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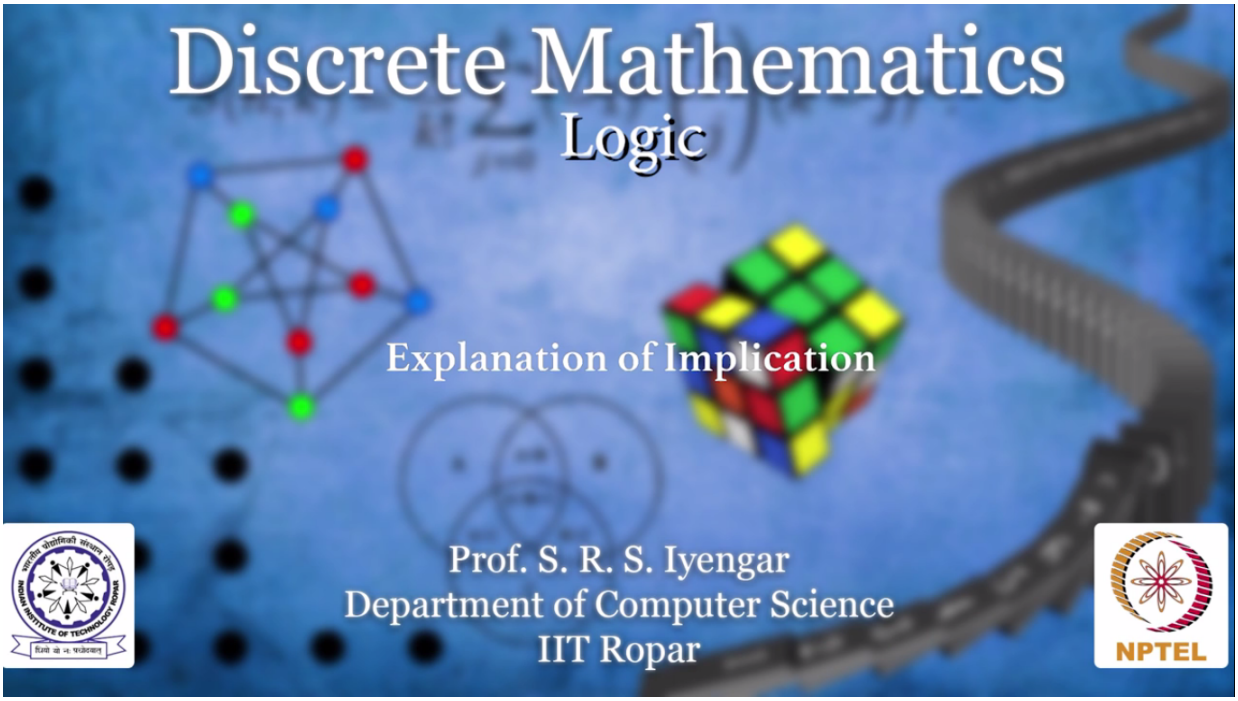

