NPTEL

NPTEL ONLINE COURSE

Discrete Mathematics
Advanced Topics

Summary

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Well, we have come to the fag end of the course. Till now we have completed some 10 good chapters, let us now finish the course by seeing a short summary of all the topics.

Week one, we had seen the chapter let us count, we started it this way
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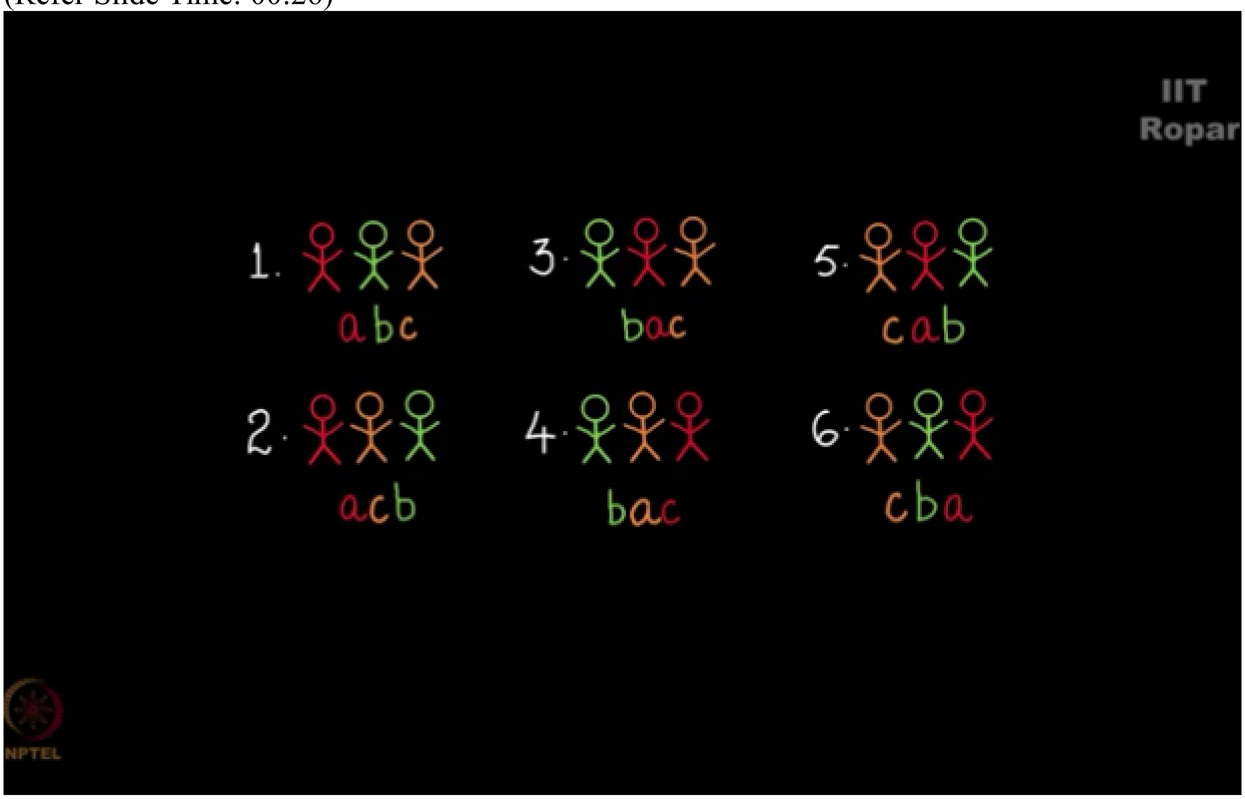

we took 3 people and we have seen in how many ways can they take photographs by all possible positions swapping, right, so here are all the 6 possibilities of this 3 people A, B and C.
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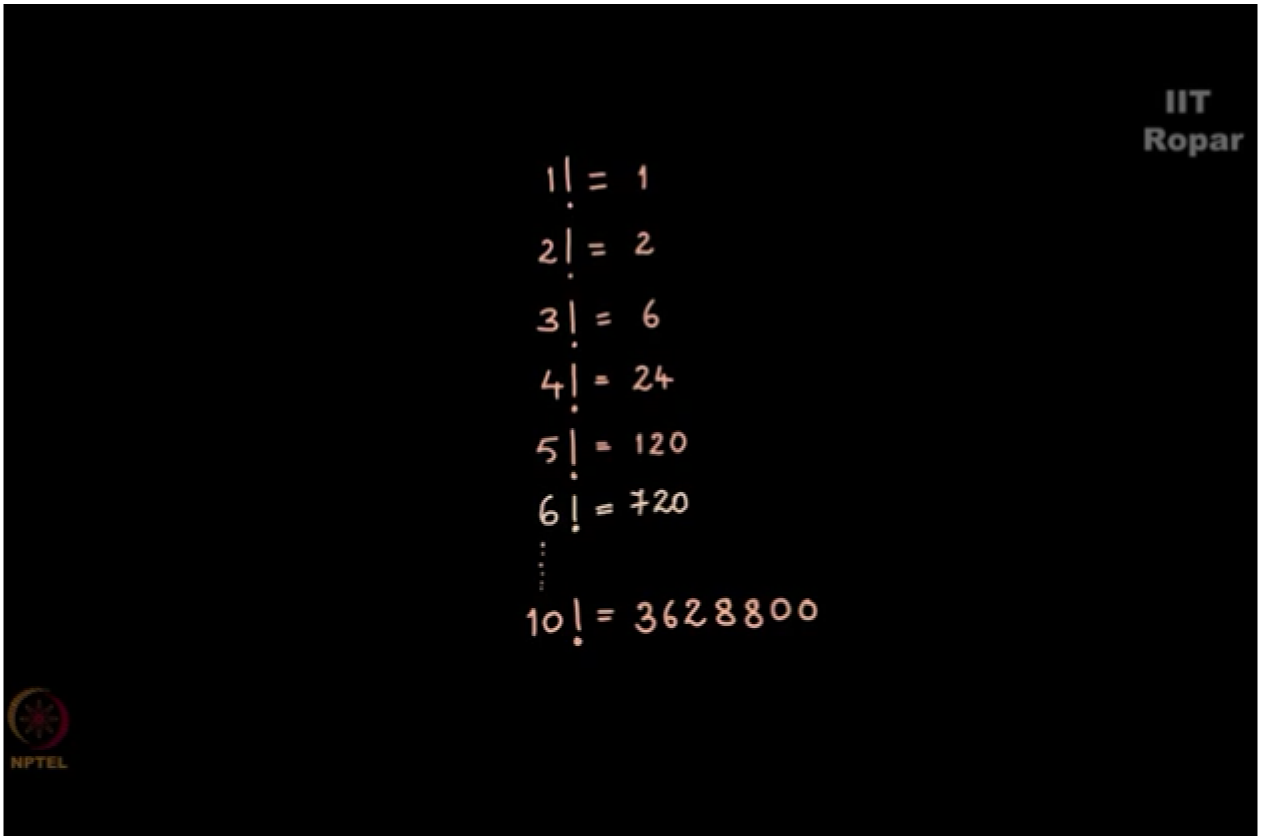

Now we then moved on to seeing what is factorial? What do we mean by factorial? The photographs example was a good motivation to learn more about factorial, 1 factorial, 2 factorial, 3 factorial so on, you see 10 factorial was a huge number, we saw that 15 factorial, 20 factorial so on end up being called as astronomical numbers, because they are quite huge, they are called as astronomical numbers.

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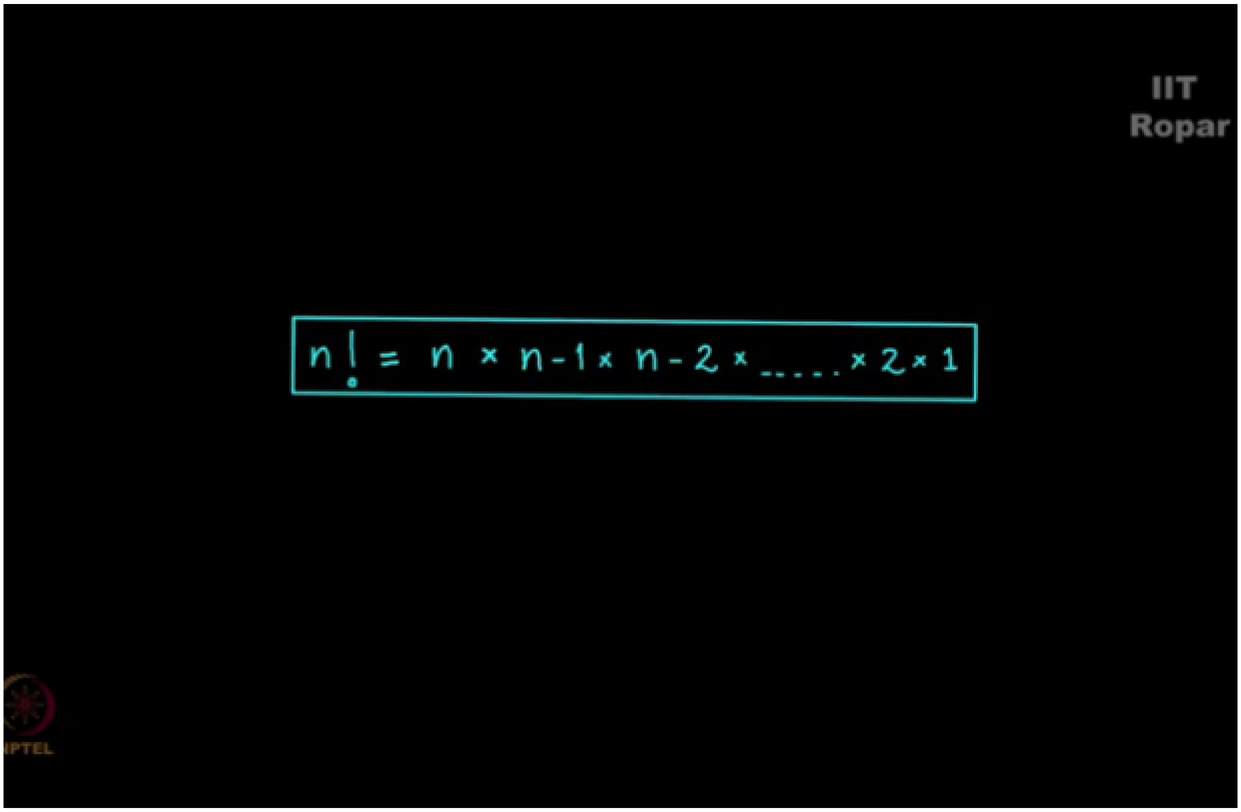

We saw what is N factorial, it is given by N x N-1 x N-2 so on up to 3 x 2 x 1.

