Computational Electromagnetics and Applications
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Lecture No. 11
Boundary Conditions

In the previous module we looked into one dimensional and two dimensional absorbing body conditions as I said during those lectures that absorbing body conditions quality is going to depend on the angle of incidence in which the impinging wave is going to see the boundary conditions so for example we are at a normal incidence you see that the reflection is going to be ideally zero where is in the two dimensional and three dimensional problem it's very difficult to get an exact boundary condition like we got in the case of a one-dimensional absorbing body condition so that being said since the Inception of the final difference method that has been a lot of development in the method itself still the absorbing conditions was always a challenging thing and people resolved this by putting the boundary at a far away distance from the scatterer .

So when you put the boundary at a faraway distance what happens is your impinging wave or the incoming wave will see the boundary almost like a normal incident that being said for that to happen we have to put the boundary at a very very far away distance that is going to increase the computational cost because we are going to simulate a larger space instead of a smaller space so this was the problem of the finite difference since its inception.

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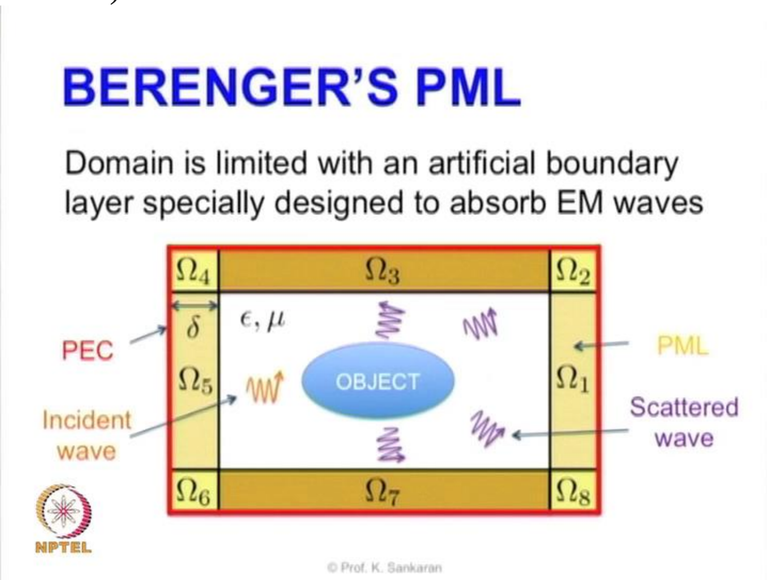

But that change in the year 1994 when the French scientist named Berenger introduced a phenomenal method for a perfectly matched layer which instead of putting the boundary at a very far away distance you can basically put the boundary very close to the scatterer or the object of our interest but by adapting the parameter inside the layer you are going to match the layer to the actual domain of interest in that sense the the layer is going to be perfectly matched to the free space or the domain that we are interested in.

Let's look into the perfectly matched layer in this particular lecture because it’s going to be very important for you to know this particular technique because we will use this more and more in other methods as well so initially when we are introducing this we are going to introduce this using finite difference method that we have learnt so far in our mind but we will apply this method later on also for other methods so let’s start looking at the perfectly matched layer with finite difference method in our mind .

Let's now look into Berenger’s perfectly matched layer I am calling this as Berenger PML for a simple reason . So in the year 1994 when Berenger introduced this was the only pml that was available so he didn’t call it Berenger PML for obvious reasons but since the Inception of the perfectly matched layer there has been so many different perfectly matched layers that came into existence that we will see those perfectly matched layers at a later stage . For now we will only look into the initial idea of Berenger of introducing the perfectly matched layer for a finite difference formulation.

So let’s look into the domain we are interested in assume that we are interested in modelling a domain consisting of an object and there are certain incident waves and there are certain scattered waves we are going to truncate this domain instead the boundary we are going to truncate it using certain layers and obviously you see here we have different types of layers we have layer1, layer 2, layer 3, layer 4, layer 5, layer 6, layer 7 and layer 8. You will see what are the similarities and dissimilarities between these layers later on but for now it is enough to know these are the layers that we are interested in and we are going to truncate the layer itself using a perfect electric conductor. And the thickness of this layer is going to be ideally delta and we will see what this Delta is going to be at a later stage.

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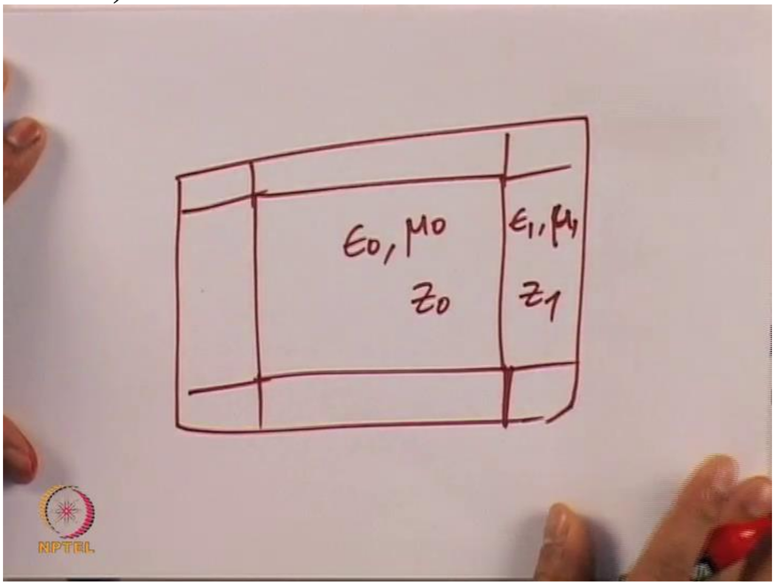

So as I said we want to have a perfect matched layer in that sense let us see that you have a domain and you have certain layers surrounding the domain as I said and you have certainly as surrounding the domain as i said there are going to be certain differences in these domains Epsilon knot Mu 0 and this particular layer is going to be epsilon 1 , Mu 1. And as I told you the inferences are going to be perfectly matched in that sense Z not should be equal to Z 1.

