Computational Electromagnetics and Applications
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Exercise No. 8
Finite Difference Method -III

In this module we are going to look into some of the modeling examples for Maxwell equations. And we are going to use Finite difference method to do this. And while doing that we are going to see some of the important aspects like modeling and slit experiment or modeling propagation with certain boundary conditions like perfectly match layer of perfect electric conductor or perfect magnetic conductor so on and so forth.

So there is lot going on in this exercise. We will take it step by step and we will look at various aspects of it as we go. One of the important aspects of this module is the boundary truncation. We have not explicitly handled perfectly matched layer in this particular example. We will do it separately in a different example. For us now there is a perfectly matched layer that we are using but we will cover it much more elaborately in a later stage. We will just give a very brief idea what it is in this example. With that introduction let us start that example that we are interested in modeling today. And we will take you step by step into the problem.

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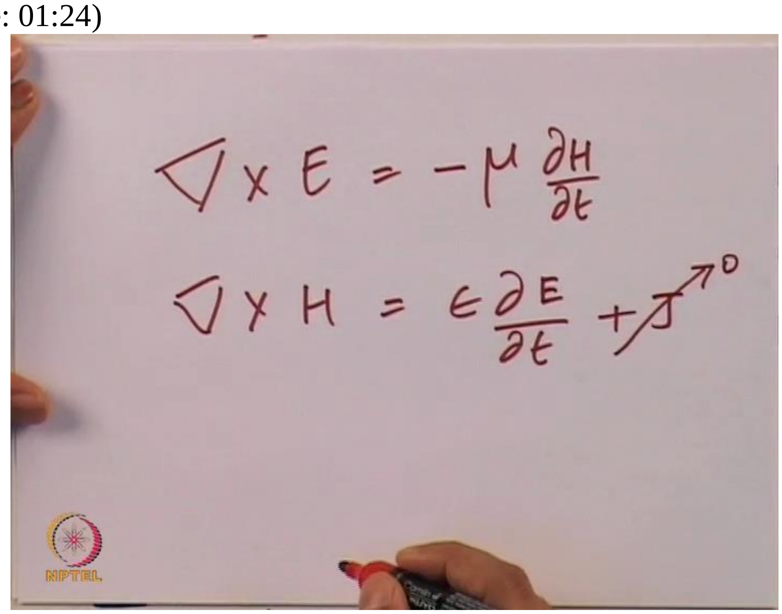

And for that first let us begin with the mathematical governing equation. So the initial governing equation that we are interested is the Maxwell equation itself, which is nothing but the curl of E is equal to minus Mu dH by dt. And then the curl of H is equal to Epsilon dE by dt plus J, of course we are not considering the J term the current density term. For us now we keep it as 0

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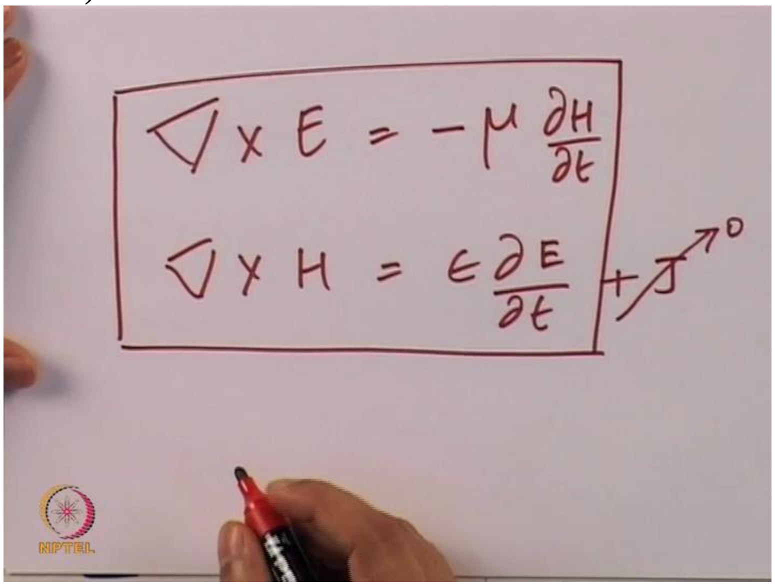

So the two important update equations that we will be focusing on in todays exercise will be coming directly from the curl equation.

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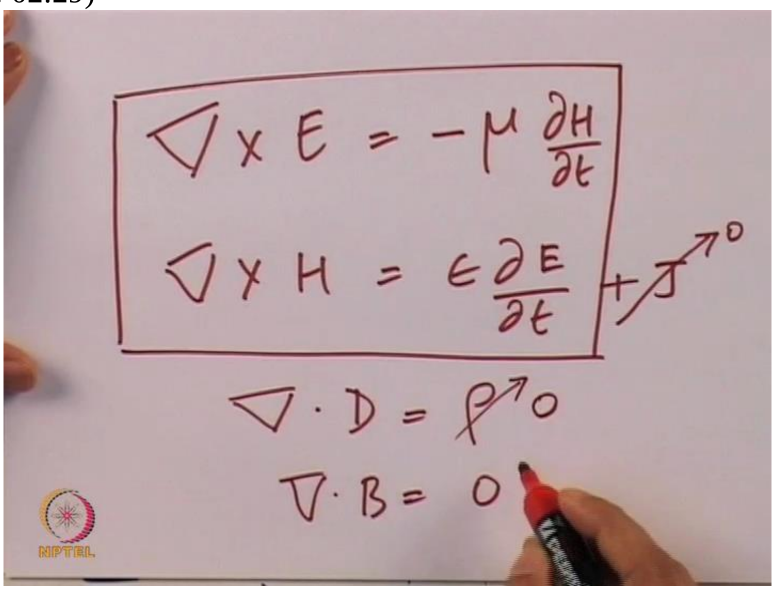

Apart from that there are other two equations which is the diversions of D is equal to Rho and diversions of B is equal to 0. For us in this problem we consider Rho to be also equal to 0, so both the diversion conditions are set to 0. In the finite difference method particularly the modeling that we are going to use which is staggered modeling.

The diversions are not explicitly handled. We are considering the diversions conditions are implicitly satisfied. So specifically forcing them in any of the update equation. With that being said it is a important source of error in most of the numerical methods which leads to spurious modes and so on and so forth. And we are not worrying about such errors in this case because we are implicitly assuming that the diversions conditions are satisfied and it is also easy to prove in some methods the diversions conditions will be satisfied.

But here that is not the focus we will just focus on the two equations which are the curl equations which I have written here. So now for us to go into problem modeling we will focus on a 2D case. So in this example we are going to focus on a transverse magnetic mode or transverse magnetic wave. So why we are using transverse magnetic because it is a easier choice you can also choose a transverse electric in a 2D model.

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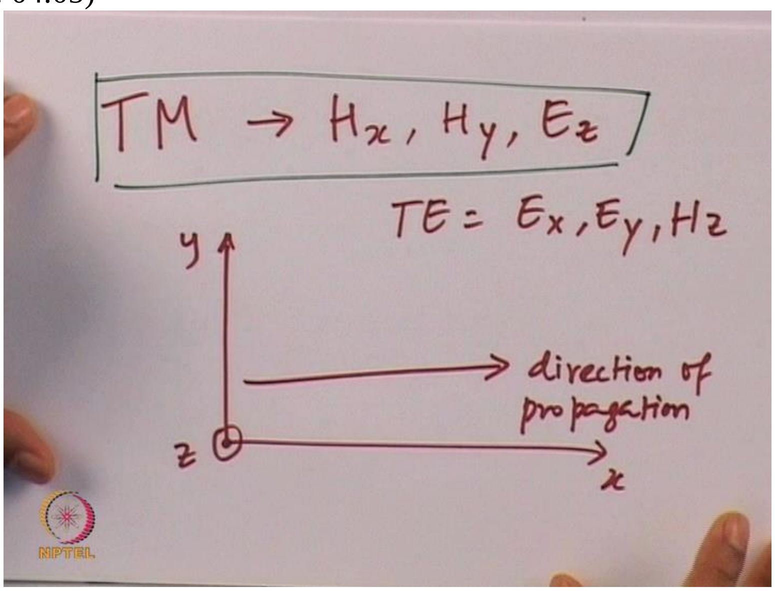

So we will start with a transverse magnetic aspect which is going to be, TM implies H x, H y, E z, if we say that the domain that we are interested is going to be x y and z is coming out of this plane. And if we assume that x is the direction of propagation. And we say the H field magnetic field is going to have components in the x and y direction. And the electric field is going to have components only in the z direction.

Different textbooks call this as different names. Different textbooks use different names. So be careful how you are defining this. For us the definition is very clear and simple, that once we define the direction of propagation and the plane of propagation which will be xy plane. Our magnetic field is purely in the plane of propagation, and the electric field is going to be perpendicular to the plane of propagation we use the definition of transverse magnetic.

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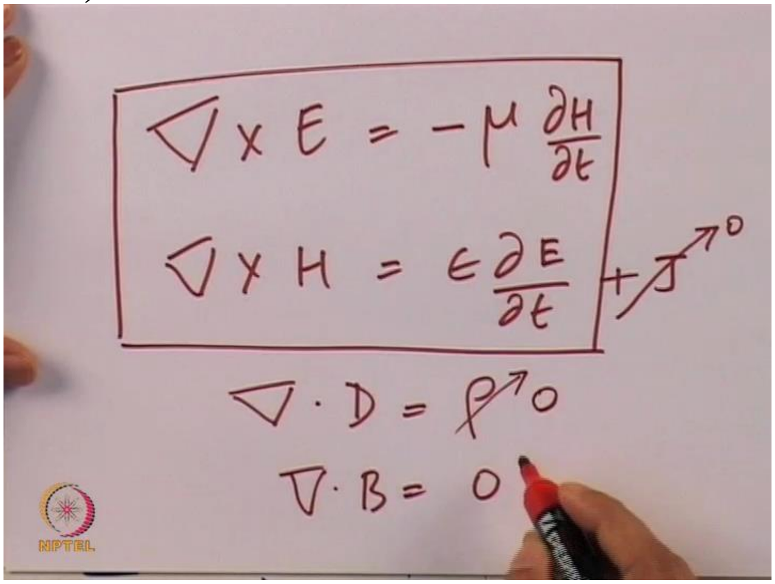

The counter part of that will be transverse electric where you will have E field in x and y direction and H field in z direction. This is not what you are going to do. In our case it is going to be transverse magnetic so let us put in the box. So with this as our models bases we are going to model the Maxwell equations, the Curl Equations that we have set here in the form of that we want.

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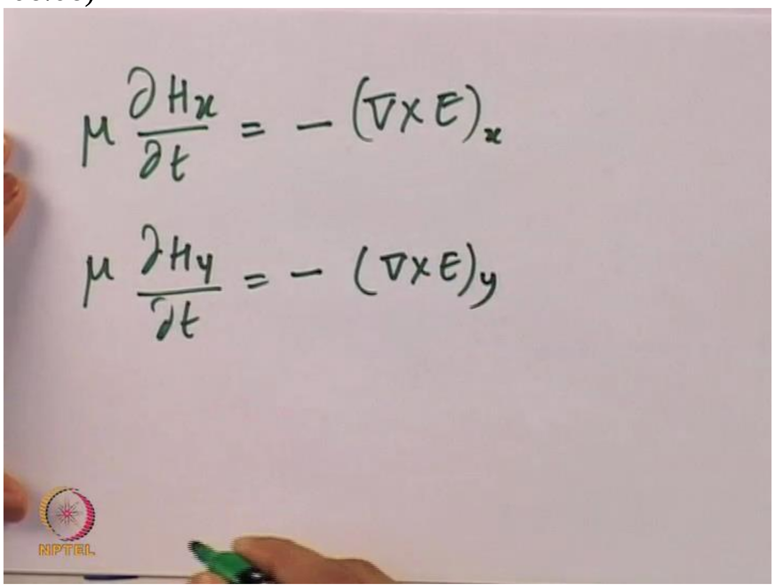

So H field is going to have x and y components. So let us write down the Maxwell equation Mu multiplied by (Doe H x by Doe t) is equal to minus (Del cross E) x, this is going to be the x component. And Mu Doe H y divided by Doe t is equal to minus (Del cross E) y component.

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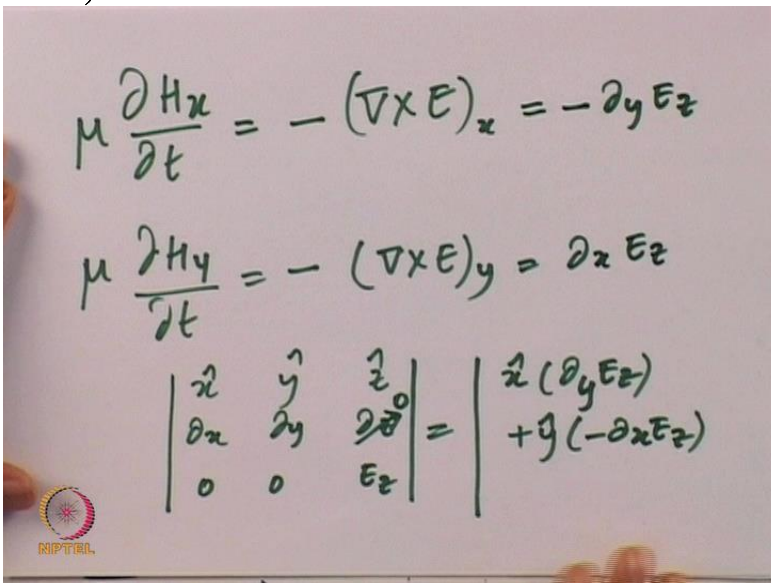

So if we write it down in individual forms what we will get is x y z, Doe x Doe Y Doe z, and we have got 0 0 E z. I said the E field is going to have only component in z direction. So we put the other components as 0. Since we have no variation in z direction. We are going to assign this also to 0.

Let us write down all the components. All components in the sense two components are there x component is going to be Doe y E z and the y component is going to be plus y it is going to be (minus Doe x E z) and then the z component will not be there.

So let us write down now the minus of the x component will be minus Doe y E z and minus of the y component is going to be minus of minus it will become plus Doe x E z.

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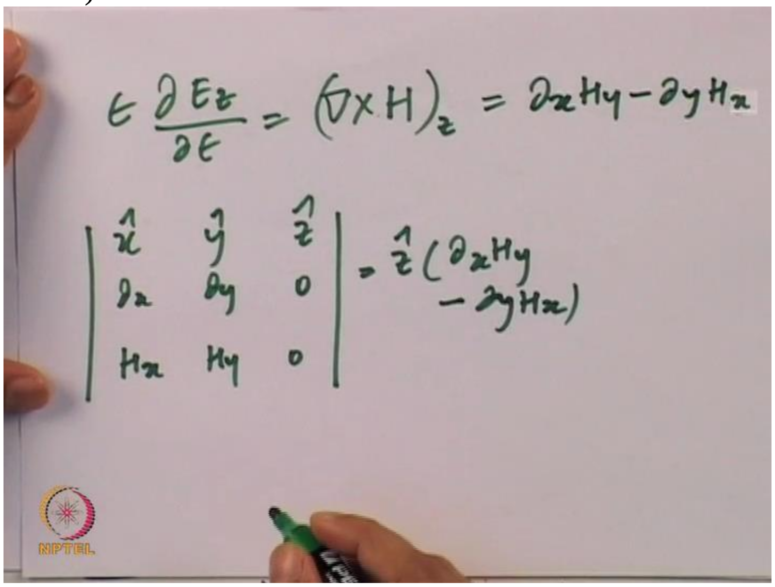

Similarly we will also have the components for the electric field. So let us write it down, Epsilon Doe E z divided by Doe t is equal to Curl of H going to have only the z component. So the z component is going to given by this expression x y z Doe x Doe y 0 and then we get H x H y 0. So this will have only the z component because the x components will have the 0 here and the y component will also have the 0.

So we only get the z component it is given be (Doe x H y) minus (Doe y H x). So let us write down this one so the z component is going to be given by Doe x H y minus Doe y H x. So this is going to be the third equation.