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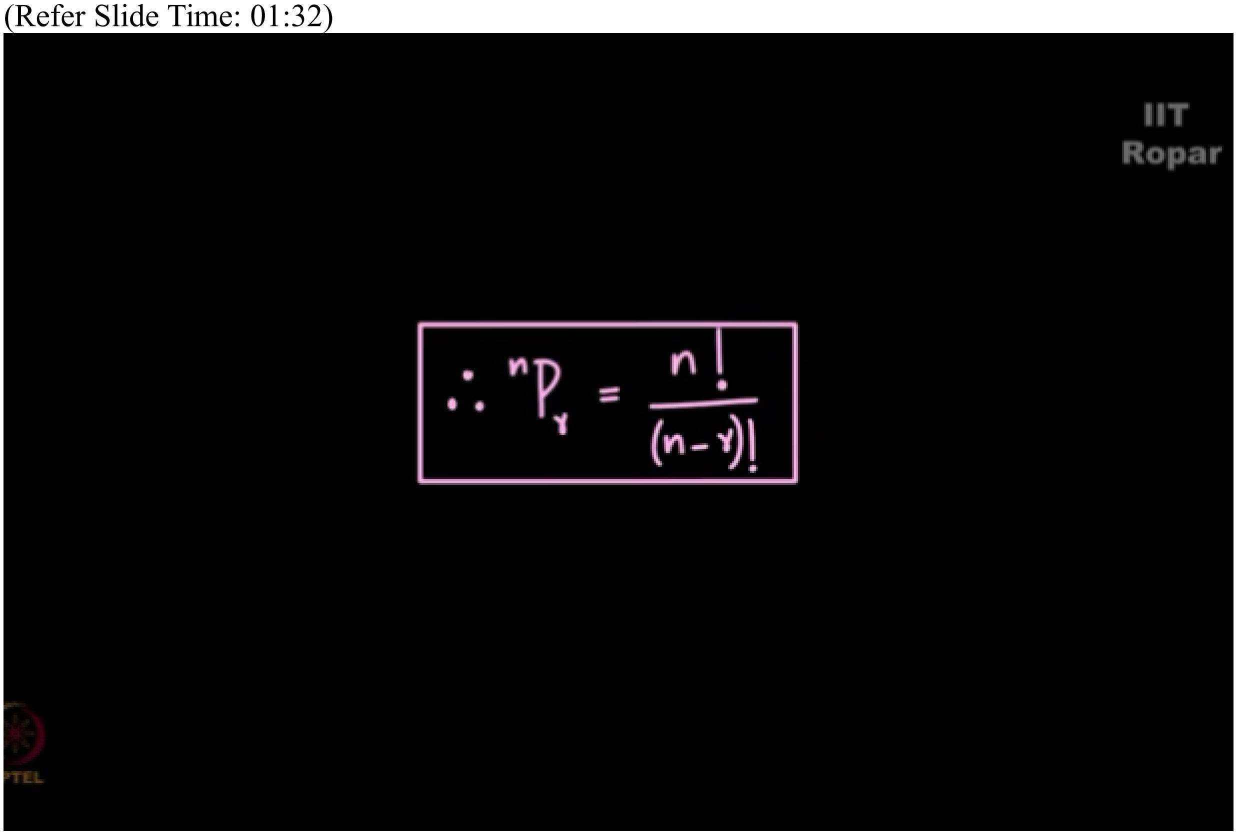

Well, having this as a basis for our learning we moved ahead and learnt what is called permutations. The formula of permutations goes like this NPR is N factorial/N-R factorial, we have solved several problems using this formula.

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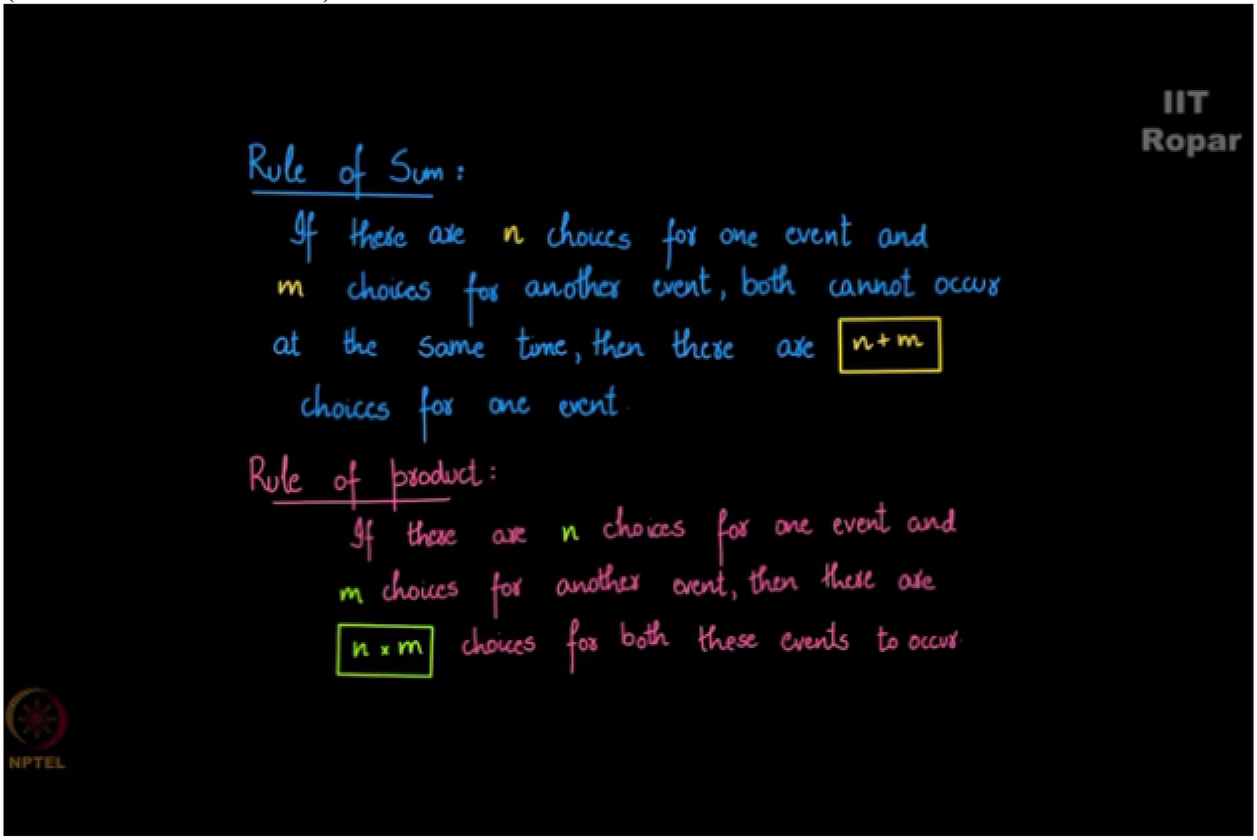

Before even heading to permutations we had also seen the rule of sum and rule of product, rule of sum goes like this, if there are N choices for one event and M choices for another event, both cannot occur at the same time, then there are N+M choices for the event to occur.

And the rule of product is if there are N choices for one event and M choices for another event, then there are N x M choices for both these events to occur, you see in rule of sum we spoke about one particular thing, one event and here we are speaking of two events occurring together, that is the difference between rule of sum and rule of product. If you remember the pizzas and the burgers of animations what we had done in week 1, then rule of sum and rule of product is really clear to you.

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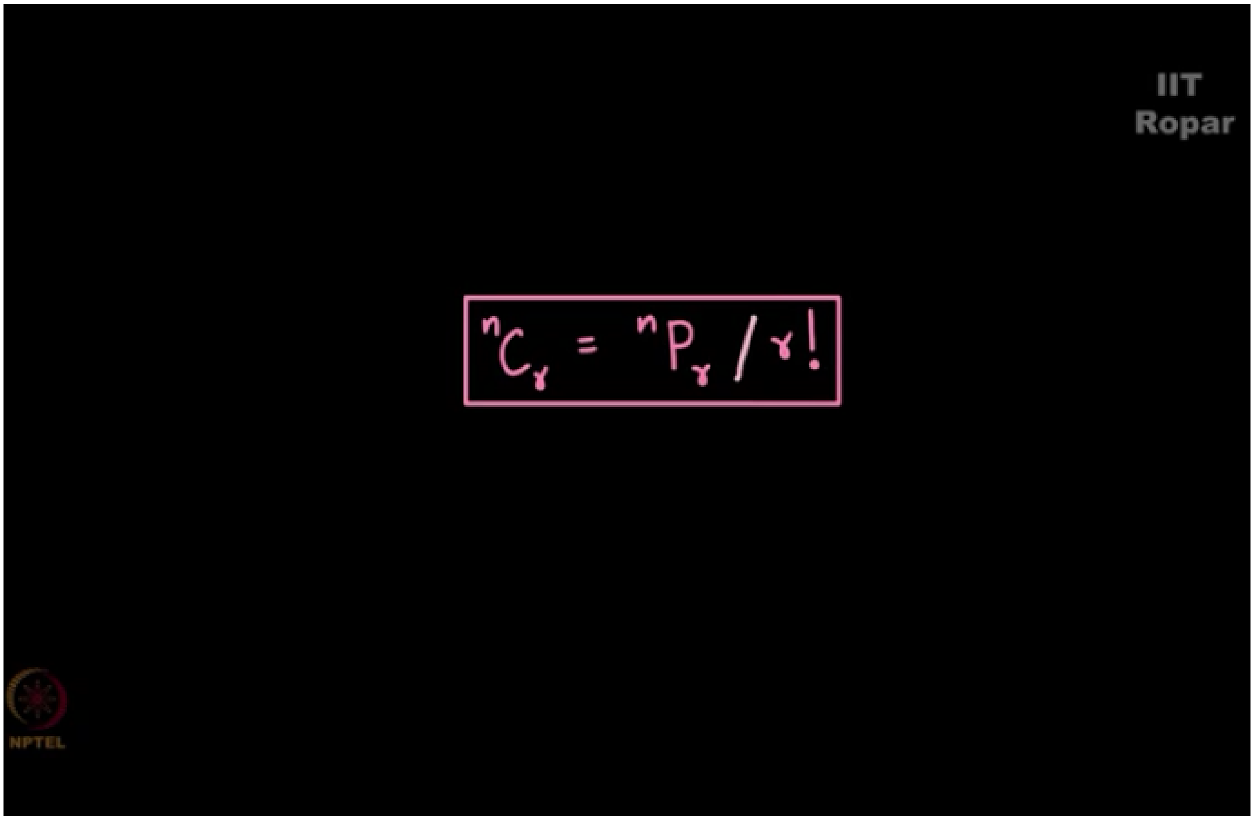

We then learnt what is called the combinations, if you remember the photographs of 5 different girls, the video of it you must have understood this concept called combinations, if they really do not worry about their positions then it follows what is called combinations, the formula for it is NCR is NPR/R factorial, well NCR is N factorial/ R factorial x N-R factorial and this is the relation between NCR and NPR, it is NCR is NPR/R factorial.

