Social Network Analysis
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Chapter - 07
Lecture - 06

Let us look at some of the epidemic models that were proposed long time back, but those models are still relevant to the current scenario, current literature ok. So, so far a quick recap. So, far we have discussed normal decision based model and also probabilistic growth model. And we have seen the problems of decision based models and how probabilistic models you know out overcome those problems.

And then we have seen cases where I mean what is the probability that when a certain epidemic starts spreading, what would be the nature of the curve right based on which we can say that this will die out or this will not die out. The epidemic will not die out and we have discussed, we have you know we have come up with a terminology called reproductive number or reproduction number right, q times d ok, which essentially indicates that what is the expected number of neighbors, which are going to be infected because of the parent, of the parent infected node right.

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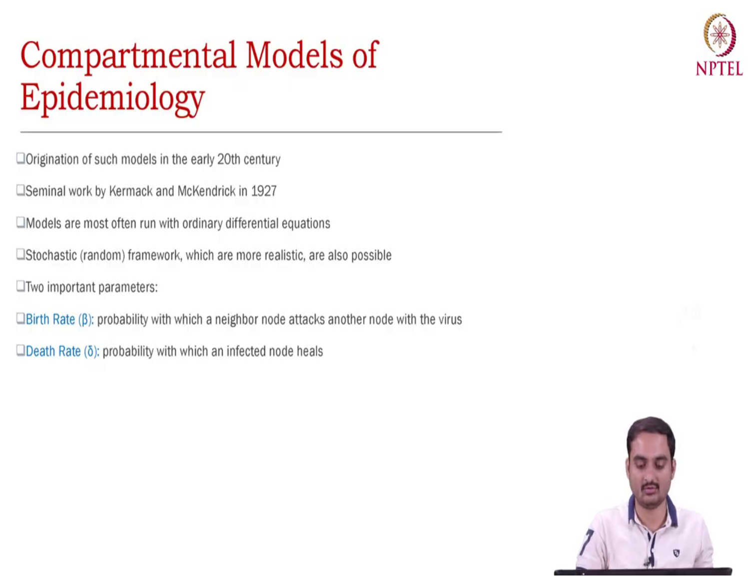

So, now we will discuss again a different epidemic model and these models were proposed long time back, 1927 – 1928 during that time ok. And the fundamental idea behind these two models, behind the model that we discussed is basically that we will use some probabilities.

The first probability is called the birth rate, which is basically the probability with which a neighbor, a neighbor node attacks another node with the virus. This is the birth rate of the virus ok; this is also called the infection rate ok. So, with probability beta a node will infect another node ok and there is something called death rate, death rate of the virus which is basically the probability with which an infected node heals, infected node cures, right gets cured.

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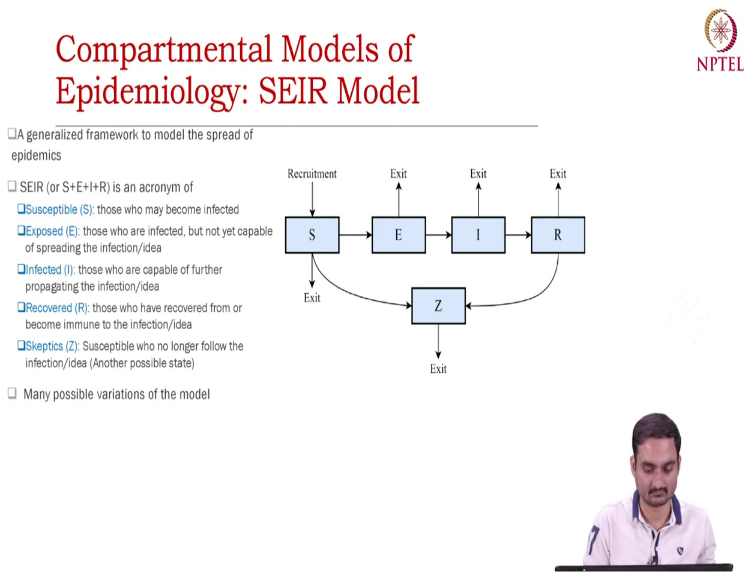

So, this is the; so this is the you know the broad epidemic model and we generally look at, we generally look at these 5 states right; susceptible, exposed, infected, recovery and skeptics ok. Susceptible state: so each node would go through one or multiple such states right. So, yes, the susceptible state is basically a state of a node which is likely to be infected right.