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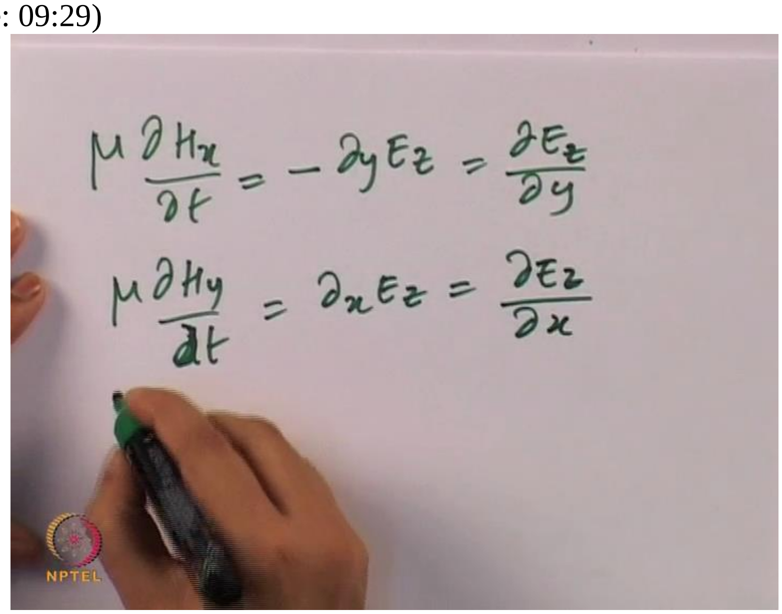

So the first two equations are namely Mu Doe H x by Doe t equal to minus Doe y E. So it is nothing but Doe E z by Doe E y. I am writing it in short form Mu Hy Doe t is equal to Doe x E z it is nothing but Doe Ez divided by Doe x.

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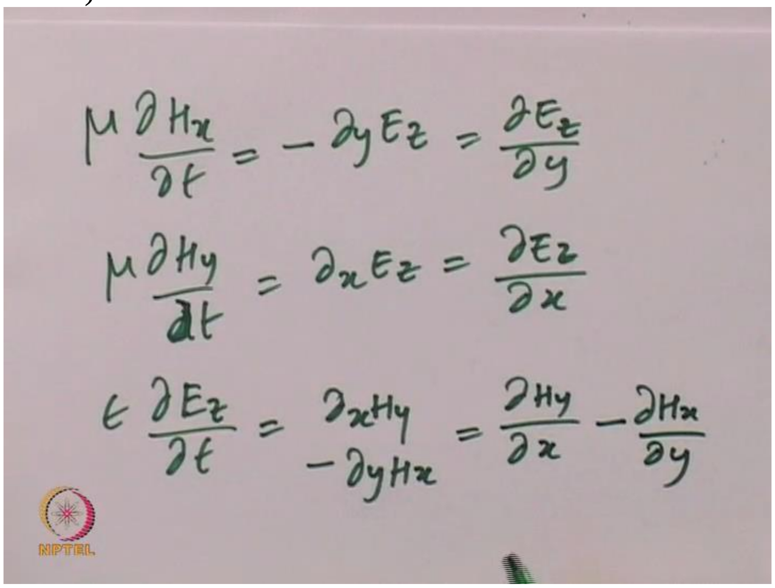

And the last one is Epsilon Doe E z divided by Doe t is equal to Doe x H y minus Doe y H x. So this is going to be written in a conventional form as Doe h y divided by Doe x minus Doe H x divided by Doe y. So this is still a continuous form. We need to discretize it using certain algorithms so what is important know here is we are going to combine forward differencing and backward differencing for the fields spatial derivatives and we are going to do Leap frogging in time derivative. So I am going to explain this step by step.

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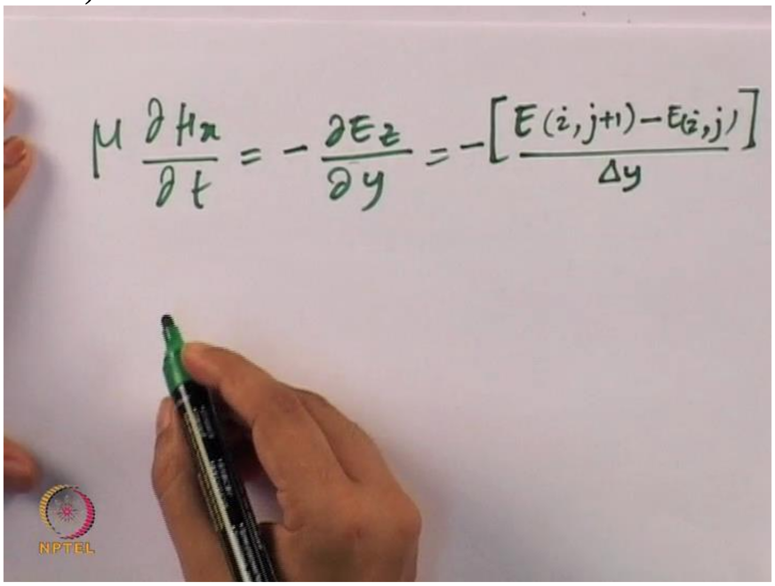

So let us take first the spatial derivative which is easy to model. So let us take the first equation what we have. Doe H x divided by Doe t is equal to so we will get minus Doe E z divided by Doe y. So we are going to do forward differencing for E z. So we are going to take since it is a two dimensional problem what we will get is minus sign is out [E (i, j plus 1) minus E (i, j)] divided by delta y. So what we have done is we have taken the forward differencing in the y direction. How do I know which direction I have to forward difference it is given by the partial derivative itself so if it is Doe y you do forward differencing or backward differencing in the particular variable? If it is Doe y you do in j if it is Doe x you do it in i.

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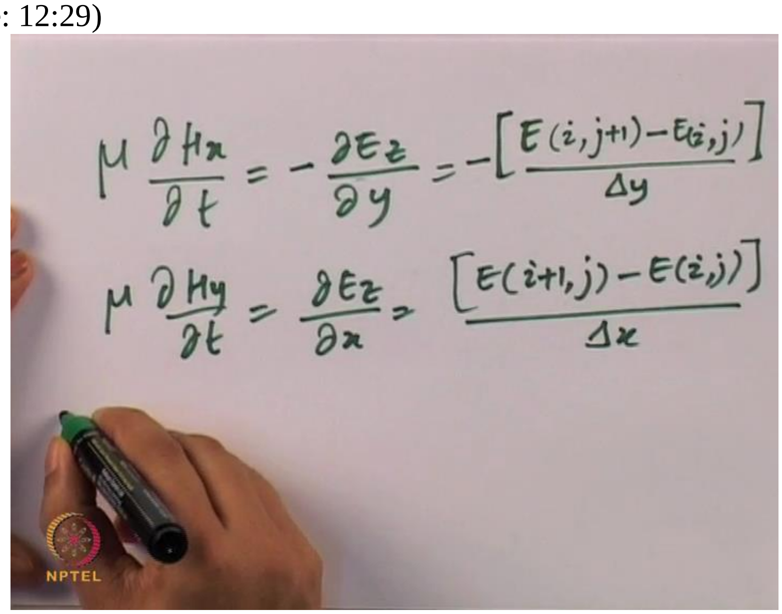

So let us say we have the second equation Mu Doe H y divided by Doe t is equal to Doe E z divided by Doe x. So in this case we will do forward differencing in x component which is i. [E (i plus1, j) minus E (i, j)] divided by delta x.

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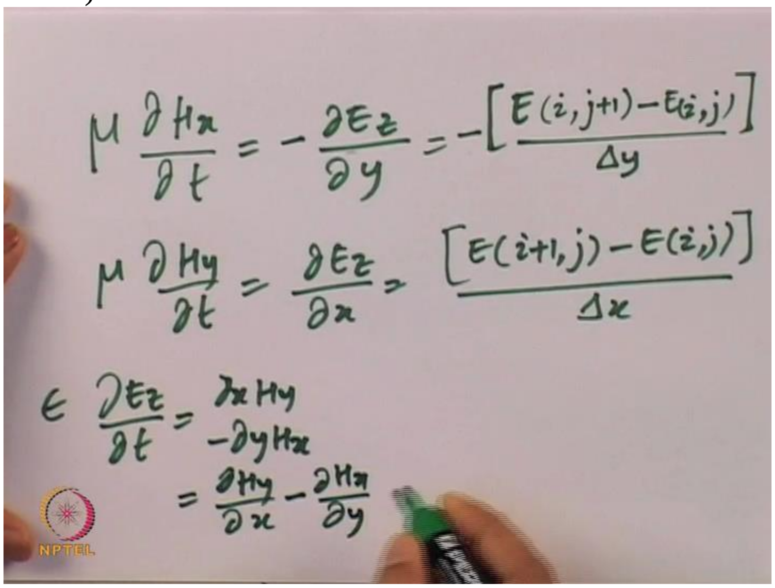

And in the case of magnetic field component you are going to do backward differencing and let us see how we are doing it. So we have the third equation Epsilon Doe E z divided by Doe t is equal to Doe x H y minus Doe y H x, which is nothing but Doe H y divided by Doe x minus Doe h x divided by Doe y.

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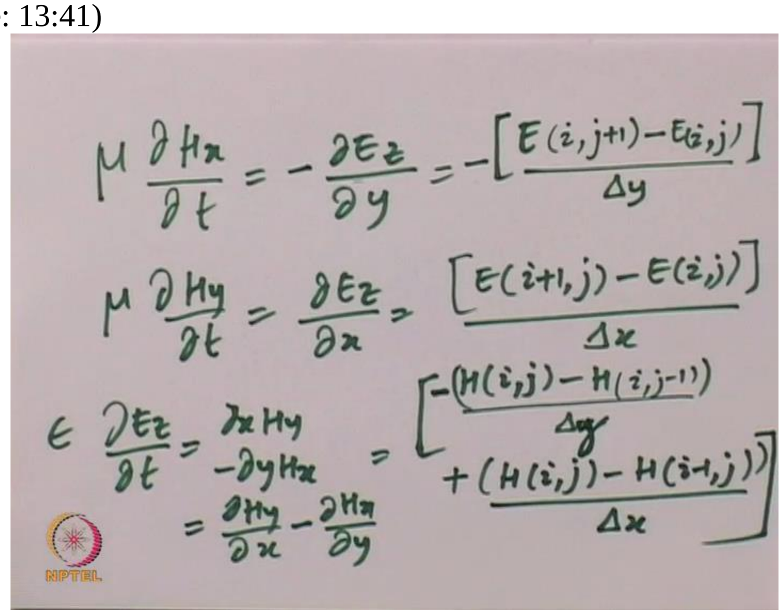

So I am going to write down this equation I have said I will do backward differencing in H. So [H (i, j) minus H (i, j minus 1) divided by delta y. So I am doing backward differencing in the J component. So this is this term, and of course I have a minus and I put a minus in the front. And since I started as the second term, I will do the first term now it should be plus backward differencing in x component which is i. So I will have Plus (H(i,j)minus H(i minus 1,j)divided by delta x)] .

So as you can see the electric field component spatial derivatives are forward difference. The magnetic field components spatial derivatives are backward difference. So let me explain that.

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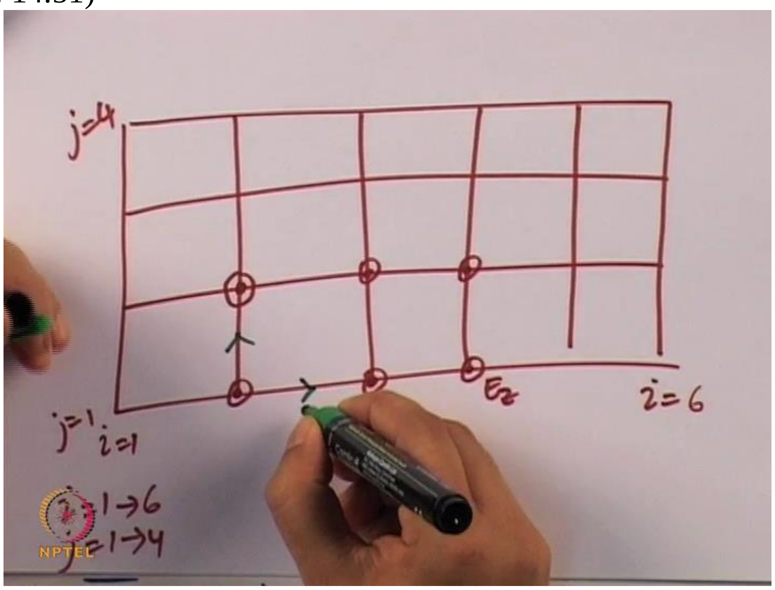

So this is your domain this is i equal to 1, this is i equal to 2, 3,4,5,6. And this is j equal to 1,2,3,4. So domain is going to have i running from 1 to 6, j running from 1 to 4. And now the magnetic fields components are going to be positioned in such a way that the electric field components will take the nodal values. So we have all the nodes will have electric field.

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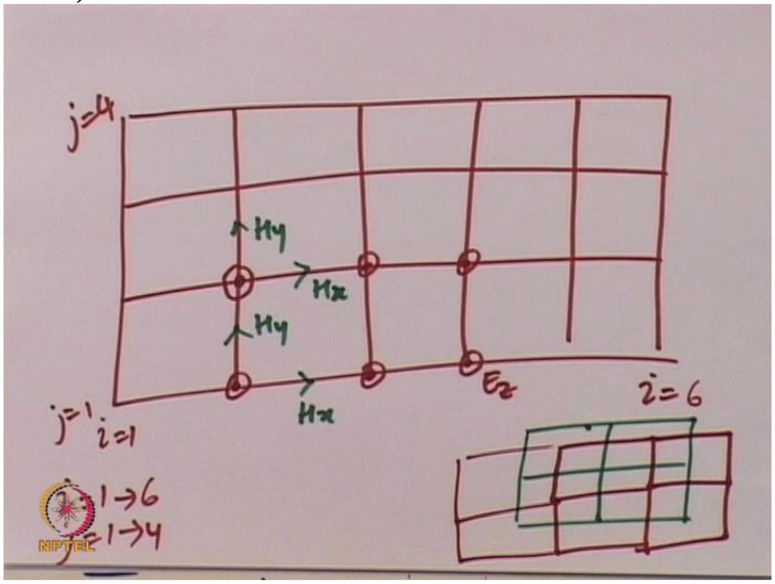

Whereas the sides are going to be components of the magnetic field, so the magnetic fields are going to be H x, H y. Similarly H x H y so on and so forth. So the staggering will look in 2D simply like this. So the red color will be a staggered grid to the green color. So the reason for doing that we discussed it already in the finite difference method. But we will give more elaborate discussion about it when we treat algebraic topological method. We will now focus on how the time stepping is going to be done.

It is going to go through a loop. So loop is going to take care of the staggering requirement what we need.

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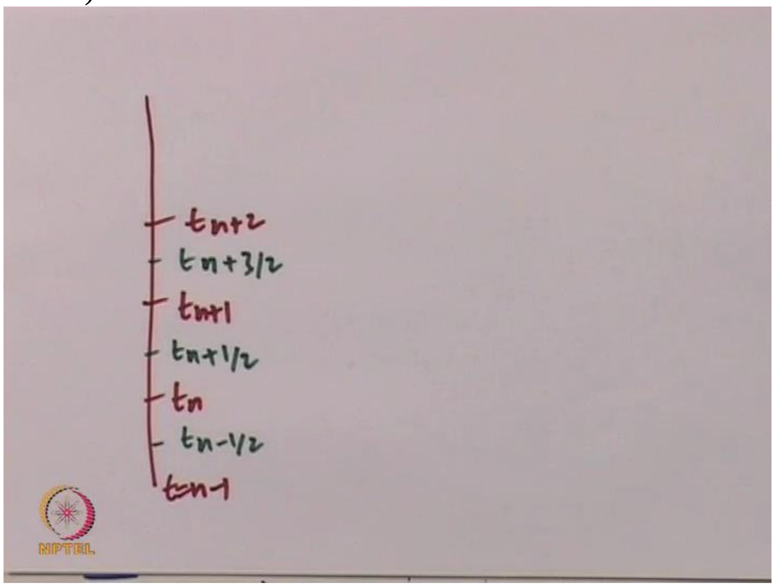

So I said this is the timeline, so this is t equal to n minus 1, t equal to n, t equal to n plus 1, t equal to n plus 2, so on and so forth. There is always going to be a intermediate point ; this is going to be t n minus 1by2, t n plus 1by2, t n plus 3 by2 so on and so forth.