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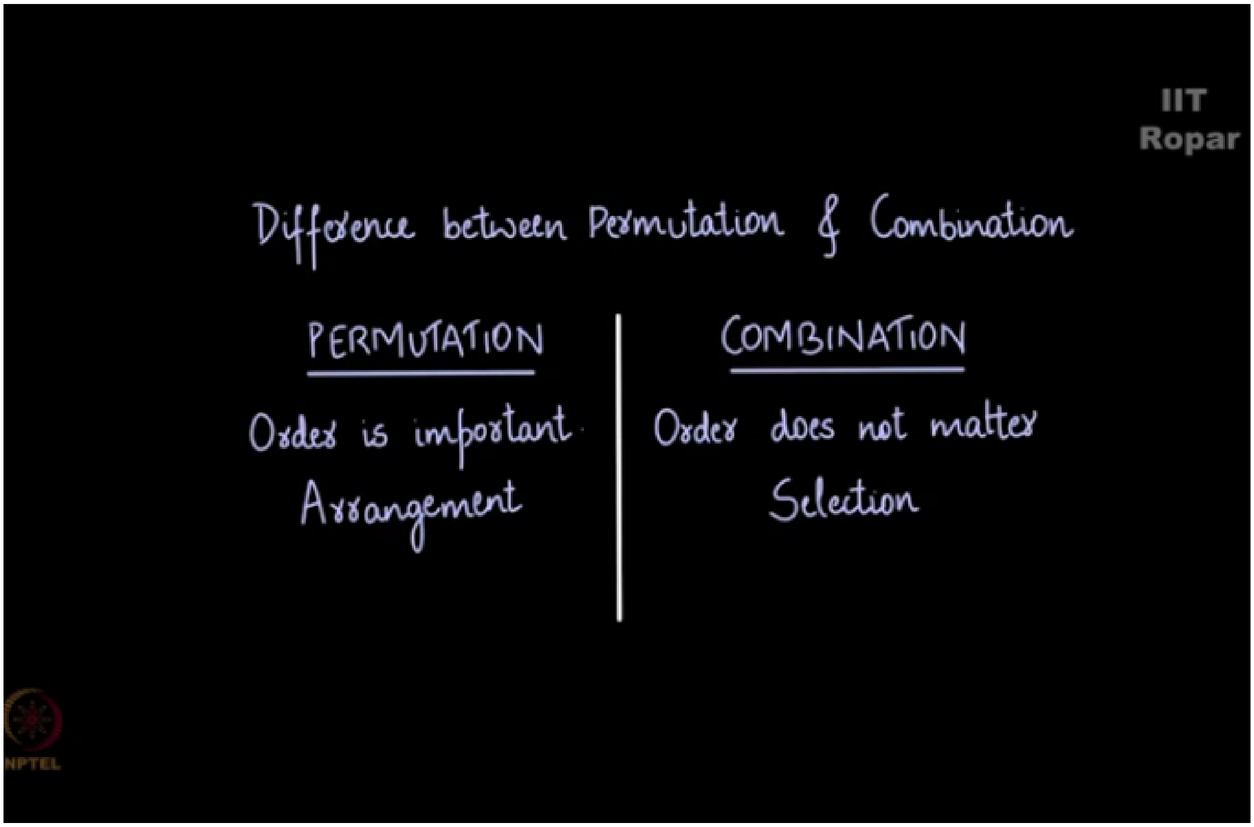

Well, we then saw the difference between permutations and combinations, permutations when we speak of it we speak of arrangements, when we speak of combinations it is mostly selections, and in permutations order is important, if you remember the video of the girl changing their positions and taking photographs they were worried more about the order, and hence there were so many pictures of them.

Whereas when we speak of combinations order really does not matter, right, and if you remember the video of combinations they were not much worried about the order, and hence there were less number of photographs in the video of combinations you might want to pause here and go back to week 1 and watch the videos once again.

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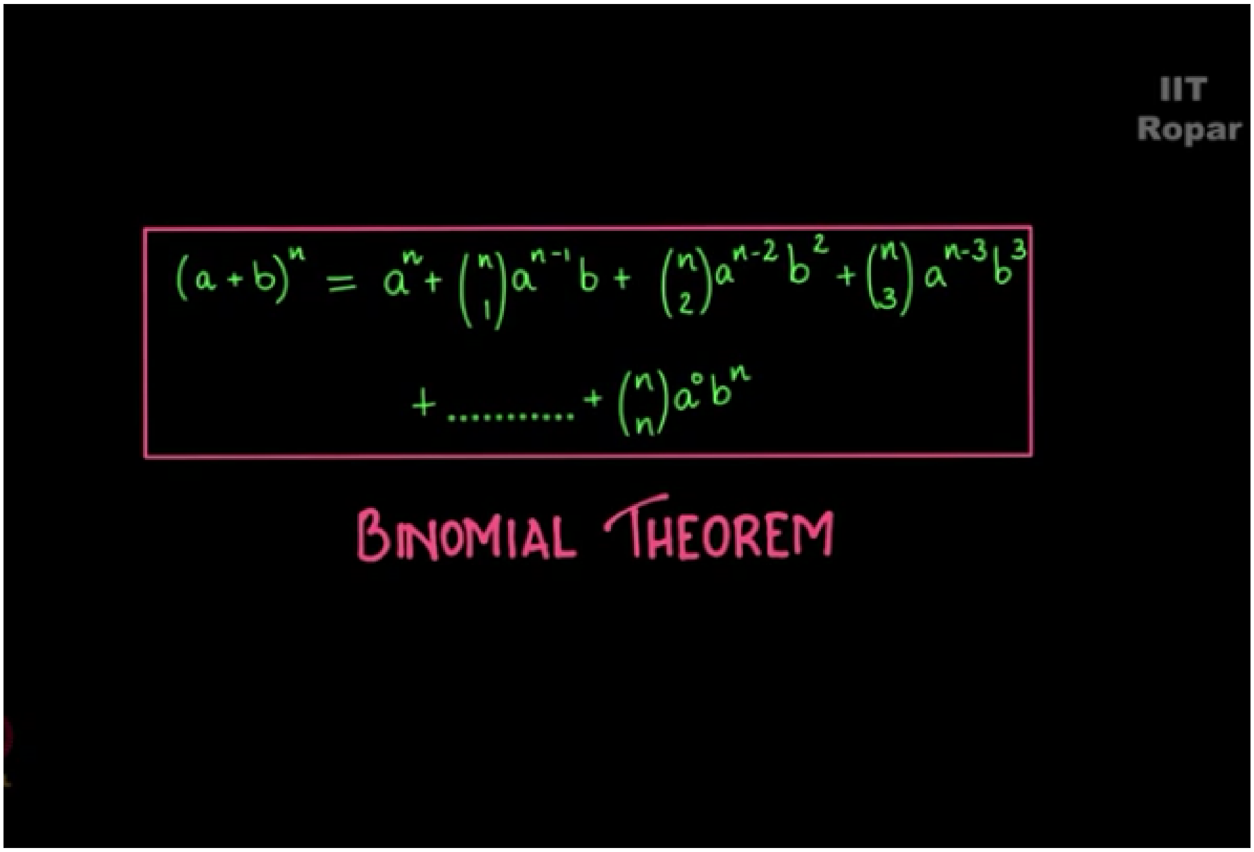

We then moved on to what is called the binomial theorem, how do we get this? We started off from A+B the whole square then we moved to A+B the whole cube so on, A+B whole to the 5 so on, A+B whole to the 10 and then we generalized to A+B whole to the N, it is A to the N + N choose 1 x N, A to the N-1 x B + N choose 2, A to the N-2 x B square so on, + N choose N A to the 0 B to the N, so this formula is called as the binomial theorem.

By now we had after covering all these topics, factorial, permutation, combination, binomial theorem we had seen several problems. The generalized version of binomial theorem is called the multinomial theorem.

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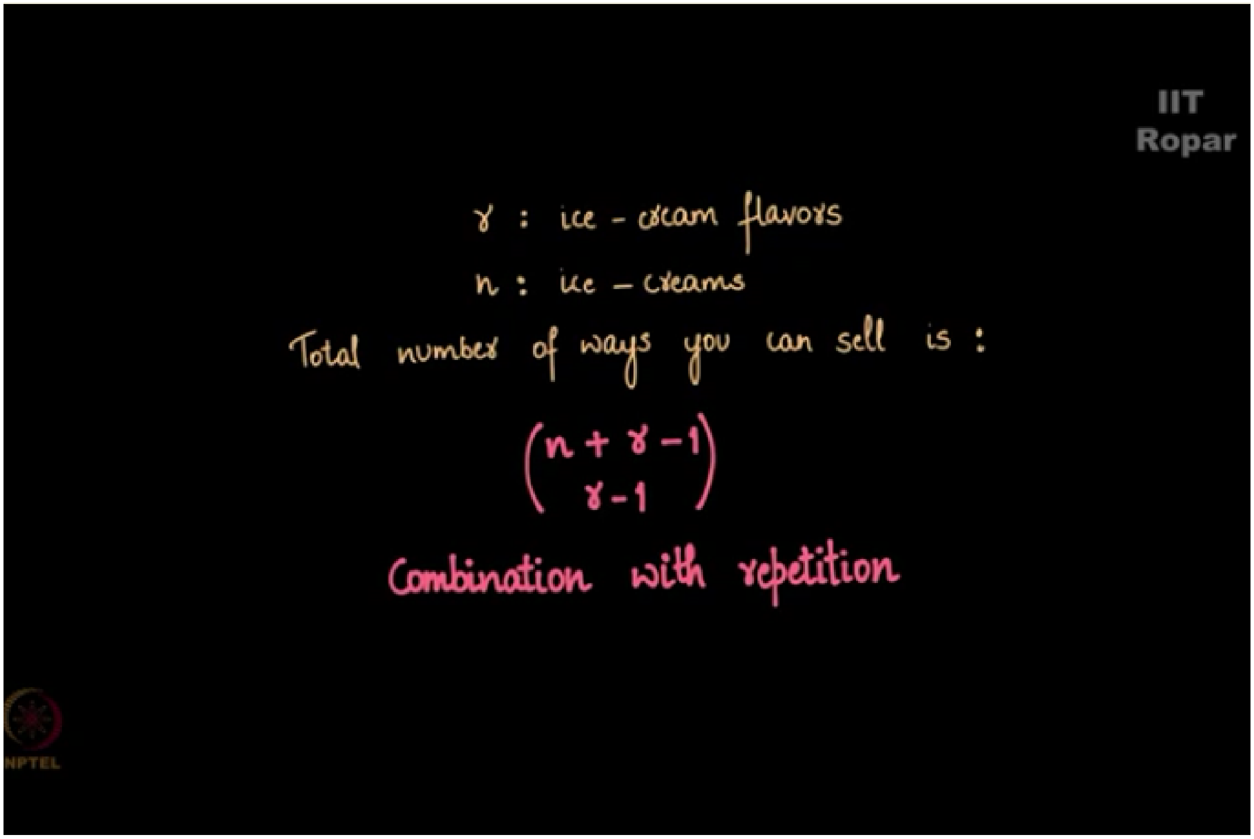

And then came the problem of choosing these ice-creams flavors, if there are R ice-cream flavors and the person wanted to sell an ice-creams and in how many ways could he sell it if you remember this video then the answer was given by N+R -1 choose R-1 and this goes by the name combination with repetition.

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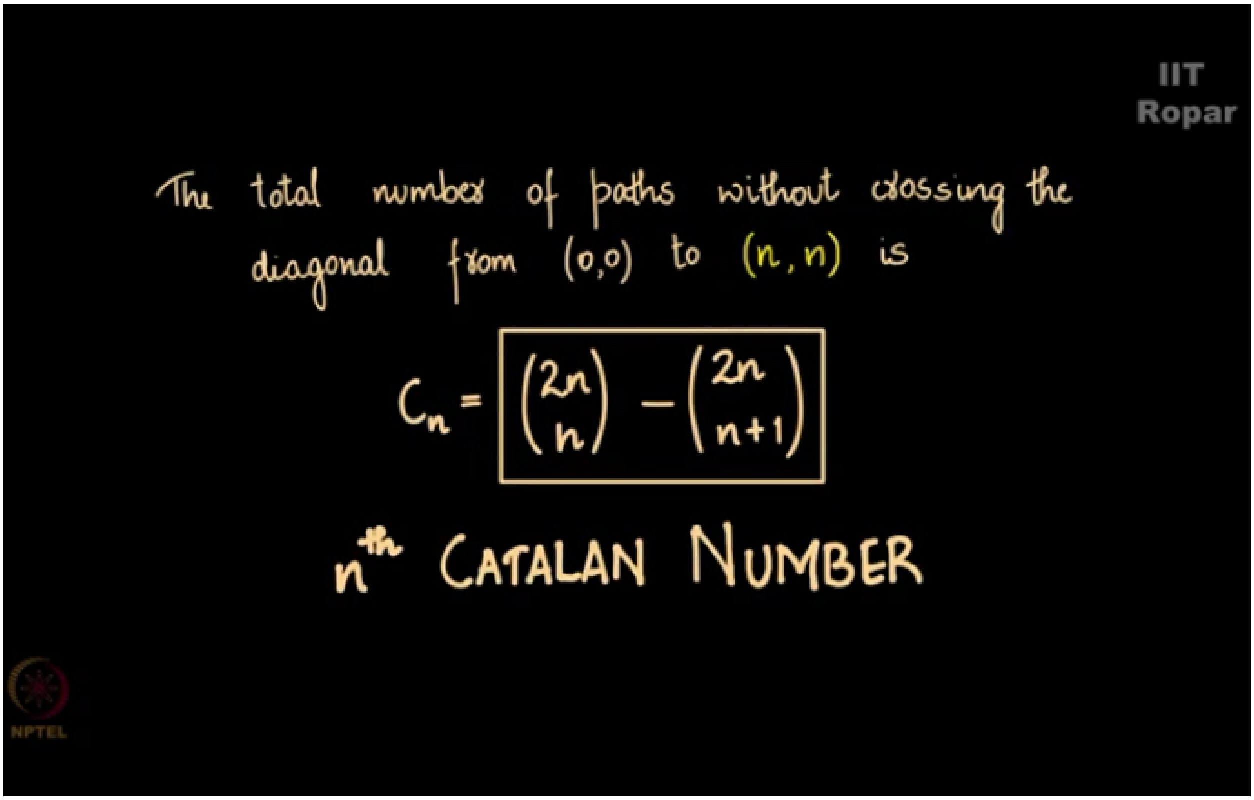

We had also seen a very important concept called the Catalan numbers, how did we start? We started by asking how many parts are there from 0, 0 to 5, 5 without crossing the diagonal. We even calculated manually and got a huge number, we then generalized to any point from 0, 0 to N, N and we see that it goes by this formula CN is 2N choose N – 2N choose N+1, this is called the nth Catalan number. We also saw the series of this N Catalan, of this Catalan numbers and we have seen several applications of Catalan numbers like handshakes across the table, the number of binary trees and so on.

