

Data Analytics with Python
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Lecture – 19
Errors in Hypothesis Testing

Welcome students in the last lecture we have seen hypothesis formulation and testing. In that hypothesis formulation and testing we have seen what is the null hypothesis and what a t-test is. In this lecture we will see some introduction about the errors in hypothesis testing. Then we have seen Z test when Sigma is known when Sigma is not known we have done a t-test also.

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Example

- We are interested in burning rate of a solid propellant used to power aircrew escape systems
- Burning rate is a random variable that can be described by a probability distribution
- Suppose our interest focus on mean burning rate
- $H_0: \mu = 50$ centimeters per second
- $H_1: \mu \neq 50$ centimeters per second



Reference: Applied statistics and probability for engineers, Douglas C. Montgomery, George C. Runger, John Wiley & Sons, 2007

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In this lecture we will go in detail about errors in hypothesis testing we will take an example this example is taken from this book a famous book applied statistics and probability for engineers Douglas C Montgomery and at all it is very interesting books I will like to recommend this book for further reading after seeing this lecture. We are interested in burning rate of a solid propellant used to power air crew escape system.

Burning rate is a random variable that can be described by a probability distribution. Suppose our interest focus on mean burning rate so null hypothesis is $\mu = 50$ centimeters per second alternative hypothesis is $\mu \neq 50$ centimeters per second.

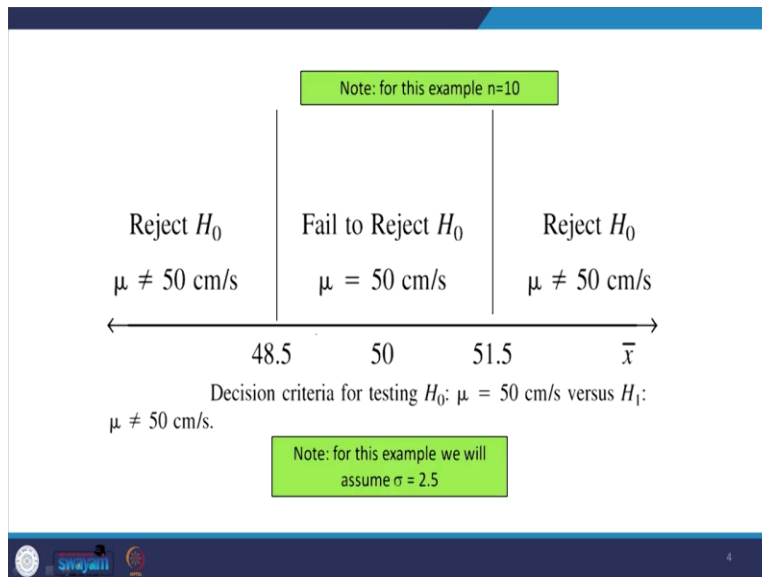
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Value of the null hypothesis

- The value of the null hypothesis can be obtained by
 - Past experience or knowledge of the process, or even from the previous tests or experiments
 - From some theory or model regarding the process under study
 - From external consideration, such as design or engineering specifications, or from contractual obligations

You see that in the previous slides we have assumed μ equal to 50 but what is the basis for that 50. There are different possibilities how we can assume the value for our null hypothesis one thing is the past experience our knowledge of the process or even from the previous test our experiments. Other possibility is from some theory or model regarding process under study even from external consideration such as design and engineering specifications are from contractual obligations we can assume the value of null hypothesis.

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Suppose I know the say I am following the say confidence interval method. So, I have found the lower limit is 48.5 upper limit is 51.5 when you equal to 50 centimeters per second. If any mean value which is beyond this 51.5 which is beyond this 51.5 if the sample mean value is beyond

51.5 I will reject null hypothesis the same time if it is below less than 48.5 again I will reject null hypothesis. So, what is happening decision criteria for testing $H_0: \mu = 50$ centimeter per second versus $H_1: \mu \neq 50$ centimeter per second. So in this example we have taken the sample size is 10 and the population standard deviation is 2.5.

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The slide is titled "Type I Error" in blue text. It contains a bulleted list of four points explaining the concept. The first point states that the true mean burning rate of the propellant could be equal to 50 centimeters per second. The second point states that however randomly selected propellant specimens that are tested, we could observe a value of test statistics \bar{x} that falls into the critical region (rejection region). The third point states that we would then reject the null hypothesis H_0 in favor of the alternate H_1 , in fact, H_0 is really true. The fourth point states that this type of wrong conclusion is called a type I error. At the bottom of the slide, there are logos for "Sreyani" and "5".

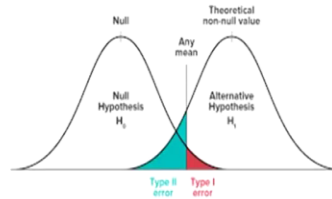
So, by using the previous example I will explain what is the meaning of this type 1 error. See the true mean burning rate of the propellant could be equal to 50 centimeter per second however randomly selected the propellant specimens that are tested we could observe a value of test statistics \bar{x} that falls into the critical region a rejection region. What in the rejection region? Below the lower limit above the upper limit.

So, if our \bar{x} is lying on the rejection region we will reject null hypothesis if the \bar{x} is lying on the acceptance region will accept null hypothesis. So, we will go to the next point we would then reject null hypothesis H_0 if in favor of alternative H_1 in fact H_0 is really true. So, this type of wrong conclusion is called type 1 error. What is the meaning of type 1 error? Even though the null hypothesis are true but the \bar{X} BAR value lying on the rejection side we have rejected null hypothesis this is called type 1 error or incorrect rejection.

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Type I Error

- Rejecting the null hypothesis H_0 when it is true is defined as a type I error



You see for example in this slide you see that there are two normal distribution one is on the left hand side another one is right hand side. So, what will happen this is a one tailed test for this value which is in the red portions sometime the value of \bar{x} may lie on the right hand side that is in the regions. We will reject null hypothesis even though the μ value is same. For example in this case μ value is 50 for example this normal distribution the mean equals to 50.