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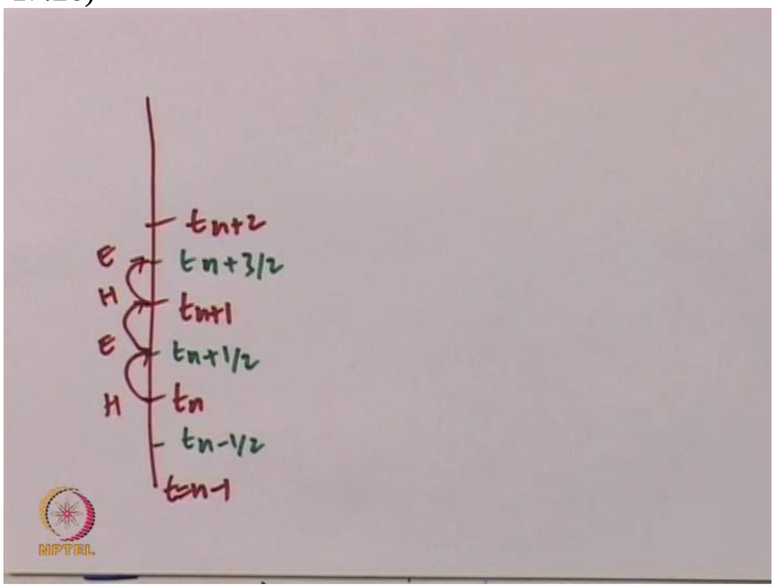

So we are going to first start with the electric field spatial derivative that we are computing to update the magnetic field. So at time t equal to n start with the magnetic field and so we start with H we go and compute E here and we go and compute H field again, we use that to compute the E field again so on and so forth. So we are going step by step, so that is why we call it as Leap frogging method.

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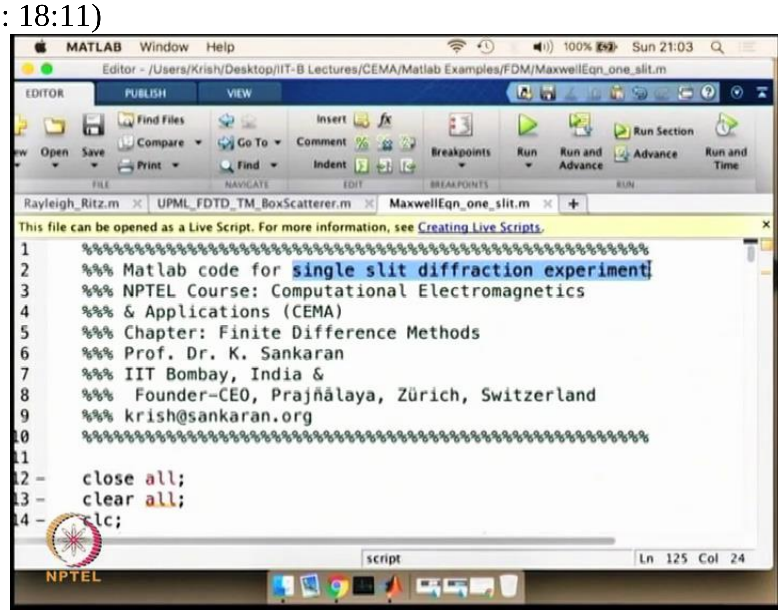

And this is going to be a single slit problem. So we are doing a single slit diffraction experiment. And we are starting with the Finite difference method. And I have already discussed how we are doing the differencing.

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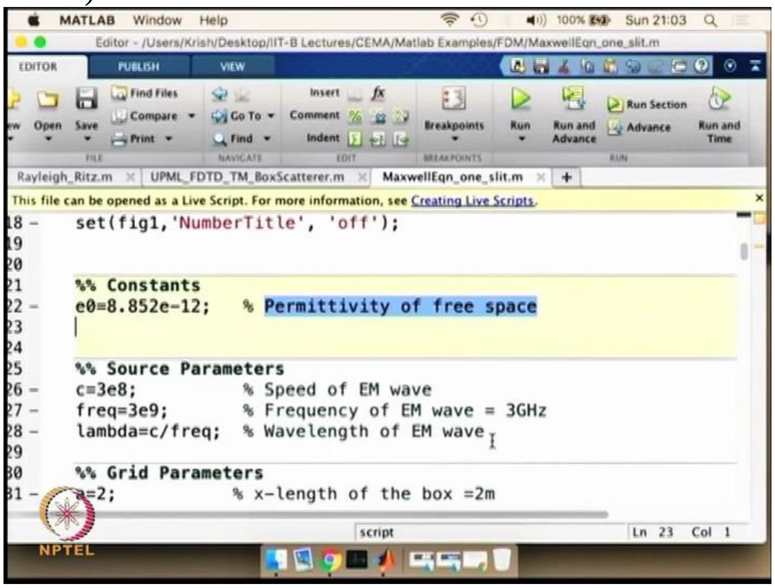

So let us start by defining certain parameters constants Epsilon which is given by 8.852 multiplied by 10 to the power of minus 12, which is the permittivity of free space.

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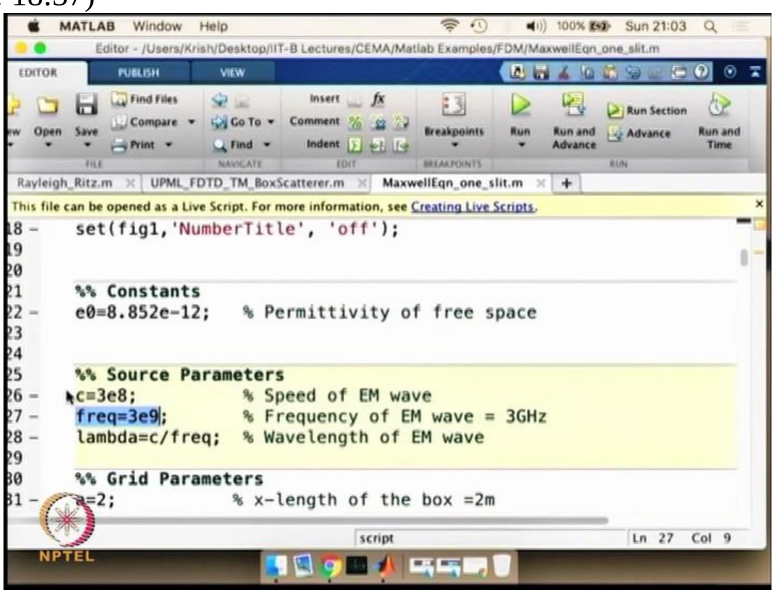

We are setting the value of the frequency that we are exciting as 3 Giga hertz, which corresponds to roughly 0.1 meter or 10 centimeter.

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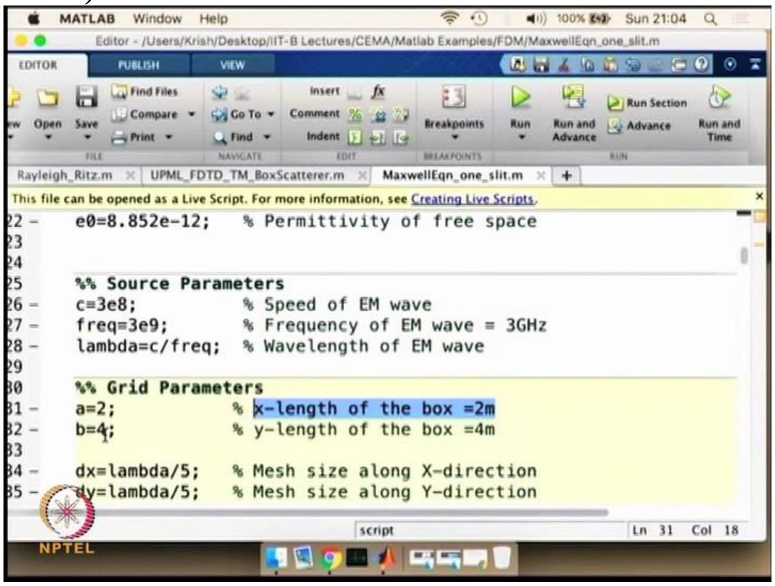

And we are setting the boundaries of our parameter a equal to 2, and b equal to 4. a is the x length of the box which is 2 meter and y length of the box is b which is 4 meters.

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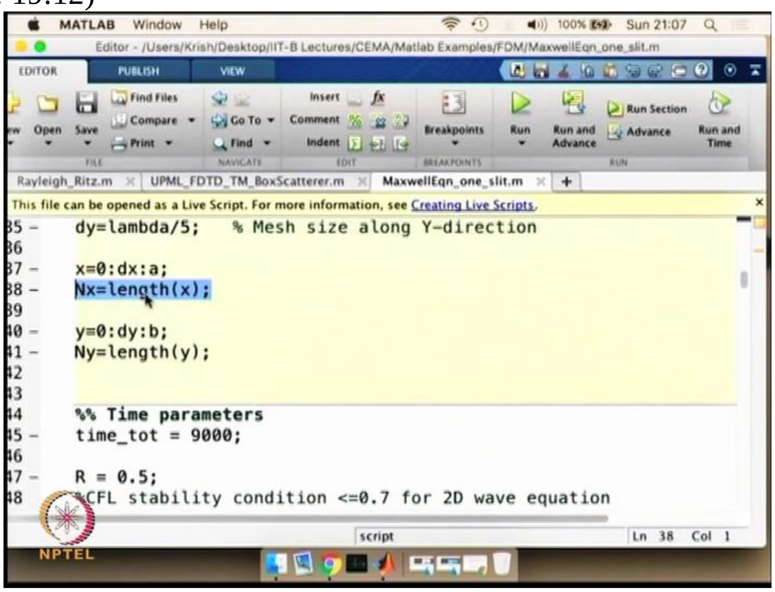

And nx is going to be the length of x space stripping. And y is going to be the length of y space stripping.

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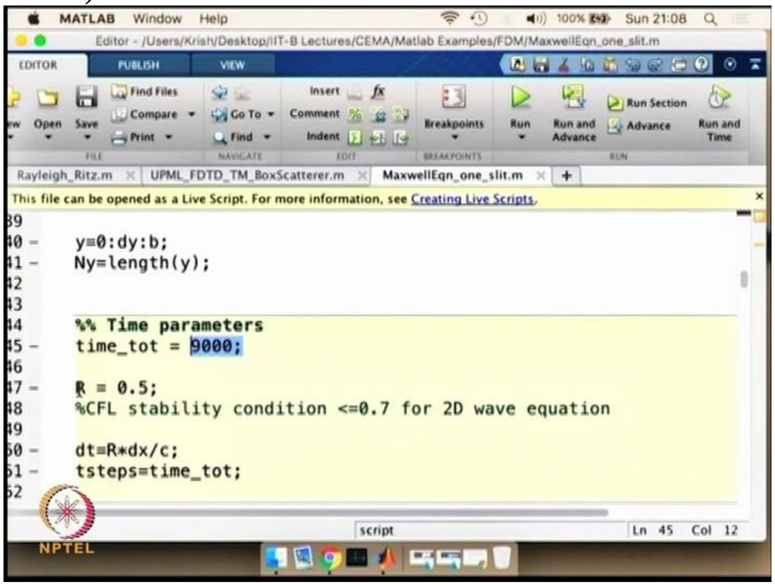

The time duration for which we are going to run this simulation is going to be 9000 time steps. We have kept r value is equal to 5 on condition that we are satisfying the CFL condition.

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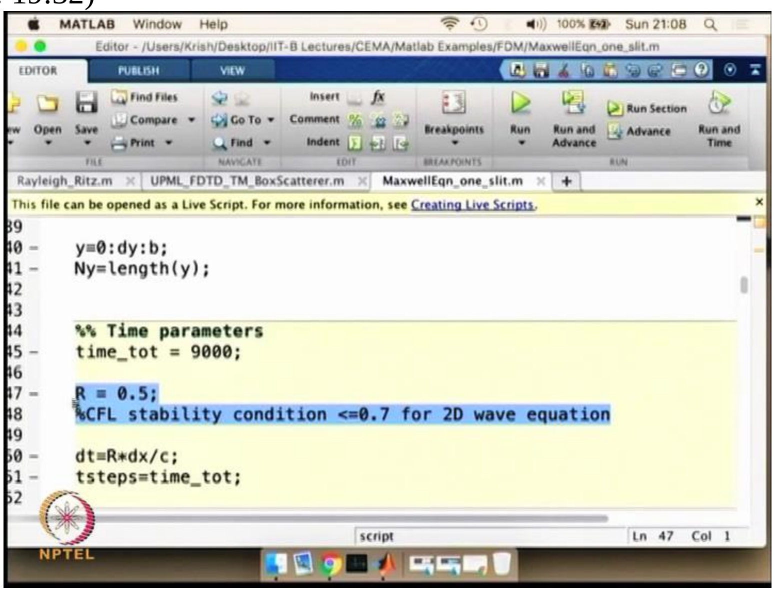

We can go r slightly above 0.5 also, but for this particular experiment we just wanted to show the proof of concept and we are not worried about the time stepping as such. So we are making sure that the stability is satisfied, so we are keeping it in 0.5.

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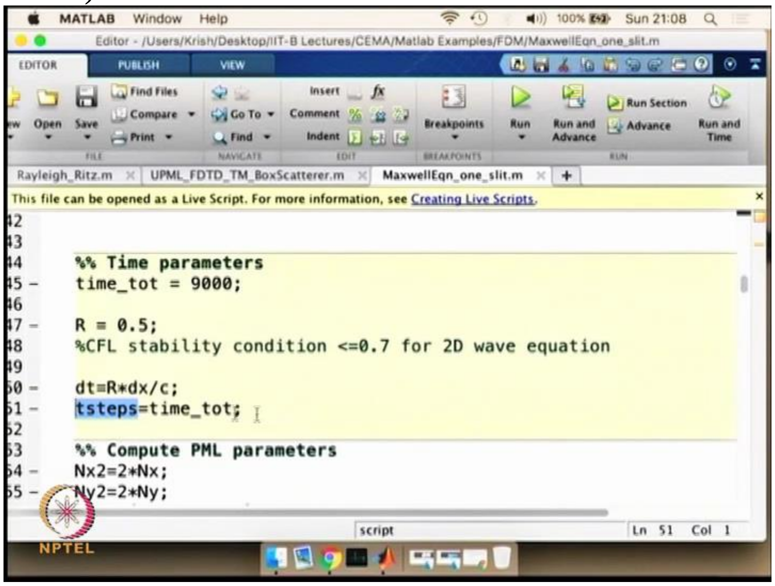

Once these parameters are fixed you can compute the value of dt using r, dx and c, and we will get the value for the time stepping. And the total time duration is already given so we call it t steps.

