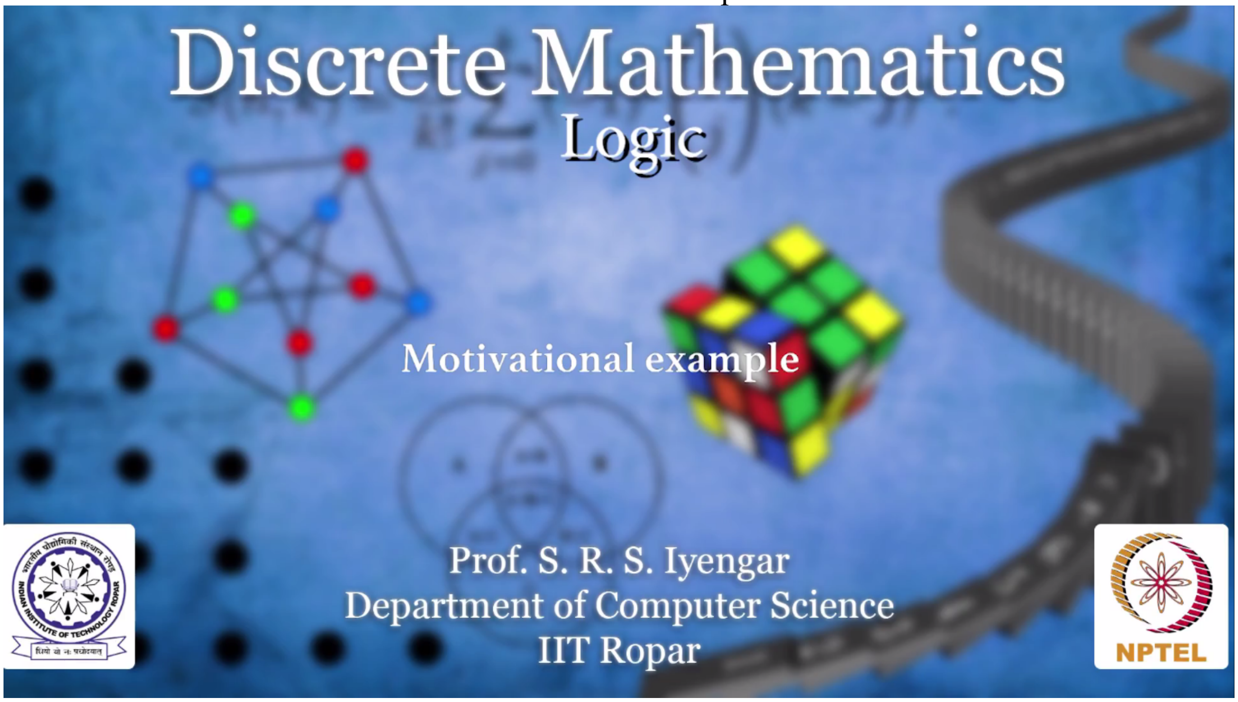
NPTEL
NPTEL ONLINE COURSE
Discrete Mathematics
Logic
Motivational example