E is the state of a node which basically indicates that the node has got exposed to that virus. I is the infected stage which basically indicates that the node has already got infected, R is the state which indicates the recovery and skeptics is basically those nodes, which is susceptible, but who are no longer you know following the infection or whatever ideas that we have. So, they are basically they are immune in some ways ok.

So, you see here, so this is the kind of a transition diagram from one state to another state. From susceptible state you can move to exposed state, from exposed you can move to infected state, from infected you can move to recovery state, from susceptible you can move to skeptic state and from skeptic you can from recovery also you can move to skeptic state when you got. So, you have got some sort of hard immunity, you will never get infected ok.

You see here, with every state there is this exit symbol which basically indicates that at every state there is some chance that the user will die ok. And that user will be basically removed from the system ok. Now, it is not the case that for all the models we will follow all these 5 states, we will follow we can take a subset of the states and then define our own model.

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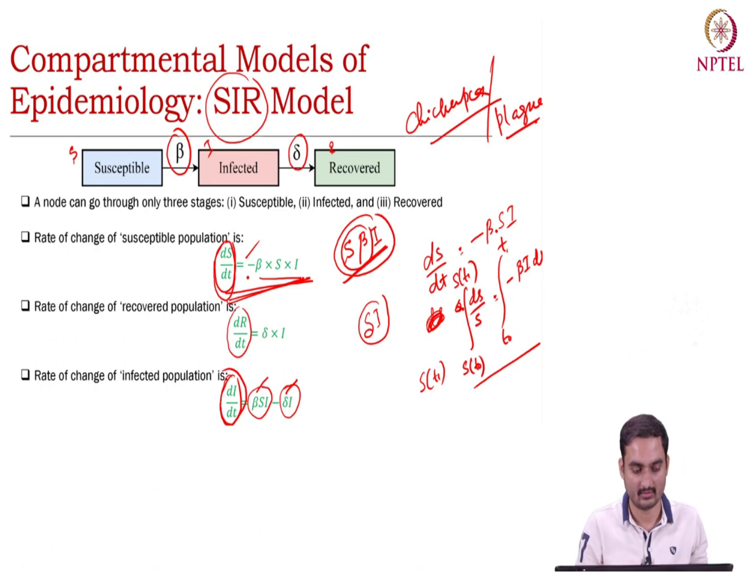

For example, the first simple model is called SIR model, Susceptible Infected and Recovery, right. So, say you are susceptible to certain disease then you get infected with probability beta, then with probability delta you get recovered right and if once you get recovered there is no chance that you will again become susceptible. This is kind of a chicken pox right, chicken pox kind of disease where I mean in which if you get infected by chicken pox, you will not be infected by it you know in the near future ok.

So, or say plague ok, now we will see the rate of change of susceptible population, infected population and recover population ok. So, the rate of change of susceptible population: so each infected so beta is the infection rate birth rate and this is the death rate, recovery rate. So, each infected node infects a susceptible node with probability beta, right.

So, there are S number of susceptible nodes. So, each infected node will infect S times beta susceptible nodes right and there are I such infected nodes, right. So, this is S, I and R right. So, I such infected nodes will infect these many new, these many susceptible users and these are new infected these are new infected users or these are the users which would no longer be a part of susceptible user set, right.

So, the rate of change of susceptible user is negative of this quantity. Why negative? Because the population will decrease the susceptible population will decrease therefore, negative. And right and what is the rate of change of infected user? The same number of users will be infected now. So, the rate of change of infected user is this, but there are some users which would get recovered. So, what would be the rate.

