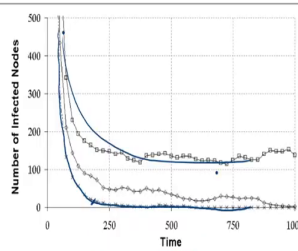


Social Network Analysis
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Chapter - 07
Lecture - 07

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**Compartmental Models of
Epidemiology: SIS Model**



Reduction of Infected nodes for different β and δ

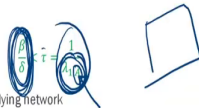


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**Compartmental Models of
Epidemiology: SIS Model**



- The epidemic dies out if virus strength $< \tau$
- τ is nothing but the reciprocal of the largest eigenvalue of adjacency matrix representing the underlying network
- The epidemic dies out if



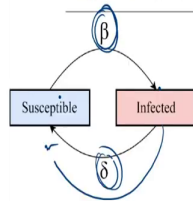
where A : adjacency matrix of the underlying network

$\lambda_{1,A}$: largest eigen value of A



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Compartmental Models of Epidemiology: SIS Model



- a node can go through the phases of 'susceptible' to 'infected' to 'susceptible' again
- Common cold can recur with a high probability can be modelled by SIS
- Rate of change of 'susceptible population' is:

$$\frac{dS}{dt} = -\beta SI + \delta I$$
- Rate of change of 'infected population' is:

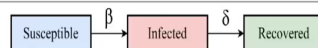
$$\frac{dI}{dt} = \beta SI - \delta I$$
- Strength of a virus = $\frac{\beta}{\delta}$
- Epidemic threshold, denoted by r



So, in the last lecture we have started discussing on epidemic models right. And we have seen models like SIR, SIS models. These are simple models that were proposed long time back, but these models are quite useful for modeling the spread of in fact Covid-19 right.

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Compartmental Models of Epidemiology: SIR Model



□ A node can go through only three stages: (i) Susceptible, (ii) Infected, and (iii) Recovered

□ Rate of change of 'susceptible population' is:

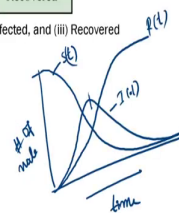
$$\frac{dS}{dt} = -\beta \times S \times I$$

□ Rate of change of 'recovered population' is:

$$\frac{dR}{dt} = \delta \times I$$

□ Rate of change of 'infected population' is:

$$\frac{dI}{dt} = \beta SI - \delta I$$



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Compartmental Models of Epidemiology: SEIR Model

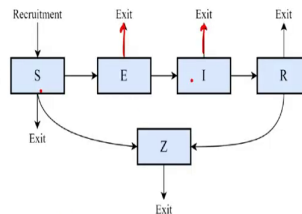


□ A generalized framework to model the spread of epidemics

□ SEIR (or S+E+I+R) is an acronym of

- Susceptible (S): those who may become infected
- Exposed (E): those who are infected, but not yet capable of spreading the infection/idea
- Infected (I): those who are capable of further propagating the infection/idea
- Recovered (R): those who have recovered from or become immune to the infection/idea
- SLEEPERS (Z): Susceptible who no longer follow the infection/idea (Another possible state)

□ Many possible variations of the model



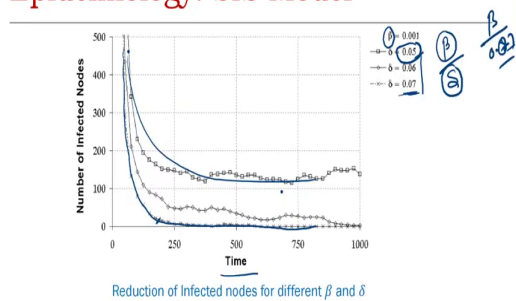
So, for example, we have seen right, we have seen this SCIR kind of generic model. Where there are 5 states right and basically it is a state transition diagram from one state you can move to another state. And with certain probability of course, now all these models like SIR, SIS these models they take a subset of the sets of the states and then they create this state transition diagram.

For example, this is SIR model we discussed last day susceptible with probability beta, susceptible you know citizen gets infected with probability delta. An infected person gets recovered and so on and so forth right. And then we derived all this differential equations ordinary differential equations.

And then we can essentially measure the number of susceptible users at a given point in time, number of recovered users as a at a given point in time and so on. And then we also discussed something called the strength of a virus which is beta by delta. Probability of infection probability of cure basically the fraction of death rate birth rate and the death rate and we have seen there is a nice correlation with the eigen largest eigen value of the graph adjacency matrix right.