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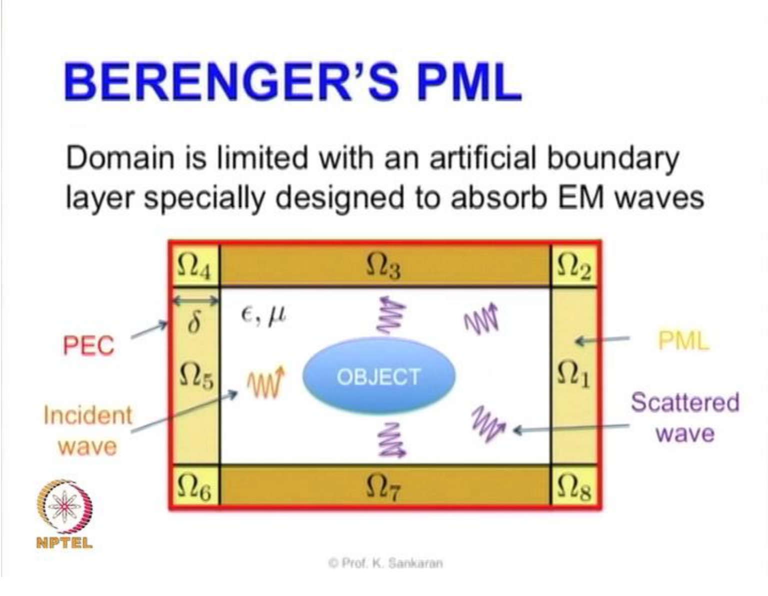

So let's see what is going to be the mathematical implication of this or in fact the physical implication of this condition.

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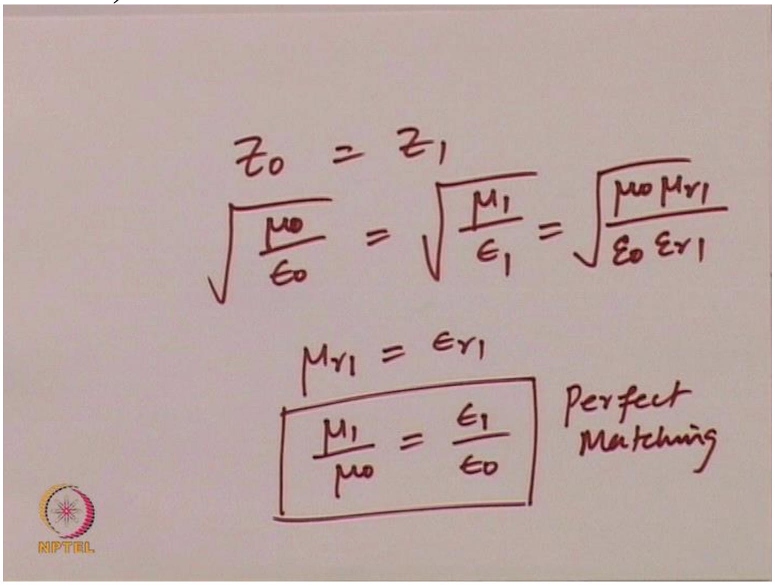

So when we say Z 0 is equal to Z 1 we are saying square root of Mu divided by Epsilon 0 should be equal to square route of Mu 1 divided by Epsilon 1 equal to square root of Mu 0 Mu r 1 divided by Epsilon 0 Epsilon r 1. Now we can see due to this condition we get Mu r 1 equal to Epsilon r 1 in other words what will happen is you will have Mu1 divided by Mu 0 should be equal to Epsilon 1 divided by Epsilon 0. This is the condition for perfect matching between the layers. So that being said let's look add two dimensional TM case for modelling this problem.

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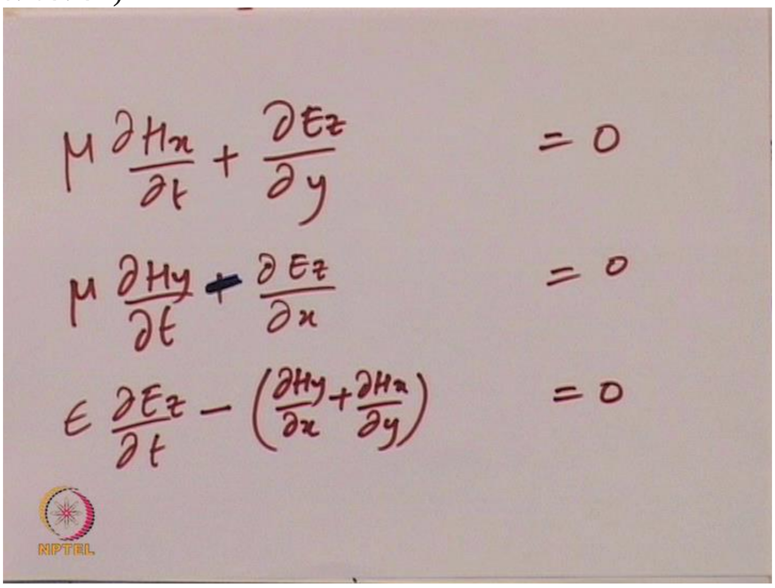

So we will start with ATM case for the system presented by these three equations we will have Mu d h x by dt + d E z by d y equal to zero. You will have Mu dH y divided by dt minus dE z divided by dx equal to zero and we have the third equation Epsilon d e z divided by dt minus dH y by dx plus dH x divided by d y equal to zero.