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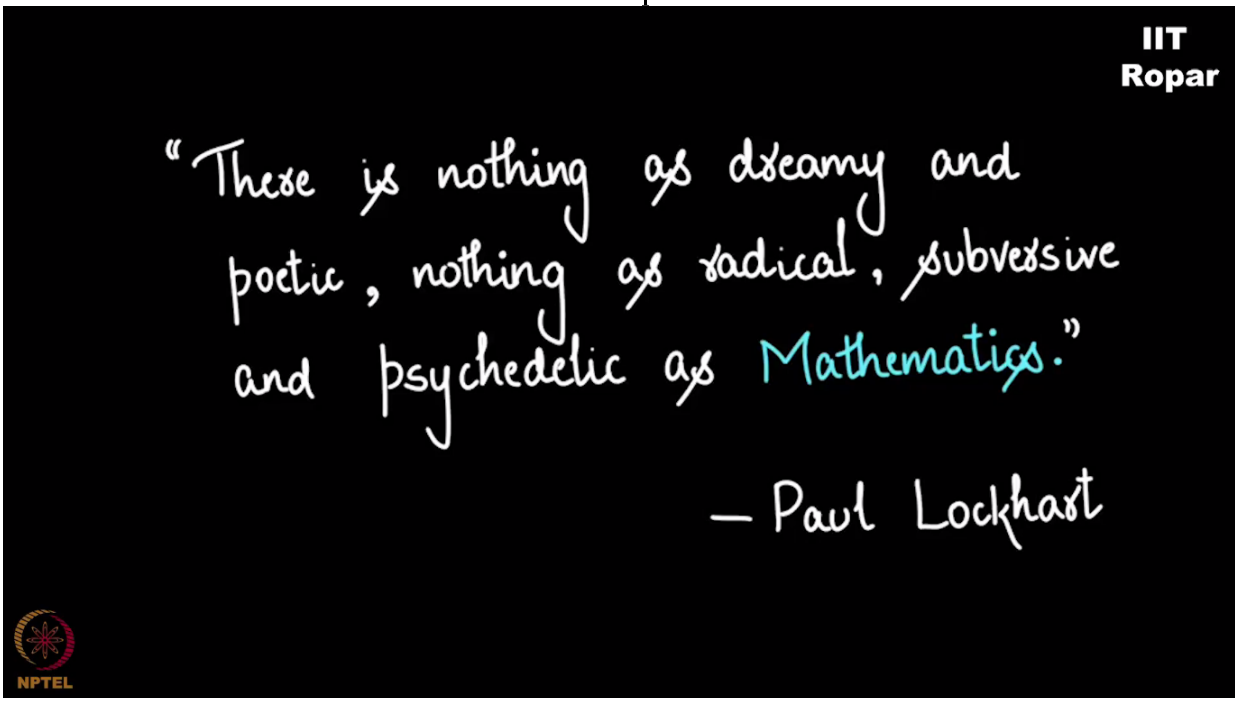