So, what is the meaning of type 1 error rejecting the null hypothesis H_0 when it is true is defined as the type 1 error. So, what will happen actually the null hypothesis is true because the sample was randomly selected the value of \bar{x} is falling on the rejection region we have rejected a null hypothesis but it is not correct, so, it is incorrect rejection so this is called type 1 error.

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Type II Error

- Now suppose the true mean burning rate is different from 50 centimeters per second, yet the sample mean \bar{x} falls in the acceptance region
- In this case we would fail to reject H_0 when it is false
- This type of wrong conclusion is called a type II error

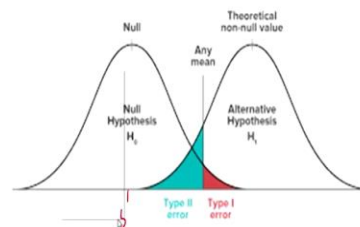
Now we will explain what is the meaning of type 2 error?. Now suppose the true mean burning rate is different from 50 centimeter per second yet the sample mean \bar{x} fall in the acceptance region what will happen this now the null hypothesis is not true but the sample mean is following in the accepting region in this case we would fail to reject H_0 , when it is false. So, what is happening the H_0 is false but still we have accepted our H_0 , so it is false acceptance so this type of wrong conclusion is called type 2 error.

So I just am saying what do you need I type 1 type 2 error see the type 1 error is incorrect rejection type 2 error is false acceptance.

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Type II Error

- Failing to reject the null hypothesis when it is false is defined as a type II error



You see this situation there are two normal distribution one is we have seen this is 50 say it is 52 now what has happened there are 2 population which are overlapped my concern is about the population whose mean is 50. But there is an another population whose mean is 52 which are overlapping. So, what will happen this much regions is not belongs to that light green one it is not belongs to this population. So, this region this region is belongs to population whose mean is 52 just because off it is lying on acceptance side of this population where the mean is 50.

We are wrongly we are falsely accepted, so it is called a type 2 error, so, failing to reject null hypothesis when it is a fall is defined as the type 2 error.

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Type 1 and Type II Errors

	H_0 is correct	H_0 is incorrect
H_0 is accepted	correct decision	Type II error (β) Incorrect acceptance
H_0 is rejected	Type I error (α) Incorrect rejection	correct decision

You see the type 1 type 2 error see the H_0 is correct but we have rejected. So, this is your incorrect rejection alpha the H_0 is incorrect but we accepted this is a type 1 error. So, that is a false acceptance this we have seen previously also the same table.

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Type I error

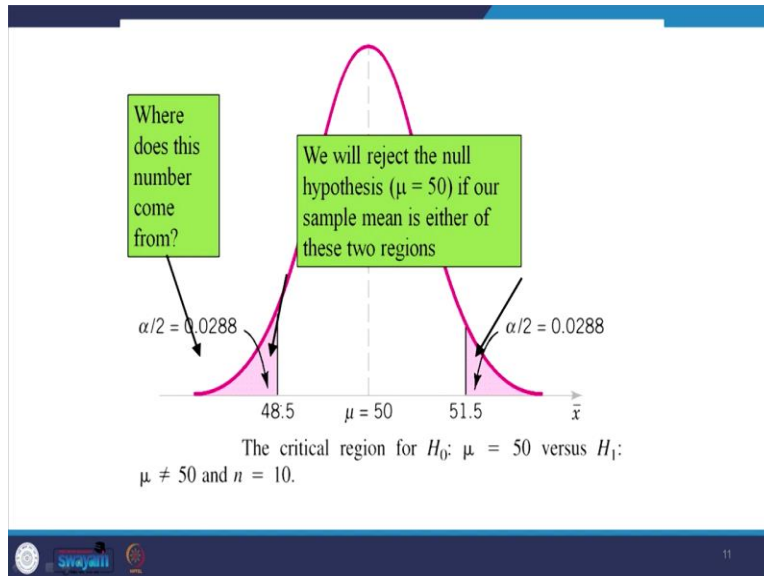
- In the propellant burning rate example, a type I error will occur when either $\bar{x} > 51.5$ or $\bar{x} < 48.5$ when the true mean burning rate is $\mu = 50$ centimeters per second
- Suppose the standard deviation of burning rate is $\sigma = 2.5$ centimeters per second and $n = 10$
- Probability distribution $\mu = 50$, standard error = 0.79.
- Type I error is

$$\alpha = P(\bar{x} < 48.5 \text{ when } \mu = 50) + P(\bar{x} > 51.5 \text{ when } \mu = 50)$$

Now you will see how to calculate type 1 error and type 2 error first we will go with the type 1 error. In the propellant burning rate example a type 1 error will occur when either the sample mean is greater than 51.5 or our sample mean is less than 48.5 when the true mean burning rate is μ equal to 50 centimeters per second. Suppose the standard deviation of the burning rate is that is a σ is 2.5 centimeters per second and n equal to 10, so the probability distribution μ equal to 50 the standard error actually standard error is over σ by root n that value standard error is 0.759.

So what is the value of the type 1 error is when the probability of \bar{X} less than or equal to 48.5 when true mean is 50 plus when the \bar{X} is greater than 51.5 when true mean equal to 50.

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You see that you see in the left hand side it is a 48.5 when the sample mean is what is the probability of sample mean to lie below 48.5 plus what is the probability of that sample mean to lie above 51.5 when you add that that corresponding probability is nothing but your type 1 error. You see that we got alpha by 2 0.0288 similarly on the right hand side you will see how this has come.