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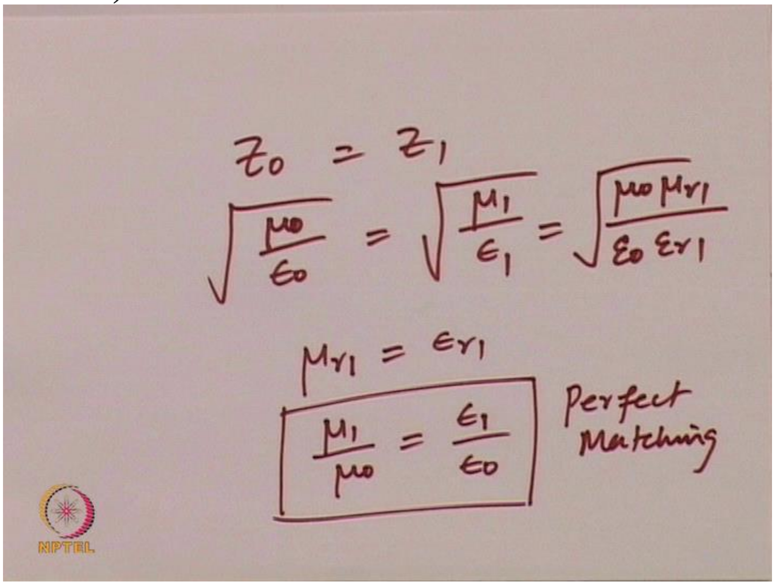

The thing about perfectly matched layer is the not only the impedances have matched as in the case we have set here we are also going to have certain losses so we want the wave to see no difference between the medium where it is coming from and the perfectly matched layer. So that way we are able to see that the wave will go through without reflection.

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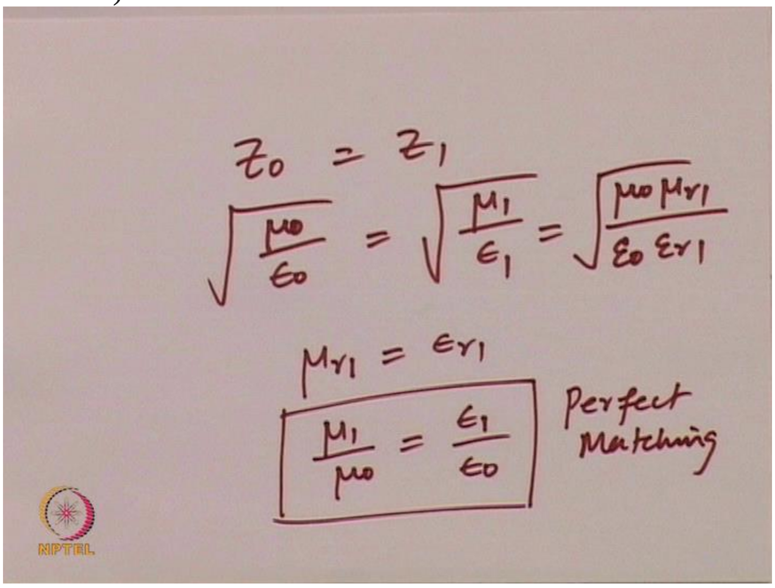

But we also want the layer to have certain losses so that the incoming wave will get absorbed by the layer. In that sense whatever is coming inside will move without reflection and while going inside the layer it can absorbed. How it is going to absorb what is the absorption position to be we will see that later on but for you to know there are going to be certain absorption that is going to happen within the layer.

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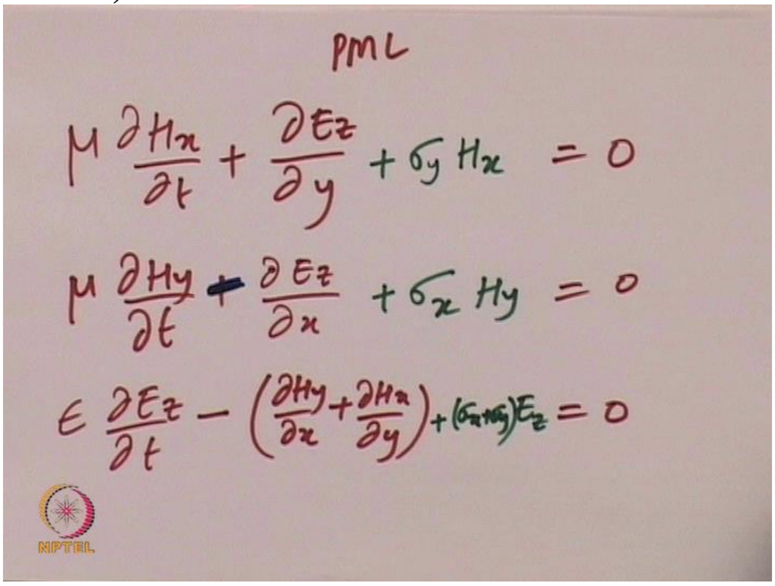

So if we have a equation like this represented by the TM formulation of two dimensional Maxwell equation we wanted to have this absorption within the perfectly matched layer one way to get the absorption is to have certain losses within the layer itself. So if this particular equation is going to be the equation for the perfectly matched layer. We can have the losses inside the layer as the additional term that we are going to add for this particular equation. So that is going to be the loss terms that we are going to have. Since it is going to have a H x we are having Sigma H x and since we have a flux component which is Y component we are going to have Sigma Y here. Similarly since it is aH y component we will have plus Sigma h y and since the flux component is a dx component we will have a loss component Sigma x.

Similarly here since it is a EZ component we will have the EZ value here. And since it has both X and Y flux components we will have Sigma X plus Sigma y. So what we have done now is basically we have got expression for the Maxwell equation within the PML using the perfectly matched condition and also the losses. So it is going to allow the wave to come in inside and it is going to absorb the wave inside the perfectly matched layer.

So now for us to model this equation in a finite difference algorithm we have to modify this equation a little bit so that is what we are going to do next for us to practically model this equation for finite difference method.