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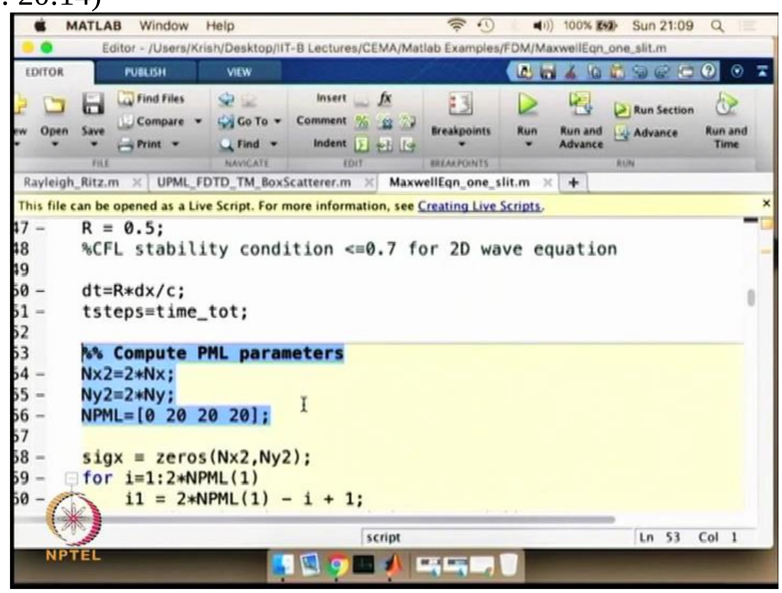

And now we are going to put the PML conditions. We are not going to discuss specifically the PML conditions in this particular module as I said before. This is going to be a uniaxial PML. I will describe the concept of PML at a later module. But for now it is enough that you know that this NPML module what I have, if I put the value for first second third and fourth component that is going to define the boundaries of the PML and the directions of the PML.

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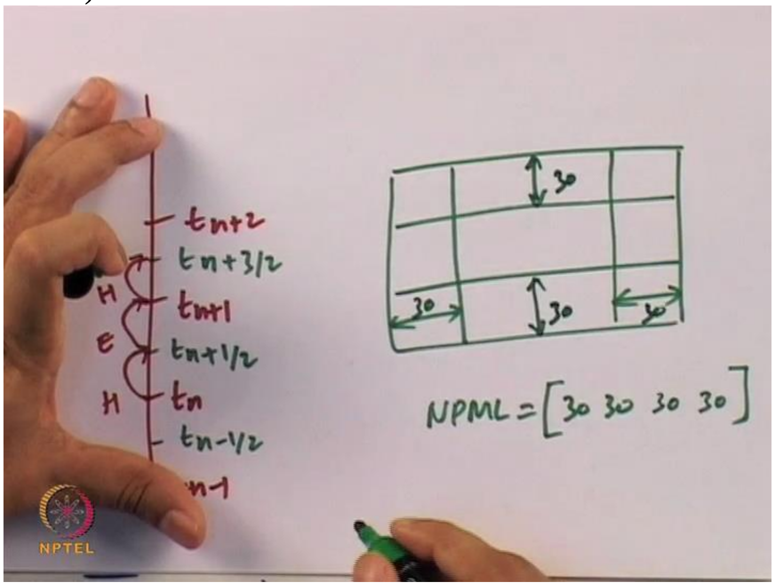

For example if I have rectangular domain and I am having PML on all the four directions. Then NPML parameter will have components that are going to give me the thickness of the PML in the both minus x direction and plus x direction. So it is going to be if I put value 30 30 that means the thickness of the PML on the left hand side and right hand side is going to be 30 space steps and 30 space steps. So 30 x space steps on the left hand side and 30 x space steps on the right hand side.

Similarly if I put 30 30 also for a third and fourth component it is going to be 30 y space steps and 30 y space steps on both the lower and the upper side of the domain. So if I put this, this is going to be 30, this is going to be 30, this is going to be 30, this is going to be 30 space steps.

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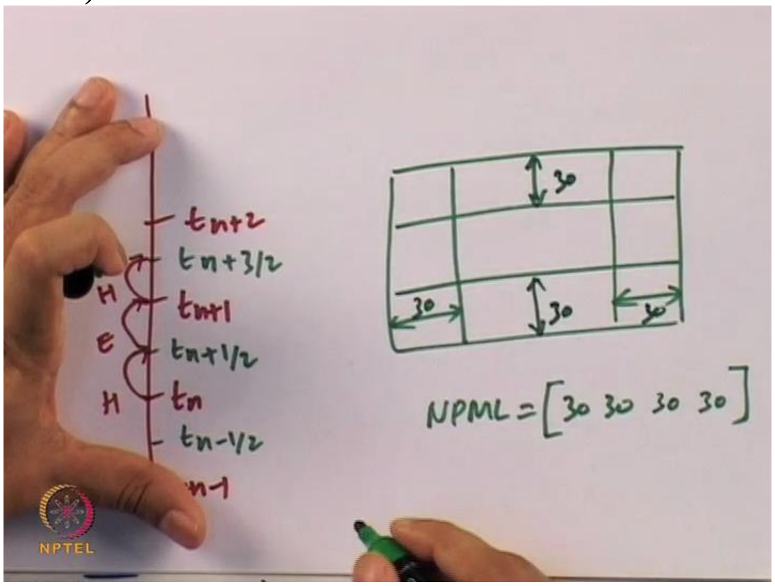

And our computational domain is going to be the one which we are marking in red. Of course I have exaggerated the figure. Your computational domain will be larger than the PML thickness itself but just for the sake of proving the just for the sake of explaining the concept I made it look like this.

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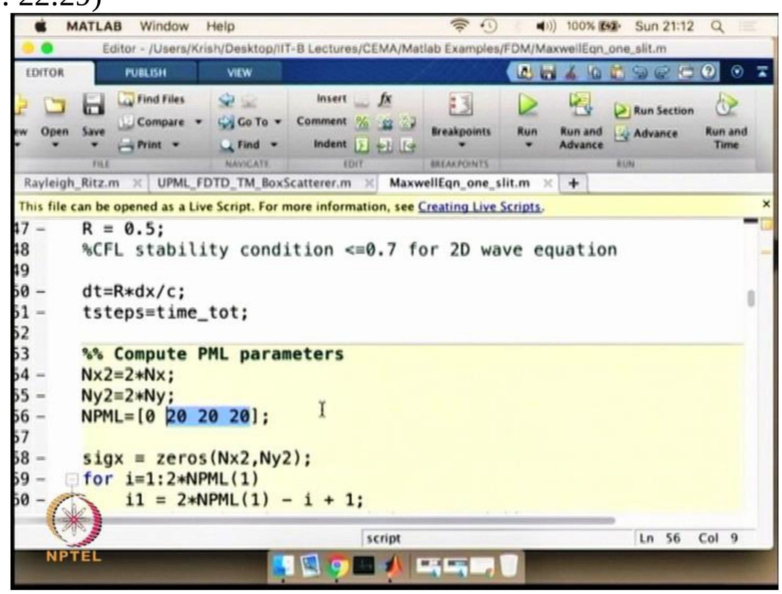

So let us go back to the code what you see is the first component is 0 whereas the last three components are going to have 20 20 20.

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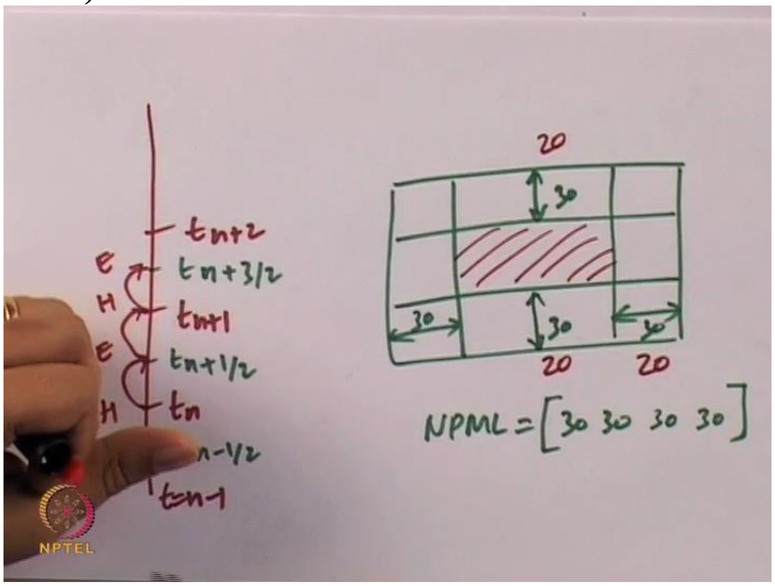

That means coming back to the figure here what you see is the left hand side there is no PML that is why it is 0. Whereas the right hand side it is going to be 20. The lower end it is going to be 20, the upper end it is going to be 20 the thickness of the PML. So that is all you need to know for now about the PML. As I said we will come back and discuss PML at length at the next module. For now we take PML for granted. So all the equations where there is PML I request you to neglect in this exercise and we will look at it at a later stage.

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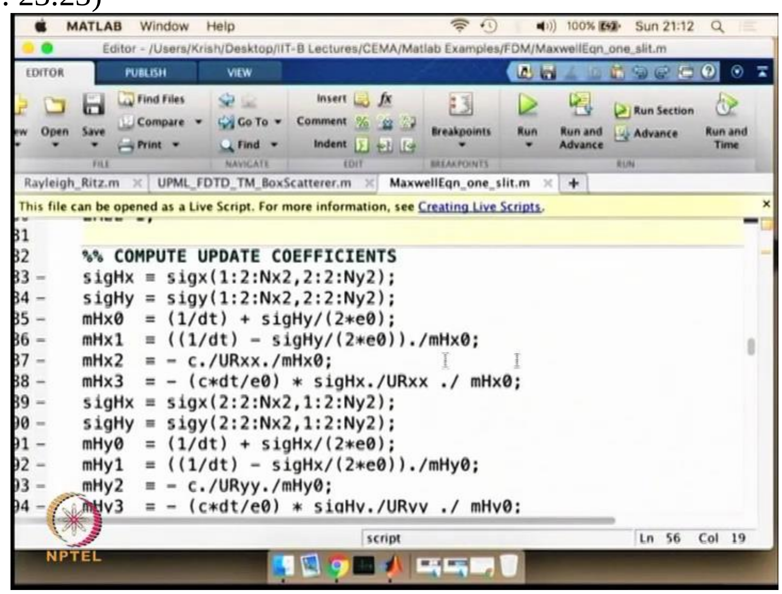

So let us go and look at the way the PML components are computed. The PML components are computed accordingly using sigma Hx, sigma Hy. All these components are related to the PML, we will not look at it at this stage. What is more important is how are we going to update the field equations.

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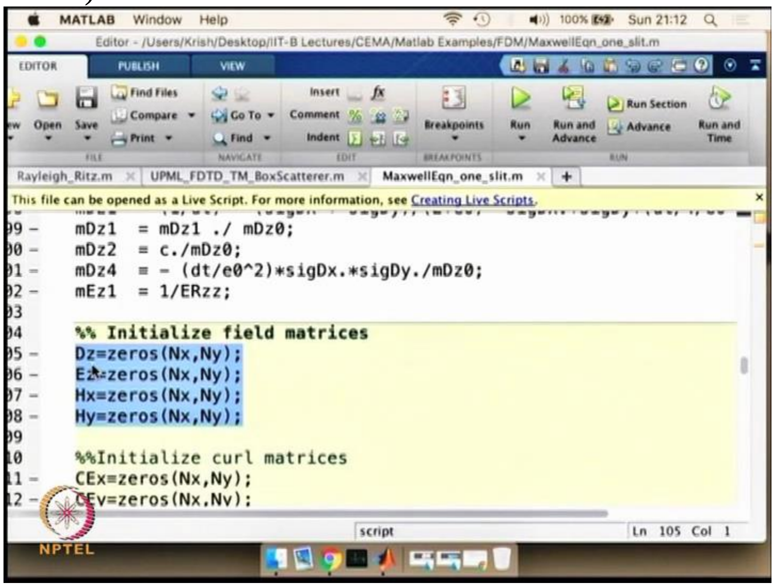

As you can see here, there are going to be four field components, Dz, Ez, Hx, Hy. Of course we could have just done it with only three components. In order to make the computational coding easier we have introduced another component Dz.

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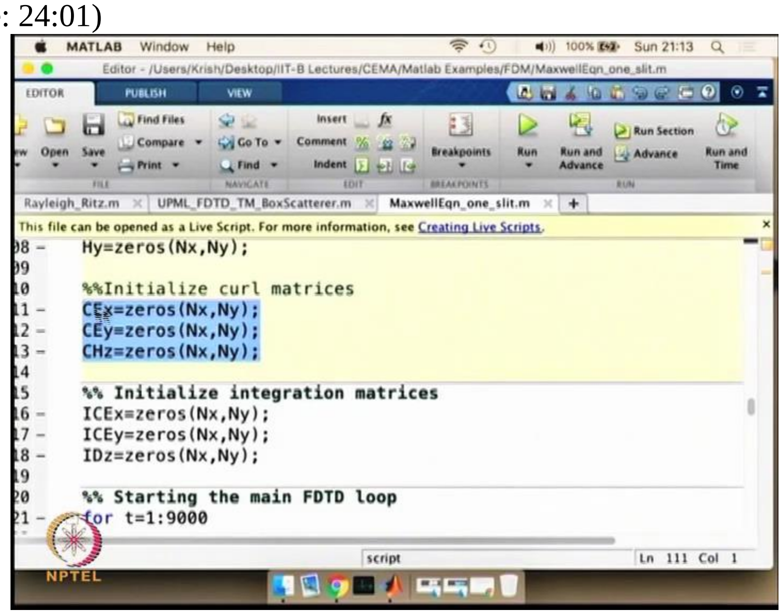

As you can see here we have three curl components. So the curl Ex component, the curl Ey component, curl Hz component. And these are the components that I have described to you while we were discussing the initial stage.

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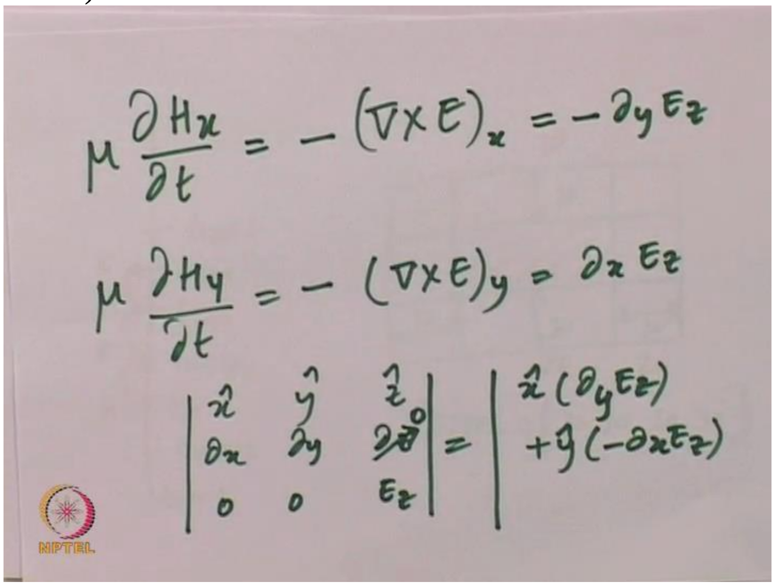

So let us look back those components here. So these are the curl Ex component, curl Ey component,

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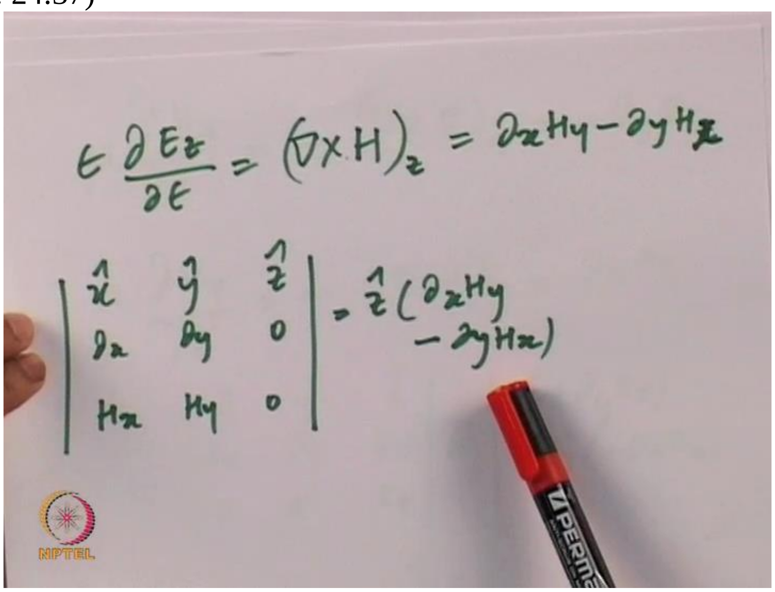

And the curl z component is written separately here. So the curl Hz component is here.

