.

Of course I would like to give you some references that you can go and look comparing pm else and also understanding the formulations and understanding their pros and cons in today's lecture I am going to focus on uniaxial PML that is due to Stephen get me which was published in 1996 I would like to show how one can understand the formulation of the uniaxial PML using a simple 2D Maxwell equation formulation and also I would like to show how one can for men at the PML and implement those PML in matlab environment so that is going to be abroad area that we are going to look into with that being said it's going to be a quite intense session but also an interesting session because once you get to know how to modeller PML.

So many different problems that one can really solve and it itself is a motivation forest to get started and look into the formulation and rest assured I am going to be there with you step-by-step discussing each and every step and manipulation that one has to do and also make sure that you are really able to formulate them in a Matlab environment so that is going to be my promise and with that Todays lets start todays module.

So let's start looking at a simple Maxwell equation in 2D when I say Maxwell equation I am going to focus on transfers magnetic mode so in that sense my magnetic field Rs components are going to be in the xy plane which is going to be also the plane of propagation and my electric field is going to be in the Z direction which is going to be in the perpendicular direction to the direction of propagation on the plane of propagation let's look at the equation and start step by step in the formulation.

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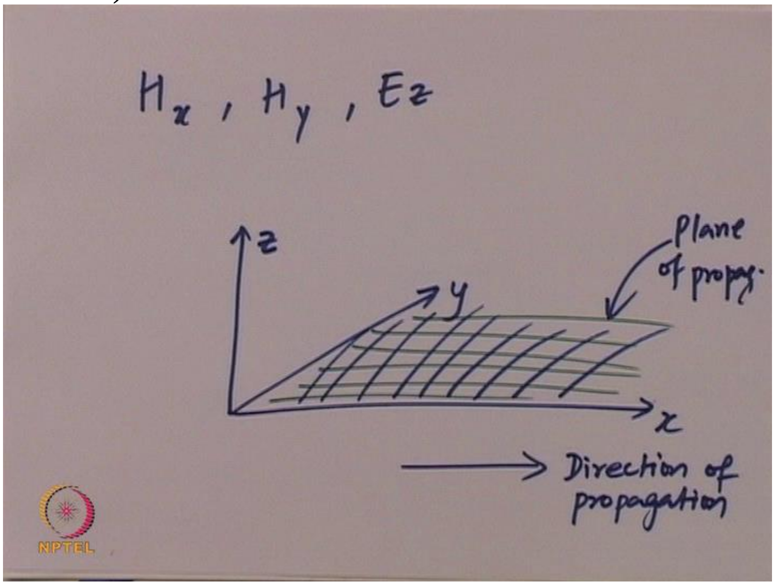

So we have got H x , H y, and E z that being said so the plane of propagation is going to be the x-y plane and the Z direction is going to be the perpendicular direction to the propagation if I say X is going to be the direction of propagation and xy plane so the entire plane here is going to be the plane of propagation few returns will argue with you can call this as a plane of propagation but I will stick to the term. What I mean is the propagation is going to stick to the xy plane and Z is the direction that is perpendicular to both the direction of propagation and the plane of propagation so this is going to be our setup point with that being said so the Maxwell equation for such a setup is going to have a very simple form which is given by the two curl equations.

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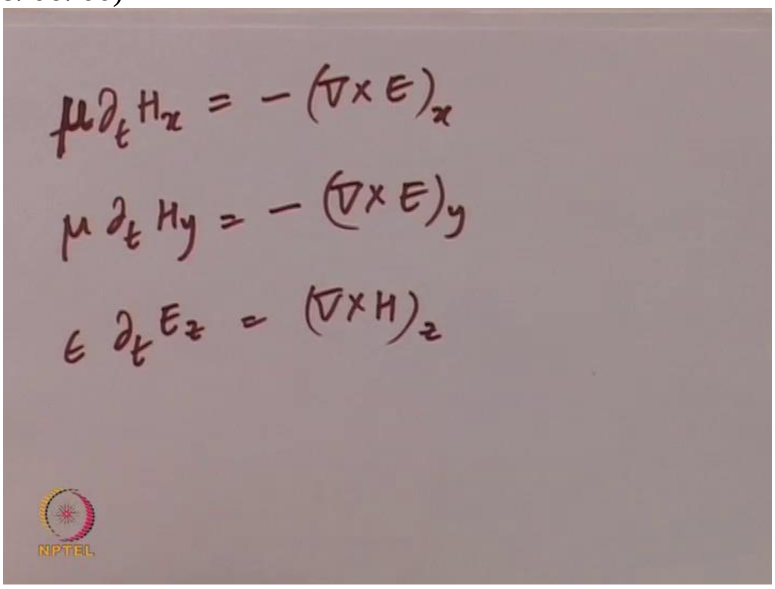

