

Social Network Analysis
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Chapter - 07
Lecture - 03

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Cascades for Infinite Chain Networks: Multiple Choice: Case II



Consider the case: $a = 5, b = 3, c = 1$

Two possible choices for node u

- Stick with strategy B, total payoff: $0 + 3 = 3$
- Switch to strategy A, total payoff: $5 + 0 = 5$
- Switch to strategy AB, total payoff: $5 + 3 - 1 = 7$

So, node u would adopt strategy AB

Two possible choices for node v

- Stick with strategy B, total payoff: $3 + 3 = 6$
- Switch to strategy A, total payoff: $5 + 0 = 5$
- Switch to strategy AB, total payoff: $5 + 3 - 1 = 7$

So, node v would adopt strategy AB

And the cascade continues!

Final state: $a = 5, a = 5, AB, b = 3, v, b = 3, B, b = 3$. $c = 1, c = 1$

Handwritten calculations:
 $A = 5 + 5 = 10$
 $B = 0 + 3 = 3$
 $AB = 5 + 3 - 1 = 7$



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Cascades for Infinite Chain Networks: Multiple Choice: Case I



Consider the case: $a = 3, b = 2, c = 1$

Two possible choice for node u

- Stick with strategy B, total payoff: $0 + 2 = 2$
- Switch to strategy A, total payoff: $3 + 0 = 3$
- Switch to strategy AB, total payoff: $3 + 2 - 1 = 4$

So, node u would adopt strategy AB

And system is stable now!!

Final state: $a = 3, a = 3, AB, b = 2, v, b = 2, B, b = 2$. $c = 1$

Handwritten calculations:
 $A = 3 + 0 = 3$
 $B = 0 + 2 = 2$
 $AB = 3 + 2 - 1 = 4$
 $A = 3 + 2 = 5$
 $B = 2 + 2 = 4$
 $AB = 3 + 2 - 1 = 4$



So, in the last lecture, we have discussed you know this we have started discussing decision based model and we have seen two cases on a infinite chain network, two cases. One setting a was 2, b was 2 and c was 1, another setting a was 5, b was 3 and c was 1 and we have seen how two different, you know two different adoption strategies emerged right over time over iteration.

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Cascade in Infinite Chain Networks: Generic Model

$a=2, b=2, c=1$
 $a=5, b=3, c=1$

- Let us consider an infinite chain network with strategy set $\{A, B, AB\}$
- We consider the scenario: $a = a, b = 1, c = c$
- Two possible cases may arise:

Case A

Case B

Now, let us make this thing generic ok. So, what we do? Here we so, we have three basically parameters so, to say right. And, let us assume let us make one parameter fixed. Let us assume that b is 1 right and a is a and c is c. How you can do that? Say let us say a is 3, b is 2, c is 1 so, we can easily make a b equals to 2 and then a would be 3 by 2 right.

So, let us fix b, let us assume that b is 1 and the other two parameters would remain same; a equals to a and c equals to c ok. And, we will discuss two cases, case 1 is like this. We will decide u's strategy depending on u's neighbor strategy, u has 2 neighbors. The one neighbor has adopted A, other neighbor has adopted B and case 2, u has adopted one of u's neighbors has adopted AB and the other has adopted B.

I am assuming that you have already you have understood the previous lecture and you have memorized the payoff table right. A quick recap again, in case you have you have forgotten.

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Decision-based Cascade Model: Two-player Coordination Game



u's decision	v's decision	Payoff
(A)	(A)	a^*
(B)	(B)	b^*
(A)	(B)	0
(B)	(A)	0

Payoff distribution for different adoption strategies
* a and b are positive constants

A = Amiseem Parone
B = Neelish

- A and B: two possible strategies that each node in network $G(V, E)$ could adopt
- Each node u will play its own independent game
- Final payoff is the sum of payoffs for all the games
- To calculate the required threshold at which a node u would decide to go with strategy A



So, for single strategy right, if both the parties have same strategy right have adopted same strategy, then the payoff would be the corresponding payoff; say both of them adopted A, then this would be a payoff, B then b. If both of them have adopted different strategies then 0 right.