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Defining function for calculating alpha value

In [6]: def z_value(x,mu,SEM):
        z = (x - mu)/SEM
        if(z < 0):
            alfa = stats.norm.cdf(z)
        else:
            alfa = 1 - stats.norm.cdf(z)
        print (alfa)

calculating alpha for different values of x,mu, and SEM

In [8]: x =48.5
        mu = 50
        SEM = 0.79

In [9]: z_value(x,mu,SEM)
        0.02879971774715278

```

Using Python actually I have pasted the print screen of the Python after running first we for finding the type 1 error will define a function. So, that function I'm going to call it as def of Z underscore value X, mu, y sem standard error of mean you see that whenever we define function there should be a colon. So, first I am finding Z value Z values x - mu by standard error if the Z

value is less than 0 that means if the value of it will be like this if the Z value is the Z values on the negative side simply the p value is nothing but the cumulative distribution function of Z.

So, when you type alpha equal to alpha I just I am naming equal to stat stat norm dot cdf Z you will get alpha value suppose if the Z values greater than 0 we have to find out the right side area so if we want to know the right side area you the whole area has to be subtracted from one so that we will get the right side area it yells : alpha equal to 1 - stat stat norm dot cdf Z so print alpha, so calculating alpha for different value of x mu and standard error of mean.

You see first I will find out the left side area so when area suppose this is X values 48.5 what will be the area? So, X is 48.5 mu this is 0 for standard normal distribution but we at present we are taking mu equal to 50 because outer community to Z scale to become 0, standard error of the mean is Sigma by root n 0.75. Now we will call that a function which we are defined. So, Z value so we have to give the value of x because that function is defined 48.5 mu is 50 standards of the mean so the left side area will get 0.0287 this value is given left side also see that 0.0288 right.

Now we have to find out the right side actually this is alpha by 2 value whatever value which we got it this value is alpha by 2. If you put the upper value that is 51.5 alpha by 2, so if you replace instead of 48.5 if you if you put 51.5 here when you replace this X and when you replace 51.5 what will happen this Z value will be positive the Z value is positive this command will be executed. So, if you want firstly they will find this left side area then from one the left side area will be subtracted then we will get the right side area so we will get another 0.0287.

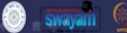
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Type I error

- Type I error = 0.057434
- This implies that 5.7% of all random samples would lead to rejection of the hypothesis $H_0: \mu=50$ centimeters per second.
- We can reduce the type I error by widening the acceptance region. If we make **critical value 48** and **52**, the value of alpha is 0.0114 (adding 0.0057 and 0.0057).

In [40]: `z_value(48,mu,SEM)`
 0.005676434117424844
- Change sample size to 16 then alpha is 0.0164.

In [41]: `z_value(52,mu,SEM)`
 0.0056764341174248

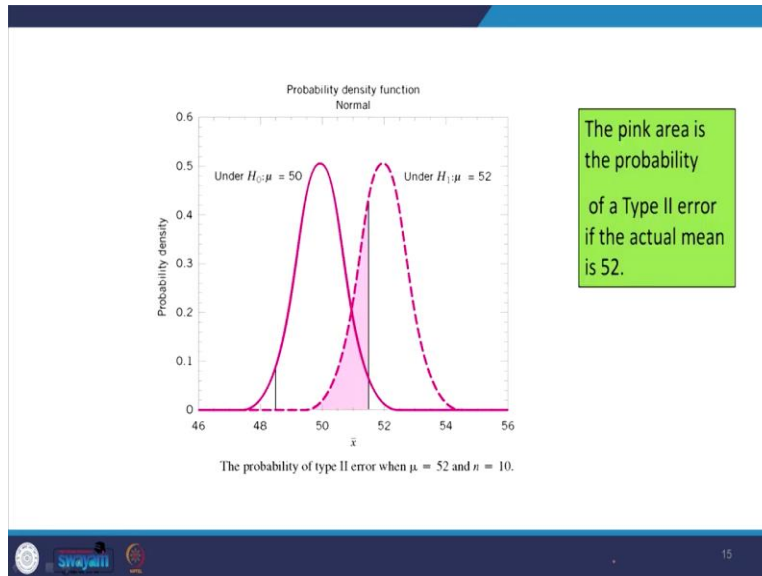

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When you add this $0.0280 + 0.0287$ you will get the Alpha value so that alpha is 0.057 this is your value of alpha error type 1 error. So, what is the meaning of this type one error this implies that 5.7% of all random samples would lead to rejection of null hypothesis that is H_0 when μ equal to 50 centimeters per second. So, there is a possibility of rejecting to null hypothesis is 5.7%, so we can reduce the type one error 1 possibility is by widening the acceptance region.

What is the widening the accept region if you make critical value 48 and 52 what will happen what is the widening of this acceptance region is suppose this is currently this one so when you increase now it is 51.5, so now you make it this right hand side 52 left hand side 48 so what is happening the acceptance region is widened yeah you see that lower side on 48 upper side 52 so what is the area cutting $0.00567 + 0.0567$ when you add it it will become 0.20114.

So, what is happening the type 1 error can be reduced by increasing the acceptance region that is one possibility another possibility is if you increase the sample size the previous problem you have taken sample size is 10, now from 10 if you increase 16 what is happening the value of alpha is decreasing that means the Alpha is decreasing means we are more accurate in making decision that is the possibility of incorrectly rejection is reduced.

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Next we will go to type 2 error first we will explain what is a type 2 error and I will take 2 example 2, 3 examples to calculate the value of type 2 error. In the previous example what has happened there are 2 population which are overlapped so this is 50 this is 52 the rejection region this is this much when the population mean is 15 there is another population whose mean is 52 which is over lapping.

So, this overlapping region is that pink one because the pink portion is lying on acceptance side we have falsely accepted assuming that that region has come from the population mean whose value is 50. So, it is a false acceptance right so this pink region is nothing but the value of your type 2 error.

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Type II Error

- Type II error will be committed if the sample mean \bar{x} falls between 48.5 and 51.5 (critical region boundaries) when $\mu = 52$. $\beta = P(48.5 \leq \bar{x} \leq 51.5 \text{ when } \mu = 52)$
- 0.2643
- When $\mu = 50.5$
- 0.8923