So we have Doe t H x is equal to minus(curl of E) x component of course we have the Mu term here. And similarly the Mu term by Doe t of H y is equal to minus(Del cross E) y term here. And the Epsilon Doe t E z is equal to minus(Del cross H) z term.

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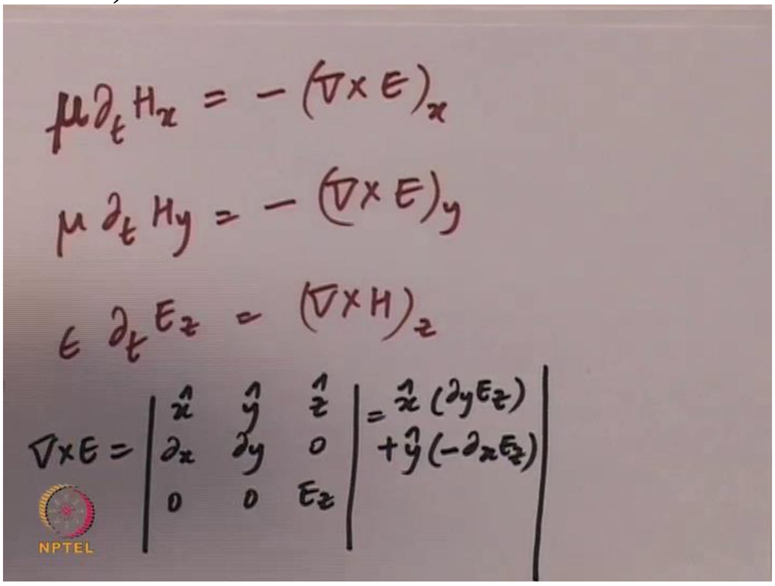

So let’s look at this individual terms step by step you are going to start with the E term So the Del cross E is equal to x y z you have Doe x Doe y that is not going to be any variation in the right direction we can put them to be zero and we have 0 0 Z if we expand it what you get is X component Doe y E z plus the Y component minus Doe x E z. So this is going to be the delicacy term and similarly the Del cross H term.

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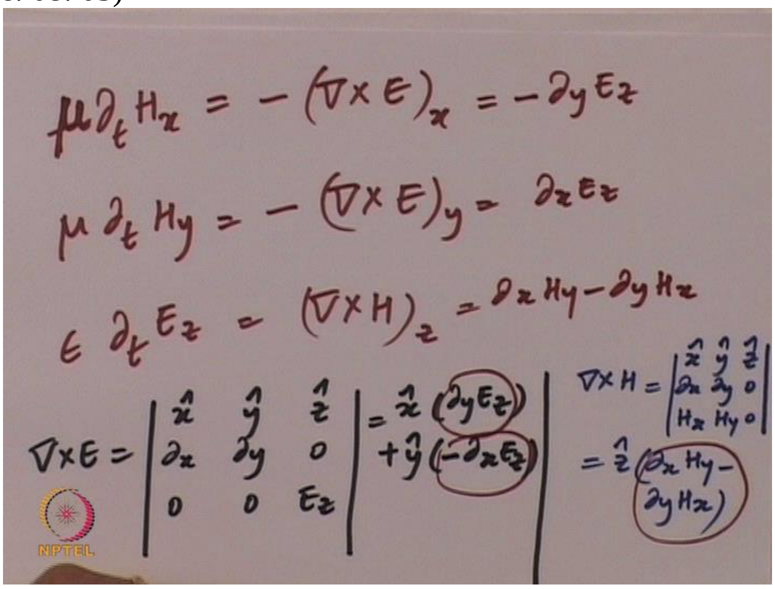

