

Social Network Analysis
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Chapter - 07

Lecture - 02

Now, let us discuss the models right the actual models. So, if you look at the models for information diffusion, they are broadly categorized into two types.

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Cascade Model: Decision-based Model

1) Probabilistic model

- Given a network, each node has the **freedom to decide** whether to adopt a contagion or not
- Originated from the idea of **local interaction models** described by Morris in 2000
- Decision at each node is influenced by the **behavior of nodes in its neighborhood**
- Nodes decide to adopt a new contagion driven by a **direct benefit or payoff**
- The **payoff** by adopting a contagion is directly proportional to the **number of its neighbors** that have adopted the same contagion
- Can be explained using a **two-player coordination game**
 - Given a number of strategies, the end goal of the players is to **coordinate on the same strategy to maximize their payoffs**



The first type of models is called decision based model right. In the decision mode based model the idea is that you basically look at your neighbors adoption strategy and based on that you have the liberty to decide your strategy ok.

Whether you adopt a particular product or decision or not right. You have the liberty you have the freedom to decide whereas, the second type of model that we will discuss later is called probabilistic model probabilistic model and here the idea is that you do not have control on the adoption process with certain probability, you will be you will adopt otherwise not right.

Now, decision based model is mostly applicable for situations where you want to be a part of a certain protest, you want to you know buy a product, you want to go for a movie or not, you want to go to a restaurant or not. It all depends on your decision you have the freedom

whereas, probabilistic model applies to cases for example, disease spreading epidemic spreading. Now when users interact right come the two individuals come closer to each other with certain probability and say one user is already infected with certain probability the other user will also be infected right.

The other user does not have the liberty to choose whether he will be infected or not right. So, this is probabilistic model. So, we will discuss about the probabilistic model later, but now we will focus on the first type of models called decision based model. So, let us look at the process right. So, given a network each node has the freedom to decide whether to adopt a particular contagion or not right.

Remember the contagion can be a product and opinion and individual and decision right and so on. Originated from the idea of local interaction models right basically depends on how you interact we will see that we mostly look at you know two party interactions. We will not look at the other interactions as a whole. We look at a two party interaction and based on that we decide.

Decision at each node it influenced by the behavior of nodes in its neighbors right. We already mentioned that depends on your neighbor's decision. Nodes decides to adopt a new contagion driven by a direct benefit or a payoff, now this is very important. So, let us say let us take the same example right subscribing to Netflix versus Amazon Prime. Now whether you subscribe to Netflix or Amazon Prime it depends on your profit benefit right or payoff, it is called payoff. In the game theory game theory literature it is called payoff.

Now, if you see say if you are; if you are more of a you know a Bollywood kind of movie lover, you may want to choose you want you may want to subscribe to Amazon Prime because of the amount of collection compared to Netflix right whereas, if you are say more of a you know Hollywood kind of movie lover you may want to subscribe to Netflix rather than Amazon Prime, now it depends on your payoff.

The payoff can also be measured in terms of money right. Say if you subscribe to Netflix you will have to charge you will have to you will have to be charged say 2000 per month I do not know the subscription charge, but let us say 2000 per month right or 2000 per quarter. Whereas, Amazon Prime says the charge is whatever say 1000 per quarter right. Now you want to choose Amazon Prime rather than Netflix right.


So, the payoff by adopting a particular contagion is directly proportional to the number of its neighbors that have already adopted the same contagion right. So, the idea is that let us say let us take another example. Let us say there are two you know voice communication apps one is WhatsApp and another is Skype right. Now which one you would like to choose? Now you would look at your neighbor's right your friends and you see that most of your friends have installed WhatsApp right.

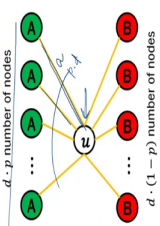
So, and you also want to communicate with your friends quite often quite frequently. So, you would also install WhatsApp not Skype right. It depends on your neighbors decision neighbors choice right. So, it can be explained using a two player coordination game, now we will discuss this thing later. So, in game theory there is this famous you know two party game or two player game multiplayer game and so on. So, in two player game you may have heard about something called zero-sum game right.