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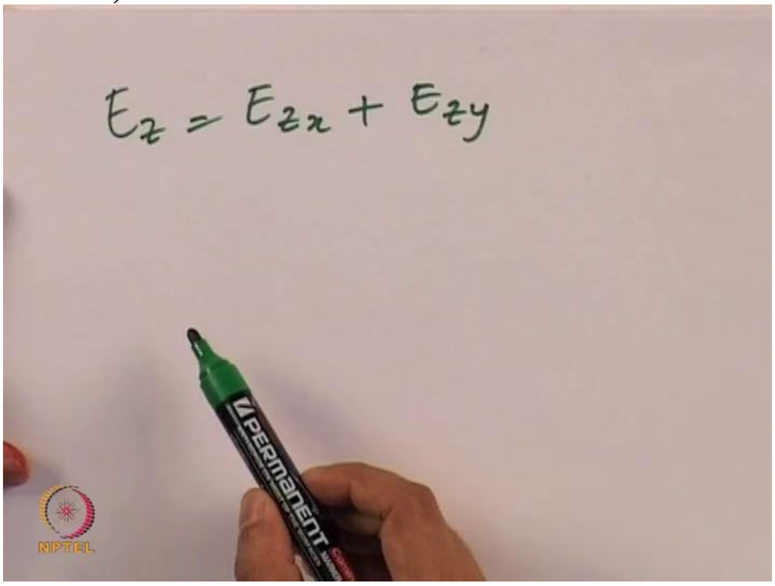

So for that what we are going to do is we are going to split the value of E z into E zx and E zy.

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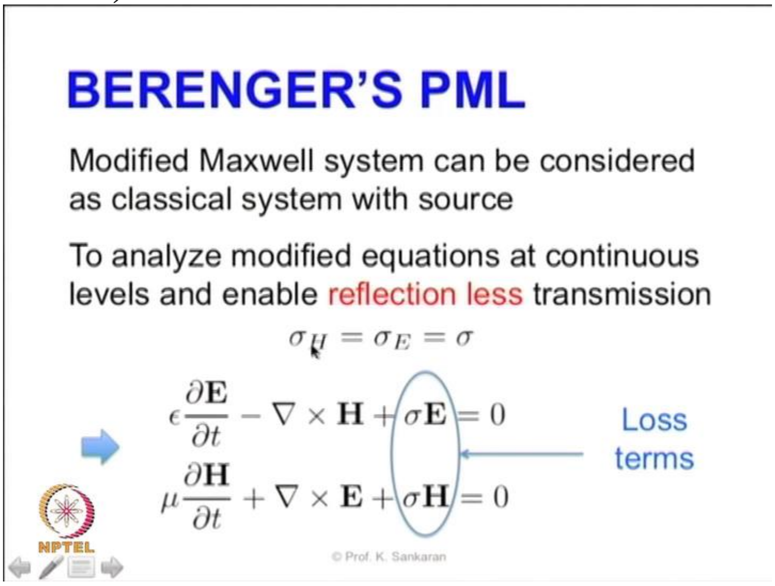

So that we can formulate it in the finite difference method and that's what we are going to see now. So as we saw as in the case of the expression we have the loss term sitting here and the Loss terms we are making them equal for both magnetic and electric case so we don't have separate electric and magnetic losses we are only having a Sigma which is loss term.

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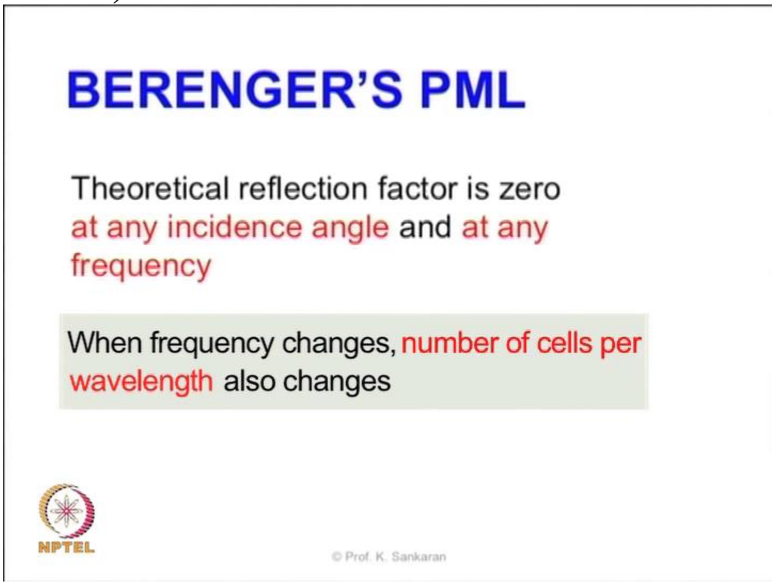

And now we want that theoretical reflection to be equal to zero for any incident angle at any frequency. The reason why we say for any frequency because if the frequency changes the number of cells is going to change accordingly we want the perfectly match layer to work for any frequency as well not only for any incident also for any frequency.

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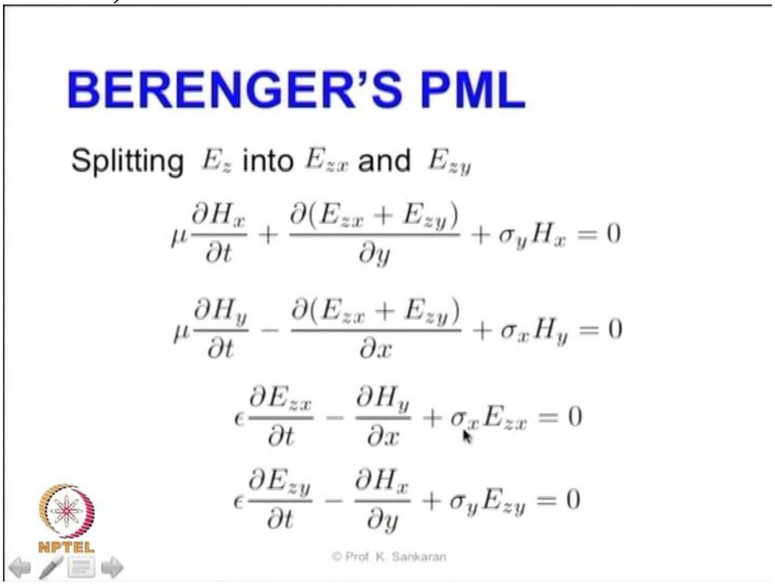

So now we are going to split the E z as I said before and the first two equations get transformed into this form and the third equation get split into two for the equations so we can split the last term Sigma component in 2 Sigma X component and Sigma Y component.