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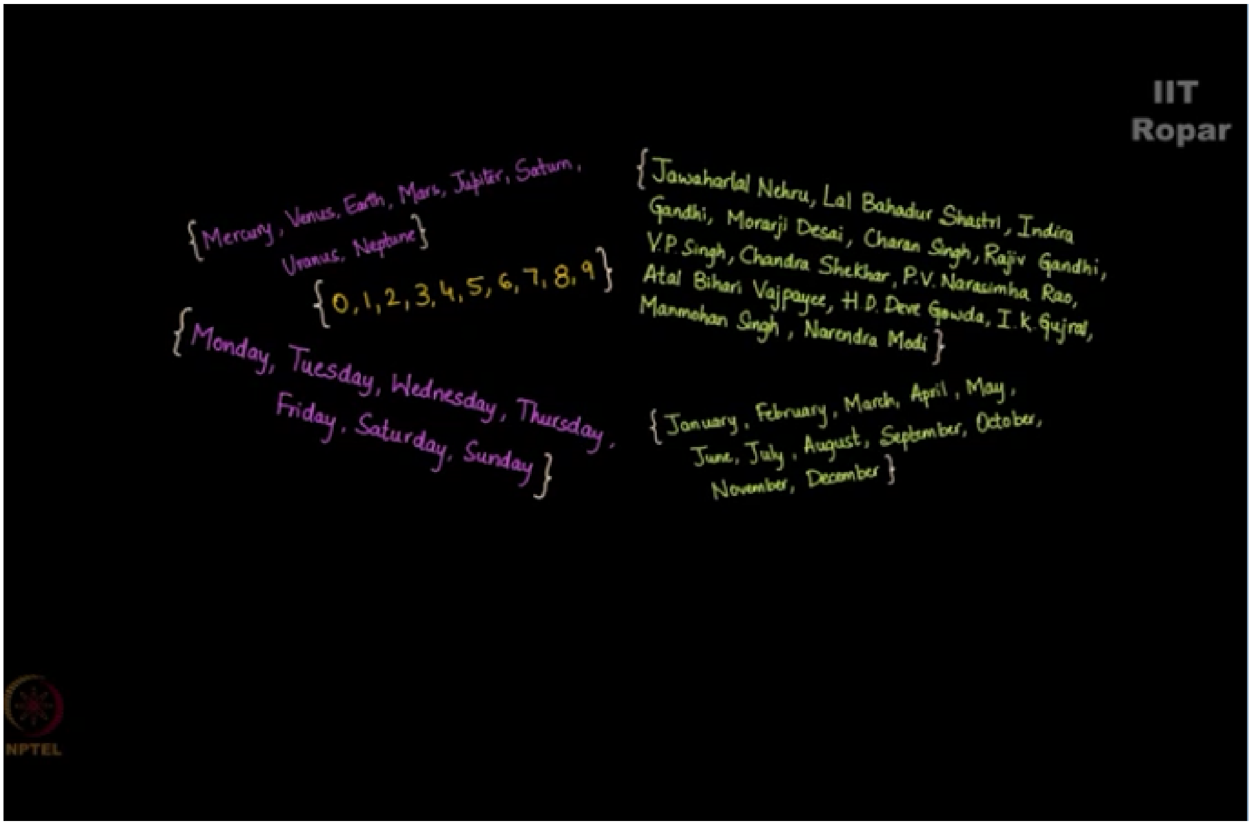

The parenthesis also was one among those examples, so this was with the counting chapter. Well in week 2 we started with some examples of set theory, then we saw what is the definition of set, it is a collection of a few objects, then we moved on to observe what is union and intersection, right, union if an element belongs to A union B,

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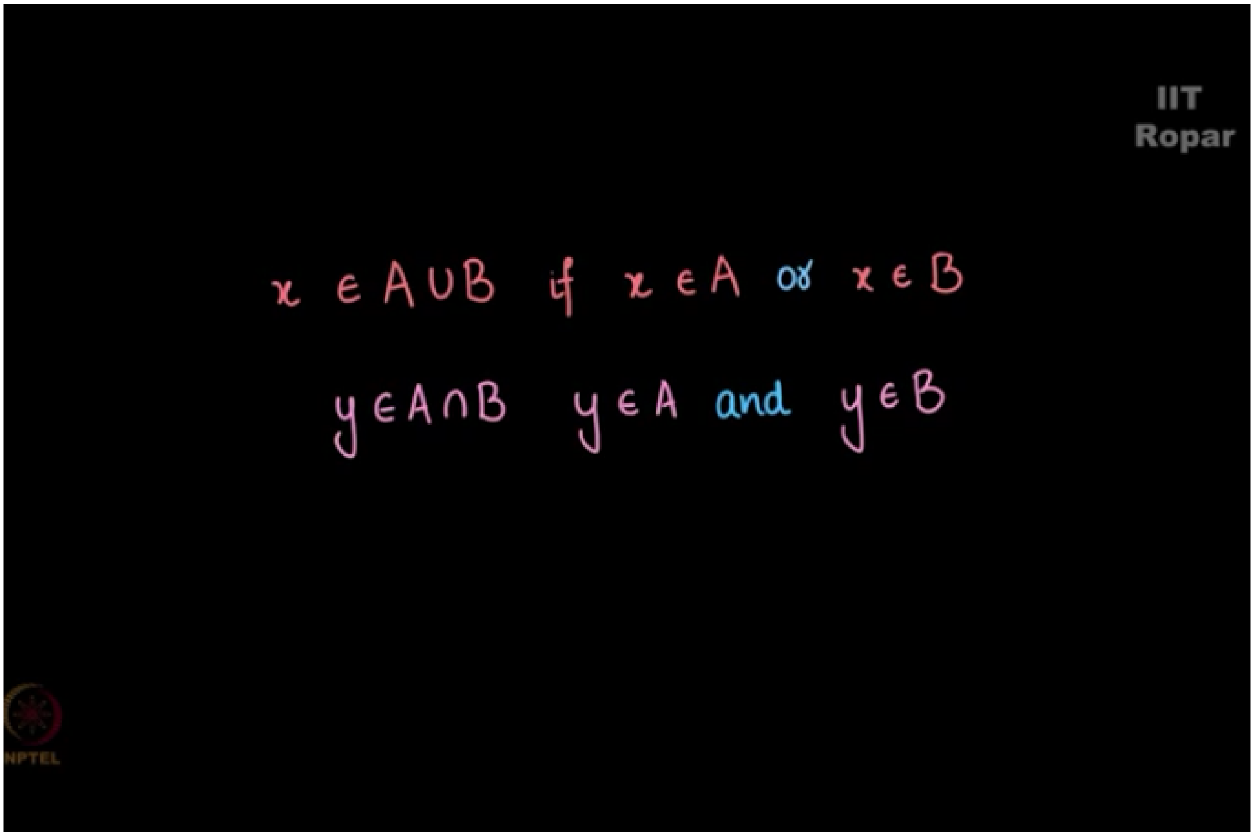

then it means that either it belongs to A or it belongs to B, and if it belongs to intersection it means it belongs to both the sets, so we saw these two definitions and we solved several problems based on this and then we have seen what is the cardinality of A union B,

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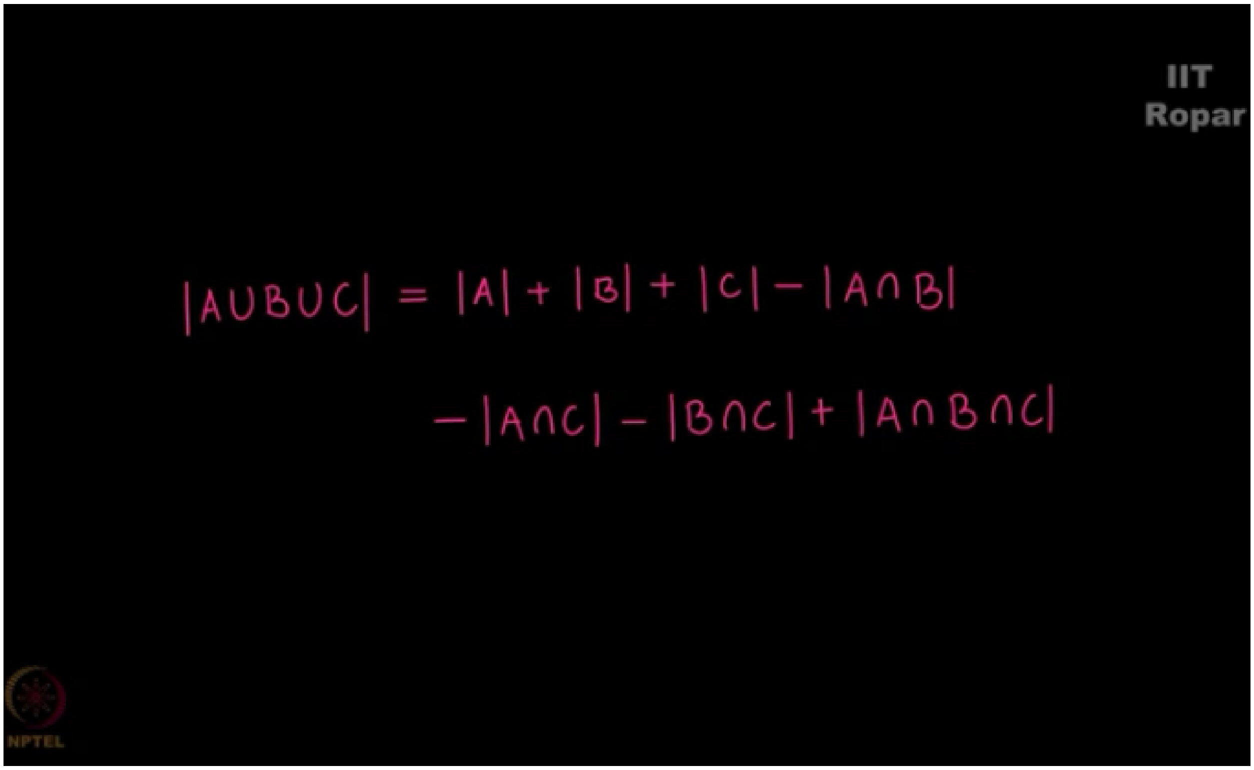

it is cardinality of A + cardinality of B – cardinality of A intersection B and then we have seen the same formula for 3 sets, cardinality of A union B union C is cardinality of A + cardinality of B + cardinality of C – A intersection B – cardinality of A intersection B – cardinality of B intersection C + cardinality of A intersection B intersection C.