Discrete Mathematics
Logic
Explanation of Implication
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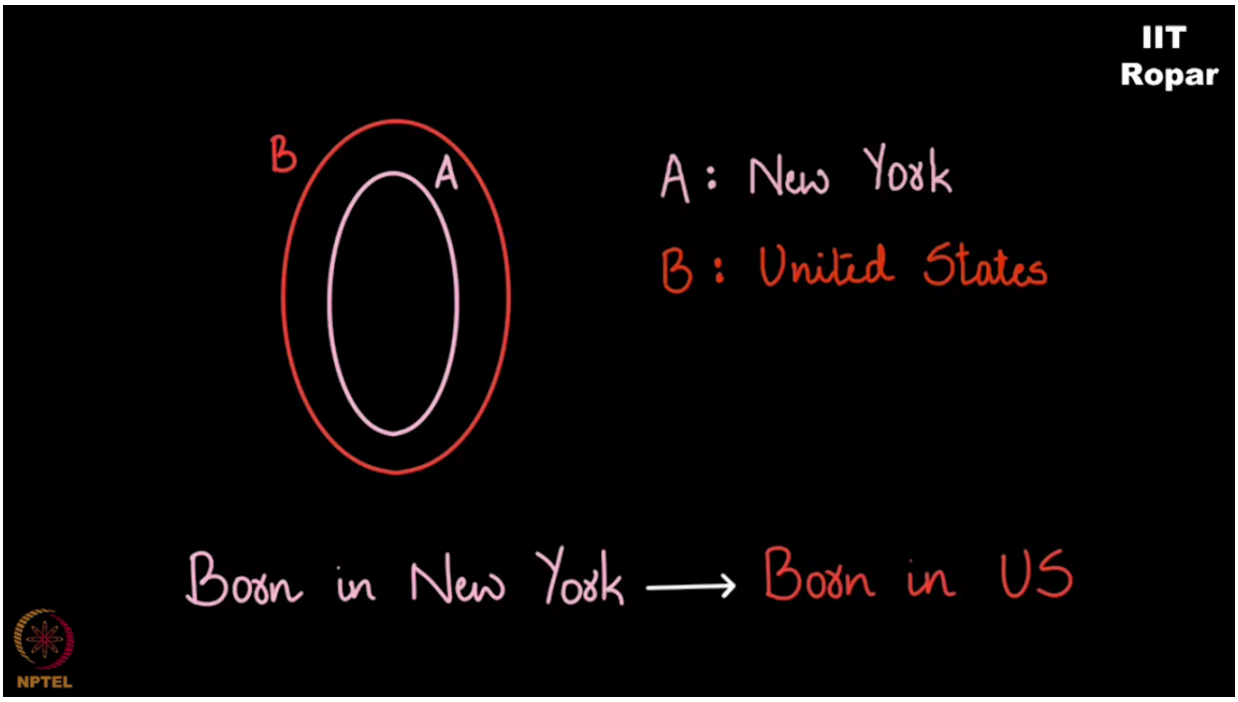
Let me start my explanation with a question. Assume there is a set A, a smaller set and a bigger set B. So let's say A is New York and B is the whole of the United States. We know New York is in Unite States.

Now if you were born in New York it implies that you were born in U.S. Let that stay in your mind. As simple as it can get what we are going to say is nothing more than just this example. We are going to formally see how to interpret this very fact with some rigorous mathematics.



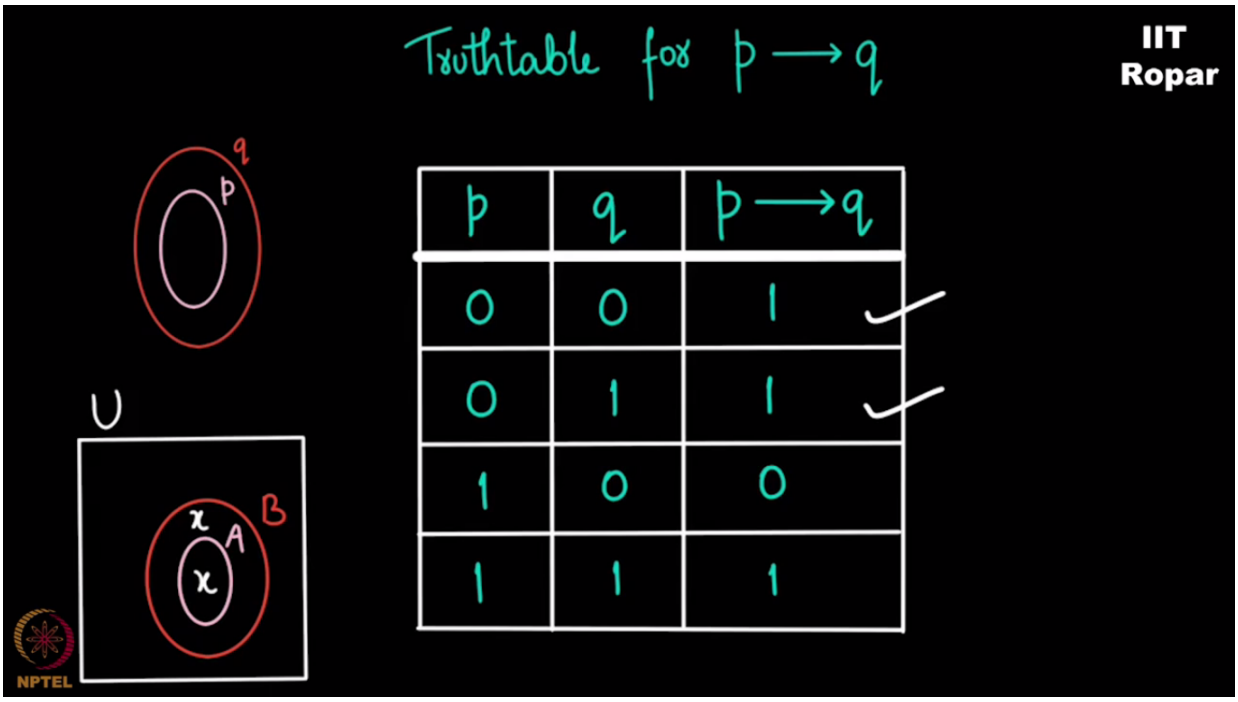
Whenever someone says statement P you will derive statement Q. Didn't you derive that person was born in U.S. the moment someone said he was born in New York. Well if you are asked to write the truthtable of P implies Q, how will you write?