So Del cross H term is going to be Del cross H term is equal to similarly x y z Doe x Doe y 0 and H x H y 0. When you do that what you get is X component will be zero why component will be zero you get only the Z component which is given by Doe x H y minus Doe y H x) so now I am going to plug all these components into the equation so what we have got is for the first equation we take the first component that’s what I have written here so this is going to be this term. Of course there is a minus sign so I am going to write it as minus Doe y E z. For the second one I am going to write this term the minus sign and minus sign gets cancelled we get Doe x E z. And for the third one this is going to be the entire time is equal to Doe x H y minus Doe y H x. So these are going to be the field components and these field components are going to be the main elements that we are going to take for modelling the universal PML so let’s look now into the uniaxial PML setup.

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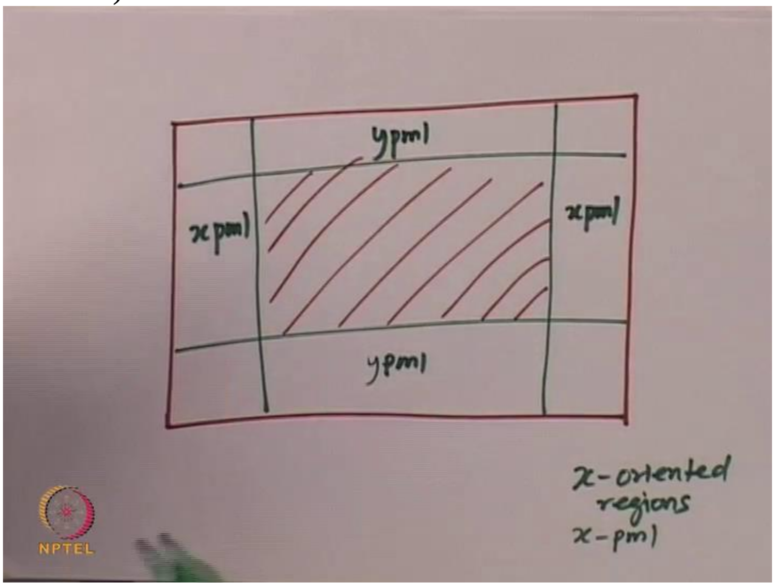

A domain that is rectangle so in this kind of a domain the uniaxial PML or any PML for the matter of fact has to be sitting in a way given below. You will have top and bottom PML you have the right and left PML as you can see real computational domain is going to be the entire domain although the entire domain is called as a computational domain the object of simulation will be mostly sitting in this area so we will be keeping all the objects that care modelling in this area but let’s look into the PML itself now. So this going to be the domain and what you can see is there going to be several reasons the first region is the Axe oriented regions they will be called as x PML similarly there is going to be 5 p.m. l so this is an x PML. Similarly y oriented PML are y PML and top PML are also y PML.

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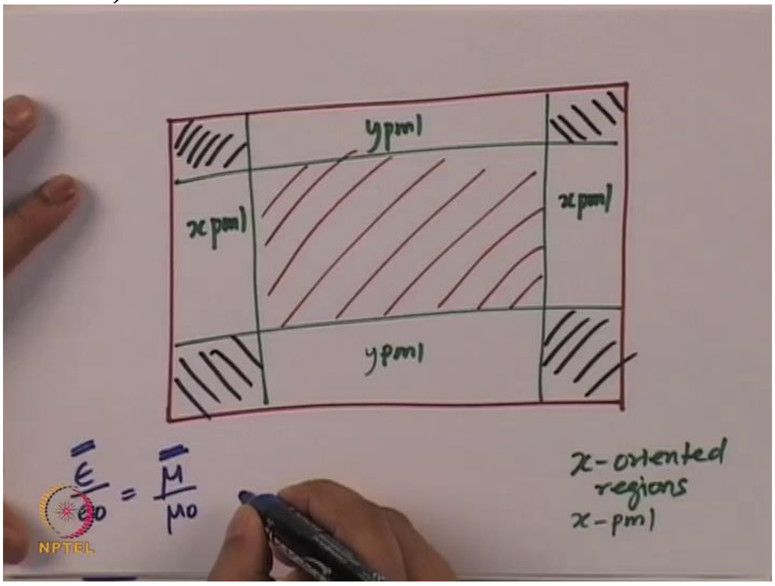

