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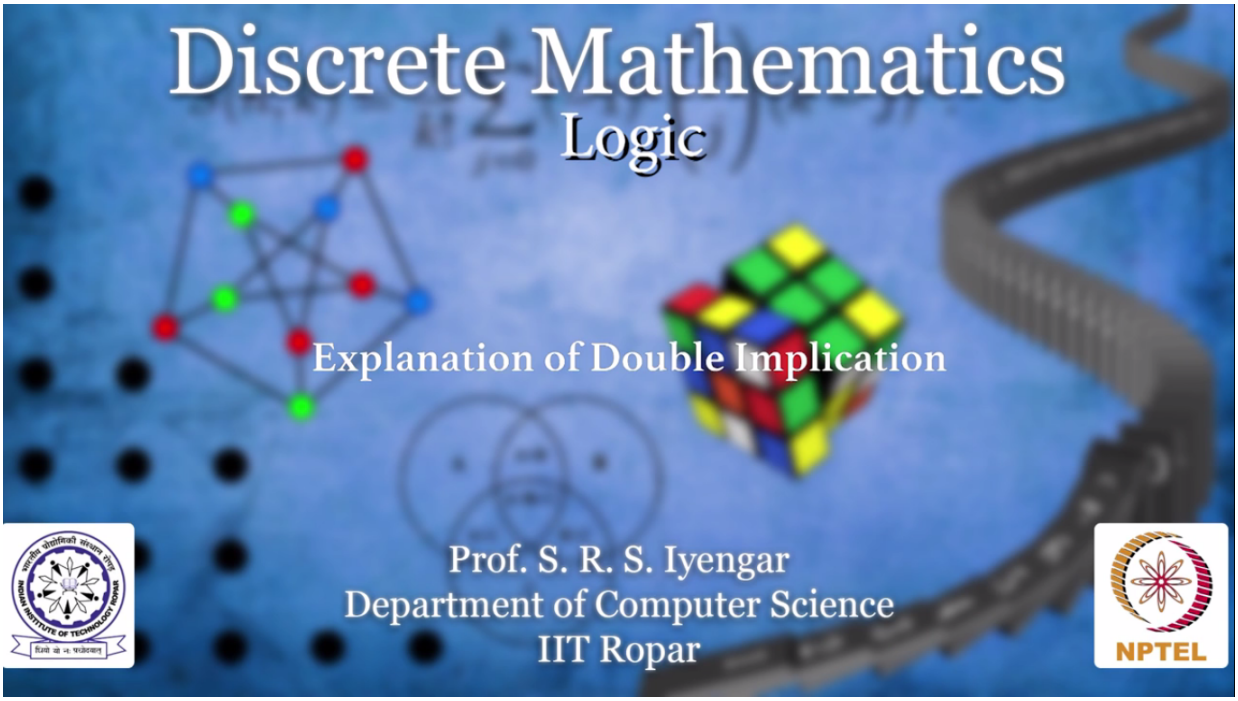