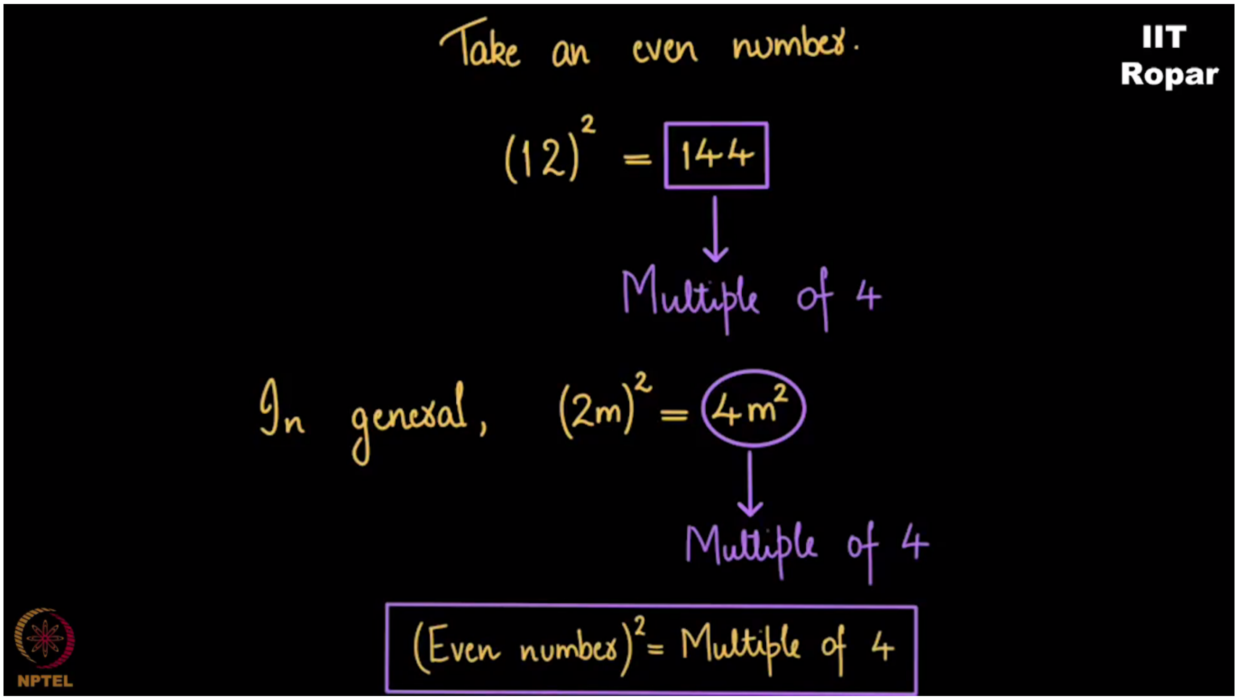
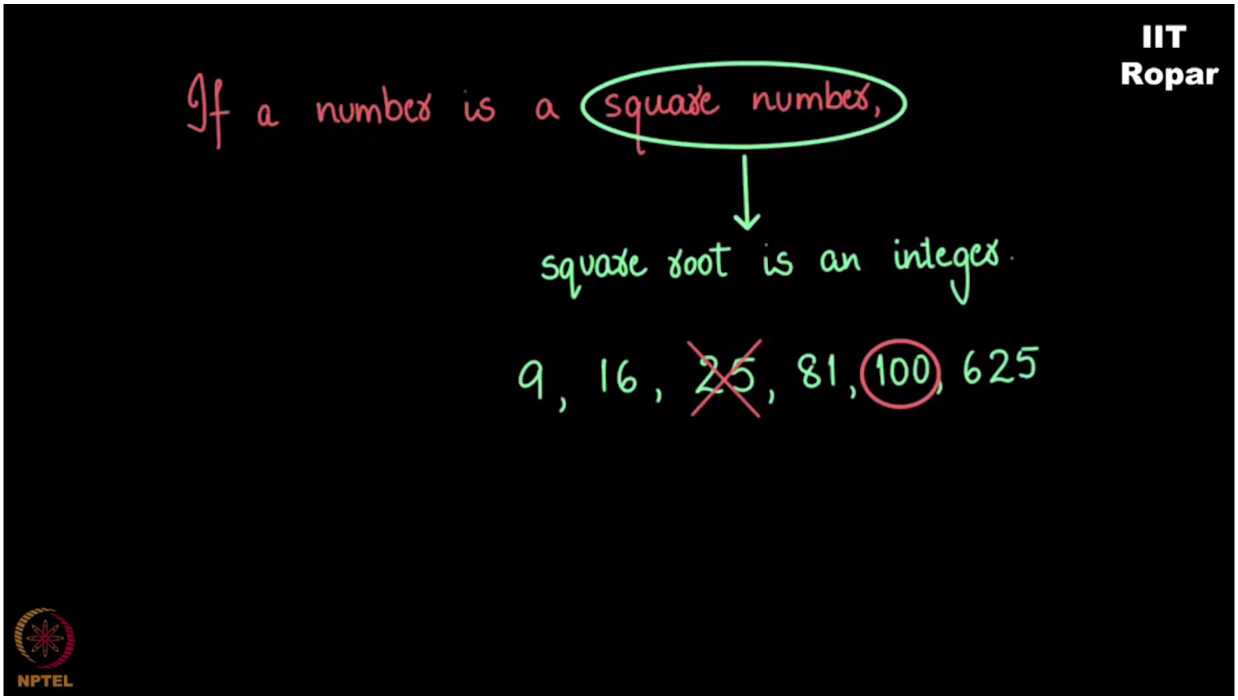