```

In [4]: beta = stats.norm.cdf((51.5-52)/0.79) #
Out[4]: 0.2639575390741593

In [5]: beta
Out[5]: 0.2639575390741593

In [8]: beta = stats.norm.cdf((51.5-50.5)/0.79) #
Out[8]: 0.892117321157791

In [9]: beta
Out[9]: 0.892117321157791

```

The probability of type II error when $\mu = 52$ and $\sigma = 10$.

Suppose how to find out this type 2 error see type 2 error will be committed if the sample mean \bar{x} falls between 48.5 and 51.5 critical region when μ equal to 52 you see that it is a 52 not μ equal to 50 when I say 52 that population is nothing to do when the mean equal to 52 population because it is something other population but it is lying on the acceptance side so I am accepting falsely I have accepted so that is our type 2 error. So, what is the possibility so this is worth 51.5.

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$z = \frac{51.5 - 52}{10/\sqrt{n}}$

So what is happening there are 2 population is overlapped. So, this is our original μ equal to 50 so this region is 51.5 this left hand side is what is that value 48.5 this is for my assumed hypothesis value in μ equal to 50 but actually what has happened there is another population

whose mean is 52, so I am extending this so this much portion this much portion is not really belongs to the first one, but lying on the acceptance side so this much portions I have falsely accepted so this portion is called the beta this portion is called alpha.

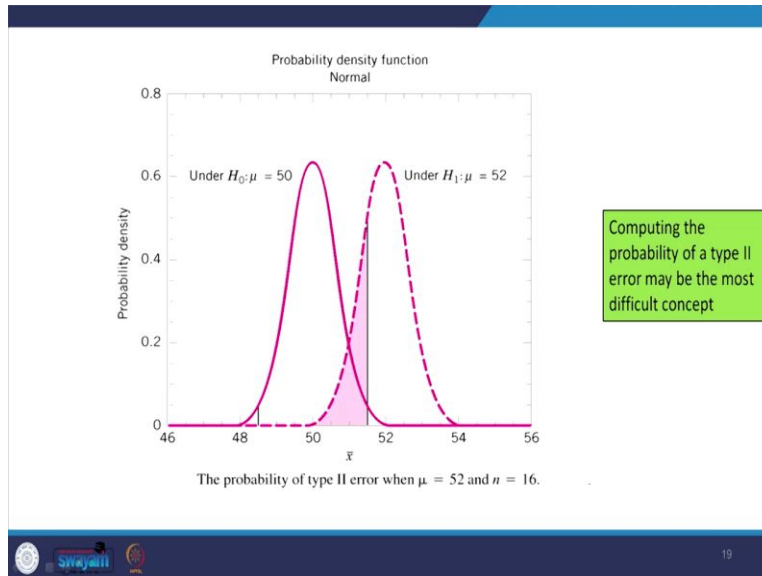
So we have to find out the value of beta and one more thing the value of alpha plus beta is not equal to one it should be very careful. So, what is happening this is 51.5 so this also 51.5 because this condition is same. Now for this population that is which is on the below we have to find out the left side area then we how do you do that first you have to find out the Z value Z value is $X - \mu$ by Sigma, so $51.5 - 52$ divided by Sigma by root n.

So, what will happen for this region when Z it is below the left hand side the corresponding value is your type 2 error. See, that I have done that also so I calling it the beta stat stat norm dot cdf cumulative distribution function. So, my x value is 51.5 minus my true mean is 52 divided by Sigma by root n so 0.7 so that beta value is 0.263. So, this 0.263 is nothing but this value 0.263 is 0.2264, so this is that pink region area is this one just I will give an explanation the code over so Python is given how to find out.

Now you see that no μ equal to 50.5 what happened again between the true mean and assumed mean the difference is becoming less. So, if I put the true mean is 50.5 you see that I have changed there 52 here I have changed in 50.5 so again beta equal to stat stat norm dot cdf of 51.5 there is nothing but $X - \mu$ by Sigma by root n that value is 89 you see that the value of beta has increased. So, one point at present you have to remember when the difference is decreasing between your assumed mean and the true mean that type two error is increasing.

We can say one example suppose there are two product goodness original another one is duplicate both are looking similar color wise texture wise quality wise, there are more possibility for committing type two error when whenever the difference between original and duplicate of product is very, very less, the same way whenever the mean the published which you assumed and the true mean the difference is less there is more possibility of committing type 2 error that I will explain.

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You see this one again when the mean is 50.5 there is a more there are more pink region that is there is more type 2 error. So, the point is when the distance between your assumed mean and the true mean is decreasing there are more possibility of committing type 2 error. Now we will see computing of this type two error already I have explained so computing the probability of type two error may be the most difficult to concept if you are doing it with the help of statistical table it will take more time that is why we go for Python that it will solve your problem very quickly. So, this area we have found out already.

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acceptance region	sample size	α	β at $\mu = 52$	β at $\mu = 50.5$
$48.5 < \bar{x} < 51.5$	10	0.0576	0.2643	0.8923
$48 < \bar{x} < 52$	10	0.0114	0.5000	0.9705
$48.5 < \bar{x} < 51.5$	16	0.0164	0.2119	0.9445
$48 < \bar{x} < 52$	16	0.0014	0.5000	0.9918

For constant n , increasing the acceptance region (hence decreasing α) increases β .

Increasing n , can decrease both types of errors.

Now look at this table acceptance region is their sample size is there you see that for the same sample size when you increase the acceptance region what is happening the Alpha is decreasing,

sample size is same you are widening the acceptance region, so the Alpha is decreasing. When alpha is decreasing you see what is happening the beta is increasing. So, the relation between alpha and beta is when you decrease alpha beta will increase it is like this as I told you previously there are two normal distribution this is your rejection region there is another normal distribution this is μ equal to 50 this is 52.