Now, here we will not talk about zero-sum game, but here the idea is that we assume that there are only two parties, two individuals and one individual has already adopted something and the other individual will decide his strategy depending on his friend strategy right and at the end of the day what is the what is the objective function? What you want to maximize? You want to maximize the payoff right. So, given a number of strategies, the end goal of the players is to coordinate on the same strategy to maximize their payoff ok.

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Decision-based Cascade Model: Two-player Coordination Game





$d \cdot p$ number of nodes

$d \cdot (1 - p)$ number of nodes

- Node u has d neighbours
- p fraction of neighbours adopt strategy A
- Rest adopts strategy B
- Total payoff for node u if it goes with strategy A = $a \cdot d \cdot p$
- Total payoff for node u if it goes with strategy B = $b \cdot d \cdot (1 - p)$
- Node u would adopt contagion A if

Handwritten notes:

$a \cdot d \cdot p > b \cdot d \cdot (1 - p)$

$a \cdot p > b \cdot (1 - p)$


$a \cdot p > b - b \cdot p$

$a \cdot p + b \cdot p > b$

$p(a + b) > b$

$p > \frac{b}{a + b}$

activation threshold



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Decision-based Cascade Model: Two-player Coordination Game



u's decision	v's decision	Payoff
A	A	a
B	B	b
A	B	0
B	A	0

Payoff distribution for different adoption strategies
* a and b are positive constants

A = Amazon Prime
B = Netflix

- A and B: two possible strategies that each node in network $G(V, E)$ could adopt
- Each node u will play its own independent game
- Final payoff is the sum of payoffs for all the games
- To calculate the required threshold at which a node u would decide to go with strategy A



Now, this will be clear in the next slide ok. Let us say; let us say you have 2 strategies strategy 1 is subscribing to Amazon Prime, strategy B is subscribing to Netflix right. Then let us say; let us say you are u ok and v is your friend. So, v has already has already adopted A right strategy A. If you also adopt A the payoff would be a small a right.

Now, this payoff would basically be benefit some sort of benefit ok. So, both u and v they have adopted the same strategy And both will get a payoff a . Let us say v has adopted B and u also has erupted B right. The payoff would be b a b these are constant, small a small b these are constant. Some sort of payoff that we will decide ok, v has adopted A and you have and u has adopted A sorry v has adopted B and u has adopted A right.

So, the strategy are different therefore, the payoff would be 0 right and the other way around A v has adopted A and u has adopted B payoff would be 0. So, let us say a game where you look at individual you look at a two party interactions right and if both the strategies, if the strategy of both the players is same then it would be a either A or B depending upon the choice of the strategy. If they are different they are 0 right.

p fraction of neighbors have adopted strategy A. Now this set of neighbors have adopted strategy A and the remaining $1 - p$ fraction of neighbors have adopted strategy B right. So, if u adopts A, if u adopts A what would be the total payoff? So, u 's payoff would be now remember for each of this adoption right you will get a payoff of a small a right and how many such links are there? There are p times d links are there ok.


So, total payoff would be a times d times P right. If u adopts b then the total payoff would be b times d times $1 - p$ right. Now what is the criteria in terms of A B right. So, that u would adopt a strategy A . So, the criteria would be this would be greater than this. If this payoff is greater than this then u will adopt strategy A right. So, a times p $1 - p$ right, so if you calculate this would be ok.


So, we are not considering the equals to conditions, it is a border line. So, P which is the fraction of neighbors who have adopted strategy A . This would be greater than this fraction b by a plus b ok, which is also quite obvious. What is this fraction? This fraction is essentially.

So, b is the payoff if you adopt b and a plus b is a total payoff. So, this fraction would be greater than the fraction of you know payoff if you adopt strategy p right. Let us look at an example and this is called this threshold is of often called as activation threshold ok.