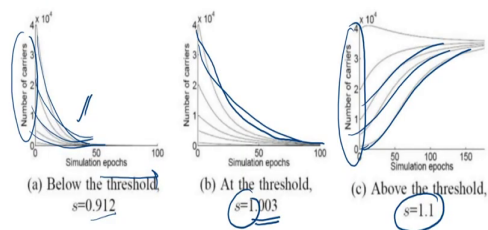
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Compartmental Models of Epidemiology: SIS Model



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Compartmental Models of Epidemiology: SIS Model



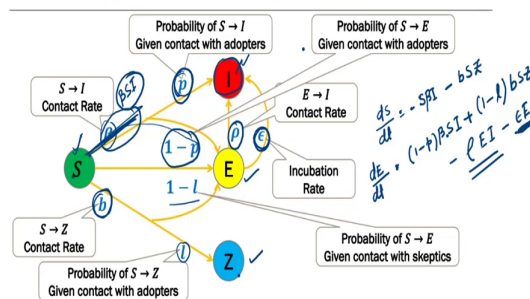
Increasing or decreasing the number of initial carriers will make no difference in extending or reducing the duration of the epidemic



So, and we have also seen you know different relations between the number of iterations versus number of carriers and so on and so forth.

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Analysing Rumour Spread: SEIZ Model



So, today we will discuss and; so, all this discussions were mostly you know based on real world epidemic spread models right. Say Ebola spread model, missile spread model and so on. So, now, we will discuss another spread model another such spread model, but this time on online social network right.

So, we will see how information like rumor or fake news spread over social media and how we can basically model it using the same state transition diagram that we have discussed ok. So, in this rumor spread model right. So, this was paper published long back in 2013, 2014 during that time.

And they basically tried to understand how rumor spread on online social network and they actually came up with four states ok. So, the first state is susceptible, this is same as the previous one, but in the context of Twitter these are susceptible users are all the Twitter users right.

The second state is infected right, these are basically those Twitter users who got who basically started believing on a particular rumor or started spreading a particular rumor and so on right. Similarly there is another state called exposed right, this exposed states state basically indicates you know those users who are already exposed to the news. For example, say if I post a rumor all my followers will be exposed to that particular post right.

My tweet will be visible on their Twitter feed right. So, they are exposed right. Now, some of them will be infected, some of them will further spread, some of them will some of them will not react ok. So, there is another state called skeptics Z right. So, skeptics so, skeptical users skeptic I mean users with this Z state, those are that kind of users who essentially you know do not react to this to the rumor. So, they are essentially you know hard immune right.

Even if you I mean even if there are say social pressure or whatever right. They would never react to any rumor; they would never react to any fake news right. So, these are four states right. And you see these states are quite similar to the state that we mentioned earlier in the epidemic model right. So, now, let us look at the transitions from one state to another state. So, from the susceptible state with probability beta you will be infected right.

Now, the amount of population that are moving from S to I, a fraction of it would actually move to E. So, among the number of susceptible users there would be a set of susceptible users who would be exposed ok. And then from exposed state they may further move to the infected state or they may not right. And there would be some fraction of susceptible users who would directly be infected.

For example, say there is a bot right, whatever I tweet the bot will immediately re tweet say there is a bot which basically re tweets Donald Trump's tweet right. So, what about Trump tweets, that bot will immediately retweet. So, that bot is basically an infected user right. So, with so, the amount of you know population that is moving from this path, p fraction would actually move to I and 1 minus p fraction would come to E ok.

And again from E, from exposed state rho fraction would move to I right. And the remaining will actually stay at the current state E. Similarly with beta probability people would move from S to Z state ok. Now, some of them would actually be you know skeptic right. They would never react. So, by default all the Twitter users are susceptible. Some of them are kind of some of them have hard immunity right. Hard behavior they would never respond.

So, that fraction is basically 1 ok. And 1 minus 1 would basically move from S to E. Susceptible to exposed right. So, and you see that there is some parameter called this epsilon, which is basically called incubation rate kind of a noise right, which can be used to control and this noise is under the control of the user right. So, you basically tune this, this parameter for you know for fitting your data points for fitting the line with the data points; whatever, curve with the data points ok.

So, now, if this is the state transition diagram let us see what happens with the rate of every population every state. So, the rate of change of the susceptible user set would be. So, the beta fraction would move from S to I right. So, each infected user would infect a susceptible user with probability beta. So, out of S susceptible user with probability beta, this many susceptible users will be infected.

And how many infection infections are there I right. So, and this would be negative, because this population will be reduced. Same as SIR model right. And there is also this part right and what is this with probability b susceptible users will move to Z right so, this one ok. Now, remember this I Z right or E these are fraction of users ok. These are essentially E by N. I am just ignoring N, I am assuming that these are basically fractions.

