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Discrete Mathematics

Let Us Count

Problems on Rule of Sum and Rule of Product

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The professor has introduced rule of sum and rule of product. Let us see more examples now. Alice goes to the library to read a book and the library has books of different genres. Alice goes to the shelf where there are seven science fictions, five mystery books, and two journals but Alice wants to read precisely one book. Now the question here is how many choices does Alice have? So you see Alice wants to read precisely one book that is very important. So she has seven science fictions in her choices, five mystery books, and two journals. So among all these she can choose precisely one. So the number of choices sums up to be seven plus five plus two. It is very obvious to see why we do a plus here because she wants to read precisely one book. So Alice has 14 choices to make.

Let us see another example. In the town of Germany there are eight daily newspapers and four weekly magazines printed. Peter wants to subscribe to exactly one newspaper and one magazine. How many choices does he have? So Peter wants a newspaper and a magazine. I am stressing upon the word and here because he wants a newspaper and a magazine. So he has eight newspapers and four magazines to choose from. And therefore, it is 8 multiplied with 4 which is 32 choices and out of these 32 choices he is going to pick a newspaper and a magazine.

Let me just change the question here slightly. Suppose Peter wants a newspaper or a magazine. You see the word and became or. Here he would have eight plus four choices which is 12.

Let us move on to another example where in a society there are eight men and six women contesting in an election. How many ways can the people choose a leader out of these. So there are eight men and six women. So the people have 8 plus 6 that is 14 ways to choose from because they wanted just one leader and hence it is a plus operation here. What if the people wanted two leaders, a man and a women? I hope it would be very obvious to see that it would be 8 into 6 which is 48 choices. If they want precisely one leader then it is 14 choices and if they want two leaders a man and a women then it is 48 choices.

Let us move on to another question. How many positive divisors does 2000 have? Before going to the answer let me make it very clear what is a divisor. A divisor is an integer which divides another number. With examples it will be more clear. Consider the number 4; 2 divides 4, 1 divides 4, 3 does not divide 4, and 4 also divides 4. So you see these numbers 1, 2 & 4 are the divisors of 4 and 3 is not a divisor of 4 because it does not divide 4. 5 divides 10, hence it is a divisor of 10. Divisors are also called as factors.

So let us get back to the question. How many positive divisors does 2000 have? Let me write 2000 as a product of its prime factors. So 2000 is equal to 2 into 2 into 2 into 2 into 5 into 5 into 5. Yes. So this is equal to 2 to the 4 into 5 cube. So this is same as 2000.

So we see that divisor of 2000 must be of the form 2 to a into 5 to the b. Now you see these a and b they must be integers where a lies between 0 to 4 and b lies between 0 to 3. So a and b lie in a particular range.

Now how many divisors do we get? We can answer this question by checking the possibilities for a and b. So you see that a has 5 possibilities; 0, 1, 2, 3, and 4 and b has 4 possibilities that is 0, 1, 2, and 3. Supposing a takes the value 0 so 2 to the 0 will be 1, then b can take any value from 0 to 3, 5, 25, 125 and also zero. So all these are factors of a divisor of 2,000 and if b is 0 then this becomes 1 and a can take the values between 0 to 4. Let me just write some divisors 2000. 1, 2, 10, 25, 125, and so on.

So we have 5 possibilities for a and 4 possibilities for billion. So how many possibilities are there for a and billion, it is precisely 5 into 4 which is 20 and this answers the question that there are 20 positive divisors for 2000.

Let us now see another example. Charlie visits an ice cream parlor to buy one and he sees that there are three types of cones, four flavors of ice creams and two toppings. So how many choices does Charlie have to buy an ice cream? Let me enumerate all of them. So let this represent Charlie. So he wants to buy an ice cream. He has three choices of cones and I'll write them as C1, C2 and C3. So for each cone he has four choices for the ice cream flavors like this and for

each flavor there can be two toppings. So this represents toppings and this represents flavors. So for this cone he can choose this scoop and this topping and so on. It continues for the others as well.

So how many choices does Charlie have? He can choose between three cones, four flavors and two toppings. So you see he must choose one each among the three. So we have by the rule of product as 3 into 4 into 2 which is 24 choices. So he has 24 possibilities of choosing an ice cream.

Let's move on to the question. In how many ways can we draw a face card from a deck. As we all know in a deck of 52 cards there are four suits; the clubs, diamonds, hearts, and the spades. And the three face cards are J, Q, and K. So for each of them we have four like this. So these are four. These are four, and these are four. And we want to draw just one face card among all of these and so there are four plus four plus four choices which comes up to twelve. So in twelve is we can choose a face card from a deck.

So let me now sum up what we were discussing all these while that is the rule of sum and the rule of product. Rule of sum goes like this if there are n choices for one event and m choices for another event and both cannot be done at the same time then there are n plus m choices for one event. And rule of product goes like this, if there are n choices of one event and m choices of another event then there are n into m choices for both these events to occur.

In the previous case when we were discussing the sum only one event can occur and hence it is a plus. In the second one we want both of the events to occur and hence it's the product. So this goes with the rule of sum and the rule of product.

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