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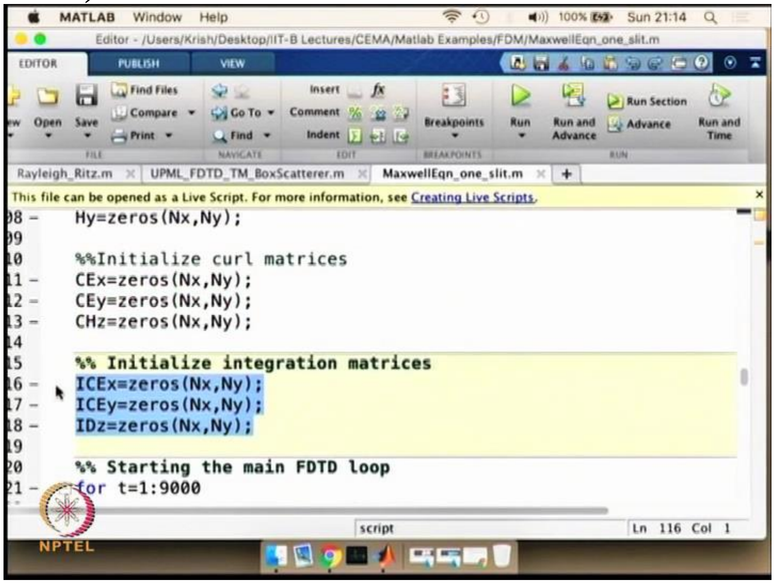

So these are the terms that we are going to use. So let us get back to the code. We are initializing these values here as zeroes. And these are the integral terms. So the integral terms are nothing but the terms that we are going to use in the update equation itself. So I have described that very shortly when we did the update equation.

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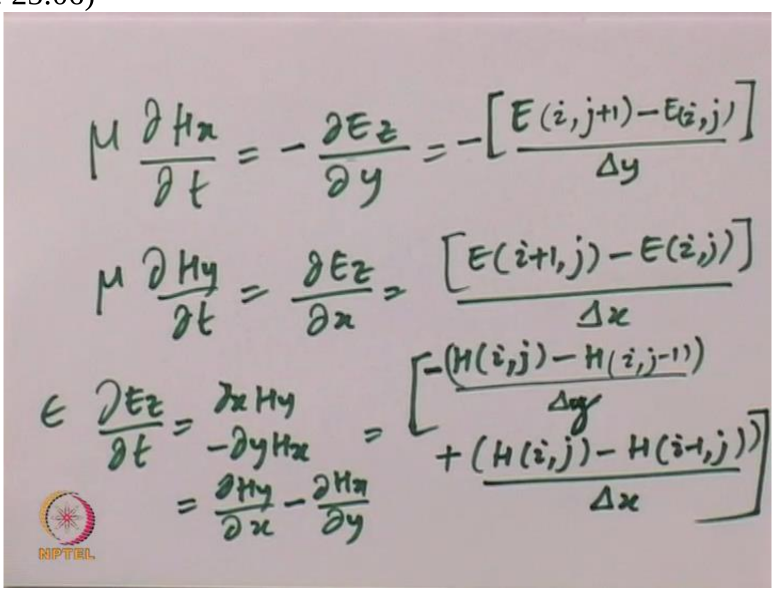

So Let us go back and look at the update equation one more time. So let us take one of the update equation and expand it in a way that it makes sense, for us to see the comparison what we have in the Matlab code.

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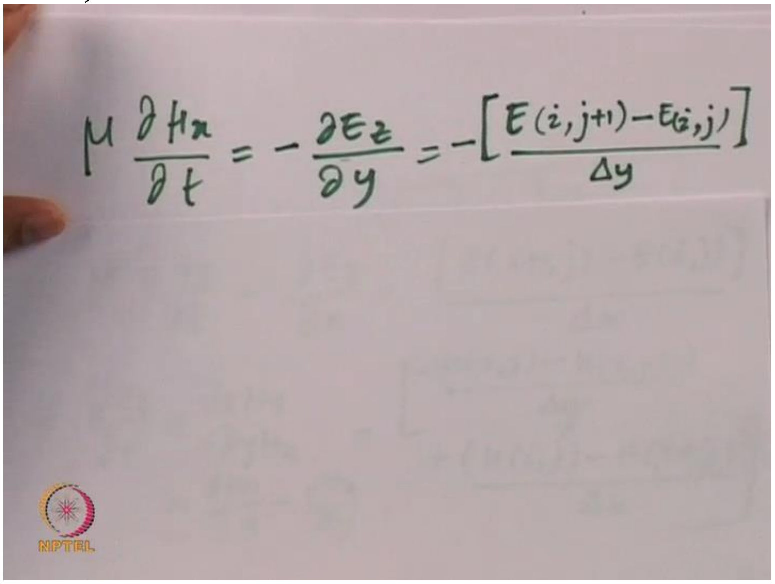

So I am going to take the first equation, and I will not do the other two. The other two you can follow through the logic what we are discussing for the first equation.

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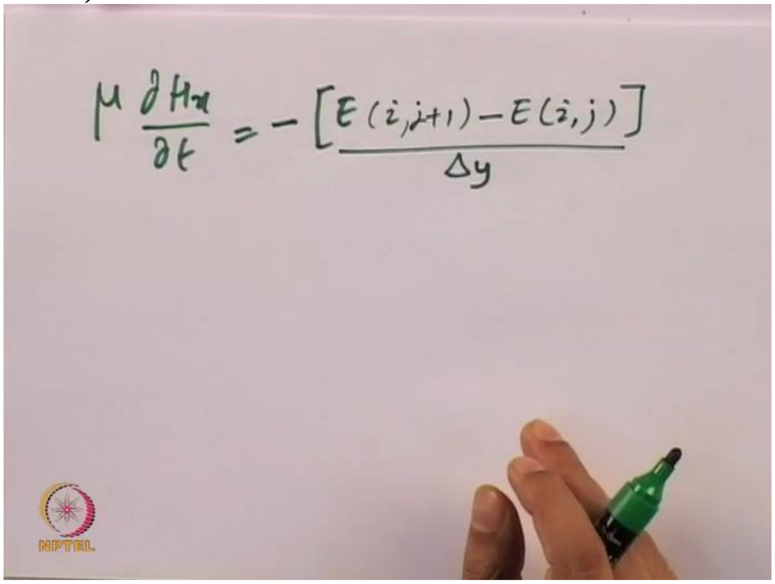

So the first equation is written as Mu Doe Hx divided by Doe t equal to minus [E(i,j plus 1) minus E (i,j)] divided by delta y.

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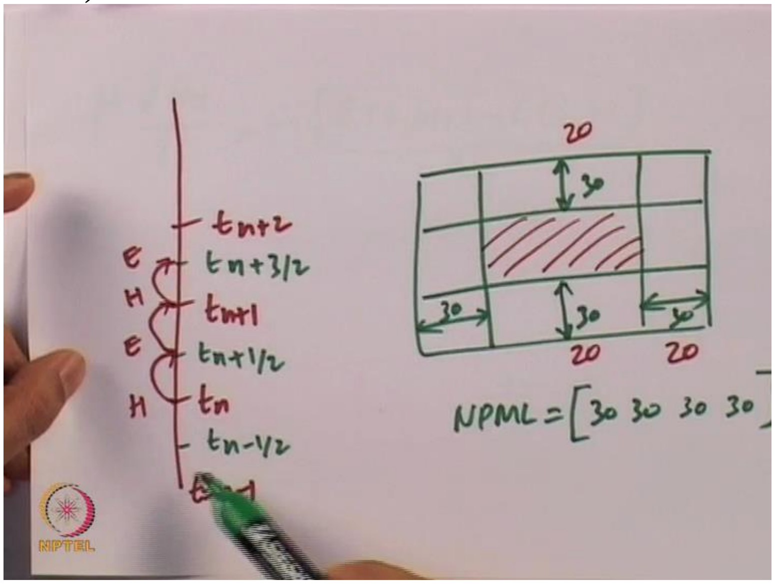

We are going to compute E at different time steps compared to the value we are going to compute H. So H we are computing at every n and E we are computing at every half steps.

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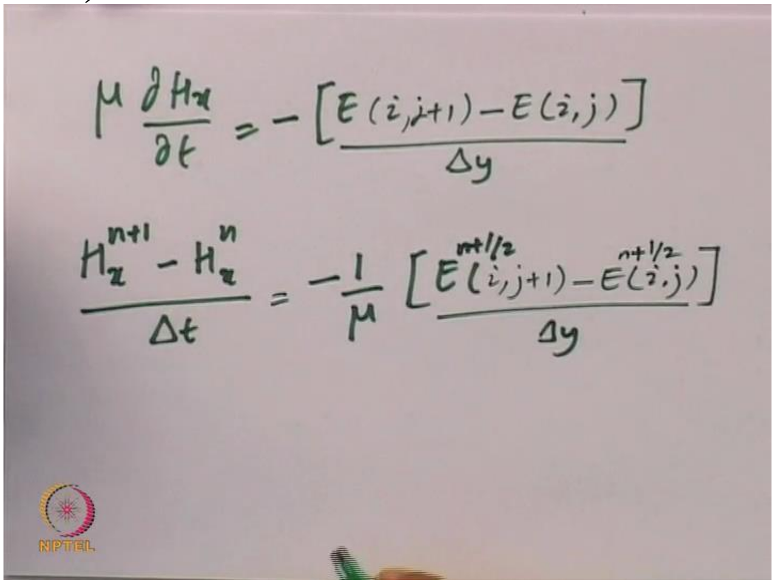

So here we have to write that and this will become clear when we expand this, so this is going to be, I am going to take the Mu on the other side, and I am going to write H x n plus 1 minus H x n divided by delta t is equal to minus 1 by Mu [E(i,j plus 1)minus E(i,j)] divided by delta y. And here they are going from n to n plus 1, so the E value will be at time step n plus 1by2 { minus 1 by Mu [E n plus 1/2(i,j plus 1)minus E n plus 1/2 (i,j)]} divided by delta y. So this is the half time step where we are computing E.

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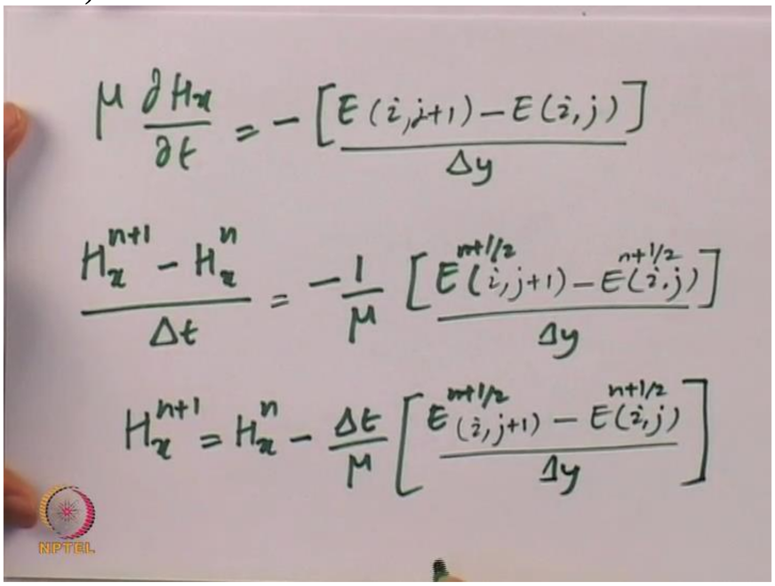

So now when I rearrange the term what I get for H x n plus 1 is equal to H x n (I take the x on the other side) minus delta t by Mu[E n plus 1/2(i,j plus 1)minus E n plus 1/2 (i,j)] divided by delta y.So this is the update equation that we will be using for Hx component.

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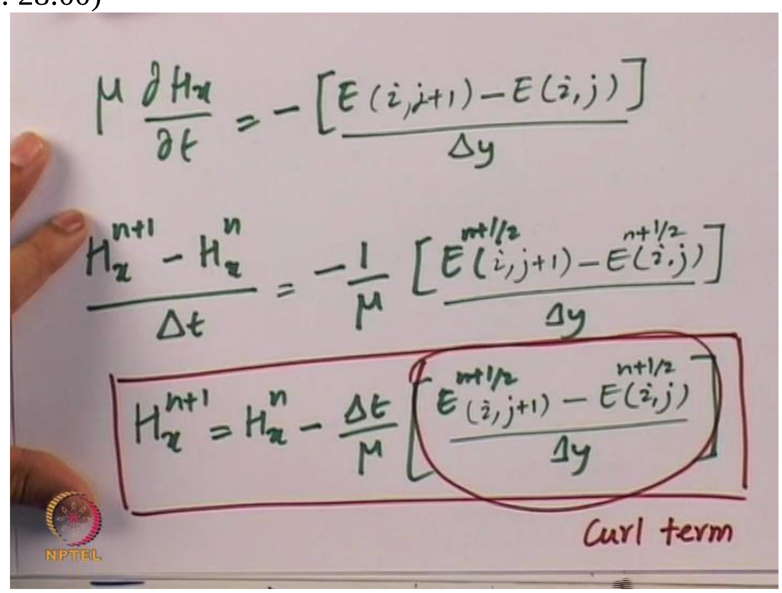

Similarly you can derive for H y n Ez component. So here the integral term what we are talking about are the terms what we are going to keep adding up. So this is going to be the curl term that we are going to add up. So this is this component will be the curl term. And every time we compute the curl term we keep adding it, we will get the integral term.

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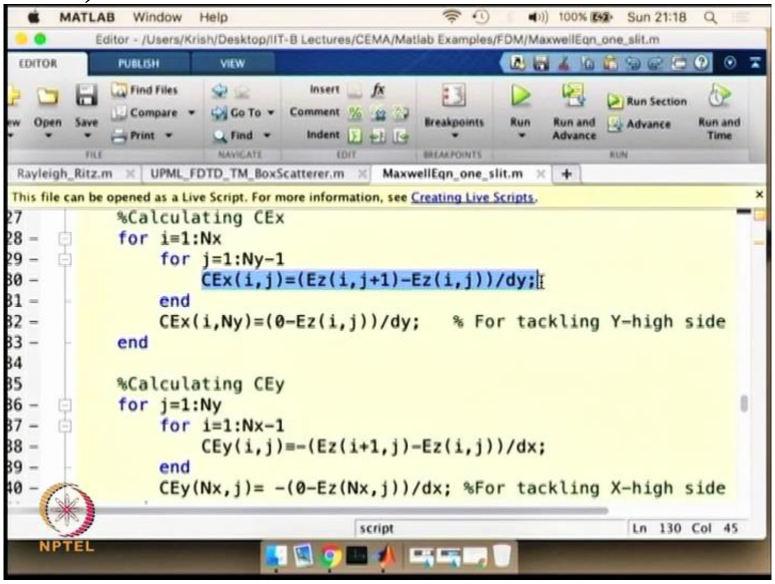

So let us look at the curl of E, So this is going to be the Ez i,j plus 14 minus E(i,j) divided by delta y . This is exactly what we have assumed in the equation what we have here.

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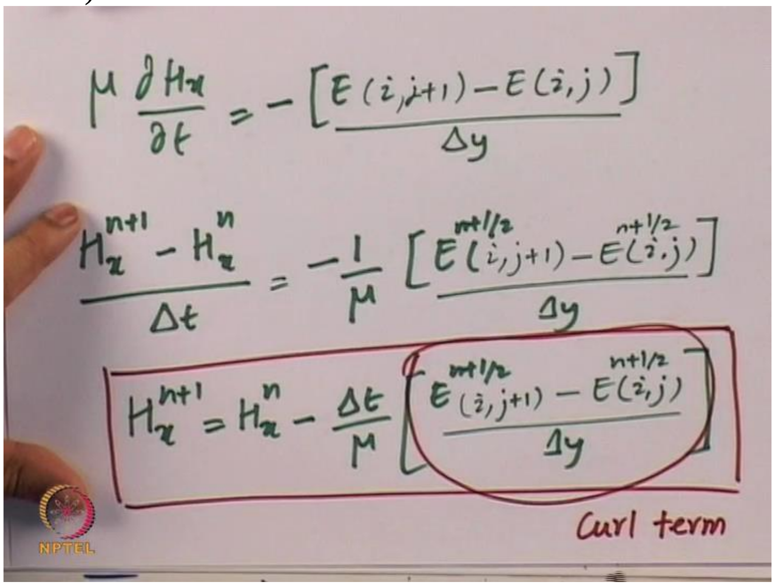

If you see on the paper so this is the value what we have set on the equation in the Matlab file here.