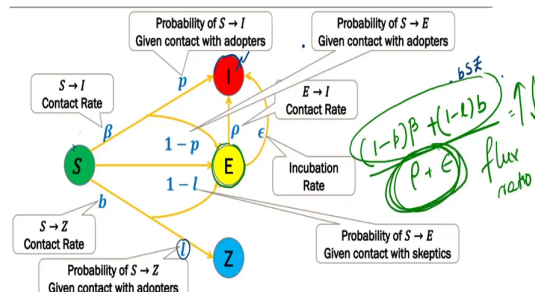
Not the actual number ok. So, similarly what is this one ok? This would be $1 - \beta S$ I right, because this part it is βS I and this is the fraction of this one and $1 - I$ right. And some of them are also moving from E to I and what is the number the number is sorry ρE I ok. And of course, this E right, this epsilon into E kind of a incubation rate ok. This is d by dt this is dE dt right.

Now, let us see what is dI dt ok, dI dt . So, this would be with ρ right. Sorry with p with p βS I. This is the fraction and this is the total population and then you have ρE I the ok. Same quantity plus right, what is dt this would be this would be $1 - \beta S$ Z ok. So, this is the these are the differential equations right.

So, if you look at it again carefully. Let me erase this part. So, they also proposed something called the flux ratio. What is the flux ratio? So, the flux ratio is the amount of population coming to the exposed state and amount of population moving out of the exposed state ok.

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Analysing Rumour Spread: SEIZ Model



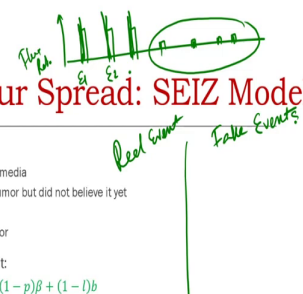
So, this would be I this would be 1 minus p ok. Times beta I am just looking at the probability the fraction ok. Plus 1 minus l b, this is a total influx. What is the outflux? Outflux is rho plus epsilon ok. So, this is called flux ratio, think about it; more the flux ratio higher this numerator. Meaning that a lot of people are exposed right, but very few of them would actually be infected right. And if it is lower it means that a lot of them are infected, because this number is high ok.

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Analysing Rumour Spread: SEIZ Model

- A rumor in many ways is like a disease
 - Susceptible: those who are active on social media
 - Exposed: those who have seen/heard the rumor but did not believe it yet
 - Infected: those who believe the rumor
 - Skeptics: those who did not believe the rumor
- A new kind of metric defined for the event:

$$R_{SI} = \frac{(1-p)\beta + (1-l)b}{\rho + \epsilon}$$
- A kind of a flux ratio between the ratio of effects entering a node which is being examined to those leaving that node
 - high for real-life events
 - low for rumors



So, a kind of flux ratio between the ratio of effects entering a node which is being examined to those leaving that node ok. And turned out and essentially they in this particular paper. They looked at some real events ok and some fake events ok and it turned out that for I mean if you just plot say y axis is the flux ratio ok. And you have different bars corresponding to different events ok. Say this is event 1 event 2 and so on and so forth. So, it turned out that for real events. The flux ratio is quite high ok.

So, you see larger bar graph for real events whereas, for fake events this ratio is quite low ok. What does it mean? It basically means that for real events people are exposed to that particular news quite often, but they do not react. Therefore, the numerator is high denominator is low.

Whereas, for fake events people are exposed as well as infected immediately right. So, they tend to re tweet whatever the c. So, therefore, fake news spread in fact, much faster and wider compared to the real news ok. So, you see that in this way you can essentially incorporate the APDB models to basically model how misinformation rumor spread on social network.

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Analysing Rumour Spread: SEIZ Model

□ Mathematical representation of the model:

$$\begin{aligned}
 \rightarrow \frac{dS}{dt} &= -\beta S \frac{I}{N} - b S \frac{Z}{N} \\
 \rightarrow \frac{dE}{dt} &= (1-p)\beta S \frac{I}{N} + (1-l)b S \frac{Z}{N} - \rho E \frac{I}{N} - \epsilon E \\
 \rightarrow \frac{dI}{dt} &= p\beta S \frac{I}{N} + \rho E \frac{I}{N} + \epsilon E \\
 \rightarrow \frac{dZ}{dt} &= l\beta S \frac{Z}{N}
 \end{aligned}$$



So, these are the equations that I just mentioned ok. So, we stop here. In the next lecture we will discuss the last model which is the IC model independent cascade model and then we will conclude this particular chapter.

Thank you.