So this side portions, so this is your alpha this side portion is beta I will change the color. So, this portion the green portion is nothing but your type 2 error the red portion is your type 1 error on our alpha and beta. So, what is happening suppose assume that I am keeping in that line where there's intersections there I am keeping a pen like this if I move towards right-hand side what will happen alpha will decrease so when alpha is decreasing what is happening to beta, beta value is increasing.

Suppose I am keeping this pen I am moving towards left hand side what will happen beta will decrease but alpha will increase so that is explained with the help of this table so the relation between relationship between alpha and beta is inverse apart from this equal to the the last column the μ equal to 52 again I am the μ through 50.5 suppose the difference is decreasing what is happening the value of beta is increasing that is one point.

Now we look at the another point suppose when you increase the sample size you keep the acceptance region as it is you increase the sample size 10 to 16 you see that when you go to 10 to 16 you see here 48.5, 51.5 sample size is 10 alpha is 0.05 for the same reason when you increase you compare this will go to say I am comparing this accepted region is same but I have increased my sample size the Alpha value is decreasing.

See initially our alpha value 0.05 now it is 0.01 look at this beta also the value of beta also you see initially it is 0.26 now it is 0.21, so what the point we are learning from here is when you keep acceptance region as the constant one when you increase the sample size both value of alpha and beta will decrease okay that is the point here let us see this. For constant n when you increase the acceptance region alpha is decreasing with the Alpha is decreasing beta values increasing.

Second point increasing n can decrease both type of error that is type 1 and type 2 there is a learning from this slide.

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Type I & II Errors Have an Inverse Relationship

If you reduce the probability of one error, the other one increases so that everything else is unchanged.

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Type 1 and type 2 errors having an inverse relationship if you reduce the probability of one error the other one increases so that everything else unchanged. So, that is a relation between alpha and beta remember alpha + beta is not equal to 1.

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Factors Affecting Type II Error

- True value of population parameter
 - β increases when the difference between hypothesized parameter and its true value decrease
- Significance level α
 - Increases when β decreases
- Population standard deviation σ
 - Increases when β increases
- Sample size
 - β increases when n decreases

β ↓

↑ α

β ↑

↑ σ

β ↑

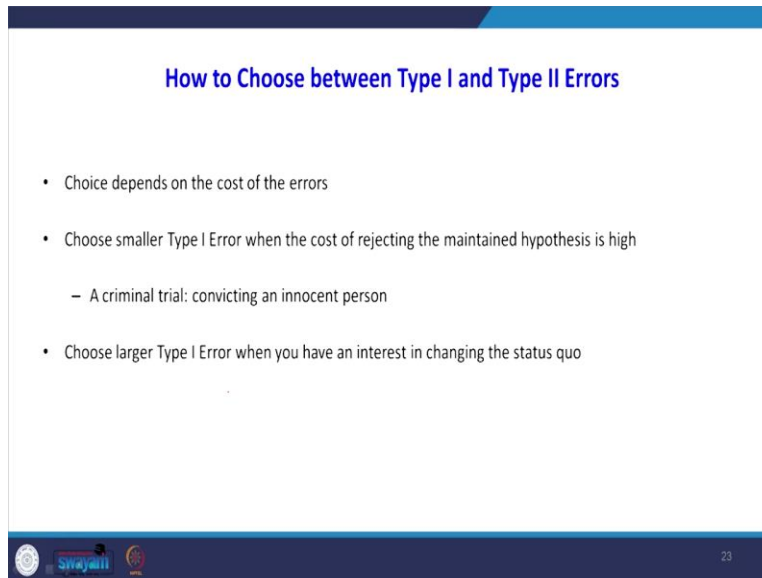
↓ n

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Now let us see a factors affecting type 2 error the true value of population parameter beta increases when the difference between this point already I told you the difference between

hypothesis the parameter and its two values decreasing, significance level alpha increases when beta decreases, population standard deviation Sigma increases when beta increases, sample size beta increases when n decreases that is the relation between your different element of your type 2 error.

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The slide is titled "How to Choose between Type I and Type II Errors" in blue text. It contains three bullet points:

- Choice depends on the cost of the errors
- Choose smaller Type I Error when the cost of rejecting the maintained hypothesis is high
 - A criminal trial: convicting an innocent person
- Choose larger Type I Error when you have an interest in changing the status quo

At the bottom of the slide, there are logos for "Sri Jayanti" and "23".

How to choose there between type 1 and type 2 error the choice depends upon the cost of error. So, the first point is choose smaller type 1 error when the cost of rejecting the maintained hypothesis H_0 for example in a criminal trial committing an innocent person is very, very costly mistake so that time the value of alpha should be very less. Choosing a larger type 1 error when you have an interest in changing the status quo so what will happen if you are willing to change the status quo if you increase the alpha value obviously there is a more chances for a rejection region either hypothesis get rejected that is the status quo is getting rejected.

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Solution:

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In [48]: def type_2(mu1,mu2,sigma,n,alfa):
z =stats.norm.ppf(alfa)
xbar = (mu1)+(z*sigma/np.sqrt(n))
z2 = (xbar - mu2)/(sigma/np.sqrt(n))
if(mu1 > mu2):
beta= 1-stats.norm.cdf(z2)
else:
beta = stats.norm.cdf(z2)
print (beta)

In [49]: type_2(8.3,7.4,3.1,60,0.05)
0.27292999450730004