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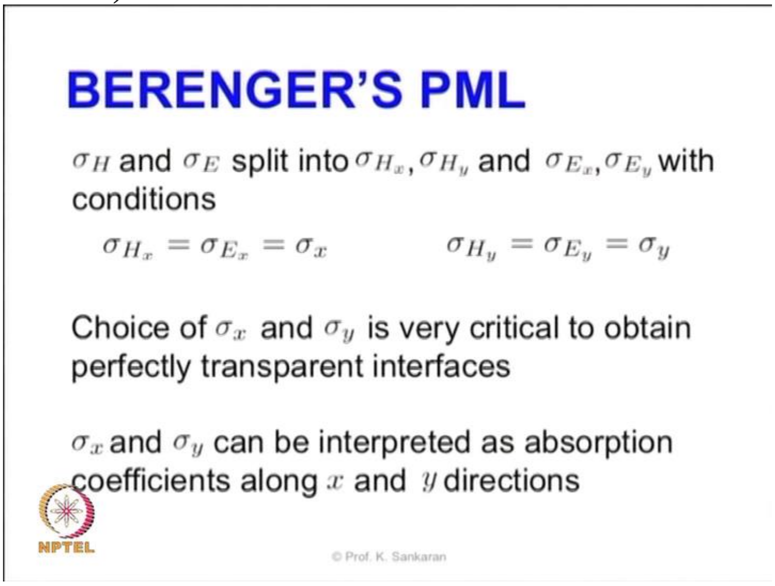

So now we are going to see that the magnetic and electric losses are going to be equal to each other and then they are going to be representing a Sigma X and Sigma y the choice of Sigma X and Sigma y are going to change within the the domain answer as I said there are going to be several layers and we are going to see what is going to be the value of Sigma X and Sigma y in different layers.

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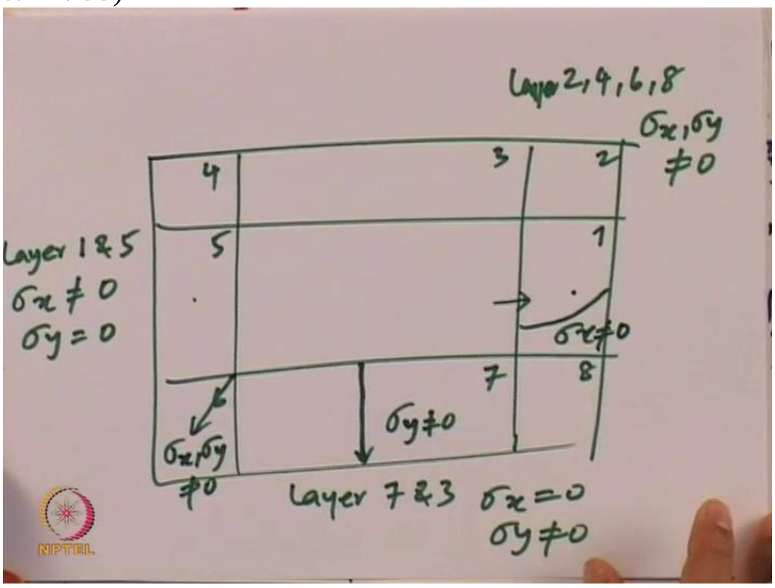

And now we will see how this is going to change so the value we said this is going to be 1, 2, 3, 4, 5, 6, 7 and 8.There are totally 8 layers so the value of Sigma is going to be different for different domains of the pml . so let's see how different it is going to be for layer 1 AND 5 so that means for this layer and this layer Sigma X is not equal to zero where as Sigma why will be equal to zero similarly for the layer 7 and 3 we will have Sigma x is equal to zero where as Sigma why is not equal to zero where is in the case of the corner domains the layer 2 4 6 and

8 both Sigma X and Sigma Y they are not equal to zero so this is the beauty of this particular formulation that we can basically using this approach we can make the wave that is going to be inside the X oriented pml absorb only using Sigma X value and the wave that is going to come in inside the Y direction will get absorbed only using Sigma y Value and the wave that is going to come inside the corner domains will get absorbed both using Sigma X and Sigma Y values so here Sigma X is not equal to zero so now we will see how we can vary the value of Sigma within a particular layer.

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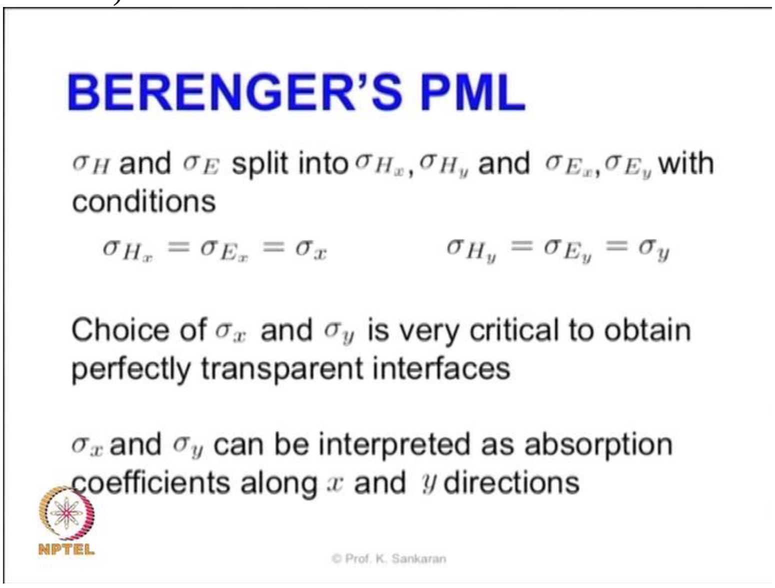

So let's take example of y oriented layer the same analysis can be done for x oriented layer.