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Decision-based Cascade Model: Illustration





Let us look at this example. Say this figure A I and G , they are kind of showing hard immunity a hard behavior and they have already adopted Netflix right and the remaining users have adopted Amazon Prime right. So, now, let us look at how the remaining users would change their strategy. Let us look at this guy right. B has 20 neighbors and let us say the activation threshold is 0.5.

So, if the if 50 percent if 50 percent or more than 50 percent of your neighbors have adopted a strategy, you would also adopt the strategy right. So, let us say node b right. So, 50 you see

that 1 out of 2 neighbors has adopted Netflix. So, B will also adopt Netflix right. What about H? 100 percent of neighbors have adopted Netflix.

So, H would adopt Netflix right. What about C? Again 50 percent has adopted. So, C would also adopt Netflix. So, you would get this kind of setting. Now let us look at the remaining iteration the remaining iterations you see that for K majority has adopted Amazon Prime. So, K would not change, D would not change U would not change and so, as F. So, this would terminate this would stop at this point, at this kind of stage ok.

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Multiple Choice Decision-based Cascade Model



- Allows a node to adopt more than one strategy/behavior
- In case a node prefers to go with both the strategies A and B, it would incur an additional cost c
- The revised payoff distribution:

u's decision	v's decision	Payoff
AB	A	a^*
AB	B	b^*
AB	AB	$\max(a, b) - \frac{c}{2}$

Payoff for a multiple choice decision model
* a and b are positive constants c



Now, let us look at a bit different game. Here you can choose both A and B at the same time ok. So, you can actually choose multiple strategies. Now what would be the payoff in this case? So, the payoff would be as follows. Say if your neighbor has chosen a payoff a and you have chosen a and b . So, your payoff would be a right let us see here let us see this payoff table.

Say you are u right and v is your neighbor, v has chosen A and u has chosen A and B. So, the payoff would be a because you do not get you would not get benefit of both A and B because your neighbor has only adopted A right. Let us say you have both Skype and WhatsApp, but your neighbor has only WhatsApp right. So, you can only communicate through WhatsApp you cannot use Skype.

Similarly, if v has chosen B and u has chosen AB the payoff would be b right. If both have chosen A and B right the payoff would be \max of a and b ok, which is quite obvious. Because say you have chosen Skype and WhatsApp your neighbor has also chosen Skype and WhatsApp.

Now you want to get benefits of either WhatsApp or Skype right and say and say you are you are currently in a state where WhatsApp is allowed Skype is not allowed. So, you would use WhatsApp you are say currently in another city, where Skype is allowed WhatsApp is not allowed you would you use a Skype right.

So, it would be \max of a and b . But important point to note is that when you choose both A and B you will incur some cost some additional cost. This cost is c small c . Now this cost can be in terms of additional money. This cost can be in terms of additional bandwidth that you need to use when you use both A and B and so on. So, the net you know payoff would be \max of a comma b minus c because c is the cost that you need to incur when you adopt both A and B ok.

Now, this cost is irrespective of your neighbors decision right. It depends on your decision. If you adopt both A and B you need to incur a cost irrespective of what your neighbor has chosen right. Now this is very important remember this particular thing constant.

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Cascades for Infinite Chain Networks: Single Choice



Consider the case: $a = 3, b = 2$
 Two possible choice for node u
 Stick with strategy B, total payoff: $0 + 2 = 2$
 Switch to strategy A, total payoff: $3 + 0 = 3$
 So, node u would adopt strategy A
 And the cascade continues...

$$\begin{array}{l}
 u \\
 \text{A} \\
 \text{B} \\
 \hline
 \text{A: } a + 0 = 3 + 0 = 3 \\
 \text{B: } 0 + b = 0 + 2 = 2
 \end{array}$$



Now, let us look at a particular setting. Let us think of a infinite chain network right. Its a infinite chain network right a chain network with infinite number of nodes and let us say small a the payoff a is 3 the payoff b is 2. So, a greater than b and let us assume that this is a single choice game you cannot adopt both a and b right.