So, each infected user would get recovered with probability delta and there are I such infected users. So, these many users will be infect will be recovered. So, these many users will move from infected state to the recover state, right. So, the rate of change of infected population is this minus this, right. And what is the rate of change of recovered user, recovered popularity? Basically this one.

So, when the popularity, when the population gets decreased; not popularity population gets decreased we will use negative symbol otherwise positive symbol right. So, now, let us see let us look at this one. So, dS dt equals to minus beta SI, right dS. So, we can say that dS by S is minus beta I dt we can integrate it right from t 0 to t from S t 0 to S t. So, you are basically looking at the number of users right.

Number of susceptible users at time t 1 say for example. So, we will integrate it and we get the value of S ti, similarly for recover state and similarly for infected state right. So, in that way you can get the number of you know number of infected users and number of recovered users and so on and so forth, ok.

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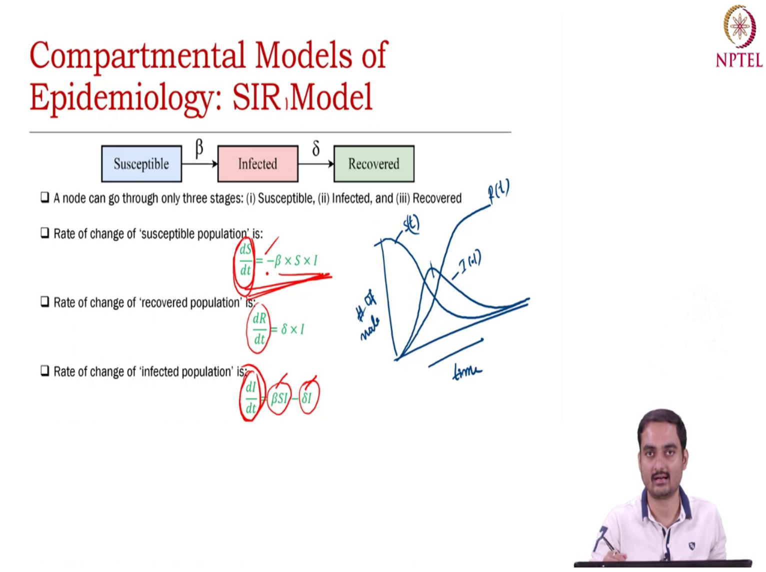

So, if we draw this population right, population S, I and R over time, you will see this kind of curve, this is time here time means iteration number of nodes. So, number of susceptible users will decrease over time, initially it is 100 percent. Similarly, number of recovered users will increase over time and number of infected users would look like this.

So, this is S of t, this is R of t and this is I of t and this is quite intuitive right, because as susceptible users increases infection increase, right. Up to certain point you see all the most of the susceptible users are infected and then infection will decrease and recovery will increase. If you do the simulation you will get this kind of patterns, ok.

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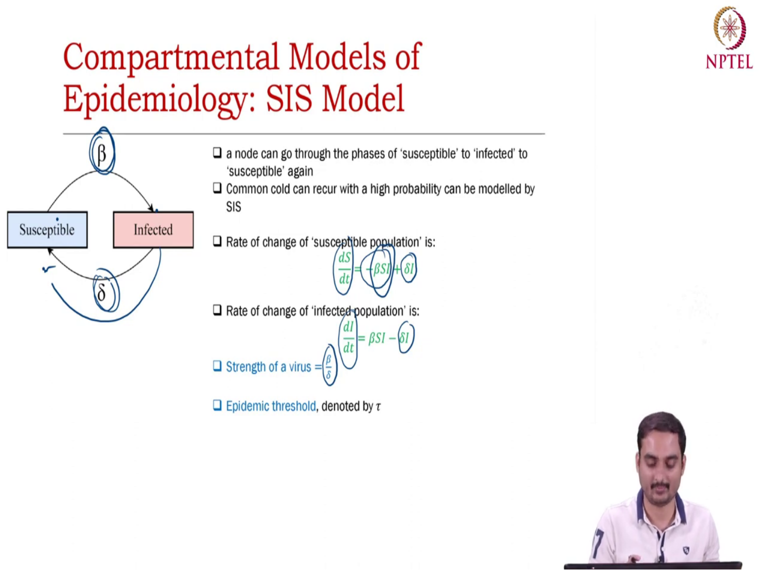