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Multiple Choice Decision-based Cascade Model



- Allows a node to adopt more than one strategy/behavior
- In case a node prefers to go with both the strategies A and B, it would incur an additional cost c
- The revised payoff distribution:

u's decision	v's decision	Payoff
(AB)	(A)	a^*
(AB)	(B)	b^*
(AB)	(AB)	$\max(a, b) - c$

Payoff for a multiple choice decision model
* a and b are positive constants c

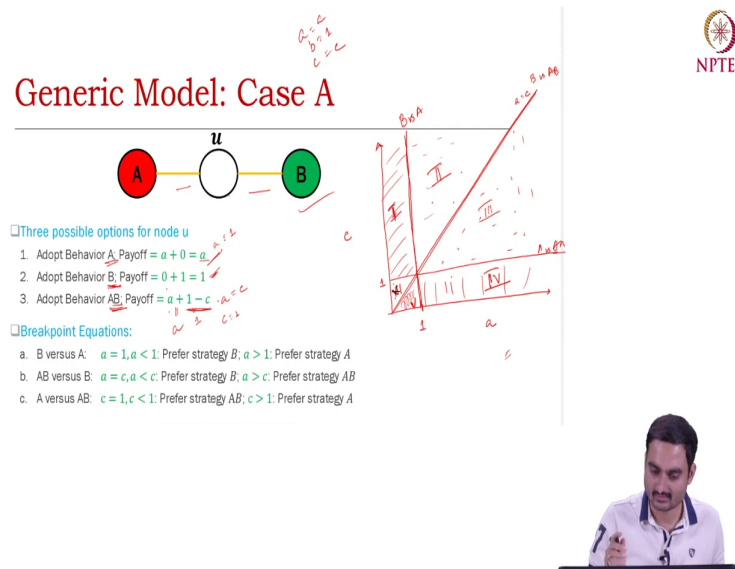
$m(a, b) - c$



Multiple choice strategy, if you have if you have adopted AB, sorry if one of your neighbors has adopted say A and you have adopted AB, then the cost would be then the payoff would be a , but the cost would be c , because you have adopted both the strategies.

If your neighbors has adopted B and you have adopted AB strategy and the payoff would be b plus the cost. And, if both of them have adopted both you and your neighbor have adopted AB, then the payoff would be max a comma b minus the cost right, that is the total payoff ok. Now, what we will do? We will see what happens ok.

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So, let us take case A ok. So, case A for node u, let us say u has adopted A, strategy A, what would be the payoff? Payoff would be a. Because, for this interaction A, this interaction 0. If u has adopted, if u wants to adopt B, this would be 0 plus 1 because remember a equals to a, b equals to 1 and c equals to c ok. If u has adopted AB, then this would be a plus 1 minus c right. What we will do? We will draw a state space diagram right.

Say this is ok, say this is a, this is c ok. Now, let us look at the you know breakout points or breakeven points whatever. Say when we will see a tie, we will see a tie between strategy A and B, if a equals to 1. If a equals to 1, then both of them would be both this and this both the payoffs would be 1. So, when a equals to 1, irrespective of c's value right will be confused between a and b.

So, through this line right, if points lie on this particular line, we will be confused between B versus A which strategy to adopt because, both of them would result in same payoff 1. If right let us say when you would be confused between AB and B ok, AB and B? So, b it is 1, when a equals to c right, this would also this would be 1, the payoff would be 1.

So, when a equals to c , you will be confused between strategy B and AB. So, a equals to c line in this particular space, a equals to c line would be this one right. This is a equals to c line and you will be confused between B versus AB. What about the confusion between A and AB right, A and AB? When c equals to 1, then this would be a , then you will be confused between A and AB, c equals to 1 means this point right. This is equals to 1 and you will be confused between A and A versus AB.

So, we know what would happen when we lie on these lines ok. These are lines on which you would be confused and you would choose either A or B or AB whatever right, depending on the case. If you look at the entire space right, this entire space ok, entire space; how many regions are there?

This is region a, this portion is region a, this is so, this is region I, sorry not a, this is region I, this is region II, this is region III ok, this is region IV, this is region V right and this is region VI ok. So, we will decide the strategies, the strategy in each of these regions ok. Let us try to understand ok.