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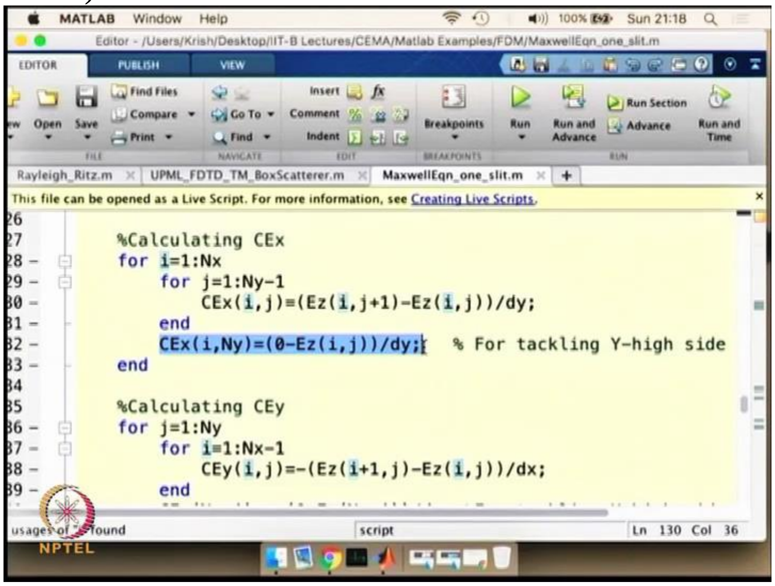

And we are also making sure the boundary conditions are satisfied.

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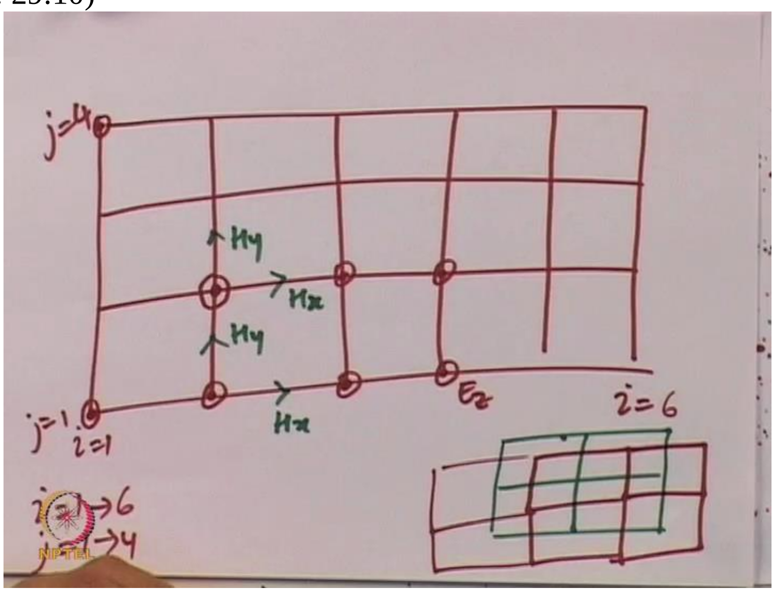

So let us look back at our computational grid. So E is going to go on the node values as I said. so on the upper side of the domain you donot have enough points for youto compute j plus 1. So the j plus 1 component does not exist. It goes only until the J. The j plus 1 component does not exist.

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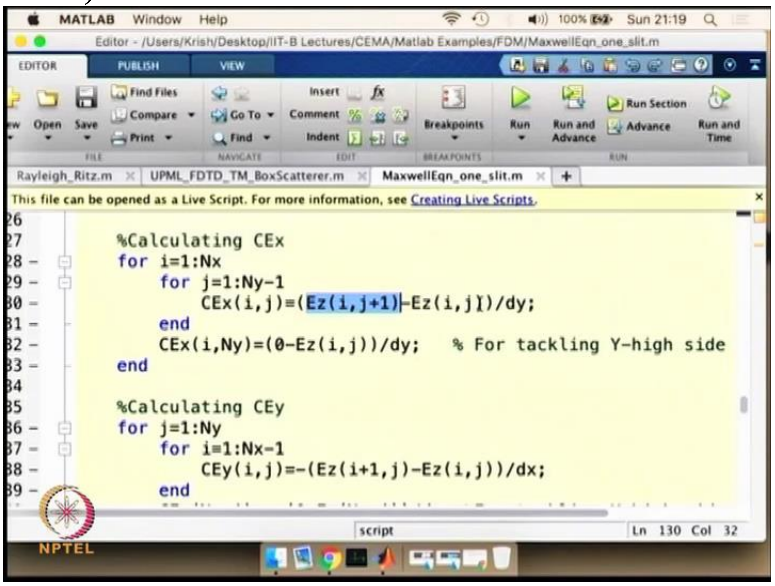

That is why for this particular case we are putting j plus 1 component as 0 and E (i,j) exist as such.

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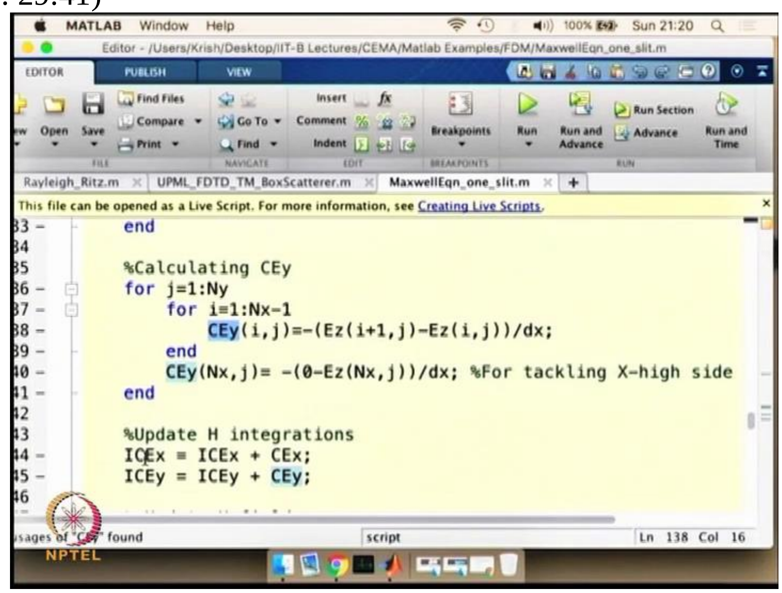

Similarlythe other components of the Maxwell field in 2Dcase so this is going to be the curl of Ey component.

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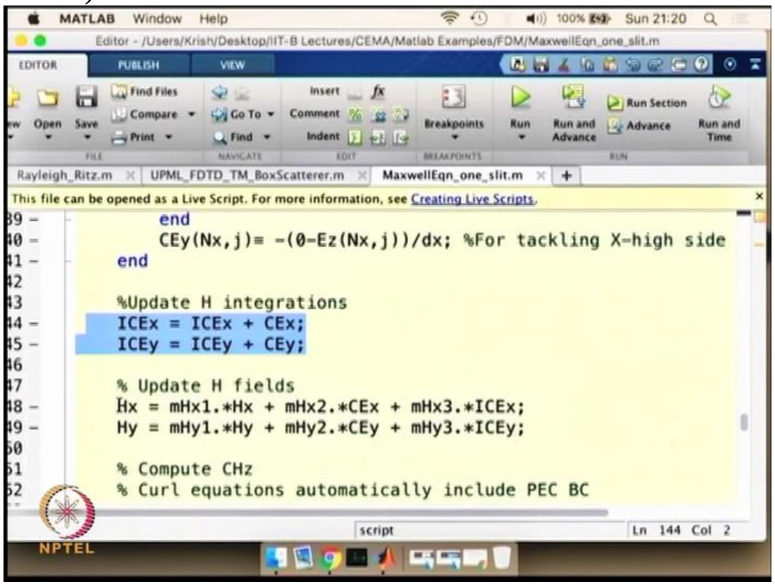

And once we have that we are going to update that into the integration term. So ICEx is the integration component of the curl, so it keeps adding up.

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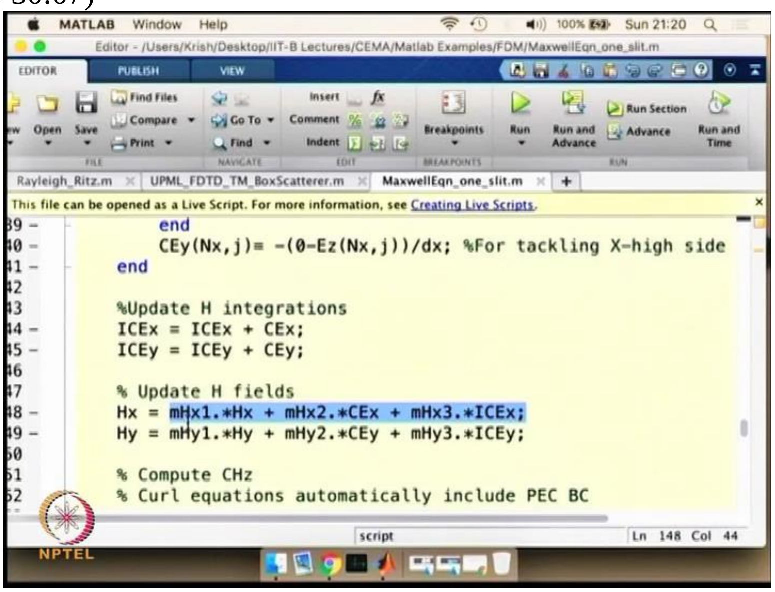

And once we have that into the equation we are able to find out the value of Hx and Hy using this particular form.

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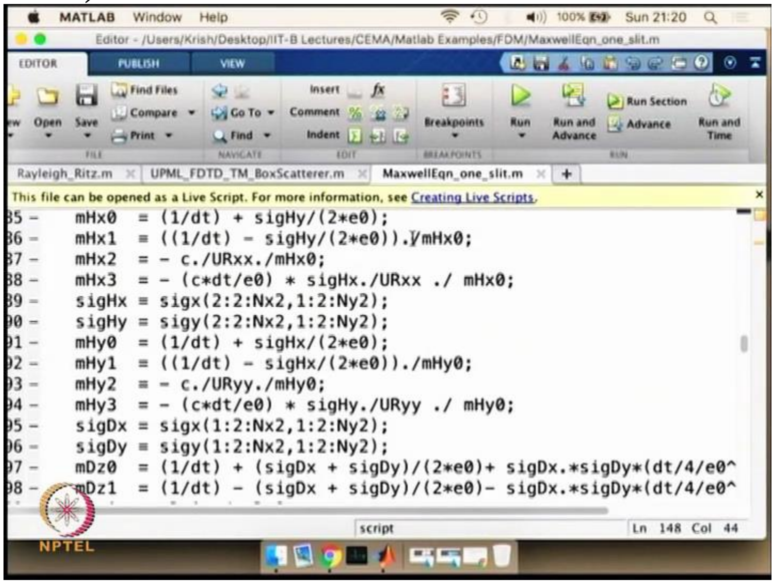

The value of mHx1 is defined in the previous steps. mHx1 is going to be 1 by dt minus sigma Hy divided by 2Epsilon 0.

So If you see when you put sigma Hy, and sigma Hx and all these components to be 0. They will reduce to the simple update equations that we have derived just now. So this is going to be the same equations what we have on the paper.

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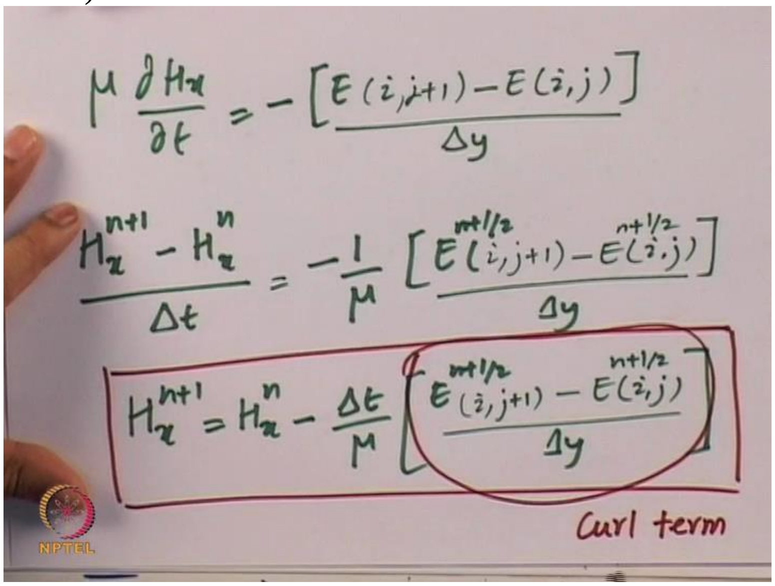

It is going to be delta t by Mu and so we are not worried about the PML factors here.

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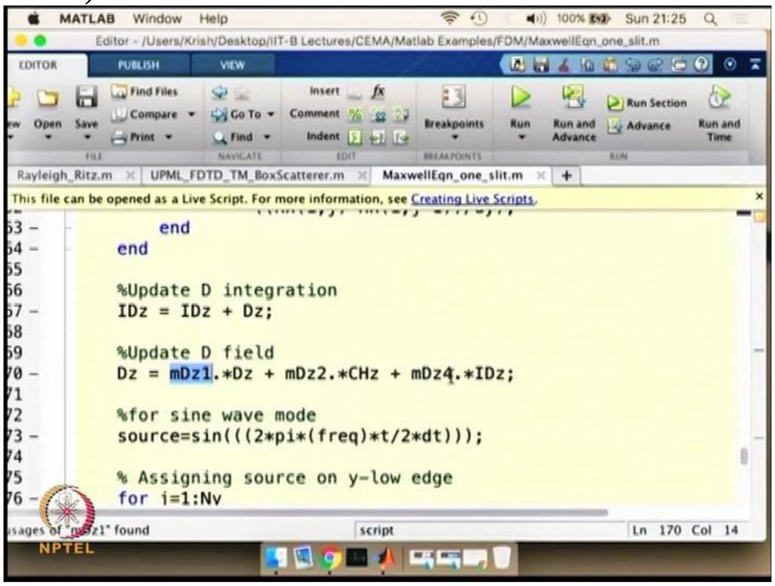

So the update equation what we are getting here is similar to the equation what we have here. The reason for having this mDz1 mDz2 mDz4 is to make sure that the PML conditions are satisfied. But other than that this is nothing but the simple update equations what we have described here on the paper.

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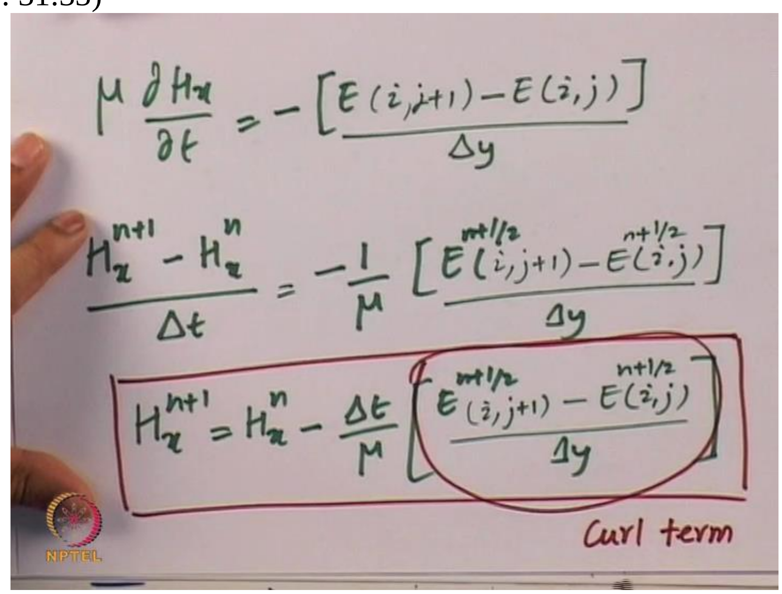

So we are now going to test this particular code for a particular excitation that we are going to have. So what we are going to have is. We are going to start with a simple sinusoidal source. And we are going to modulate this source using a mode. So I am going to explain that step by step on the paper.