However there are also for mixed DML so these reasons so I am going to Mark them in black so these are Areas where we have both x and y p m l so why am I splitting the PML in the in this way is the question that needs to be answered the reason for splitting the PML in this way this because the material composition of those regions that we are having in the PML domain are going to have a special property and the property is called as an isotropic property what I mean by that is for x oriented PML the permittivity and permeability are going to be not a scalar quantity like Del cross H term or mu but they are going to be value that is a tensor. So there are tensor because the relative permittivity which is given by the value Del cross H term divided by Epsilon 0 and Mu. Mu 0 is going to be a complex tense are also so these are the relative permittivity and relative permeability they are going to follow a pattern which is a an isotropy pattern.

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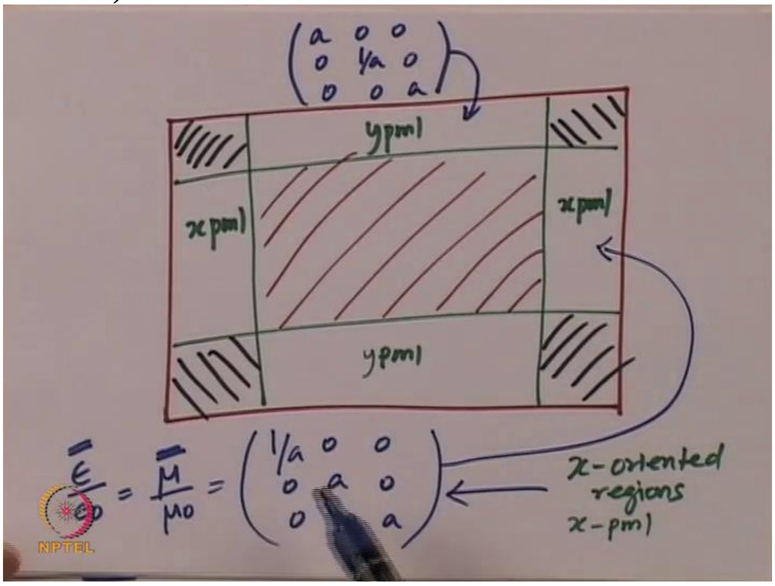

If it is a x oriented PML we are going to have a form like this(0 0 0 a 0 0 0 a). I will come to what a is but it is important to know both the relative permittivity and relative permeability are going to have a special form this is for the X oriented PML where is for the Y oriented PML they are going to have a very different form it is going to be (a 0 0 0 1 by a 0 and 0 0 a). So this is going to be for the Y oriented PML. So this is going to be for the x oriented PML.

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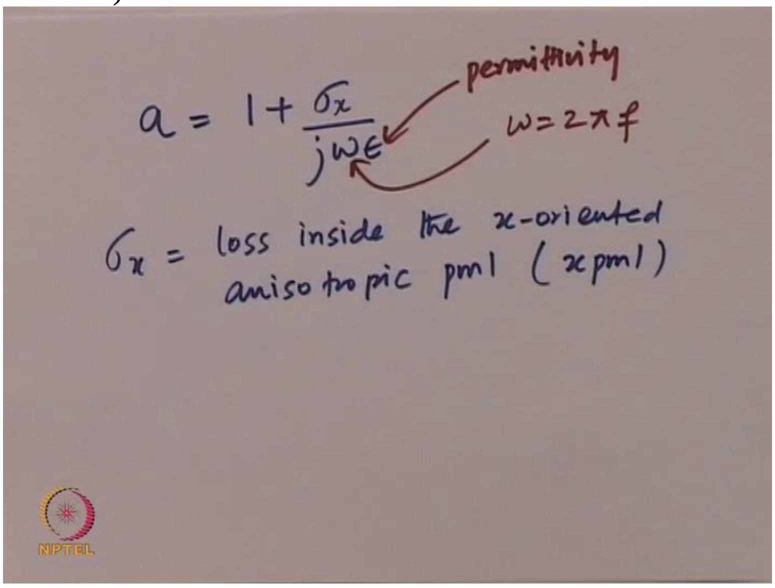