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Generic Model: Case A

Three possible options for node u

1. Adopt Behavior A; Payoff = $a + 0 = a$
2. Adopt Behavior B; Payoff = $0 + 1 = 1$
3. Adopt Behavior AB; Payoff = $a + 1 - c$

Breakpoint Equations:

- a. B versus A: $a = 1, a < 1$: Prefer strategy B; $a > 1$: Prefer strategy A
- b. AB versus B: $a = c, a < c$: Prefer strategy B; $a > c$: Prefer strategy AB
- c. A versus AB: $c = 1, c < 1$: Prefer strategy AB; $c > 1$: Prefer strategy A

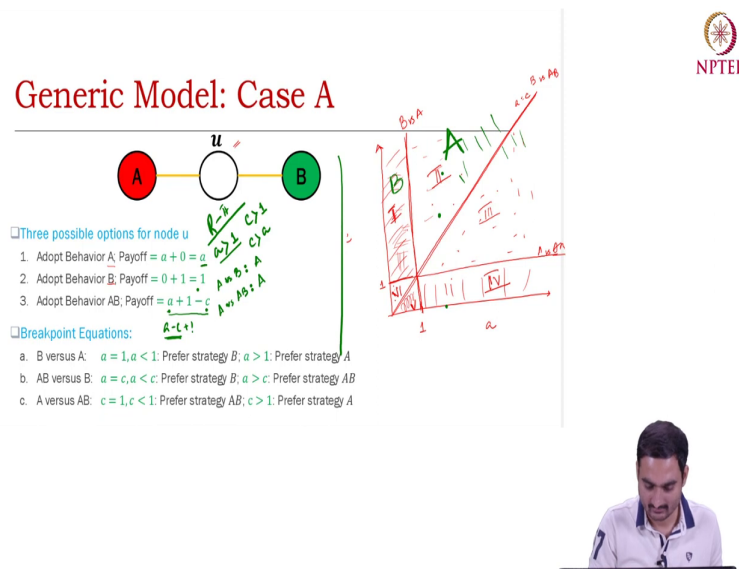
Let us look at let me first erase ok. Let us look at region I ok, I am going slow and so, that you also you understand it clearly right. If you are confused, you know draw it on yourself and you will get it ok. Region I so, region I and this is your setting ok.

You know that if you adopt A, payoff this, adopt B this and payoff c, if you adopt AB payoff is this one. Region I is what? Region I is this part which is c greater than 0 and a also this part right. So, a less than sorry c greater than 1 and a less than 1 ok. This part ok. So, if c greater than 1 and a less than 1, let us look at things between A and B ok, which one will be chosen, which one is higher.

If we choose a, it is a otherwise 1 right. Now, a is less than 1, a is less than 1. So, between A and B, B will be preferred. So, A versus B, B will be preferred because b is 1 and a is less than 1. Between, B versus AB what would happen? Let us see AB, a is already less than 1 right and c is greater than 1. So, this part a minus something which is greater than 1, this would be negative right.

And, you are subtracting you are subtracting a value from a, this would be even less than a right and a is already less than 1. So, between A and AB, B would be preferred ok. And of course, we would not need to look at A versus AB, because A is already bad right, A is already a bad choice. So, in this region B will be preferred ok.

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Now, let us look at the; let us look at the; let us look at the region II ok. In the region II, what is the case? The case is a is greater than 1, c is also greater than 1 and so, this portion right, this portion c is greater than a, is not it? If you look at say this point right, this point you see that c's value is higher than a right and below this point a is greater than c. We are looking at this part right. Let us look at what happens between A and B, A versus B ok.

a is greater than 1 right, a is greater than 1 ok and b is 1 so, A will be chosen. So, now, B is cancelled out. What about A versus AB? Look at this one ok, look at this one; c is greater than a right. So, a minus c a minus c plus 1 right, a minus c would be negative right. You are subtracting something from 1 so, that would be less than 1 whereas, if you choose a, it would be; it would be a which is greater than 1. So, a which be a will be chosen. Therefore, in region B, A will be chosen ok.

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Generic Model: Case A

□ Three possible options for node u

1. Adopt Behavior A; Payoff = $a + 0 = a$
2. Adopt Behavior B; Payoff = $0 + 1 = 1$
3. Adopt Behavior AB; Payoff = $a + 1 - c$

□ Breakpoint Equations:

- a. B versus A: $a = 1, a < 1$: Prefer strategy B; $a > 1$: Prefer strategy A
- b. AB versus B: $a = c, a < c$: Prefer strategy B; $a > c$: Prefer strategy AB
- c. A versus AB: $c = 1, c < 1$: Prefer strategy AB; $c > 1$: Prefer strategy A

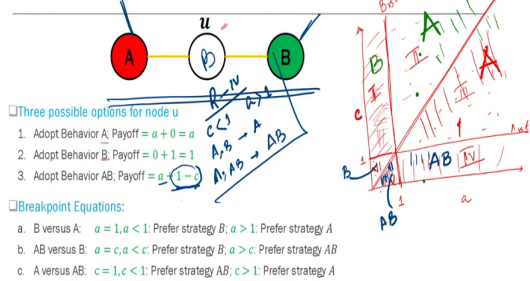
Similarly, if you think of, let me remove it. If you think of region III, what are the strategies, what are the conditions? So, c, this part right ok, c is greater than 1 ok, a is also greater than 1 ok and right and a greater than c ok. So, what would happen in this case? Think about it.