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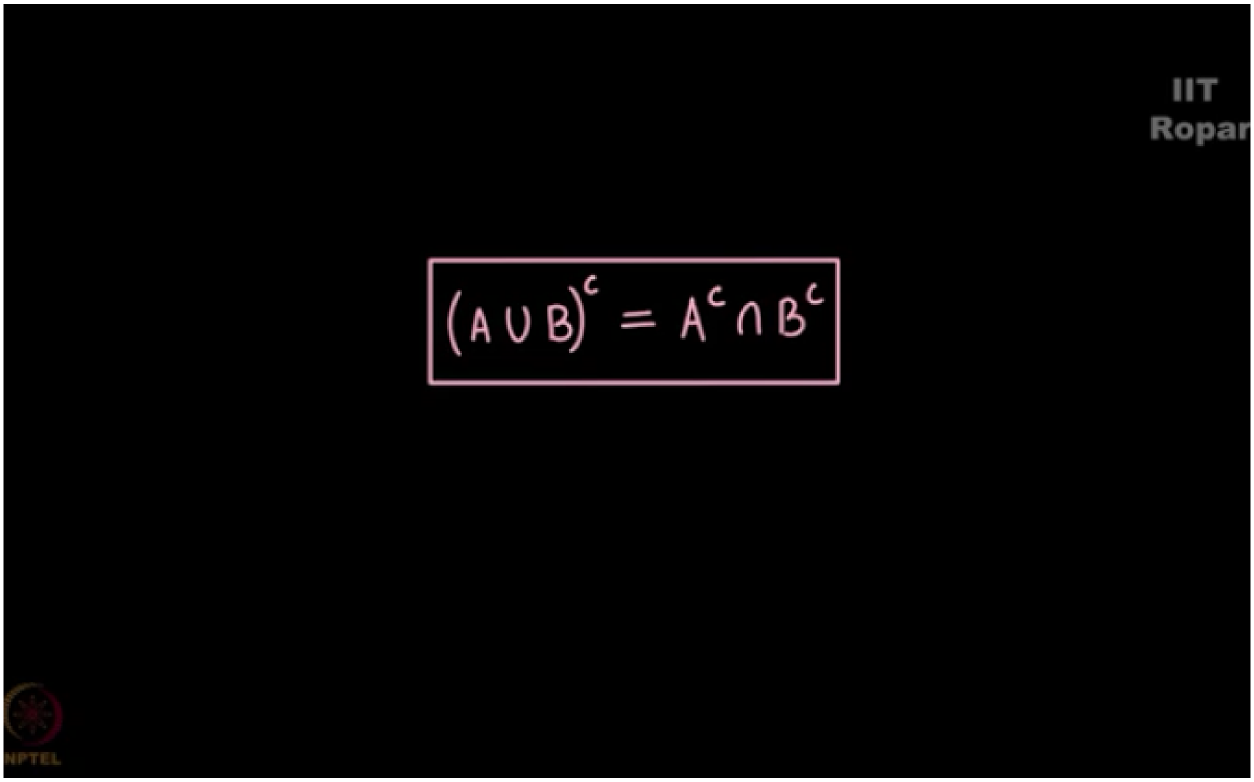

We have seen these two formulas and we have solved some problems. Well, what comes next is the De Morgan's laws we saw that A union B whole complement is A complement intersection B complement, and this is the set difference, A–B is A intersection B complement,

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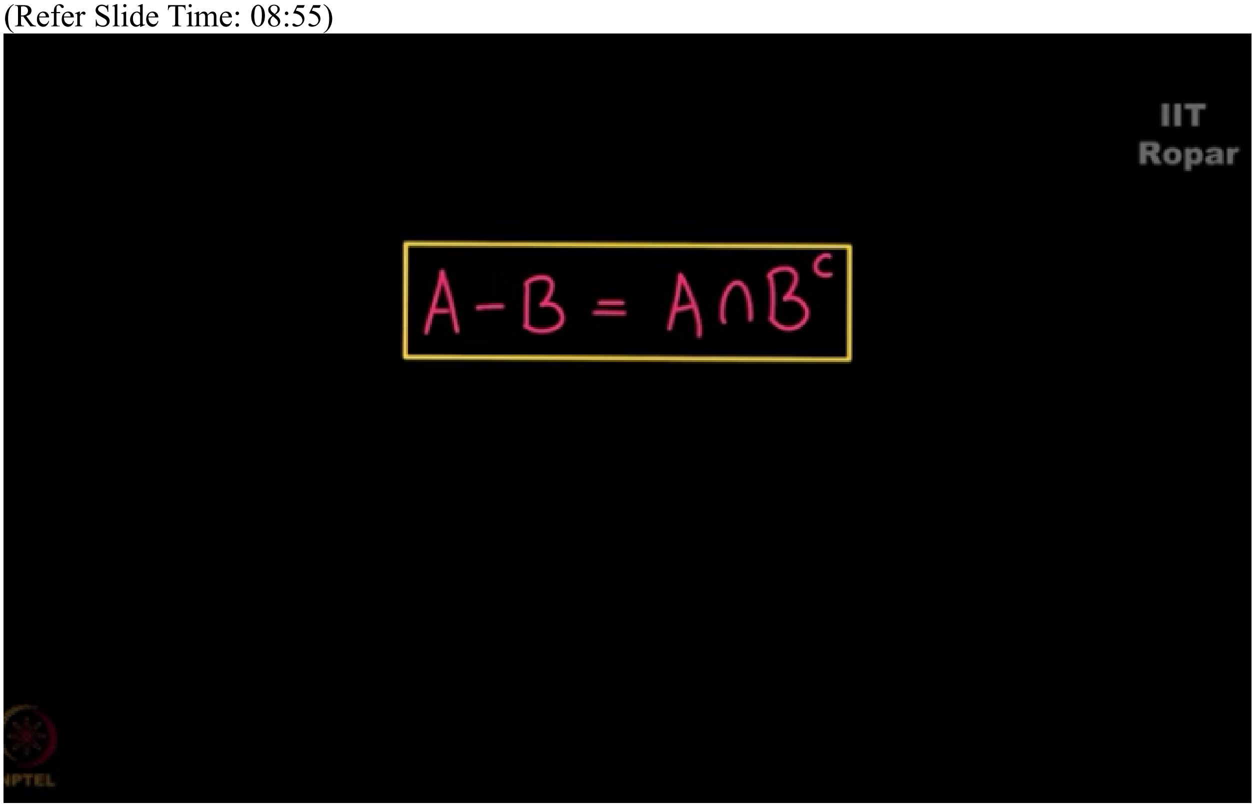

you can verify all this using Venn diagrams. We ended the chapter with the topic of symmetric difference.
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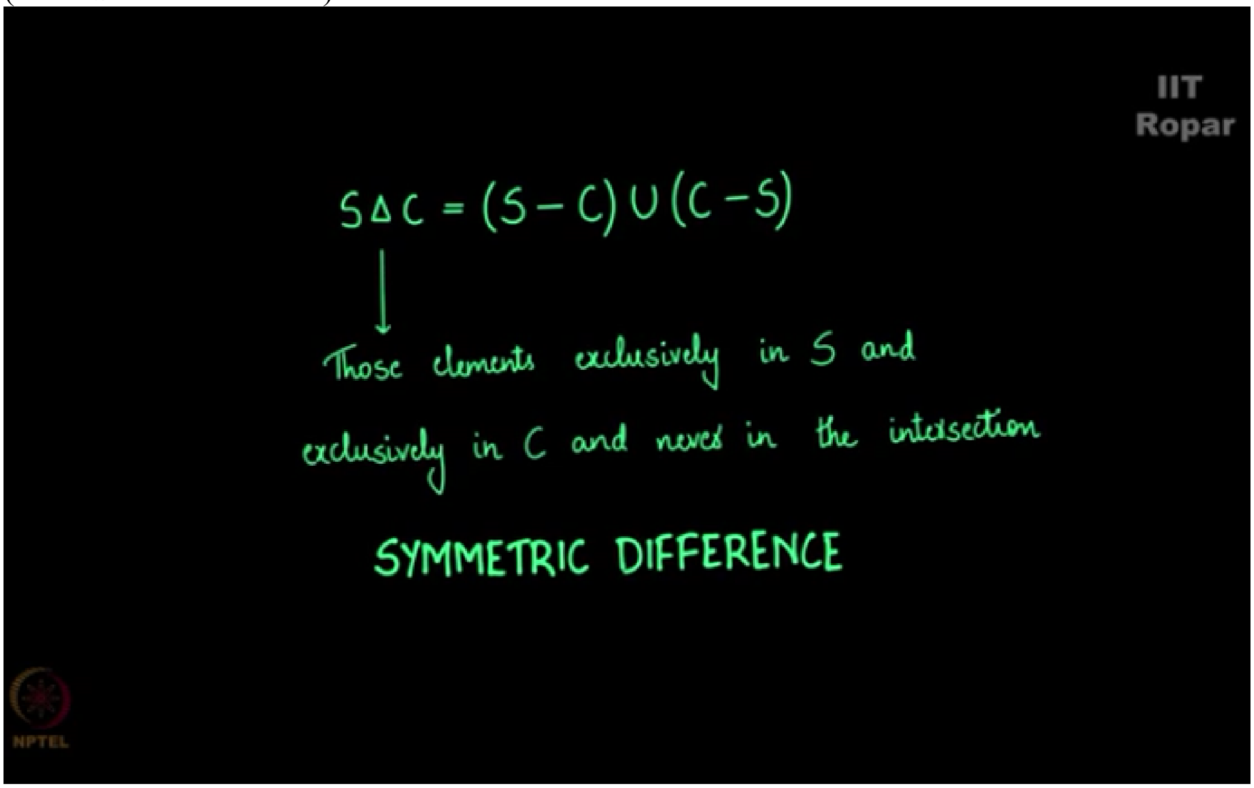

The symmetric difference S, difference C is S-C union C-S, and those are the elements which are exclusively in S and exclusively in C, we do not consider those elements which are at the intersection of S and C this is called symmetric difference. So the chapter set theory was having all of these topics sets, subsets, union intersection and we saw the definitions of them, De Morgan's law, set difference and symmetric difference.