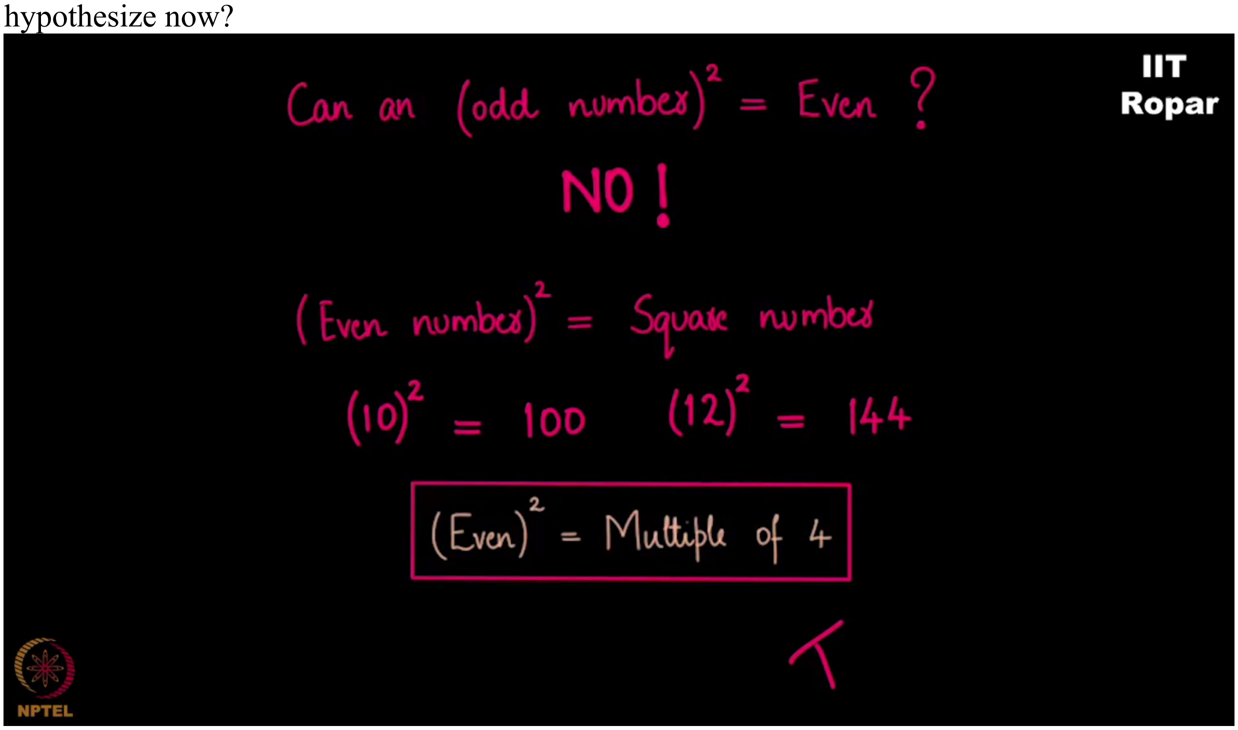
Let me start with a nice story. The story is in different pieces. We will put together all the pieces in a while. So the first piece is you take any number and even number. Let's say 12. And then square it. What you get? 144. Note that 144 is a multiple of 4. Okay. In general when you take an even number it's in the form 2m and when you square it it becomes the form 4m2. You observe that it is always a multiple of 4. So what did we observe? Even square is a multiple of 4.