So, this is a right. Now, between A and B, the clear winner is A. What about between A and AB? a minus c, a minus c is not positive right and you are adding some positive number with 1 ok. Now, 1 plus that positive number whether these would be greater than a or not, a is also greater than 1 ok, a is also greater than 1. So, what would happen? A would be chosen here also. So, this would be A ok.

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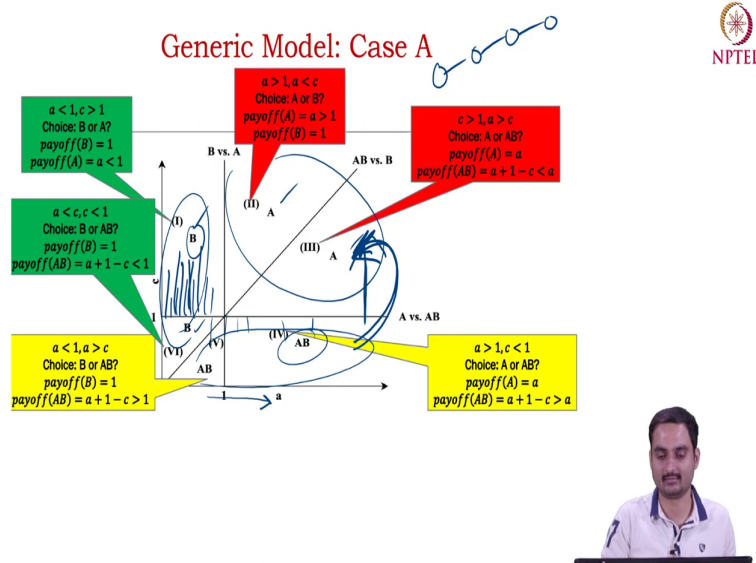
Generic Model: Case A



Now, let us look at region IV, R-IV ok; c is less than 1 this part, a is greater than 1 right. So, between A and B, A will be chosen, between A and AB what would happen? Let us look at it. So, let us look at it here right. So, $1 - c$ ok, c is less than 1 ok. It would be negative. So, this would be positive. So, a plus some positive component, this would be greater than a . So, AB will be chosen. This would be AB ok.

Similarly, if you look at this part ok, I am not deriving it. Same thing in this part what would happen? In this part in this part a is less than 1, c is also less than 1 right. You will see and you will realize that this would also be AB and this would be B, if you calculate in the same manner ok.

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So, this would be the space diagram. This would be the relation between a and c when b is fixed ok. So, you see that B here, B here, A here, A here. So, this part would be A always, this would be AB always and this part would be B always. What does it mean? From this space diagram, what did you infer? Ok. So, initially your strategy was B, if you remember right, if you remember the setting here right.

Initially, your strategy was B and this was also B and this was A right. So, and let us say and the strategy A is something which actually has come due to this hard behavior ok. So, if the cost is so, look at look at this region right. Now, what you are trying? You are trying to propagate A, you are trying to move, you are trying to spread your strategy A. Because, majority of the users have already adopted B, only a few adopted A and you want that A would now spread right.

So, if you want to spread A and if your cost is too high, if your cost is greater than high, nothing would happen right. People will still adopt B right. If you gradually increase, if you gradually decrease your cost right, gradually decrease your cost and increase the payoff of adopting A right. You insist you convince your customers to adopt both A and B. You say that hey you use you are using Amazon Prime that is fine, you also use Netflix. And, look if you use Netflix, you have a lot of benefit, lot of payoff right.

And, you also need to incur a small cost. You need to spend small amount of money for this. So, people will gradually adopt both A and B. And, then what you need to do? You need to

jump from this region to this region by increasing the quality a lot, quality of your strategy a lot, quality of Netflix a lot. So, that even with the increase of cost, people will adopt, people will stick to A and people will forget about B.

Because Netflix, now since now people will start browsing both A and B and B and C that ok, A is better right. So, I can spend money for A, that is not a problem ok. So, then you actually move from this region to this region ok. So, this is the basically the meaning of right the this space diagram.