First of all, not everything can be written in the form of a truthtable. But P implies Q being such a important logical statement it merits for one to write its truthtable. Now please note this is the confusing most of all truthtables.

Let us start. P Q. P is 0, Q is 0. 0, 1, 1, 0, 1, 1. All possibilities. P implies Q. always see this as P is a smaller set. Q is a bigger set like the New York and U.S. example. Whenever you say P some statement is true you infer that Q is also true. This P implies Q and New York and U.S. doesn't seem to be the same but you will soon realize they are the same. Let's see how.

When something is not in P can you conclude that something is not in Q is what we are asking here. Do you think when P is 0 Q is 0? stated differently all I am asking is can it so happen that inside this set there is no element. Given an element X which probably is in the universal set. Forget about A and B. A is a smaller set. B is a bigger set containing A and there is a universal set U. If I give you an element X and that element is not in A can you say that element is not in B? Not necessarily. But yes it can be true. Look at this statement, when something is not in A it could be in B but not necessary is all I am saying.

Now look what is the next statement. An element is not in A can it be in B? Not necessarily but yes it can be true same statement. You see. Now what is the next statement? 1 and 0. An element E is in A but not in B. Is that possible? That is impossible. First one is not necessarily true but yes it can be true. Second one is also not necessarily true but can be possible at times. Third one is impossible. Fourth one is an element is in A implies the element is in B also. Very true. This is always possible actually. You see first one is not impossible. Second one is not possible. Third one is impossible. And fourth one is let us put it under the not impossible category. Correct?



Now when it is not impossible we write 1. When it is not impossible always possible we write 1. When it is impossible we write 0. This is the convention that we are using to develop the truthtable. I repeat it is extremely confusing why the entries are all 1s here. In fact, I took up this exercise of going through all the videos online and in the comment section of most of the YouTube videos I observed that many people have this confusion that I understand when P is 1, Q is 1. P implies Q is 1. I understand when P is 1 and Q is 0, P implies Q is 0. But I don't understand how when P is 0 and Q is 1 P implies Q is 1. The explanation is with the subsets that I told you right now better explanation is please don't break your head. It is more of a convention to call it 1 simply because it maybe possible. It is not impossible. At times it is possible that P is not true but Q is true. That is why we put 1 there. This truthtable is more of a convention. The only thing that you should concentrate on is 1 implying 0 is never true. A word of advice, if you didn't understand what is P implies Q stare at this truthtable many times if possible even memorize it. With time you will get the wisdom of what exactly one means by P implies Q.