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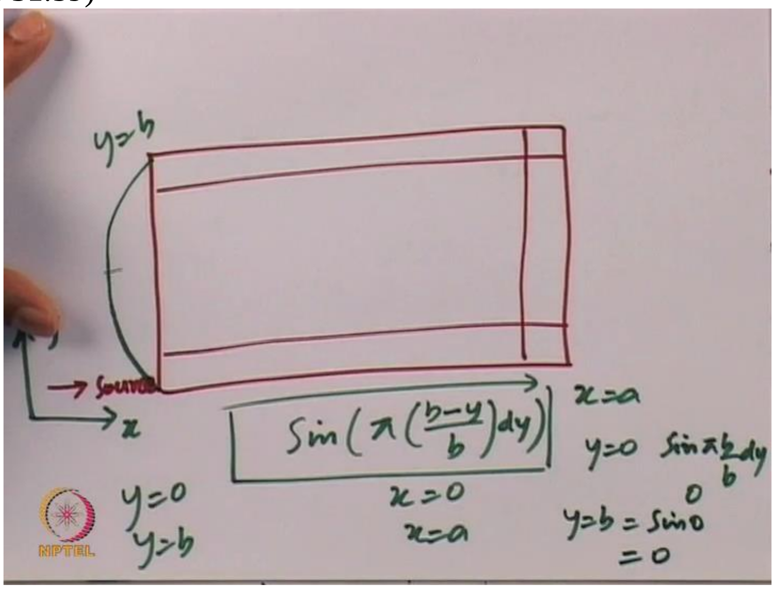

Boundary is going to have PMLs on the three sides. There is going to be a PML here. There is going to be a PML here,There is going to be a PML here, and these are all uniaxial PMLs and this is the direction of the source. The source is going to have both sinusoidal factor and a mode factor. So it is going to have maximum at the middle and minimum at both edges.This is we are going to achieve by using the mode function. So what we are going to have is this is the y direction this is the x direction and from the y direction we are going to say it is going to behave with a function Sin (Pi(b minus y divided by b)dy) x equal to a.

So the mode is going to have a profile on the y direction. So our y goes from y equal to 0 to y equal to b. And our x goes from x equal to 0 to x equal to a. And I said it is going to have a mode on the y direction. And the middle will be the maximum and we are achieving this by using this particular equation and we are multiplying this equation with the source function.

So this equation will have two functions you will see when you put y equal to 0.This equation will have a y equal to 0. This will become sin (Pi b by b multiplied by dy, which is going to be 0;side of Pi is 0and when y equal to b this is equal to b minus b which is 0. So sin 0 will be again

0. So it is going to have 0 at both the ends of the domain and it is going to have maximum at the middle.

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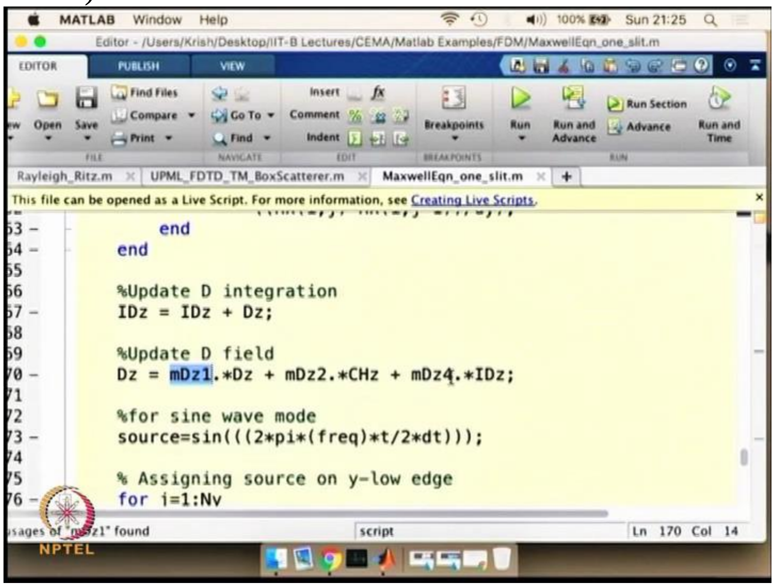

So the source is going to be 3 Giga Hertz, so we are going to write the source as also sin function.

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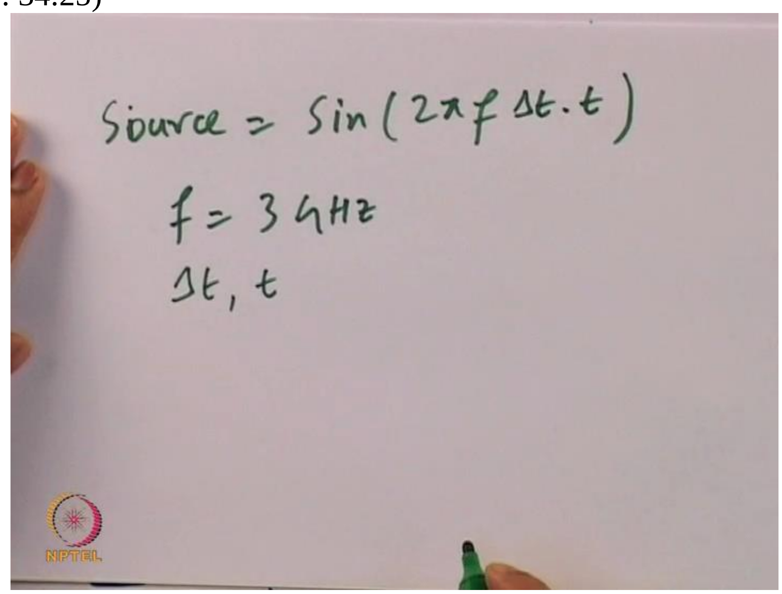

So the source function is going to be given by sin (2 Pi f delta t multiplied by the time step t itself). So this is f is going to be 3 Giga Hertz. And delta t and t are computed accordingly to the equation. And we can substitute that into this particular formula. We will get the source.

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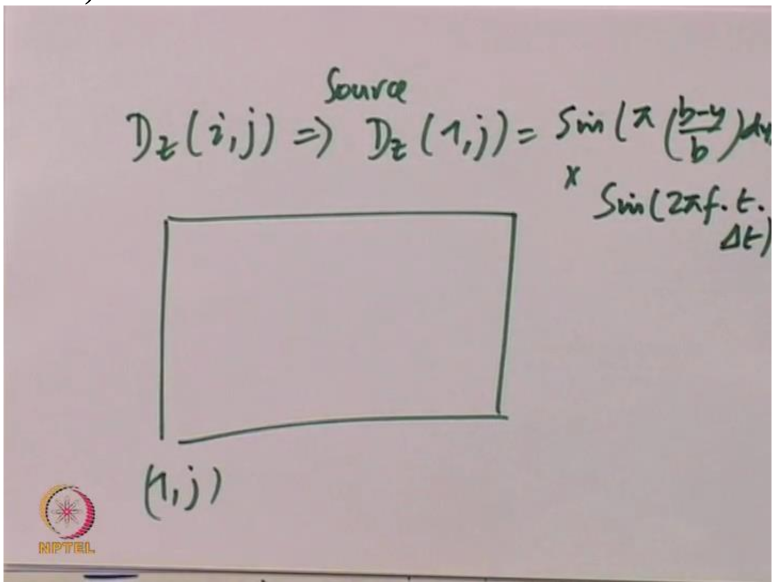

So the Dz value is given by (i,j) and we are setting the source exactly at the first line. So this is going to be 1,j line. So we are going to set for the source Dz(1,j) is equal to the mode function. So sin (Pi (b minus y divided by b)delta y) multiplied by the source function which is sin of (2 Pi ft multiplied by delta t).

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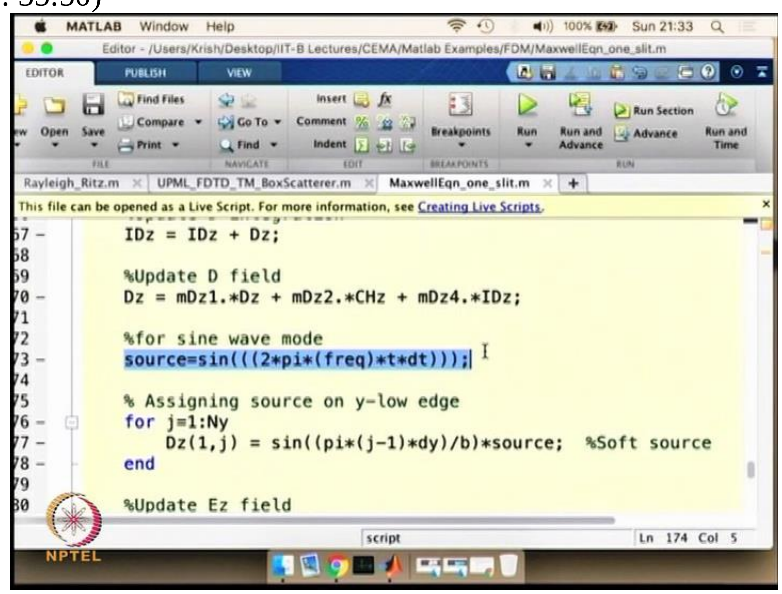

So once that is understood we will see here in this equation that this is the value we are feeding in.

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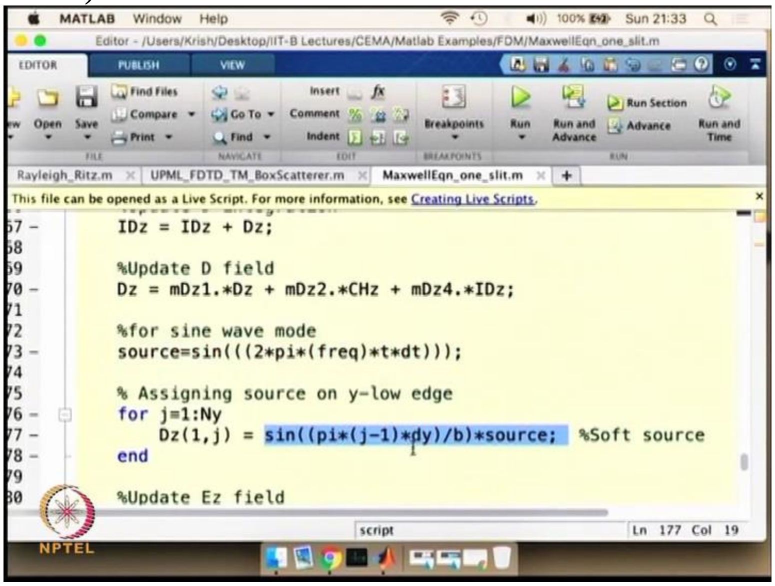

And we are multiplying it using the mode function as I explained.

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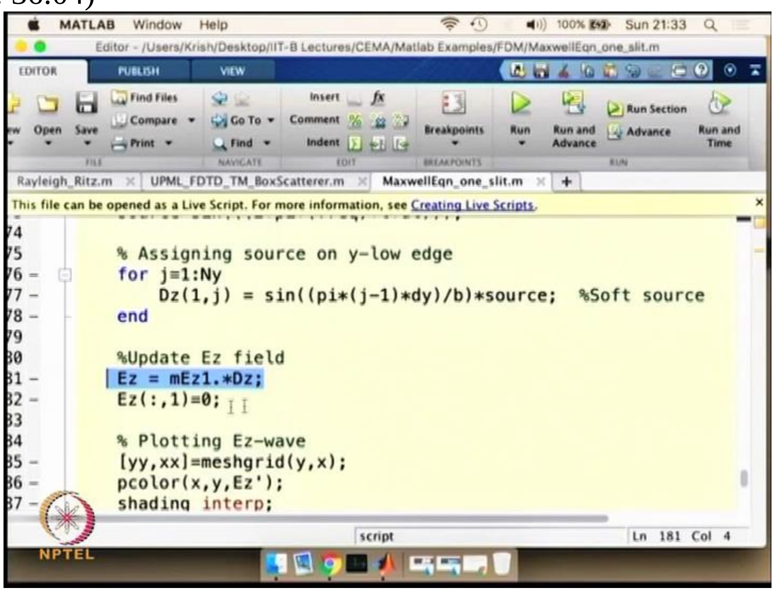

And we are updating the Ez field from the source field. The Dz field is just used for simplifying the mathematics. You can also directly do that with Ez field.But we have used here Dz field.

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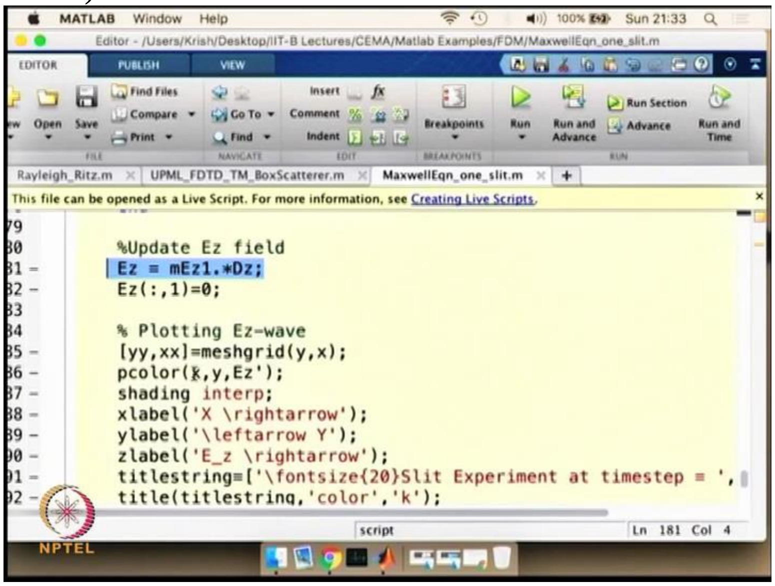

And now we are going to plot the Ez field for x and y.

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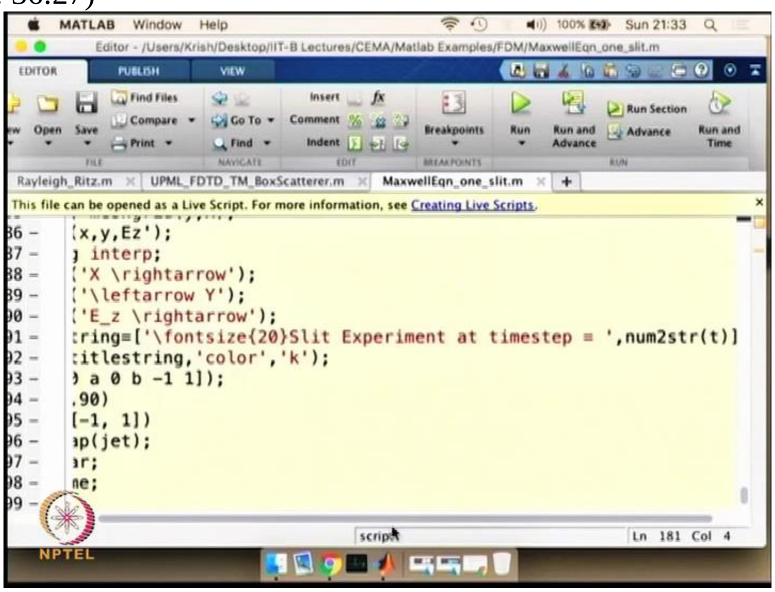

And we are going to simulate it using the slit experiment.