Essentially you are following this payoff table and remember. So, memorize this payoff table ok. If both the payoffs are same you get either A or B depending on the strategy. If they are different then you get 0 ok. So, and let us say these two nodes the left most two nodes they have already adopted a kind of hard behavior, They have already adopted A and the remaining nodes can decide what to adopt right. So, let us look at this node u ok. So, this is u and u has two neighbors these guy has already adopted A this guy has adopted B ok.

Now, if u adopts A if u adopts A what would be the payoff for u? Payoff would be for this interaction payoff would be a and for this interaction payoff would be 0 right. So, 3 plus 0 equals to 3. If u adopts B then for these interactions this would be 0 and for the other one this would be B. So, 0 plus 2 it would be 2 ok. So, this is maximum. This is higher than this one. So, a node u will choose A right.

So, the strategy now has moved from this node to this node. Similarly when you look at this node same thing will happen. This guy will also choose will also move from B to A right switch from B to A. So, the strategy A will move here and so on and so forth. So, over time you will see that all the nodes will adapt A and it will stop ok.

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Cascades for Infinite Chain Networks: Multiple Choice: Case I

- Consider the case: $a = 3, b = 2, c = 1$
- Two possible choice for node u:
 - Stick with strategy B, total payoff: $0 + 2 = 2$
 - Switch to strategy A, total payoff: $3 + 0 = 3$
 - Switch to strategy AB, total payoff: $3 + 2 = 5$
- So, node u would adopt strategy AB
- And system is stable now!

Let us look at a multiple choice strategy ok and remember we will follow this table ok, try to memorize it again. If I choose A and my neighbor has chosen A I will get a. If I choose B my neighbor has chosen B I get b. If I choose a B my neighbor has chosen A I would get a. I choose AB my neighbor has chosen B I will get b. I choose AB and my neighbor has chosen AB, I will get $\max(a, b)$ but I will incur a cost c. In fact, in last three cases I will incur a cost c right, cost c not a cost c and c is a constant ok.

Let us see the same setting a equals to 3 the payoff b equals to 2 and the cost is 1. Let us look at this node ok. So, this is A and this is B ok. So, if this guy u has if this guy chooses A right due to this interaction it will get 3 this due to this one this will be 0. So, 3 for B 0 2 2. If AB if this is AB then what would happen? Due to this interaction it will get A which is 3 due to this interaction it will get 2, but since u has adopted both the strategies it will incur a cost c which is 1. So, the net would be 4 ok.

Remember one thing this cost is age independent. It means that when we look at this interaction and when we look at this interaction, this would not happen that we will incur a cost here one time and when you look at this interaction you will again incur a cost on that time this would not happen ok. So, this cost is independent of the interaction, you will incur cost only one time ok. So, this guy will adopt AB ok. Now let us see what happens ok.

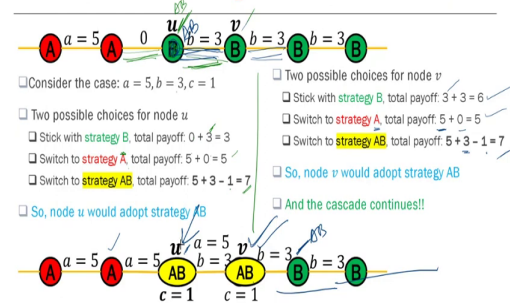
Now, let us see what happens with v ok. So, node v has two neighbors. This guy has adopted B, this guy has adopted AB already ok, we have seen. If v adopts A then what would happen? Due to this interactions it would be 3 a small a due to this it would be 0. So, 3, if it adopts B 2 plus 2 right 4 ok, b payoff small b payoff for this you also get small b payoff. What about AB? For this you get $\max(a, b)$ which is 3. For this you get B which is 2 for and you would also incur a cost right one.

So, this would all be 4. So, you see a tie here ok. So, since v has already adopted B. Now v is choosing now we will not you know switch the strategy because you know there is no point in switching. If which switches then we will get the same payoff right. So, we will not switch ok. So, ultimately what would happen? We will see that the cascade the AB cascade actually would stop at this point, it would not move further with this setting the cascade AB B cascade would not move further ok. It will stop here alright.