Let us look at the second model, SIS model right, Susceptible, Infected and again when you recovered, so there is no infected there is no recover state as such, but when you get infected then you again can become susceptible. This is kind of a flu, flu kind of epidemic, a flu kind of virus spread right. Someone is susceptible he will be infected and it is again likely that he will again be susceptible and then infected right.

So, here again with probability beta susceptible user becomes infected with probability delta infected user becomes susceptible. So, what is the rate of change of susceptible user? This quantity right which indicates a decrease in sustainable population; whereas, this quantity indicates this one, increase from infected to susceptible right.

What about this one? Rate of change of infection this is essentially the so, those users, which are essentially moving for susceptible to infected this quantity, but in a positive sense minus those users were moving from infected to susceptible, this is this one ok.

So, here we define a quantity called the strength of a virus. So, strength of a virus is beta by delta infection rate and recover rate ok and for every epidemic we essentially measure this beta by delta ratio.

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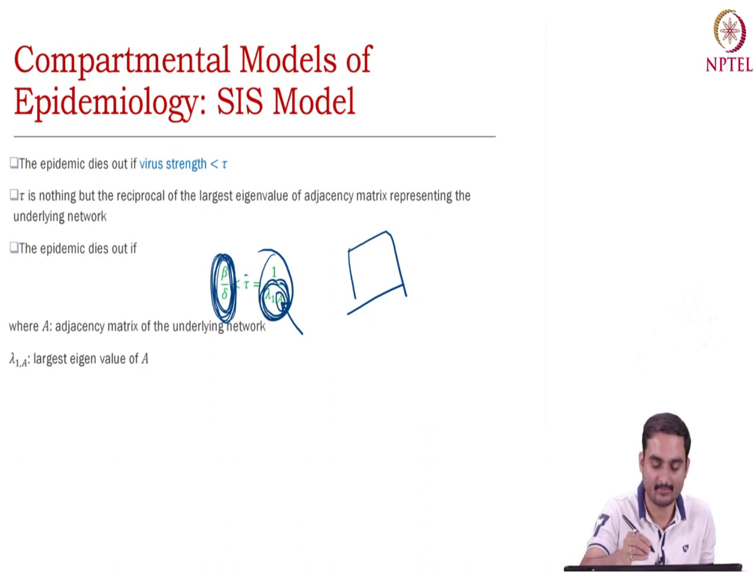

And it turned out that there is a nice relation between the graph adjacency matrix ok and this strength ok, it turned out that beta by delta is always less than this one, 1 by 1 by this lambda 1 A. What is this lambda 1 A? Lambda 1 A is the largest eigen value of the adjacency matrix A.

So, if you have the adjacency matrix A, you measure the largest eigenvalue, you take the reciprocal you know reciprocal value of this and that would be always greater than this beta by you know delta value. So, what does it mean? It means that the network has an effect on the strength of the virus, right.

In fact, you can actually modify the network; you can change the structural property of the network in such a way that the strength will you know will become lesser than a certain threshold. Because A is something which is on your hand you can change the network structure, let us say, let us say you are Facebook administration, I mean Facebook moderator, right.

So, you can think of which users to delete, which links to break and so on and so forth. So, that is there in your hand. So, you can control this and therefore, you can also control this one. In an offline case you can control this by imposing say lockdown or you know imposing this mass varying mass constraint and so on. In that way you can modify adjacency matrix user interaction network. And therefore, this will be controlled ok.