```

An error will be made when $Z \geq -1.645$, for that will fail to reject H_0 .

$\beta = 0.2729$

We will take another problem to find out type 2 error so I am assuming μ equal to 8.3 alternate hypothesis a μ less than 8.3 it is a left tailed test. Determine the probability of type 2 error if the true mean is 7.4 at 5% significance level when σ is 3.1 and n equal to 60. See that I have drawn this one my assumed mean is 8.3 the question is asked if the true mean is 7.4 what is the value of β ? You see that if any portions which are going on left hand side I will reject it so this side I will reject it but the right hand side I will accept it.

But what is happening the true mean is 7.4 it is lying on acceptance side of where the μ equal to 8.3, I actually I have to reject this since it is lying on the acceptance region so this much portions this much portion I have falsely accepted so that β value is nothing but your type 2 error. See that the value of C is constant for this population and this population so we have to find out this right side area. So, for this purpose I have developed a function because this function is very useful first you let us understand.

So I am going to define your function I am going to call it as a type underscore 2 so what are the parameter which I am going to take μ_1 my assumed mean μ_2 , true mean σ population standard deviation n sample size α significance level colon. So, for example the first one

which is in the topper 1 the normal distribution and find I'm finding out what is the Z value in this location what is the Z value I know what is α value so Z equal to stats dot norm dot cdf

of alpha value if we substitute what will happen I will get the value of Z. If I know the value of Z I can find out X bar how I can find out because Z equal to in this relationship if I know Z value I can find out X bar. So, X bar is when you mu multiplied by mu plus when you bring this left hand side Sigma Z multiplied by Sigma by root n that is it an X bar equal to mu 1 + Z star Sigma divided by np because from numpy that is kernel numpy dot square root of n.

So, I will get X bar this is the value of Z now I have to find out. The Z 2, this said to this what will be this Z 2 this this X bar this X bar this value is X-bar so corresponding normalized scale is Z 2, so what will happen Z 2 is X bar - mu 2 that is for the from this population what is the mean that is a 7.4 X bar - mu 2 divided by Sigma by n p dot square root n. You see the condition if mu 1 is greater than mu 2 what is mu 1, mu 1 is now 8.3 this is 7.4 in this case yes mu 1 is greater than mu 2 what will happen I will get the positive Z value if the Z is positive if I want to know the p value from one iota subtracted.

So the Z value is positive the beta equal to 1 - stats dot norm dot cdf of Z 2 if Z value is negative just finding the left side value beta equal to stats dot norm dot cdf Z 2 beta. So, when you type this in Python type underscore now we have to give this value of mu 1 suppose what I am going to do a good to find out the beta value the mu 1 is the zoom in 8.3 mu 2 is true mean 7.4 Sigma is 3.1 sample size is 60 alpha equal to 0.05. so, now what will happen here if it is a positive value the corresponding probabilities 0.2729 that is why that value yes this 0.729.

So, the right side beta will is 0.2729 so when will commit to type 2 error and the error will be made when Z values greater than so now is here the corresponding Z value is - 1.645 whenever you crossing - 1.645 on the right hand side then you will accept that that is nothing but your value of type 2 error.

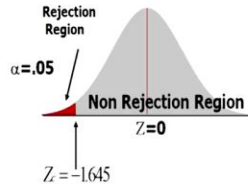
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Solving for Type II Errors: Example

$$H_0: \mu = 12$$

$$H_a: \mu < 12$$

$$\begin{aligned} \bar{X}_c &= \mu + Z_c \frac{\sigma}{\sqrt{n}} \\ &= 12 + (-1.645) \frac{0.10}{\sqrt{60}} \\ &= 11.979 \end{aligned}$$



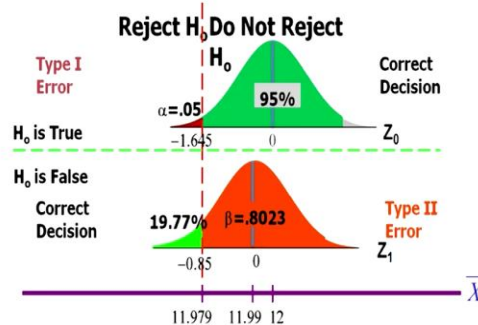
If $\bar{X} < 11.979$, reject H_0 .

If $\bar{X} \geq 11.979$, do not reject H_0 .

I will take another example solve you for type 2 error one more example μ equal to 12, μ less than equal to 12 we know that \bar{X} equal to $\mu + Z_c \frac{\sigma}{\sqrt{n}}$ so here assumed means 12, Z value is because the left side you when α equal to 0.05 corresponding Z is - 1.645 Sigma values given the \bar{X} is 11.979 if the value of \bar{X} is below 11.79 we will reject it if the \bar{X} is above 11.979 I will accept my null hypothesis.

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Type II Error for Example with $\mu = 11.99$ Kg



Now we look at this graph you see that my assumed mean is 12 suppose my true mean is 11.979 what are you with the value of type 2 error you see that when α equal to 0.05 - 1.645 I know that so what will happen if the true mean is 11.979 what will happen Z value will be $\bar{X} - \mu$

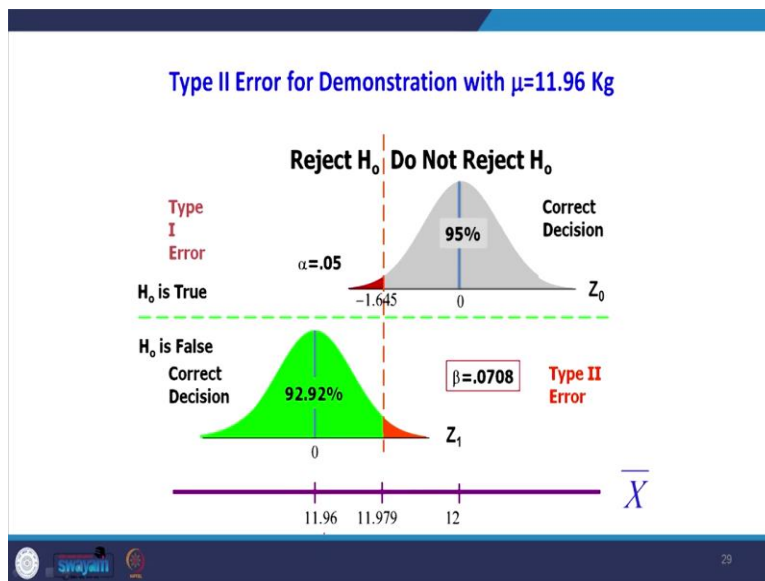
so $11.979 - 12$ Sigma by root n right, we will get Z value so corresponding right hand side area is my type two error. So, beta equal to 0.8023.