As you can see the value 1 by a is going to change its location along the leading diagonal so with that being said let's look into what is this a and how can we understand its implications in modelling a perfectly matched layer using the universal concept. So the a value what we call it as frequency dependent term it is a complex value and its value is going to be given by the equation a is equal to 1 plus sigma divided by j omega Epsilon. And of course if it is going to be an x oriented PML it is going to have a subscript X here if it is going to be a y oriented PML it is going to have a subscript for Sigma y.

So let's now look at it only from a x oriented PML point of view we can similarly think of counterpart for Y oriented PML so when a is equal to 1 + Sigma x / J Omega Del cross H term where Sigma x is a loss factor or lost inside the X oriented anisotropic PML. So when we have the form like this what we also see is it is going to depend on the frequency where Omega is equal to 2 Pi f. It is also going to depend on the permittivity inside the medium. So there are going to be lot of factors it is going to depend on the loss it is going to depend on the frequency it is going to depend on the permittivity.

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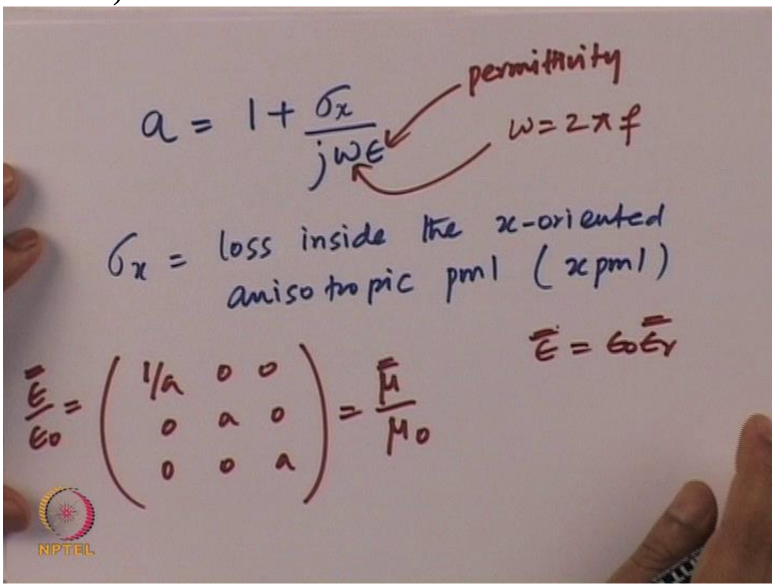

So now let's take this equation equal to 1 plus Sigma x divided by Omega Del cross H term and plug it inside the equation what we have got. So I will get the value for Epsilon divided by Epsilon Not equal to (1 by a 0 0 0 A 0 0 0 a) and that is also we going to equal to Mu divided by a 0. Don't forget excellent double bar on the top is going to be given by Epsilon 0 Epsilon r Where Epsilon r is also a tensor and this is a tensor we are talking about so now we are going to multiply this particular form to get an equation that we are going to model for the universal PML

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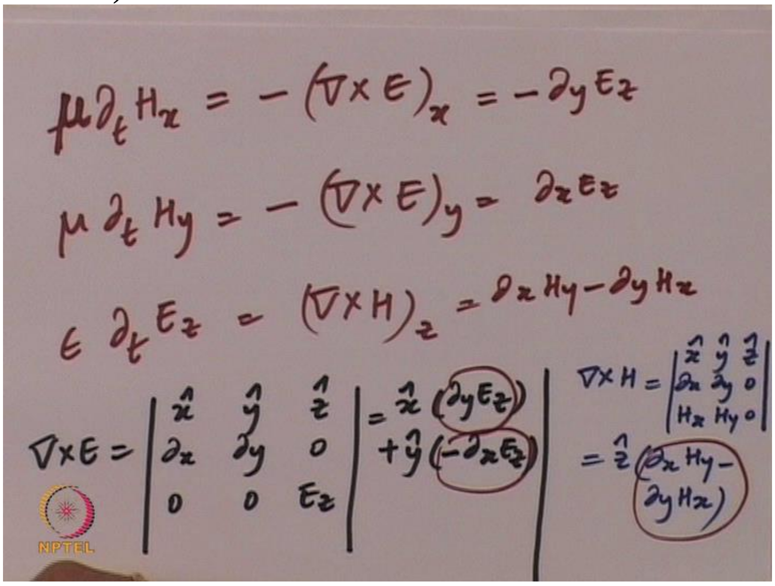