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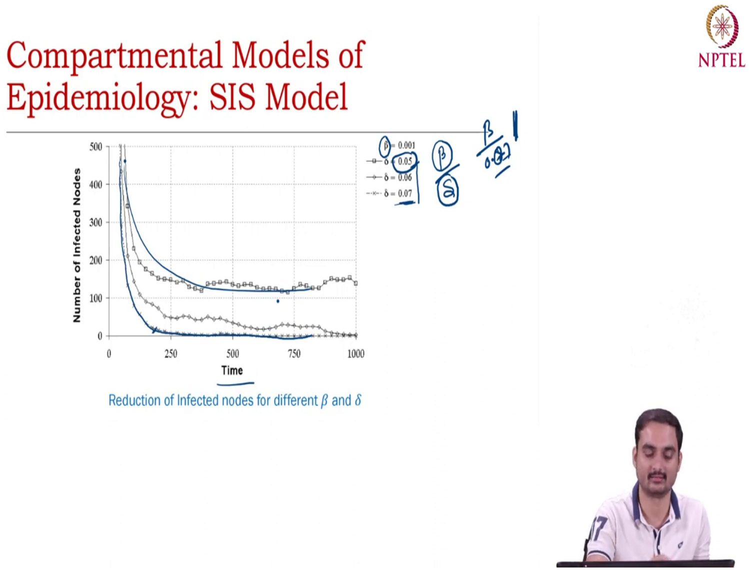

So, and this is an example right here, you see that over time right how the number of infected nodes increases decreases with different values of this strength value right. So, here you fix the birth rate and you change the death rate right and you see that with the change of death rate how this basically changes ok. So, you see that when say look at this one right.

When death rate is point 0.07 right, versus when death rate is 0.05 right. So, when death rate is 0.05. So, this is constant right and this is lower, when you have this versus when you have this is higher. So, this is lower ok and you see that in that case the strength will decrease right therefore, number of infected nodes will decrease over time ok, this is a simulated result.

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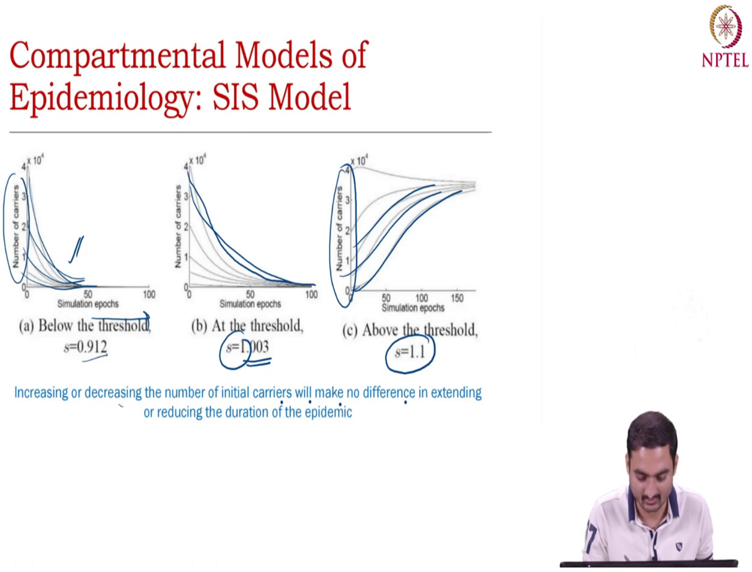

Similarly, if you look at other kind of observations for example, when you have certain number of carriers, certain number of initiators right with a threshold of say with a strength of 0.9, you see does not matter because now eventually number of carriers right will die out as you increase the number of iterations or times right.

Similarly, when you increase the threshold a bit right, when the threshold strength is 1 right, you see that it takes longer time than this one to die out right, but when the strength is above 1 right, you will see that irrespective of the number of carriers it will never die out it will always increase right, it will always increase.

So, the increasing or decrease in number of initial carriers will have no difference in the extending or reducing the duration of the epidemic ok, alright. So, in the next lecture we will discuss another such model which is aci z model ok.

Thank you.