If a number is a square number what do you mean by square number? A number whose square root is an integer. You see 9 is a square number. 16 is a square number. 25 a square number. 81 a square number. 100 a square number. 625 being a square of 25 is a square number. You take any square number that is E1. So you see 25 doesn't qualify. I want a square number that is E1 , let's say 100 is E1. Whenever you pick a E1 square it's a multiple of 4. Why is that? Can you think about it? Observe.




Can an odd number square become even? No, which means whenever you have a square number it should be obtained by squaring any one number. Correct? So 100 you obtained 100 by squaring 10 which is an even number. You obtained 144 by squaring 12 which was an even number. So whenever you have a E1 square you got it by squaring an even number and when you square an even number it gives you a multiple of 4. So what did we observe? What can be hypothesize now?



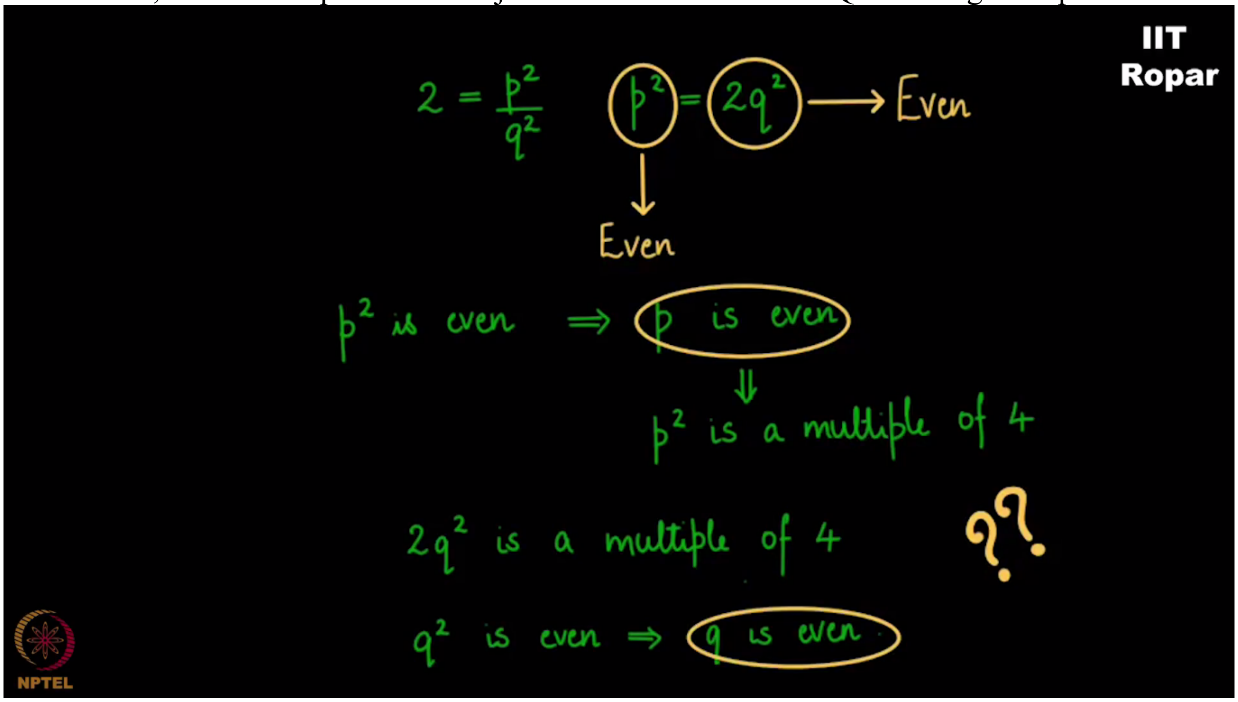
We can say that whenever you spot a even square it is always a multiple of 4. Think about it. Let us now take this number square root of 2 and show that it is irrational. Most of you know the proof of this I believe. But for the sake of being self-contained in this course, let me tell you the proof of root 2 being irrational. Now let me assume square root of 2 is rational. By that I mean it can be written as P/Q which means 2 is equal to P square by Q square.

Now please note whenever I write root 2 is equal to P/Q it means P and Q are in its simplest possible form. For example, 12/16 is same as 3/4. 3/4 is called the simplistic form. 12/16 has things in common so you should cancel things off and then write it in its simplest form namely 3/4. The idea is write in such a way that they do not have any common multiples, common divisors.

Okay. so I write P and Q obviously if 2 is a multiple of P Q cannot be a multiple of P. Note this. If it were to be a multiple of P then you would cancel 2 off. For example, if you had something like 60/70 you would write this as 30/35 and then again 30/35 gets simplified as 5 six times and 5 seven times this gives you 6/7. So I can assume that P and Q are what is called relatively prime. Their GCD is 1. Don't break your head. All I am saying is both P and Q cannot be a multiple of 2.