So let's plug this one into the equation what we have earlier derived and start deriving step by step the uniaxial PML formulation.

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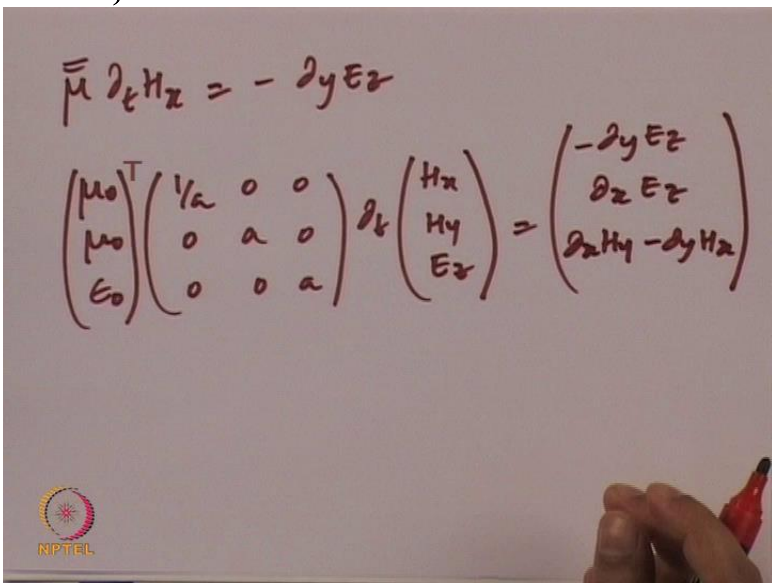

Let's start with the basic equation. So we have got Mu dt H x equal to minus dy E z and I am going to plug in the value for new as I said new is not going to be the complex number so it's going to be so now I am going to multiply Mu 0 Mu 0 Epsilon 0 into 1 by a 0 0 0 a 0 0 0 a into dt multiplied by the term H x H y E z equal to the terms of the right hand side what we have got from this equation so the first time will be going to be equal to minus dy E z second time is going to be d x E z third time is going to be d x H y minus d y H x.

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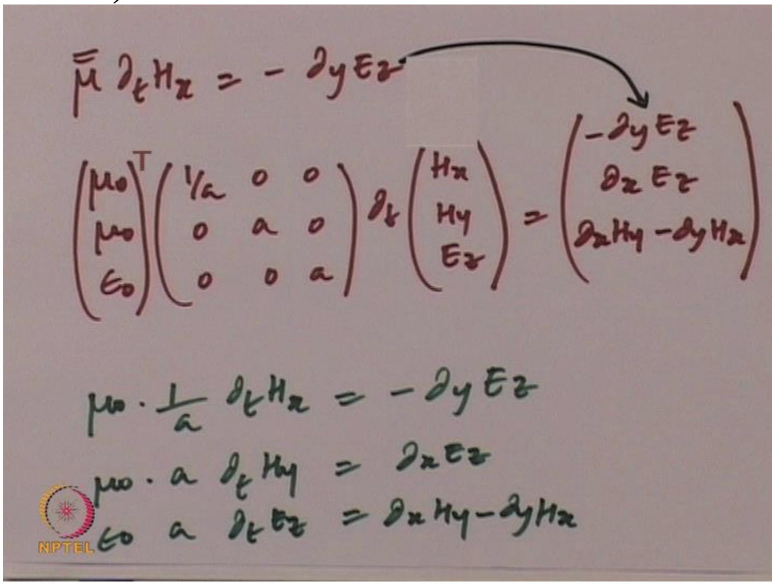

So what you are going to do is we are going to go step by step engineering this equation for example we can start with the first equation and we can move to the second equation so on and so forth the first equation is exactly this equation what we have got here so now we can write this equation in an expanded form and and see what we get so what we get is Mu 0 multiplied by 1 by a dt H x equal to minus dy E z similarly the second one will be Mu 0 multiplied by a dt H y equal to d y E z, the third one will be Epsilon 0 a dt E z equal to d x H y minus d y H x.