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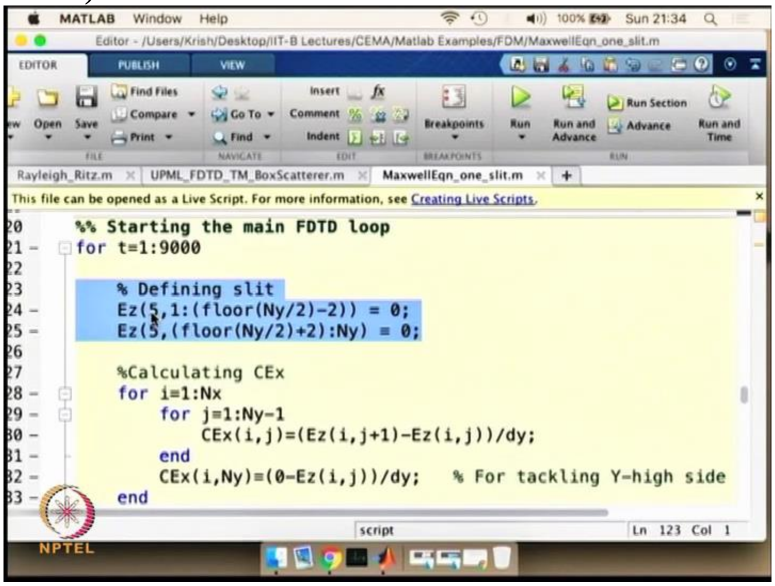

And the slit location is going to be at the point defined by these two steps. So it is going to be a point that is between Ny/2 minus2 and Ny/2 plus 2. So that is going to be four cells minus 2 to plus 2 is going to be the four cells where we are going to keep the slit. And it is going to be at the line x equal to 5.

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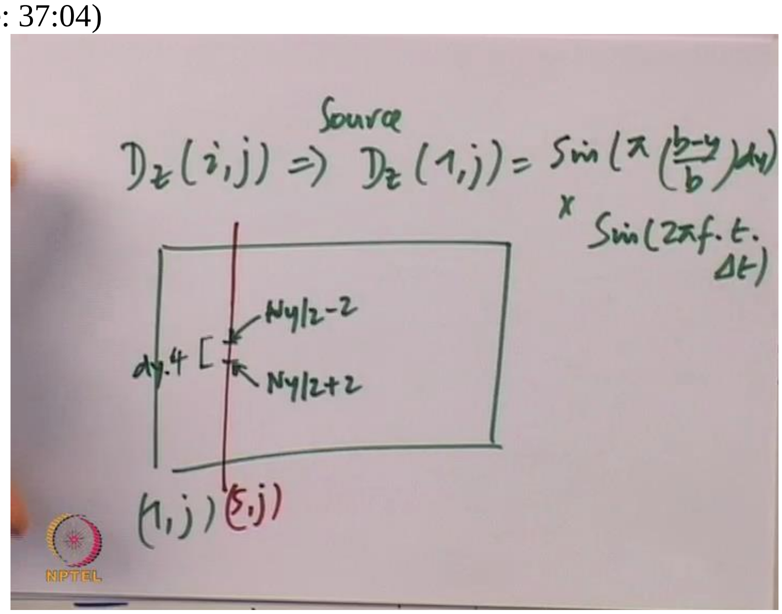

So if this is x equal to 1, so this is going to be 5,j). So this is the place where we have the slit and the slit is basically in the point given by this value Nyby 2 minus 2. So this is the upper point is Ny by 2 minus 2, the lower point is Ny by 2 plus 2. So you have a distance of 4 dy. So 4dy is going to be your slit dimension and we will see that while we are simulating.

So let us start working on this problem. So we have set all the parameters. So let us start running it.

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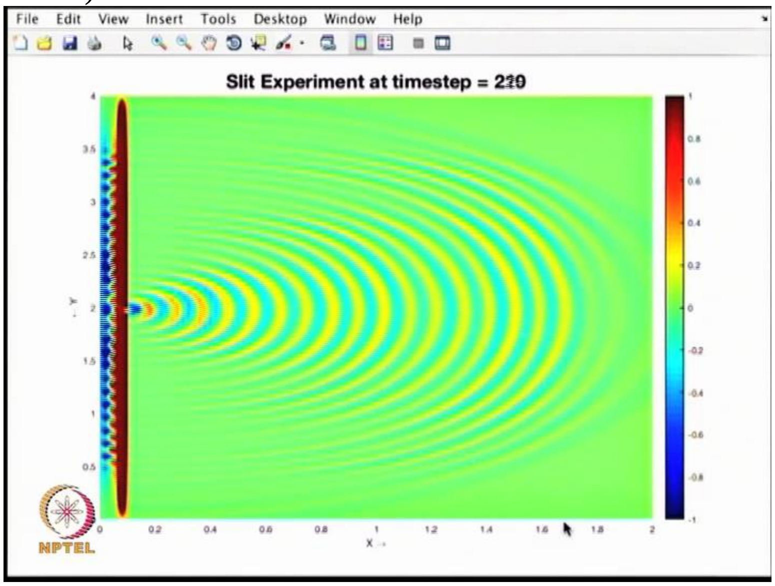

And while running it I will zoom it and you will see that the propagation of the wave is coming. So it is just a PEC that we have put here. It is going to reflect all the waves except the points here. And this is the point where there is a slit which is roughly 4 delta y. And you are seeing that the mode is getting leaked through the slit and it is coming out. And whatever is coming out you see that both in y direction and in the x direction it is getting absorbed. So you can see the waves are getting absorbed. We have a pc condition on all the three sides. So this is a source side so the PEC is here, PEC is here. And the PML is going to be here. So the PMLs are mainly there on the all three sides. And the PMLs are truncated by PEC conditions.

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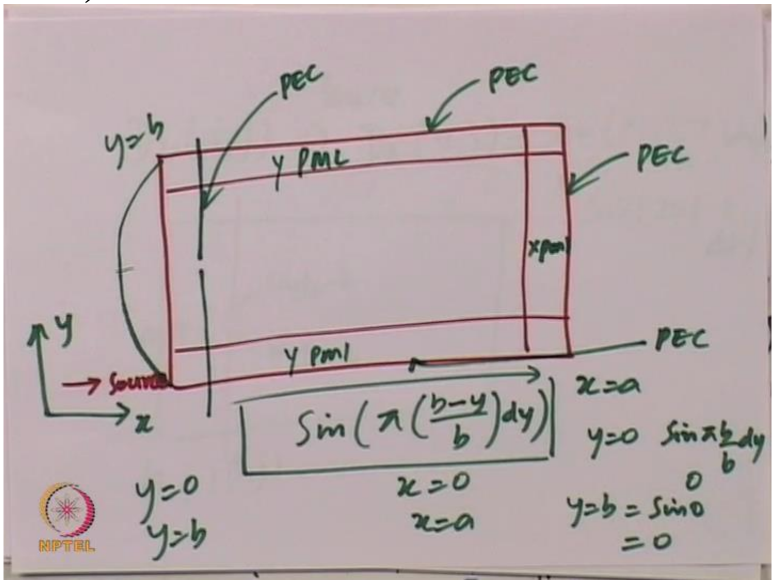

So let us look back. What we have is so this is going to be PML so this is the y oriented PML this is the y oriented PML, this is the x oriented PML and we have PEC truncating the PML. So this is important for you to know and again what we have done is we have put a small slit here and this is also a PEC. And you can see both the PMLs y oriented PML and x oriented PML is absorbing the waves as expected. And the slit experiment is behaving the way we want.

I want you take this particular example as a test case and code yourself the PML and also the slit experiment.

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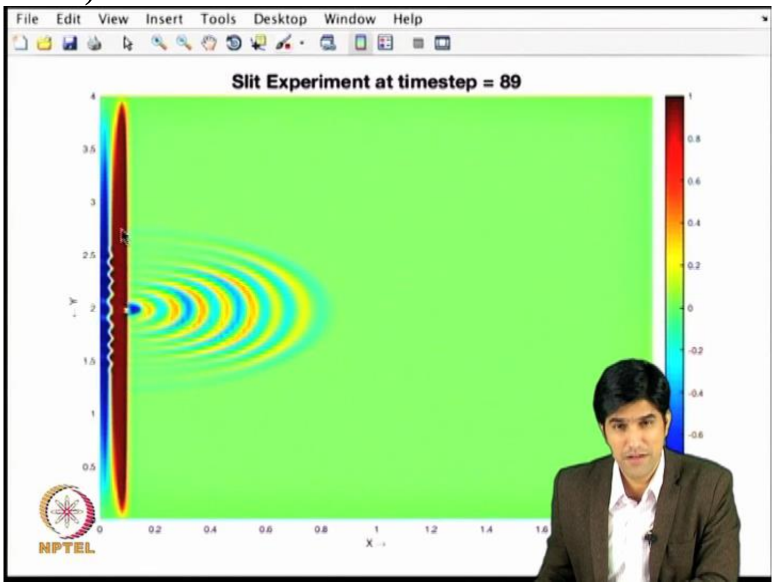

So we looked at one of the interesting problem of simulating Maxwell equation using one slit experiment. It was very fun experiment to try. I encourage you to try I donot know how much of you have really going to try, but I really encourage you to try that particular experiment. And now we are going to complicate this a little bit by having two slits. It is nothing difficult to code it in the finite difference program what we have.

So I am going to show you step by step what we have done. What you will see is it is exactly the same code except for the point where we have defined the slits.

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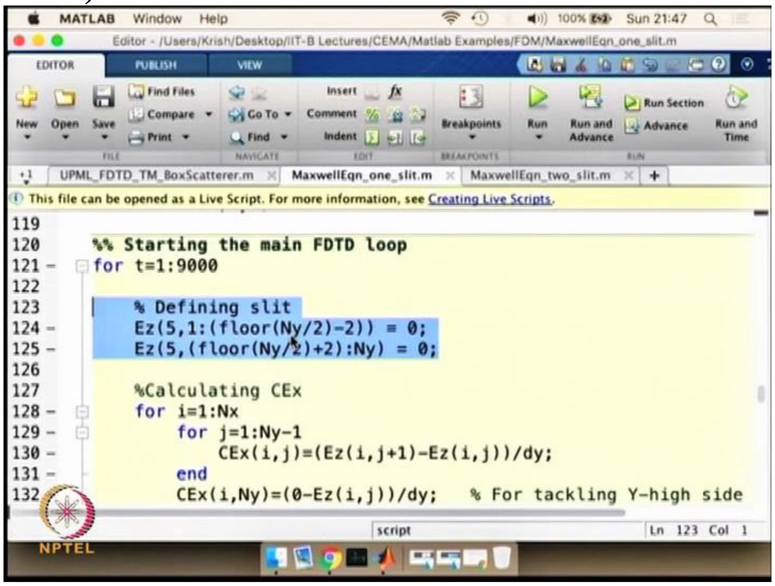

So this is the place where we are defining the slits. So if we look at the one slit experiment there is only going to be one slit.

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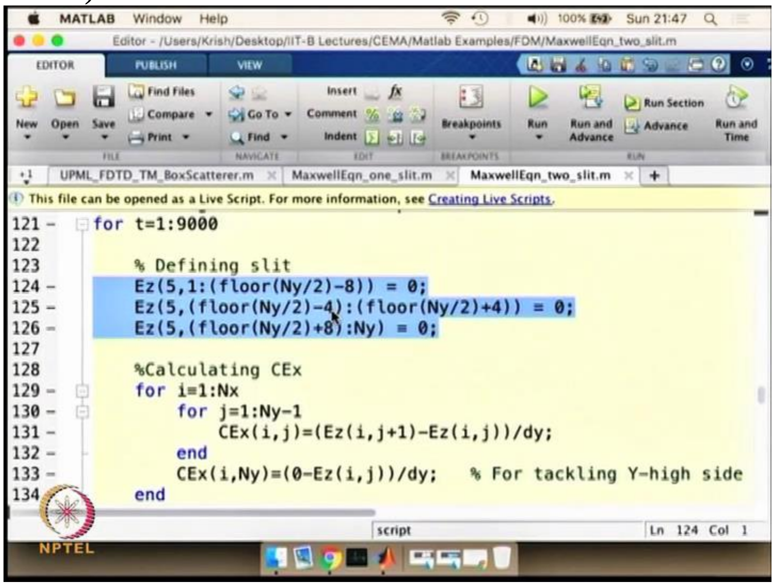

Whereas in the two slit experimentswe are going to have 2 slits and whose dimensions are given here. It is going to run from Ny by 2 minus 8 to Ny by 2 minus minus 4. And then again Ny by 2 plus 4 to Ny by 2 plus 8. So there are going to be two slits of this dimension. And they are going to be in the same location.

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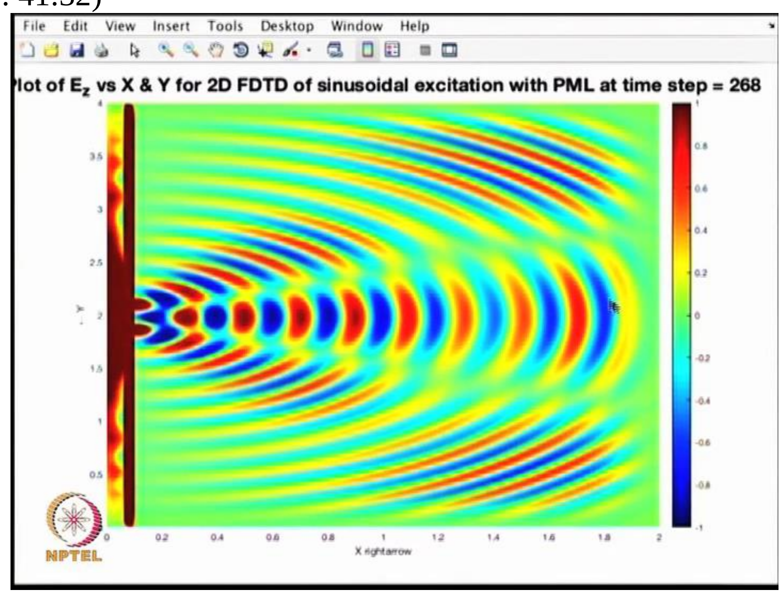

So let us run this one. What we are going to see is the mode is going to come in and you see that there are going to be two slits that are leading to two waves. They act like a point source here and they are going to have constructive and destructive differences in the wave frence. And what you see is a kind of a line straight line basically joining the center of two slits and they are going to radiate out.And you also see the maximum and minimums on the wavefronts. So these are the minimums where you have the destructive wavefronts and the maximas are the wave you have the constructive wavefronts. And the wave is propagating nicely and you see that the PML is absorbing whatever is coming in.

This is very beautiful way to test two things. One the understanding of your Maxwell equation model in finite difference method and also to test various boundary conditions. In this case we are using two boundary condition one is a simple boundary condition which is the PEC boundary condition. The second boundary condition is the source boundary condition which we are going to give as a hard source.And then there is going to be a boundary layer which is a PML. We have not discussed it here; we will describe this later on.

So I encourage you to test such problems to practice various applications of Maxwell equations. And once you know how to model that kind of problem you can test various examples, for example you can model a horn antenna. You can feed them using a particular mode and then see the radiation pattern so on and so forth.

So with this we come to the end of this particular module. I have shown you a practical way to simulate Maxwell equation using finite difference method. We have taken a simple example of slit experiment and we have slightly complicated it using two slits in order to see the maxima and minima the constructive and destructive interference so on and so forth. And we also see that how the wave is propagating and how the boundaries are behaving.

So this is an excellent example for you to try out some of your programming skills and also to model Maxwell equation related problems in a 2D finite difference method.

So please use such codes to learn and model simple problems so that when we go into advance method like finite element method, finite volume method,method of moments.You will be able to appreciate various aspects and various dimensions of modeling much more easily. So finite difference method gives you that foundation so with that I come to the end of this particular module.

Thank you!