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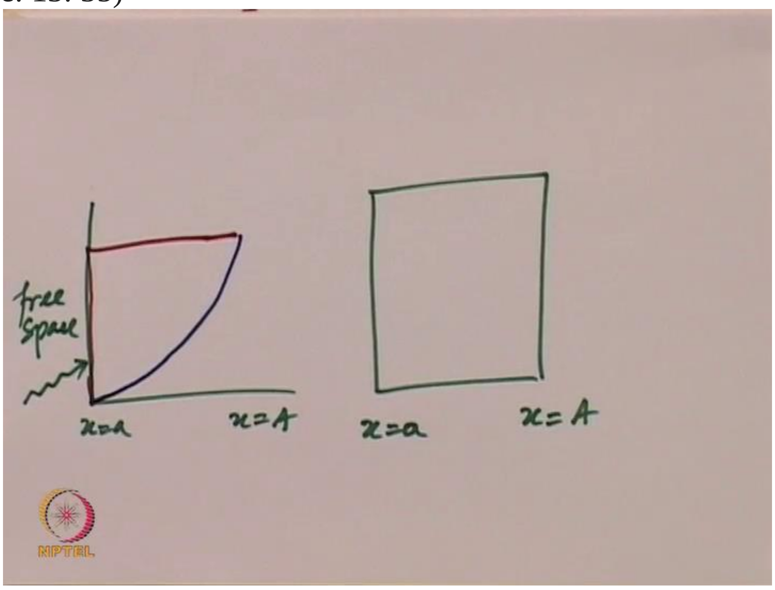

So let's say my x value is going from x equal to a t o X equal to capital A so when X goes from a to capital A I am going to change the value of Sigma I have different choices so let's say this is x equal to a and this is X equal to capital A I can make the value of Sigma as a step function like this so that means this is the free space and when the wave is coming inside it is going to see a step function of the Sigma it is OK if we are having a very very very very fine discretization is not going to be as fine as it should be for this case to be working so in order to make it working for a finite discretisation so we have to increase the value of Sigma from certain value to certain value so ideally we are going to increase it from zero to certain value and that's what we are going to see here.

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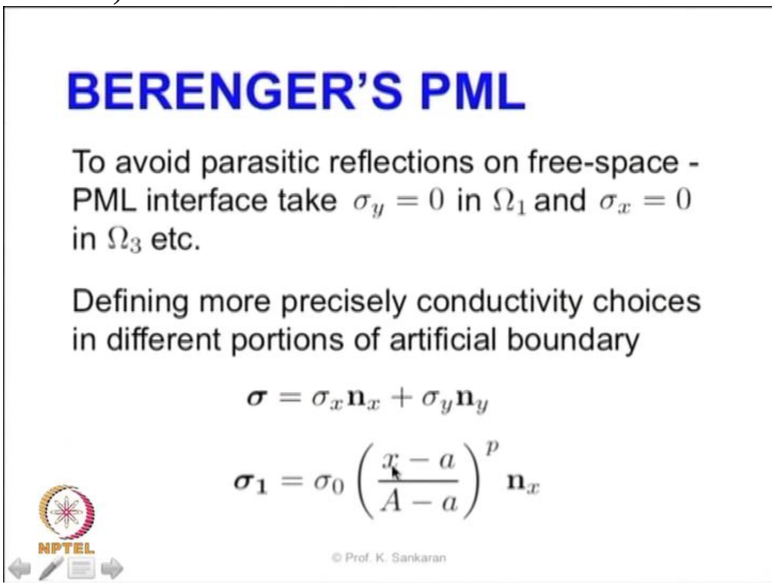

So when the value of x is equal to small a this equation will become zero; whereas when the value of X equal to capital A this value will have the maximum value. And the value of P is going to be the order of the profile for most practical applications we use quadratic profile is parabolic in their form.

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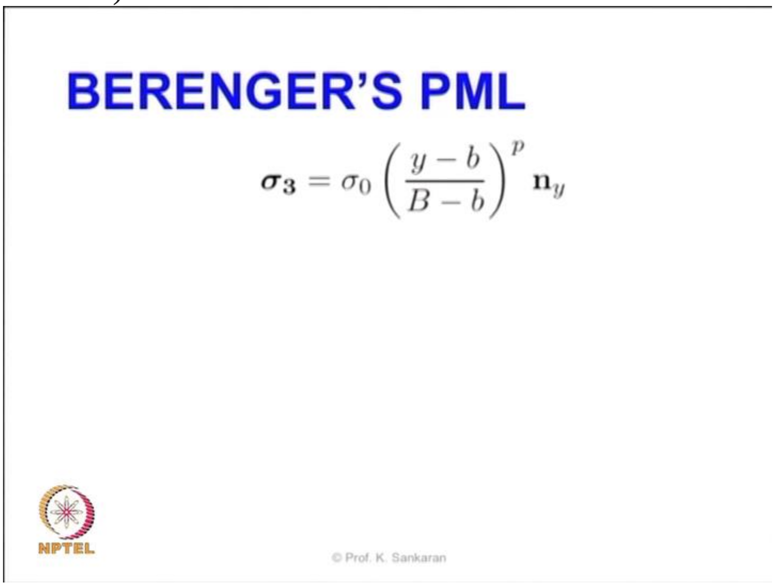

So similarly the counterpart for Y oriented pml for the third region.