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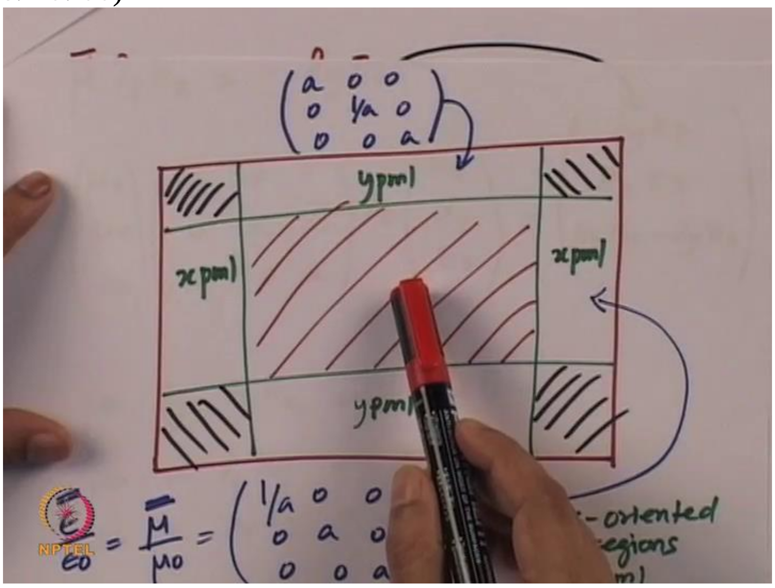

So you might wonder why I am I multiplying this for the Maxwell equation so the Maxwell equation the curl equations are only this why am I multiplying this particular time I am doing that because the Maxwell equation is only valid in this domain where I am working as read when we go into any of these PML domain the Maxwell equation will follow the anisotropy model that we have accepted so this is going to be the permittivity and permeability values so because of this if I have to model PML and model the Maxwell equation in the PML I have to inevitable start with the setup we have given here.

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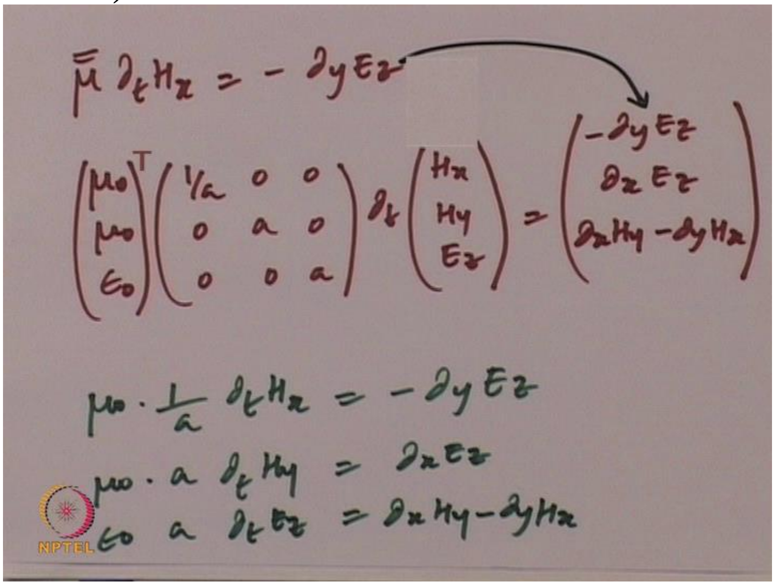

So let us stop here we will come back and derive the equation step by step as we have guessed it here and how you can take it to the Matlab model as we go so this is going to be the next step we will look at it in the next module thank you!