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In [50]: type_2(12,11.99,0.1,60,0.05)
0.8079200023112734
```

I have done some code for this so type two error so twelve is what is that my assumed mean 11.99 is my true mean this is my standard deviation Sigma value this is my n this is my alpha, so it is 80 that is why this much here, here so what we are getting we are getting 80790 approximately 80%.

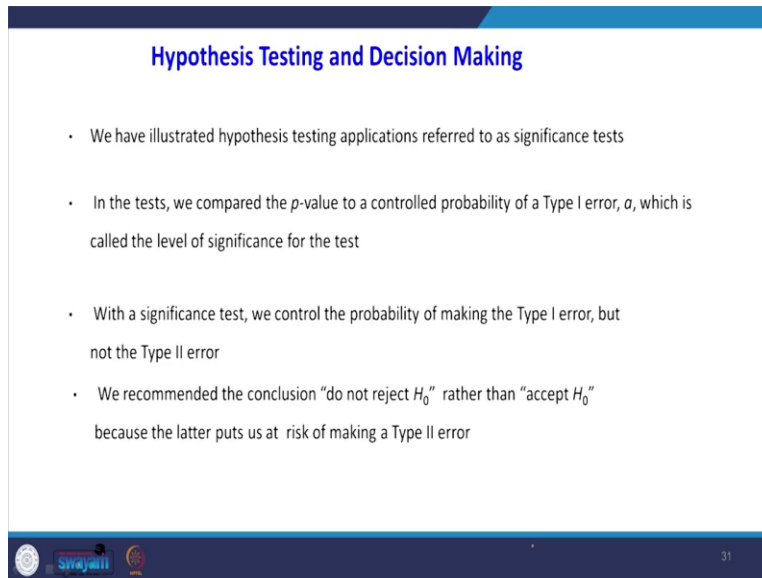
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I will go to see another problem the true mean is 11.96 little far away from the 12 what is that we the true mean is going towards left hand side now what has happened to the value of beta, so

this portion is my beta type two error. So, by use already we have developed your function for that will substitute this 12, 11.9 6.1 60.05 so what is happening the value of beta is becoming very less. Again I am stressing this point you the difference is bigger the value of beta is very less if the difference is closer the earlier beta is very high.

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Hypothesis Testing and Decision Making

- We have illustrated hypothesis testing applications referred to as significance tests
- In the tests, we compared the p -value to a controlled probability of a Type I error, α , which is called the level of significance for the test
- With a significance test, we control the probability of making the Type I error, but not the Type II error
- We recommended the conclusion "do not reject H_0 " rather than "accept H_0 " because the latter puts us at risk of making a Type II error

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Now hypothesis testing and decision making we have illustrated hypothesis and testing applications. Now let us see the; what is the application of this type 2 error we have illustrated hypothesis testing applications referred to as a significance test. In the test we have compared the p -value to a controlled probability of type 1 error alpha which is called the level of significance for the test. What do we have done to accept or reject a null hypothesis we have considered the p value that is compared with alpha.

The p value is smaller than the Alpha we have rejected it the p value is greater than alpha we have accepted it. So, we will go to the next point with the significance test we controlled the probability of making type 1 error but not the type 2 error. We recommended the conclusion do not reject H_0 actually we have to use accept H_0 but very cautiously we have used do not reject H_0 rather than accept H_0 because the letter puts us at risk of making type 2 error.

Why we are not accepting there is no proof that the value which have assumed in a null hypothesis correct so that is why we are saying do not reject it now in this example what we are

going to do what should be the value of our null hypothesis. So, that the something called the power of test can be improved we will see the definition of power of test. With the conclusion do not reject H_0 the statistical evidence is considered inconclusive you are not able to say anything.


Usually this is an indication to postpone a decision until the forces for the research and testing is undertaken. But in many decision-making situations the decision-maker may want and in some cases may be forced to take action both the conclusion do not reject or the conclusion reject H_0 in such situation it is recommended that a hypothesis testing processor be extended to include consideration of making type 2 error.


So, what we are going to say what in whenever you do the hypothesis our testing we have to see the possibility of committing type 2 error also that I will show you with the help of an example.

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Power of a test

- The mean response time for a random sample of 40 food-order is 13.25 minutes
- The population standard deviation is believed to be 3.2 minutes.
- The restaurant owner wants to perform a hypothesis test, with $\alpha = 0.05$ level of significance, to determine whether the service goal of 12 minutes or less is being achieved.



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That point is called power of test ok, suppose there is a restaurant is there suppose in the restaurant when you order some dosa and you order some coffee or soup many times it will take different sometimes they because they were to prepare it. Assume that the owner of the restaurant has the target of service goal of 12 minutes or less whether it can be achieved or not so what is the different possibility of committing type 2 error if you assume μ equal to 12. You will see that the problem detail.

The mean response time for a random sample of 40 food order is say 13.25 minutes the population standard deviation is believed to be 3.2 minutes the restaurant owner wants to perform a hypothesis test with alpha equal to 5% a significance level to determine whether the self-service goal of 12 minutes are less is being achieved. Now what is happening first you have to start null hypothesis so null hypothesis is the status quo.

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Calculating the Probability of a Type II Error

Hypotheses are: $H_0: \mu \leq 12$ and $H_a: \mu > 12$

Rejection rule is: Reject H_0 if $z \geq 1.645$

Value of the sample mean that identifies the rejection region:

$$z = \frac{\bar{x} - 12}{3.2/\sqrt{40}} \geq 1.645$$

$$\bar{x} \geq 12 + 1.645 \left(\frac{3.2}{\sqrt{40}} \right) = 12.8323$$

We will accept H_0 when $x \leq 12.8323$

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The status quo is mu less than or equal to 12 alternative hypothesis is mu greater is equal to 12 so what will happen what kind of test this is this is right tailed test. So, this value is 12 if anything goes this side I will reject it so when alpha equal to 0.05, so the corresponding value is 1.645 if any value Z value goes beyond 1.645 I will reject it. So, we will substitute this value into our Z formula so Z equal to X bar - 12 root of 40 if it is greater than 1.645 I will reject it.

So from this relationship I will bring the value of x bar okay we are finding the value of x but that is a 12.83 so what will happen we will accept H 0 when the value of x bar is 12.83, so this value will 12.83 if anything value goes that side we will accept H 0 if anything goes below this will reject it.

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Calculating the Probability of a Type II Error

Probabilities that the sample mean will be in the acceptance region:

Values of μ	$z = \frac{12.8323 - \mu}{3.2/\sqrt{40}}$	β	$1-\beta$
14.0	-2.31	.0104	.9896
13.6	-1.52	.0643	.9357
13.2	-0.73	.2327	.7673
12.8323	0.00	.5000	.5000
12.8	0.06	.5239	.4761
12.4	0.85	.8023	.1977
12.0001	1.645	.9500	.0500

So, what will happen here in this we assume you said assumed we have assumed μ less than or equal to 12 H 1 μ greater than 12 now the question is how what is the logic behind this 12. So what I am going to do instead of this 12 I am going to supply different values of this new value say I'm going to supply 14 13.6 13.2 12.00 so value of μ so Z it is $12.83 - \mu$ 2 by 3.240 in this Z formula when you substitute the value of μ 14 this is the Z value.

So when the Z value is - 2.31 what is the value of type two error 0.01 so 1 - beta so this 1 - beta is nothing but power of a test. Power of a test is rejecting a null hypothesis when it should be rejected. So now instead of 14 if I make 13 so again the Z value is - 1.52, so corresponding beta will is 0.64 you see that when the difference is becoming closer to 12, what is happening you see that the value of beta is increasing whenever value of beta is increasing the power of testers decreasing.

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In [20]: type_2(14,12,3.2,40,0.05)
0.010499750448532241

In [21]: type_2(13.6,12,3.2,40,0.05)
0.06457982995225997

In [23]: type_2(13.2,12,3.2,40,0.05)
0.2336575101104159

In [22]: type_2(12.8323,12,3.2,40,0.05)
0.49995065746353273

In [27]: type_2(12.8,12,3.2,40,0.05)
0.5254013387545549

In [24]: type_2(12.4,12,3.2,40,0.05)
0.8035262335707292

In [26]: type_2(12.0001,12,3.2,40,0.05)
0.9499796127157129