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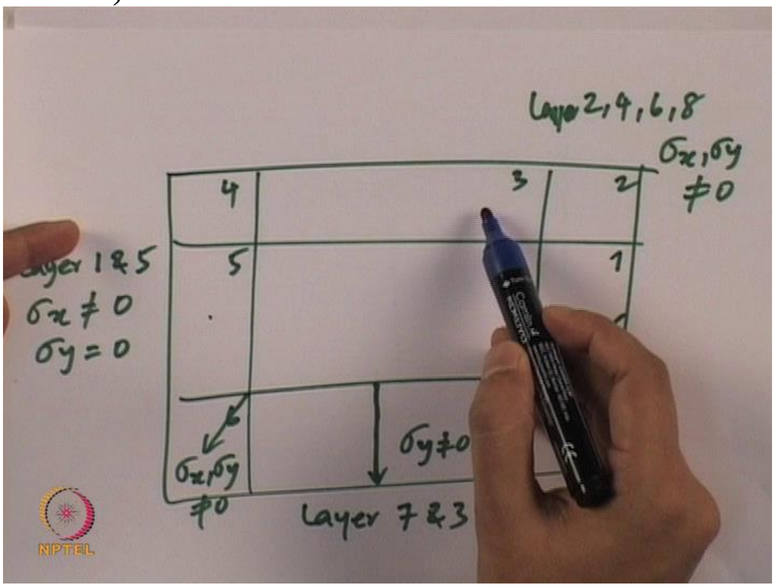

So this is for this particular region

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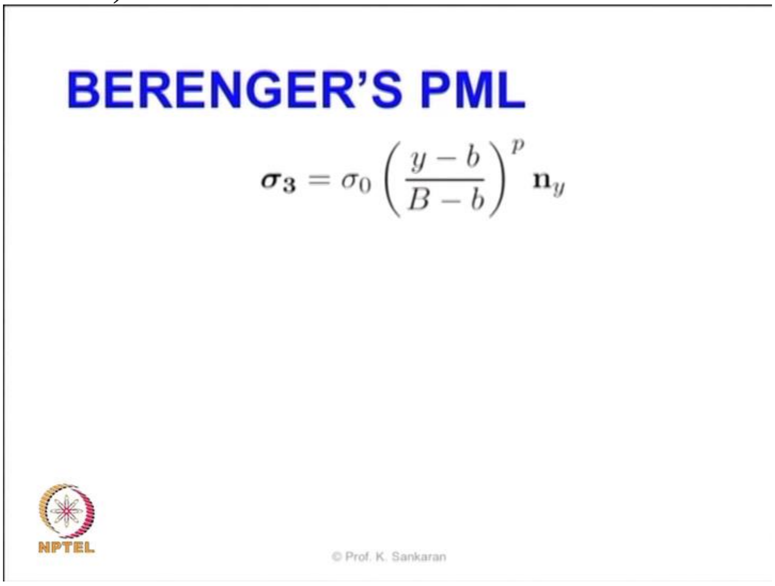

so this is going to have the form as shown here in this equation where y goes from small b to capital B and Y is equal to small b this is at the interface between the free space and the pml the value of Sigma 3 will become zero and the value of Sigma 3 will be maximum when why is equal to capital B.

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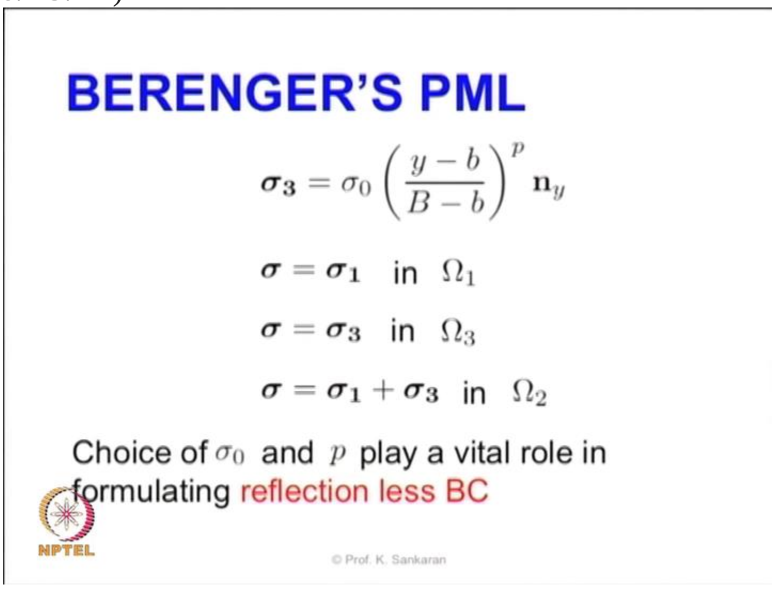

Similarly in the corner domain you have both the X and Y values using the same profile that we have used for Sigma 1 and Sigma 3. So the choice of Sigma 0 and Sigma p which are the maximum value of the Loss Inside the pml and the profile of the pml itself is going to play vital role in the accuracy of the perfectly matched layer.

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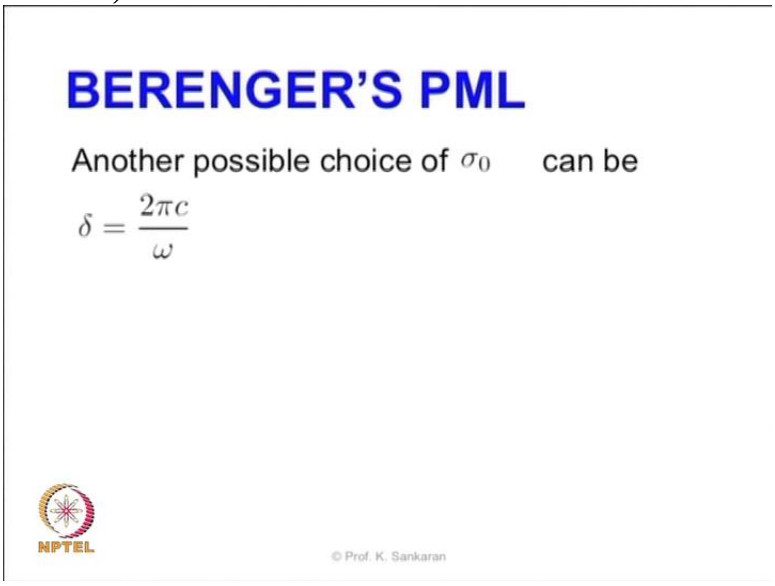

I also told you that the value of the reflection should be frequency independent but in practical applications it has certain frequency dependence. So for most practical applications.