The third week was the propositional logic. We started by learning the statement, what is the statement? It is the sentence which is either true or false it is also called as a proposition.
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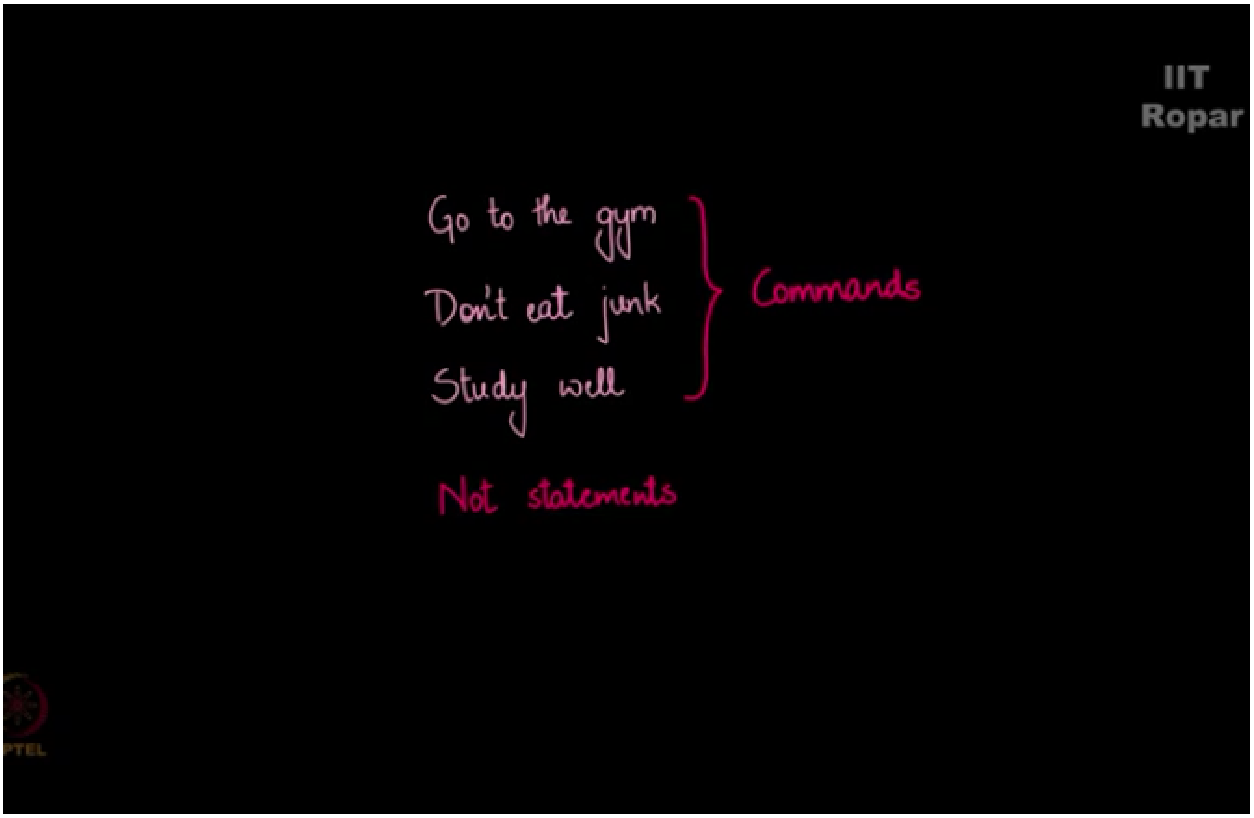

Here are a few examples of sentences which are not statements.

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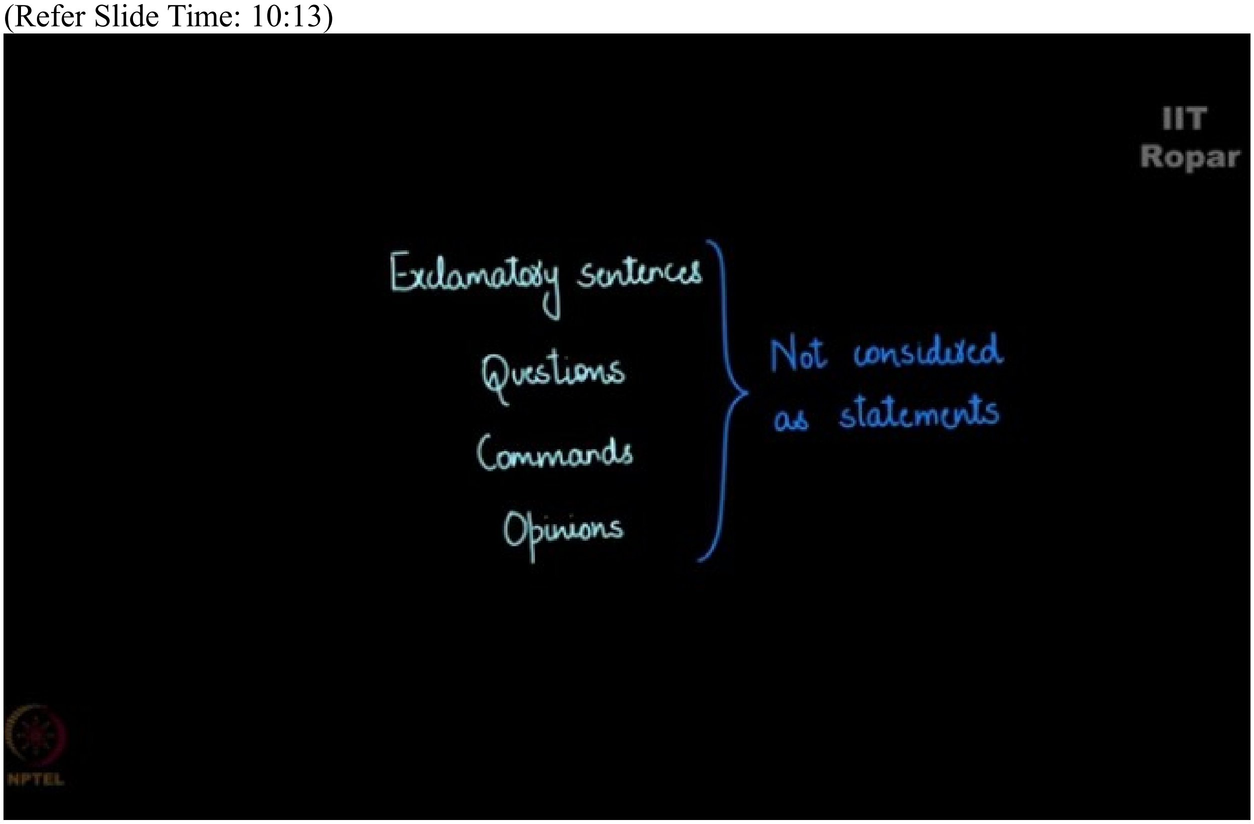

Commands like these and exclamatory sentences, questions, opinions, they are not considered as statements.

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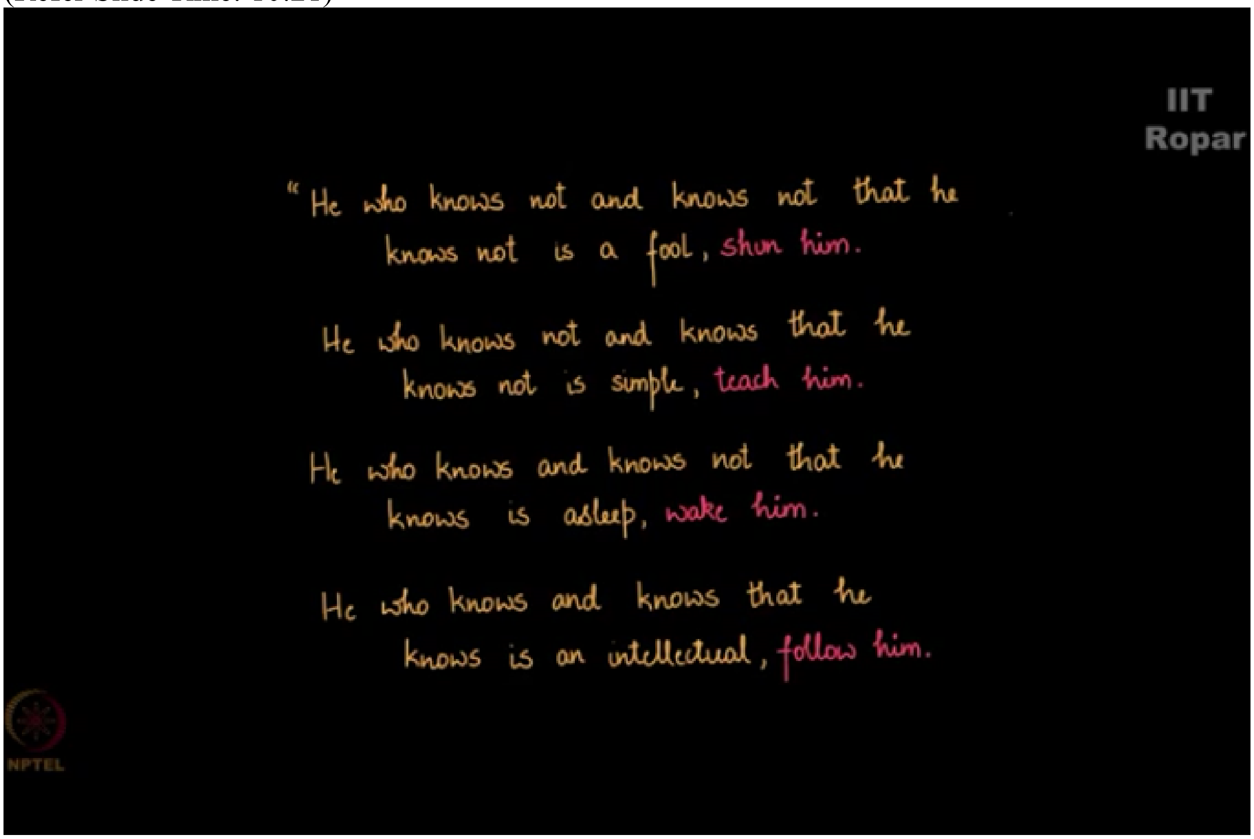

We then moved on to learning what is called the operators, we begin with OR operator, this motivational quote was the first one, he who knows not and knows not that he knows not is a fool, shun him. He who knows not and knows not that he knows not is simple, teach him. He who knows and knows not that he knows is asleep, wake him. He who knows and knows that he knows is an intellectual, follow him.

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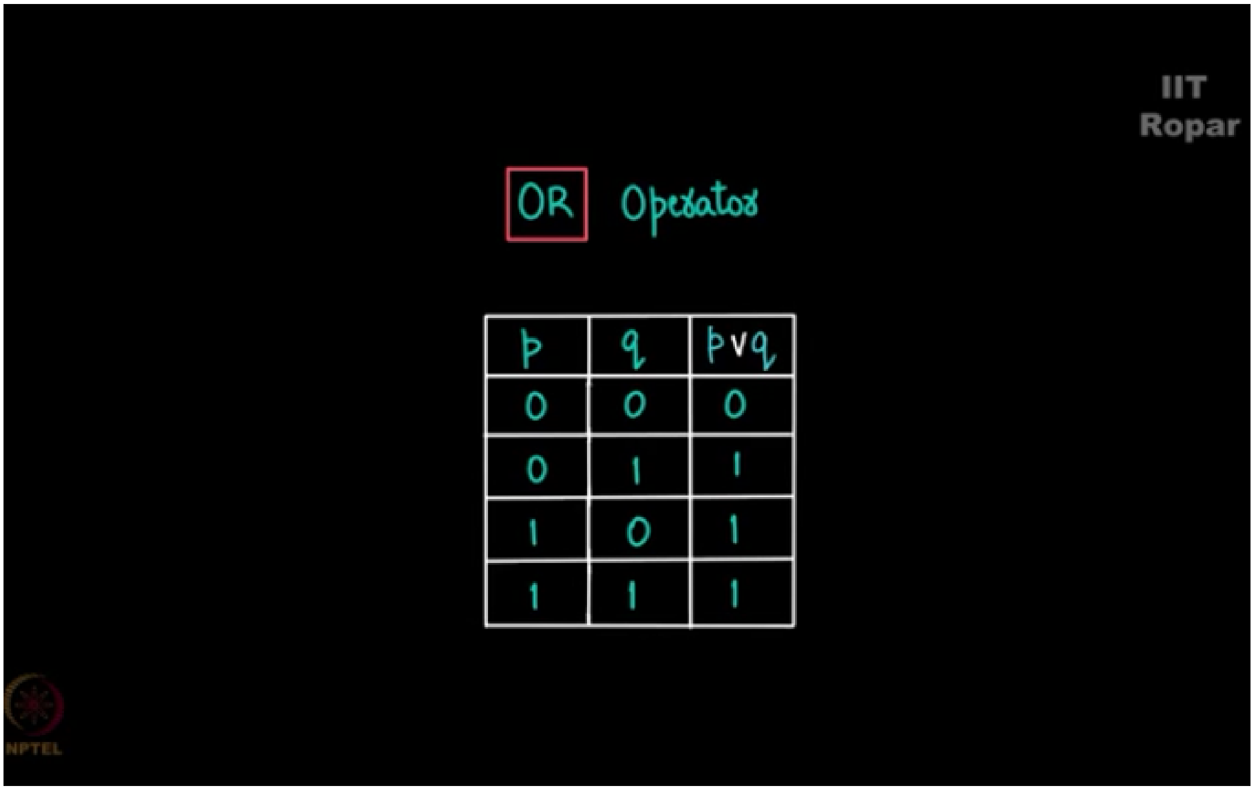

Then we learnt that this is the symbol for OR operator and we saw the truth table of the OR operator. You have two statements here P and Q, you see that either P, you have options for P and Q here 0, 0, then you get P or Q as 0, 0 1 is a 1, 1 0 is 1, 1 1 is a 1, so in an OR operator is either of them is true then it is enough for the statement to be true.