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Generic Model: Case B

Three possible options for node u

1. Adopt Behavior A: Payoff = $a + 0 = a$
2. Adopt Behavior B: Payoff = $1 + 1 = 2$
3. Adopt Behavior AB: Payoff = $a + c$. if $\max(a, 1) = a$

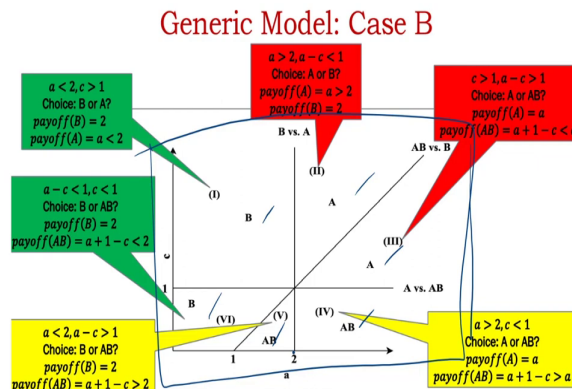
Breakpoint Equations:

- a. B versus A: $a = 2, a < 2$: Prefer strategy B; $a > 2$: Prefer strategy A
- b. AB versus B: $a - c = 1, a - c < 1$: Prefer strategy B; $a - c > 1$: Prefer strategy AB
- c. A versus AB: $c = 1, c < 1$: Prefer strategy AB; $c > 1$: Prefer strategy A

Handwritten notes: $a = c + 1$, $a - c = 2$

Similarly, if you look as case B, where this is AB, this is B. Earlier, it was A, now this is AB, this is B. And, you are choosing the decision, choosing the strategy for u right, the same way. I am not deriving it, but the same way.

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If you do this thing, you will see this kind of space diagram where here you will see 6 regions, but here things are little different. Why? Because, of this cost function, extra cost function. So, initially we had this, this, 1, 1 and this. Now, we will see from 2 ok, like this. Why? You will understand it, because you will see that when you calculate the breakpoint right, between A and B AB and c and A and AB, you will realize that this line is not now c equals to a, this is something different ok.

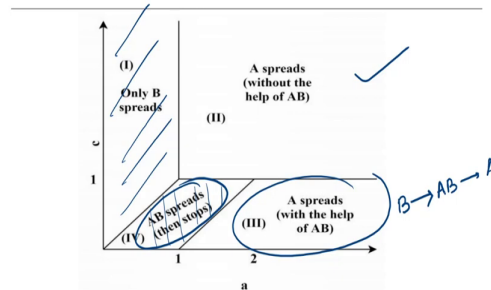
And, this two lines so, this line is c equals to 1, this line is a equals to 2 right. You see here, if a goes to 2, then this would be same ok. And, here if you see that if c equals to a plus 1, c equals to a plus 1 right, c equals to a plus 1, then you will be you will basically be confused between B versus AB ok; sorry not c equals to minus 1, a equals to c plus 1 ok.

If a equals to c plus 1, then a minus c would be 1 right, a minus c would be 1, you and you also have 1 here. So, it would be 2, then you will be confused between A and AB ok. And, if you derive in the same manner, you will see this kind of things emerging B, A, A, AB, AB and B.

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Generic Model: Combined



If you club, if you merge these two strategies, these two cases, case A and case B, you will see this kind of combined behavior ok. And, if you carefully look at it, what does it mean? In this region, you cannot do anything, people will adopt B, people will remain with B right.

With this region, at this region AB, the strategy AB will start spreading, then we stop. I hope you remember last lecture, we had this kind of setting right, where this one, where although u has switched from B to AB, but then AB has not moved further right. And, this kind of regions, these kind of things are described right in this region ok.

This region indicates that A spreads with the help of AB. What does it mean? It means first all the nodes were B right, then people started switching from AB to B to AB, then AB to A. This is something we also discussed right here, in this case when a goes to a was 5 and b was 3 right. So, A will eventually spread, but with the help of AB and that would be the ideal situation.

So, you cannot penetrate the market directly, you actually have to you know convince your customers, that why do not you also adopt my product and gradually see what happens ok. And, at the final stage, A would start spreading without the help of AB right. So, this generic model actually gives you an idea about how strategy spread and how things, product products will be adopted.

Your company would penetrate the market and what would be your optimal strategy, depending upon the payoff and depending upon that you also decide the payoff a, b, c etcetera, payoff and cost ok. So, we stop here today. In the next lecture, we will discuss a case study about the decision based strategy. And, then following this, we will discuss other models which are based on probabilistic theory ok.

Thank you.