```

So, how we got this 0.0104 I have done in the next, next slide see I am calling that function which I previously used so type underscore 2 if true mean is 14 assumed mean is 12 Sigma is 3.2 n equal to 40 alpha equal 2.05, so my beta is 0.01 this is for my this is 14. Suppose if it is a 13.6 right substituting 13.6 or other value my beta well is 0.06, so that is this value so when it is 13.2 what is the beta value so in substituting 13.2 the beta value is 0.23 and 0.23. If it is 12.8 again beta value is 0.5 and so on.

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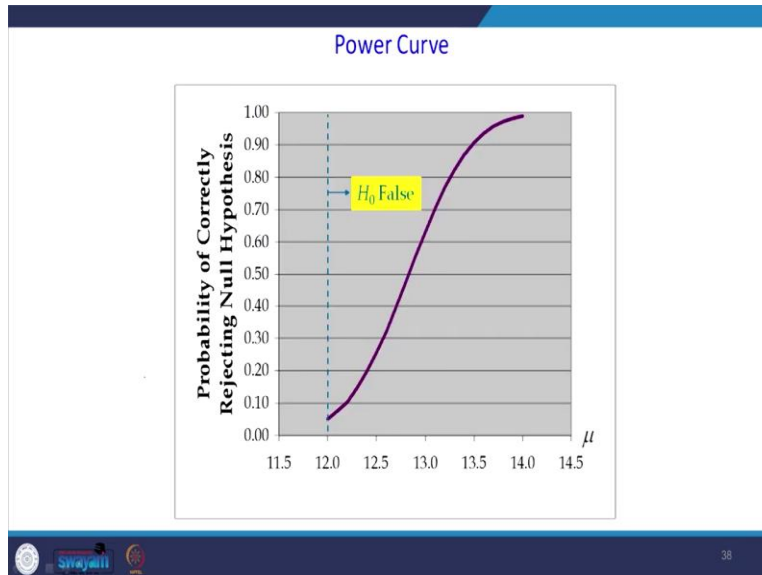
Power of the Test

- The probability of correctly rejecting H_0 when it is false is called the power of the test.
- For any particular value of m , the power is $1 - b$.
- We can show graphically the power associated with each value of μ ; such a graph is called a power curve.

So, if I plot this okay now we will come to the for plotting I will define what is this power of test the probability of correctly rejecting null hypothesis when it is false is called the power of test. For any particular value of mu this is mu the power is 1 - beta you call it is capital B that is

convenient. So, we can show graphically the power associated with each value of μ such graph is called power curve.

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So, the application of power curve first I will explain what is the element in this power curve. Here in the x-axis the true mean which I have assumed in the y-axis the value of $1 - \beta$ probability of currently rejecting null hypothesis what is happening when the difference is increasing between true mean even your assumed mean there is a more chances you will correctly reject your null hypothesis. So, this power of test says what should with the value of μ you see that when you feel if he assume you equal to 12 power is less when the power is when you assume you equal to 14.5 this power is more.

So this power curve is helping us to decide what is the possibility of committing type 2 error at the same time how much value of null hypothesis we can have so that we can improve our power of a test otherwise we can decrease the beta. Dear students in this lecture we have seen different types of error while doing hypothesis testing. We have taken one practical example I have explained what is the meaning of type 1 error and type 2 error.

And also we have calculated value of type 1 error and type 2 error at the end we have seen a power of a hypothesis testing we call it as a power curve. So, what we have done we have suggested what is the possible value of μ and corresponding beta value are a corresponding

power of a test. So, with that we will conclude in this lecture in the next lecture we will go for a two sample hypothesis testing, thank you very much.