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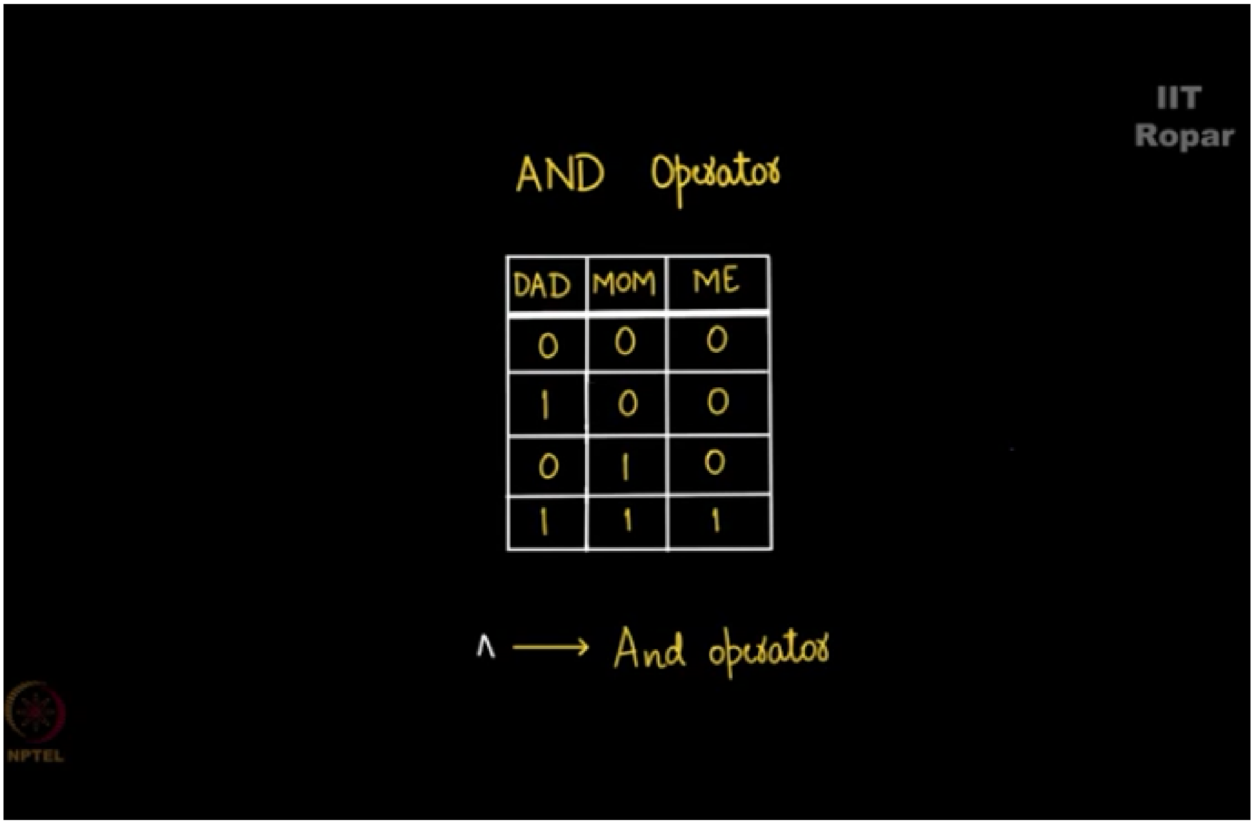

Then comes the AND operator, if you remember the boy had to go and talk to his parents that both of them should come for the college fest, right, he wanted his mom and his dad both of them to come only then he would feel happy, so there comes the AND operator, 0 0 is a 0, 1 0 is a 0, 0 1 is a 0, and 1 1 is a 1, that represents both his parents coming for the college fest, so an AND operator is written like this, only when both P and Q are true is the entire statement true, so this is the truth table of an AND operator.

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This is the truth table for an XOR operator, right, it is called the Exclusive OR. What happens here is both of them, if both of them follow the same value then it is 0, if it is both 0 then it is 0, and if both are 1 then it is a 0, 0 1 is 1, and 1 0 is a 1, this is an XOR operator.

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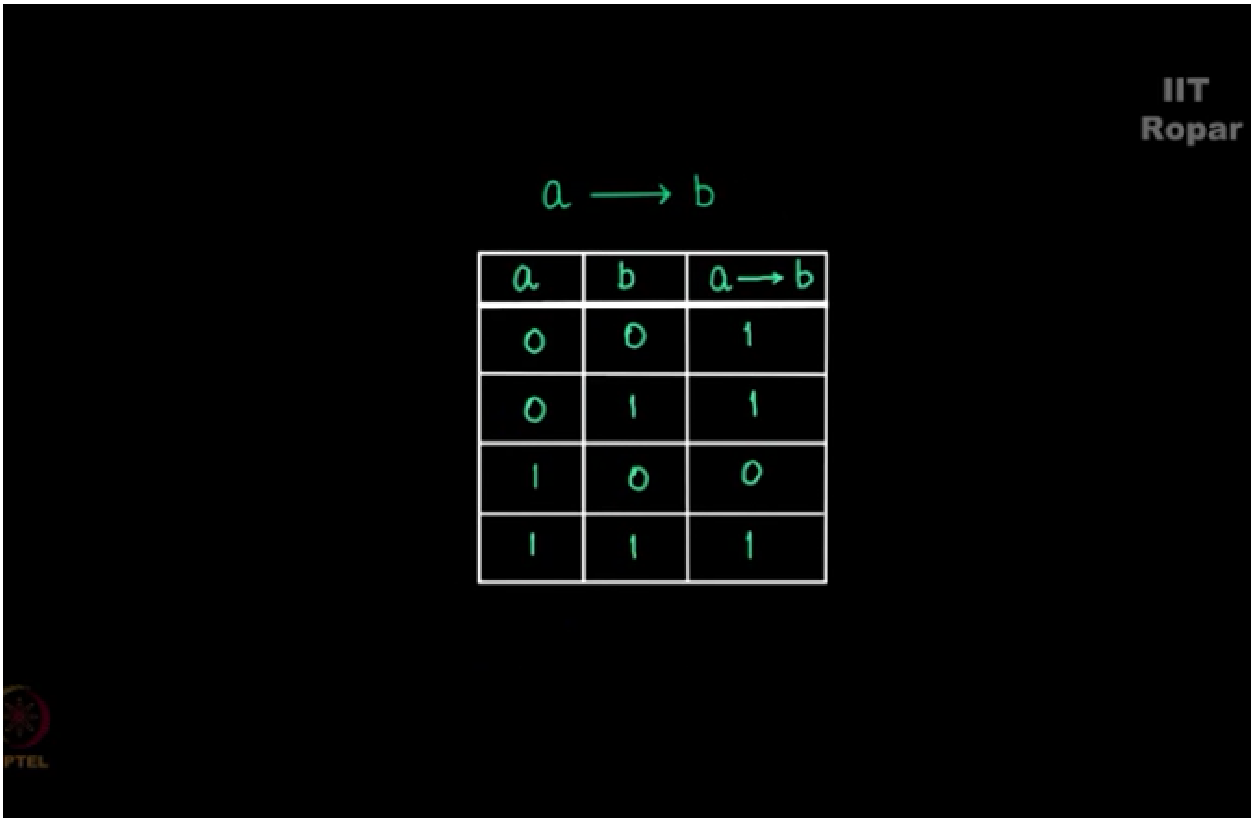

What comes next is an implication A implies B, if A and B are both falls then A implies B is 1, if it is a 0 and a 1 then A implies B is 1, if it is 1 and 0 then it is 0, 1 and 1 is a 1, right. What do you understand here? If A is true and B is not true then it is not true that implication does not hold, right?

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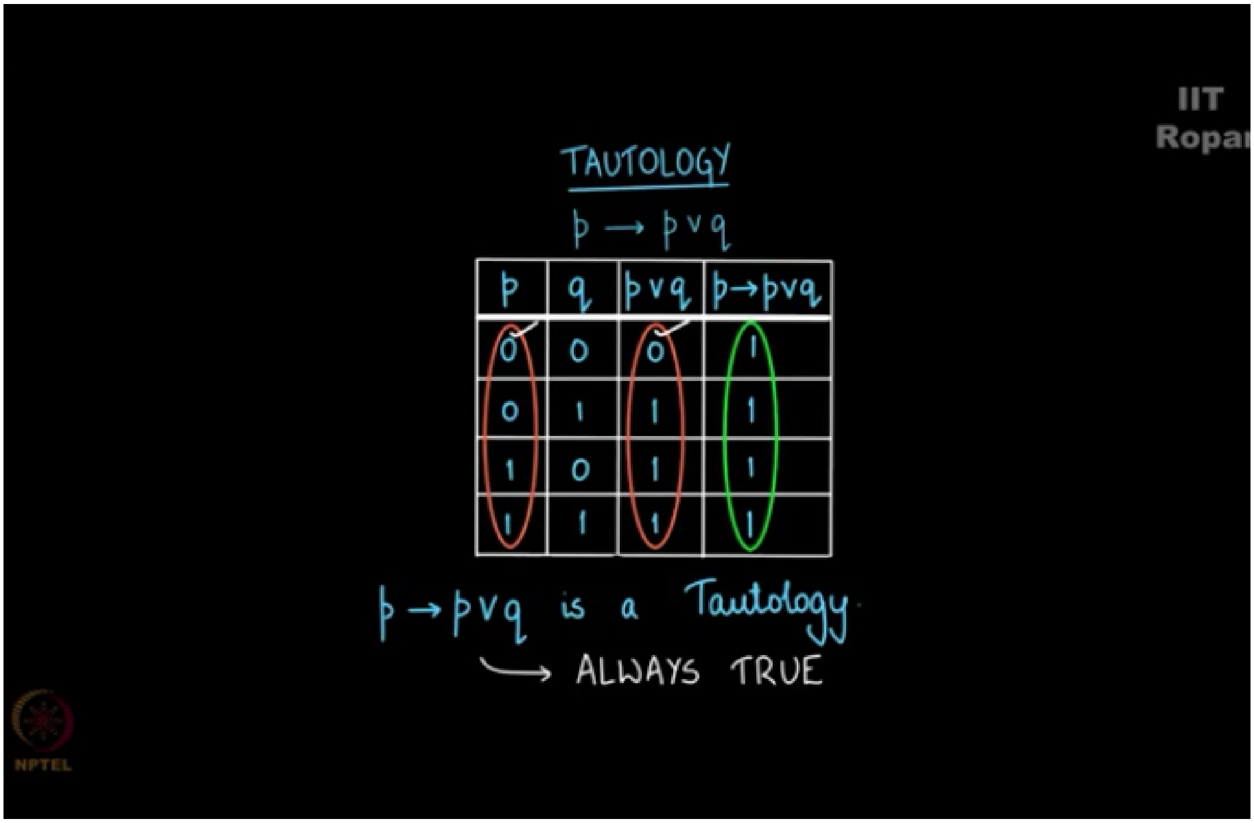

We also learnt something called a tautology. Here is the truth table for it. We have P column, Q column, P or Q, and P implies P or Q. A tautology is always true you see the truth table here we are concerned mainly with this column P and this column P or Q, and we apply and implication on it we see that 0 0, 0 1, 1 1 1 1, right, we apply implication and what we get as the final answer, look at this fourth column it is 1 1 1 1, right, you see that a tautology is always true.