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Cascades for Infinite Chain Networks: Multiple Choice: Case II



So, now let us look at another setting a different choice of payoffs different values of payoffs and cost. Initially in the earlier case a was 3 b was 2 c was 1. Now a is 5 b is 3 and c is 1 ok. Let us see what happens 5 3 and 1. So, let us see for u the same setting. So, if u chooses B due to this interactions it would get b small b payoff which is 3 and due to this interaction it will get 0 so 3. If u chooses A due to this interactions it will get 5 due to this it would get 0 therefore, 5.

If u chooses AB if u chooses AB due to this interactions it will get, due to this interaction it will get 3 and cost is 1. So, total is 7 ok. So, u will adopt AB the same as earlier right with different payoffs. So, that is fine, you will adopt AB. Let us see what happens with v . Now remember this is now AB ok, this is now AB. So, if v if node v chooses strategy B due to this interaction it would get 3, due to this interaction 3 so 6. If it chooses A due to this interaction it would get 5, due to this interaction it would get 0 so 5.

If v chooses AB due to this interactions this interaction it would get max a b which is 5, due to this interaction B which is 3 and cost is 1 so 7. So, here you see that the base strategy is AB. So, B will switch from B to AB ok. So, if B will switch to AB then similarly for this node it will also choose AB and so on. But now interesting point to note is that let us see what happens with this guy u , because now for u has two nodes two neighbors this guy has adopted A and this guy has adopted AB.

When I calculated earlier for u the set the setting was different. Now we have a different setting. Let us look at whether you would still you know retain AB strategy or you would now move, you would now switch let us see ok. So, let us look at u right, let us draw it separately ok.

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Cascades for Infinite Chain Networks: Multiple Choice: Case II



Consider the case: $a = 5, b = 3, c = 1$

- Two possible choices for node u
 - Stick with strategy B, total payoff: $0 + 3 = 3$
 - Switch to strategy A, total payoff: $5 + 0 = 5$
 - Switch to strategy AB, total payoff: $5 + 3 - 1 = 7$
- So, node u would adopt strategy AB

- Two possible choices for node v
 - Stick with strategy B, total payoff: $3 + 3 = 6$
 - Switch to strategy A, total payoff: $5 + 0 = 5$
 - Switch to strategy AB, total payoff: $5 + 3 - 1 = 7$
- So, node v would adopt strategy AB
- And the cascade continues!!

Handwritten calculations: $A = 5 + 5 = 10$, $B = 0 + 3 = 3$, $AB = 5 + 3 - 1 = 7$



Let us use a different color this is u. AB u has adopted AB earlier and this is A, this is AB ok. If u has adopted I say if u switches from AB to A what would happen? If it is A then due to this interaction it would be 5, due to this interaction this would be also 5 right. So, payoff is 10. What about B? If this is B then due to this one this is 0, due to this one this is 3 right small b 3.

What about AB? The current strategy due to this one this would be 5, due to this one this would also be 5, but the cost 1 so 9 right. Interestingly see here carefully. The base strategy is A. So, initially u has switched from B to AB, now when one of this neighbor has switched from B to AB he or the node u will now switch from AB to A right.

So, in the next iteration this would be A. So, AB will move behind AB A will also move. So, all the nodes will switch from B to AB then from AB to A right. It only happened because of the choice of A and B payoff a and b right. It is very interesting remember this it happens because the choice of A and B. Initially we have seen that we have different choice of A and B and we have got a different result altogether and now we have got a we have got a different different result ok.

We stop here, but I strongly suggest you guys to remember this you know this payoff matrix right and how we choose the strategies ok. In the next lecture we will see how we can make this thing generalized irrespective of the choice of A and B because in a real situation think about it in real situation we may not know the exact value of A and B right. When you devise a; device a setting device a model you have to choose a you have to you have to consider a generic value of A and B.

And depending on that you have to decide certain choice and let the user let the person you know assigns A and B and automatically your strategy your model will be adopted ok. So, in the next lecture we will discuss how we can generalize these two settings ok.

Thank you.