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Discrete Mathematics
Logic
Tautology, Contradiction - Part 2
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Let us now look at an example of a Boolean expression that is a tautology. So what do we do? We should just observe that the entries below that expression is all 1s. You should not even spot a single 0. Okay, let's see. Our example is going to be P implies P OR Q. Now how do I write the truthtable of this? First I write down P, then Q and I now have P OR Q. what is P OR Q? 0, 1, 1, 1. Correct. This is how the OR works. Now P implies P OR Q. How does this work? Look at the P column. Look at the P OR R column. Simply do implication of this with this. So 0 implies 0 is 1. You remember how A implies B works? 0 implies 1 is 1. 1 implies 1 is 1. 1 implies 1 is 1. And so I get all 1s in this column corresponding to P implies P OR Q and hence this expression, this truthtable, this P implies P OR Q is a tautology which means it's always true. So do we really need to use these truthtable to say that this is always true? Not really. P implies P OR Q let me think about it. When it is not a tautology? When P ends up being 1 and P OR Q ends up being 0. That's the only place where an implication can go false. If I can show that this is impossible that P being 1 and this being 0 is impossible obviously it is a tautology. Isn't it? Think about it and how do I do that? Let me say P is 1 then this P becomes 1. When this P becomes 1 irrespective of what Q is P OR Q will be 1. So you can never have 1 implies 0 ever which means this implication will always be true which means it is a tautology. You see the ingenuity of this argument. I did not write a truthtable but still showed that ever entry here will be 1. It can never be 0.

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