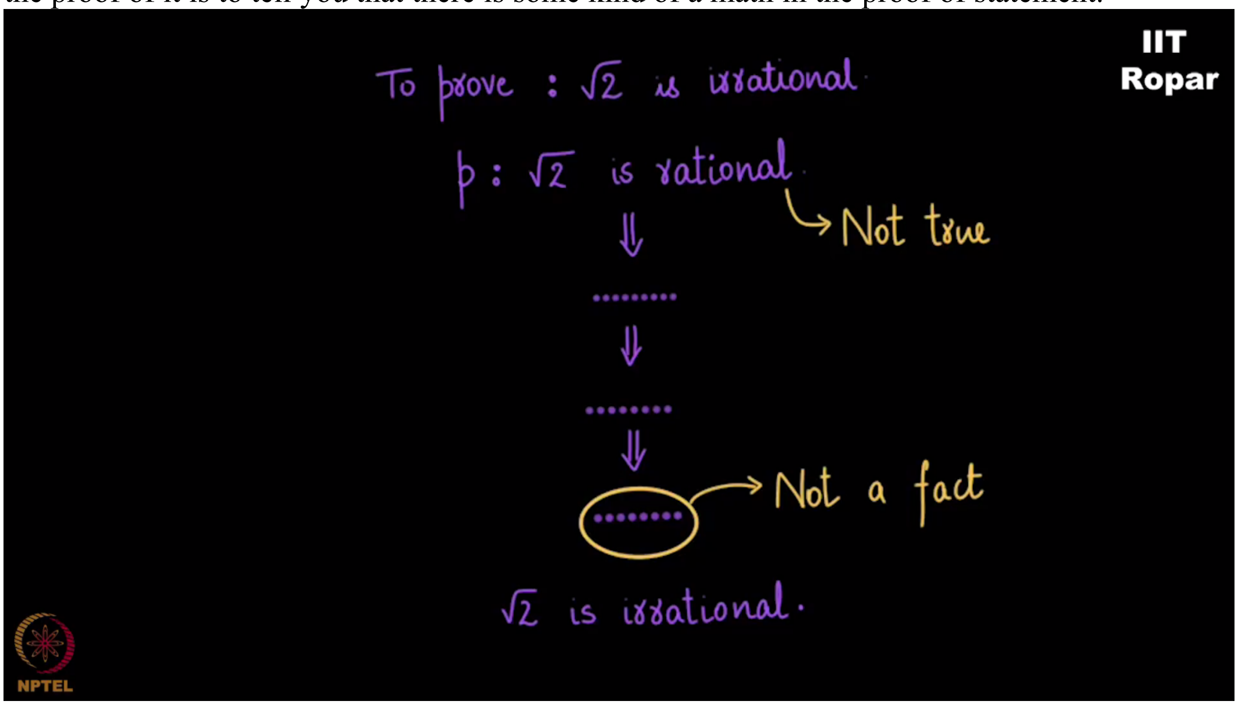
Now 2 is P square into Q square which implies P square is two times Q square. You see 2Q square is even so P square is even. P square is even implies P is even. Why? We just saw. Whenever a square number is even. Then it's – that number whose square is this number is even. 100 is even implies 10 was even. 144 is even implies 12 is even. Correct? It's only obvious. We saw the proof. So P square is even implies P is even and you see when P is even P square is a multiple of 4. Correct? We even saw that. P is even. Square is an even number. You get a multiple of 4 which means 2Q square is a multiple of 4 which means Q square is even. Why is that? 2 times something is a multiple of 4 means that something should be a multiple of 2. Correct? Q square is even implies Q is even.

Now round this off Q is even, P is even which means P is a multiple of 2 and Q is a multiple of 2. Come on, is this even possible? We just started off with P and Q not being multiples of 2.



Both cannot be multiples of 2. Correct? That's who we started. So what just happened here? To show the square root of 2 is irrational to prove this statement we assume square root of 2 to be rational. Let me call that statement as P root 2 is rational and this implied something implied something implied something and finally we arrive at a fact that was not a fact. That was some weirdity. If P implies weirdity then P should not be true, correct, which means P which is root 2 is rational is not true which means root 2 is irrational.

So this was the story which I thought I will tell you all. Observe this story closely. You would have seen this proof in your high school days or even before. It's a very standard proof. You will see it in many books. It's also sort of fun math that many people like teaching and learning. Root 2 is irrational. So the point we are trying to make in teaching you why root 2 is irrational the proof of it is to tell you that there is some kind of a math in the proof of statement.




So understand carefully, the way we proved that root 2 is irrational is by using some math. Correct? The usage of this math has what is called logical deduction. Logical deduction. So how is it done, what is all about, what is logic, how do you deduce a piece of truth is what makes this chapter called mathematical logic.

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