Discrete Mathematics
Logic
Explanation of Double Implication
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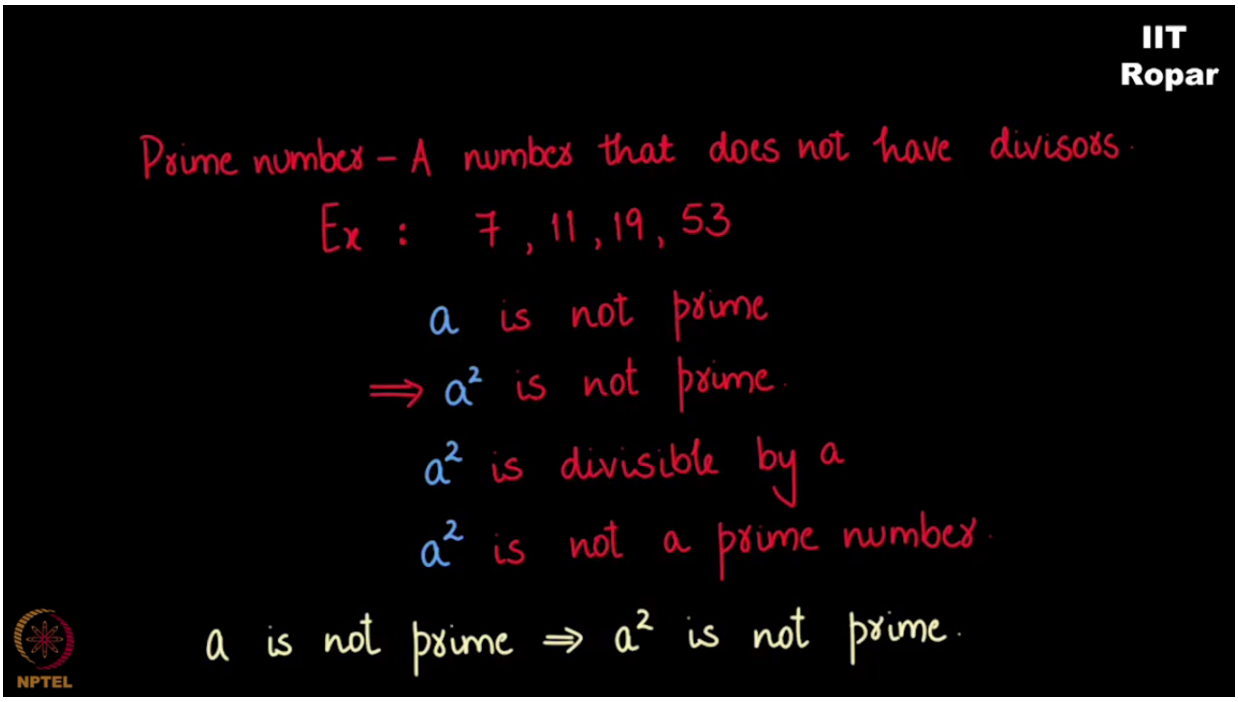
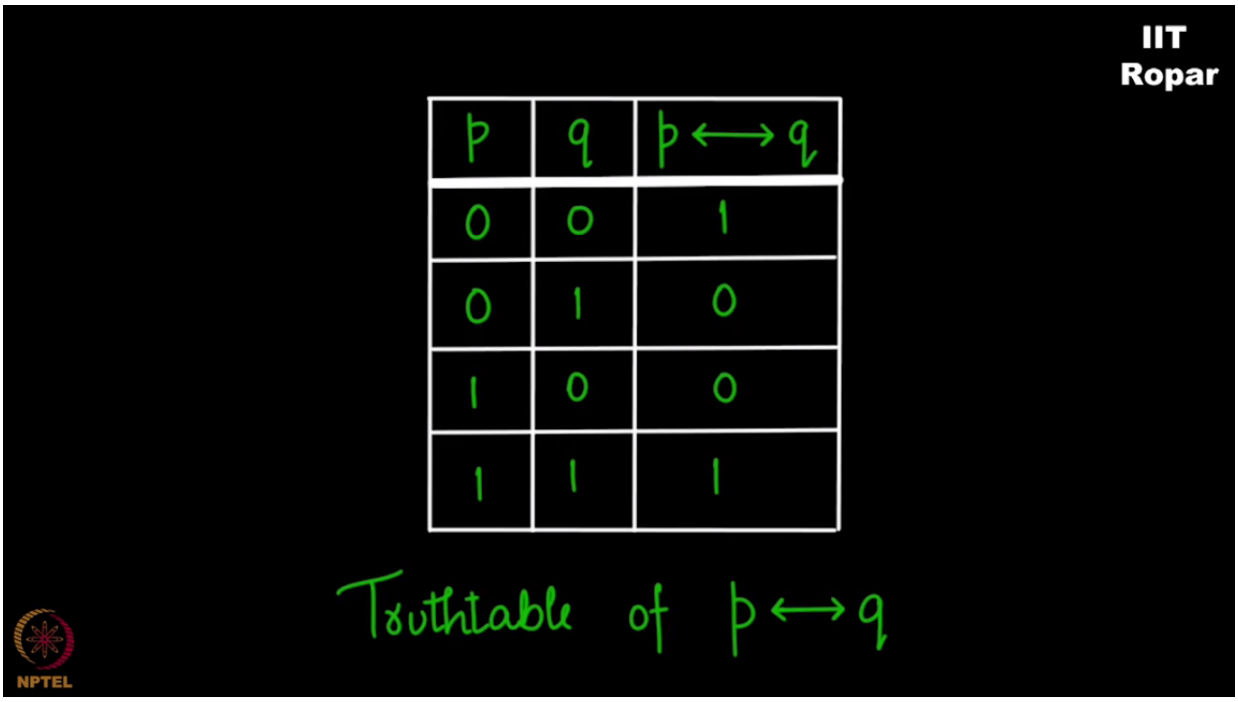
Let us look at some properties of numbers and try to see if we can understand implication properly. You see what is a prime number? A prime number is something that doesn't have any divisors. 7 is a prime number. 11 is a prime number. You cannot find a number which divides 7. You cannot find a number that divides 11. 19 is a prime number. 53 is a prime number. You cannot find a number that divides 53.

So now whenever you take a number that is not prime, let's say A is not prime. Take any non-prime number. It implies that A square is also not prime. Correct? It goes without saying. When you take some number and take its square. The square will always be divisible by A. A square is always divisible by A. And hence by the definition of prime number A square is not a prime number. A is not prime implies A square is not prime.



Let me take another example, second example. A is an even number implies A square is an even number. Correct? Why? Whenever A is even, A is in the form 2m. A square will be of the form 4m2 which obviously is an even number can you see. You have a multiple of 4 which is always even. So A is even implies A square is also even.

Now let us ask a question. If you are given that A square is even, does it imply that A is even? A square is even indeed A is even why if A were to be odd square of an odd number should be odd. If A square is even then definitely that particular A whole square is giving you A square and hence becoming and you observe that it is even means that A is even. Think about it for a minute. Take a pen and paper and then work it out. A square is even implies A is even but then we saw A is even implies A square is even. If P implies Q and Q implies P we say P two way implication symbol is how we write this as two way arrow. P implies and is implied by Q is how we say. By that we mean P and Q are equivalent because when P is implying Q and Q is implying P they both are one and the same. Remember our subset example that we said, P is a condition which implies that Q is also true. Q is a condition which implies P is true. So whenever I said A is within B and I said B is also known to be within A, A subset of B and B subset of A implies A is equal to B. This is only obvious. Correct?



So B implies and implied by Q is written with this bidirectional arrow and it simply denotes the fact that P implies Q is true and Q implies P is true. And the truthtable of this P implies and implied by Q looks like this. If P is 0, Q should be 0. Both are of the same parity that's what this is bidirectional thing means. So it is true. If P is 0 and Q is 1 that is impossible. It is false. If P is 1, Q is 0, false. If P is 1 and Q is 1 it is true. So this is the truthtable of P implies and implied by Q you will observe that if you were to write the truthtable of P implies Q and Q implies P and do an AND of it it will resemble this 1, 0, 0, and 1.