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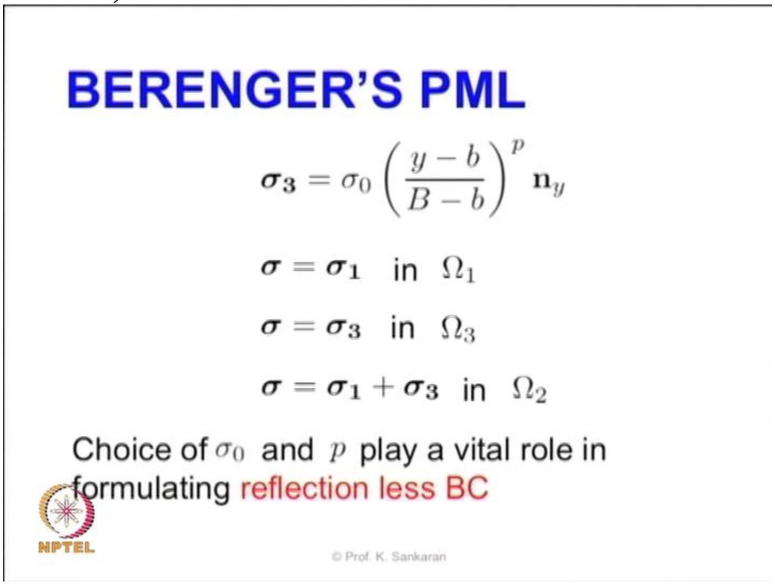

We are going to set the value of pml equal to 1 lambda. The lambda of source will be the minimum lambda of the source or the frequency that we are interested for simulating.

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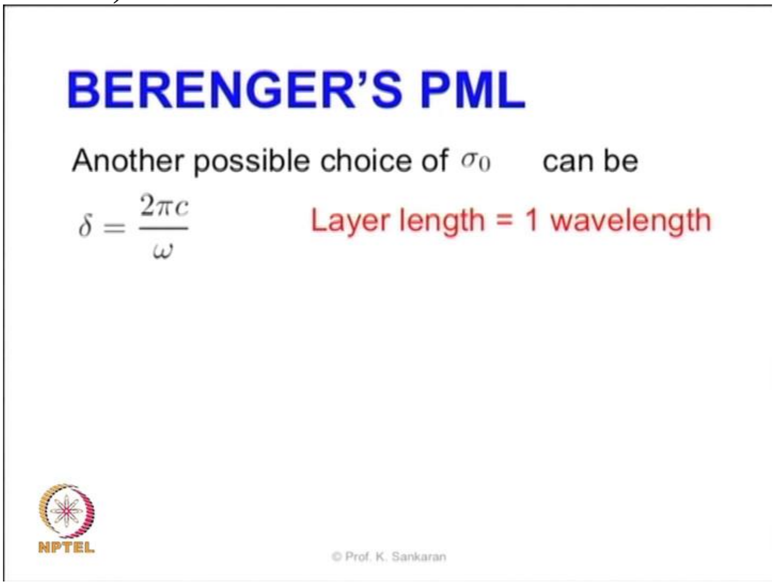

So in this case you see that The Delta value will be equal to one wavelength.

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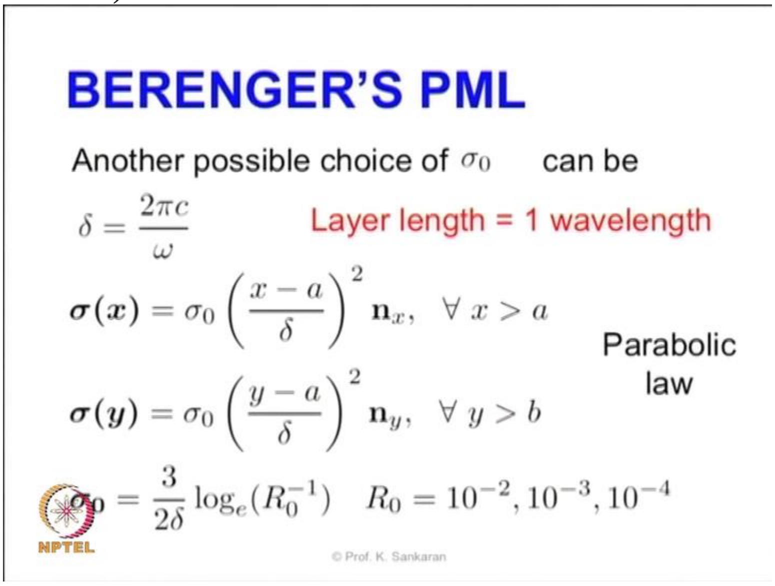

And the Sigma X and Sigma Y will have a parabolic profile as we discussed before.

So now we can see that the maximum value of Sigma 0 is also given by this equation it obeys certain law so here we are taking a national logarithm I am these are verified using numerical results for finite and finite volume approach you can choose a value for R k n o t equal to 10 power minus 2 or 10 power minus 3 or 10 power minus 4. And you can test this for various applications.

So now will stop at this point and come back in the next module to simulate the perfectly matched Layer and absorbing boundary conditions. Thank you.