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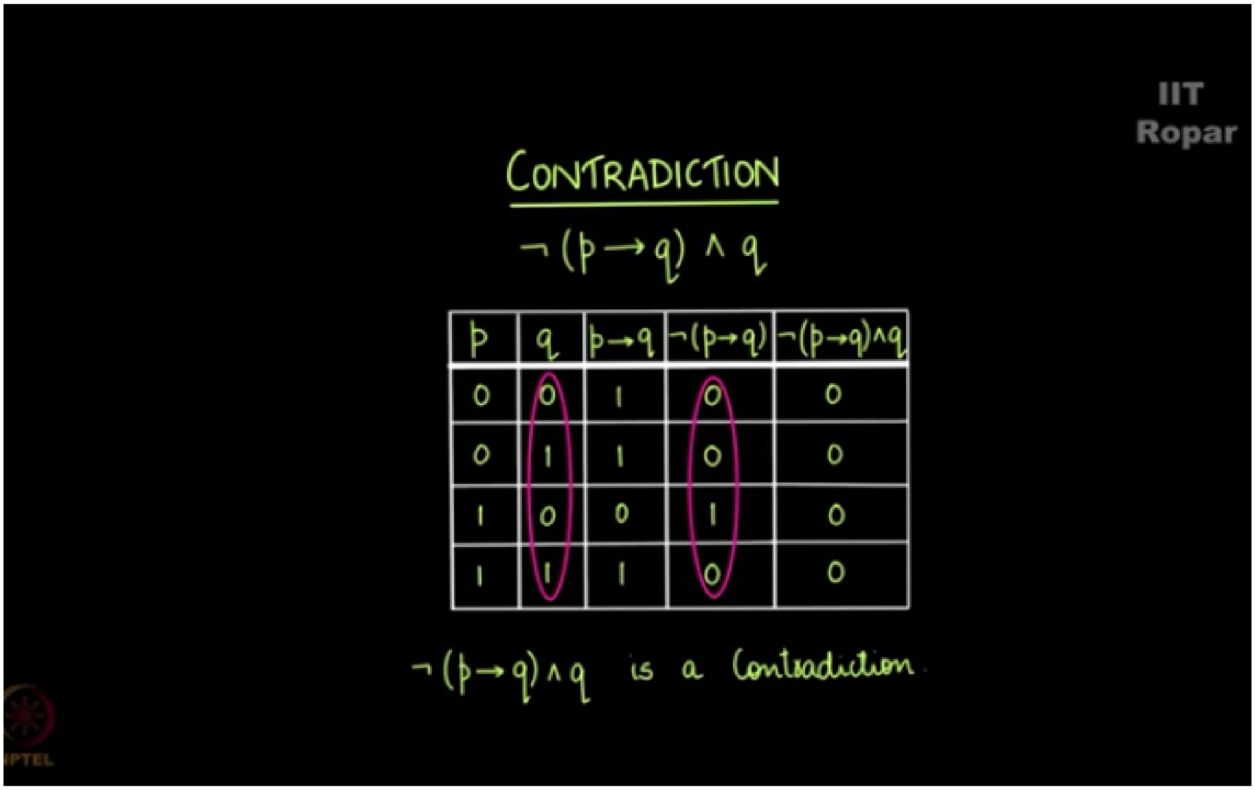

What comes next in line is the contradiction, a contradiction is always falls. Negation P implies Q and Q, right, this is the truth table we have P, Q, P implies Q negation of P implies Q, and negation of P implies Q and Q, we have these columns here. Look at the columns which are circled in pink, it is a 0 0 which is a 0, 1 0 is a 0, 0 1 is a 0, and 1 0 is a 0, why? Because it is an ADD operator, only if there is a 1 1 will you get a 1, and hence this is not true, a contradiction is always falls.

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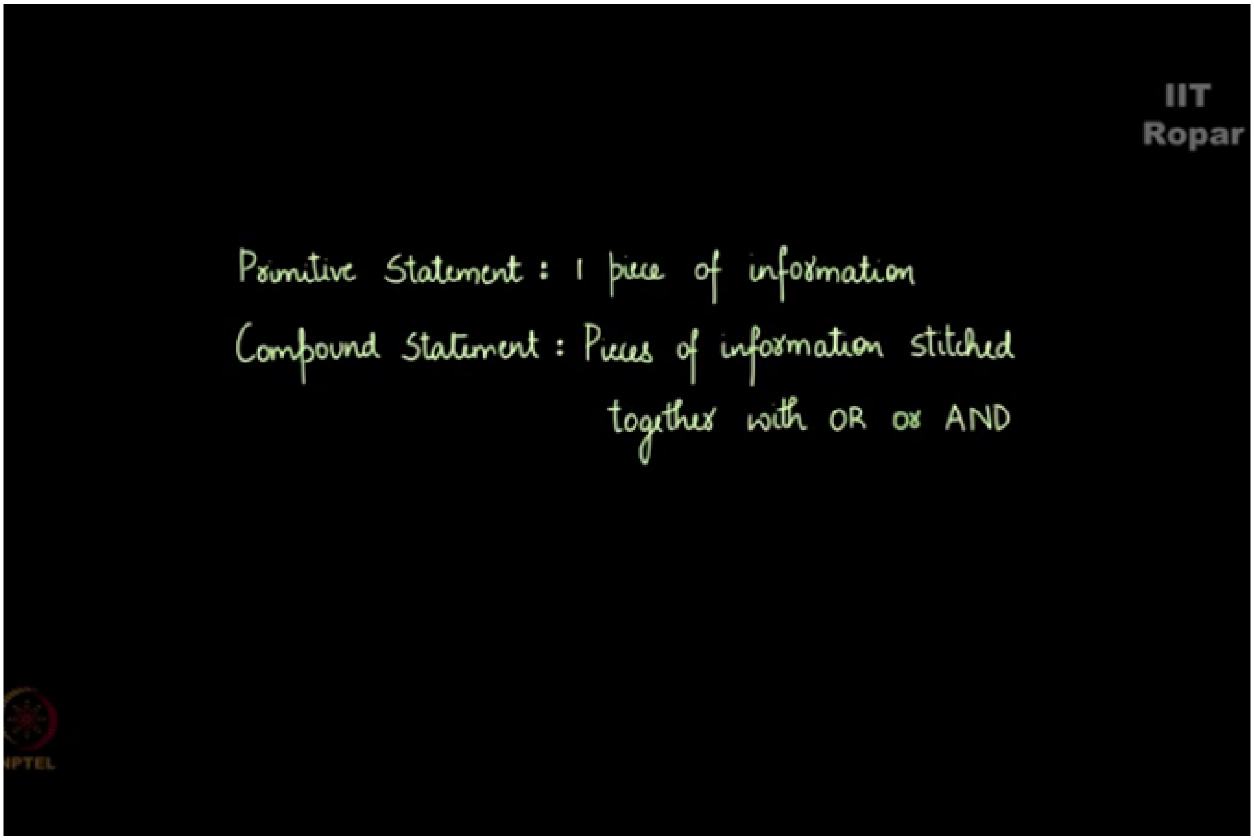

We also studied these two categories of statements, the primitive statement and the compound statement, a primitive statement is just one piece of information it is a very basic statement. A compound statement is pieces of information stitched together with OR or AND, you have either of these operations in middle or in between, the two piece of information will be stitched with these two operators, such a statement is called a compound statement.

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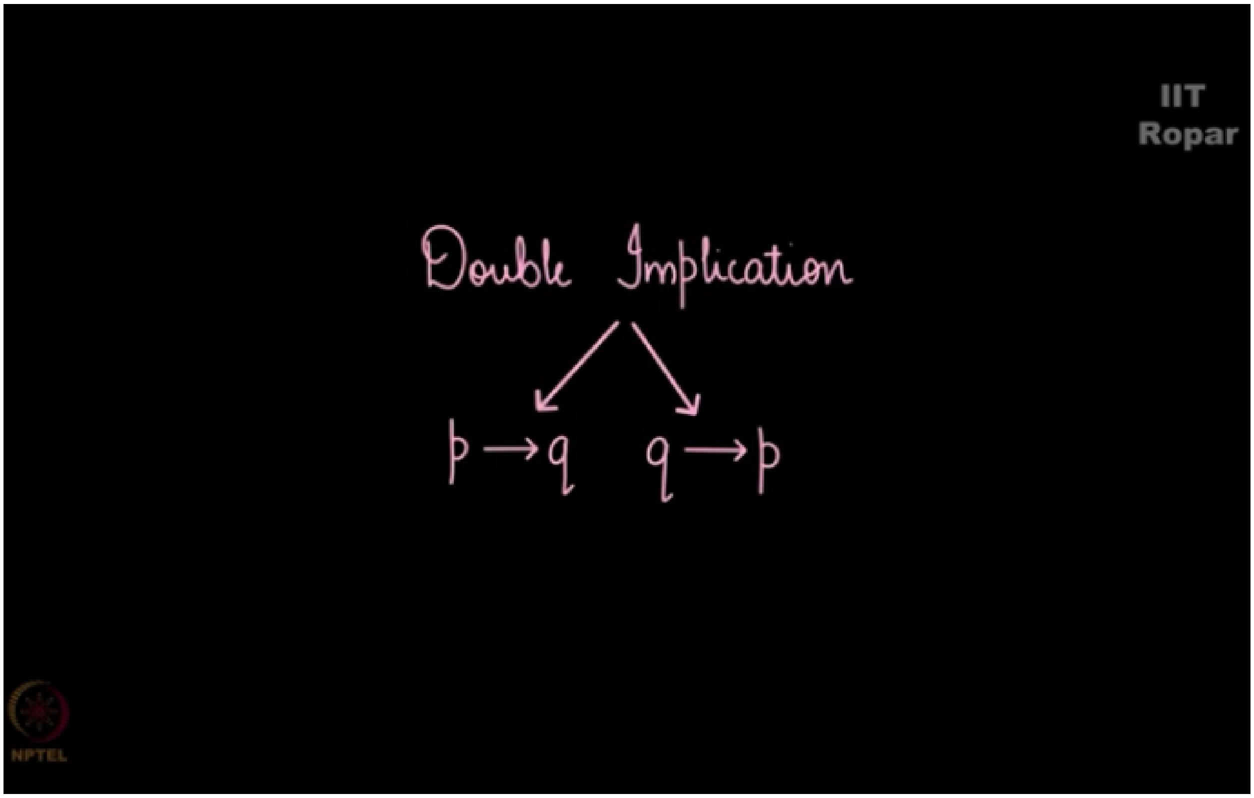

What comes next is the double implication, you had learnt already implication, here is the double implication, you had learnt what is P implies Q, and the truth table of this. If Q also implies P then it follows the double implication, if it is called the if and only if, right with the double headed arrow.

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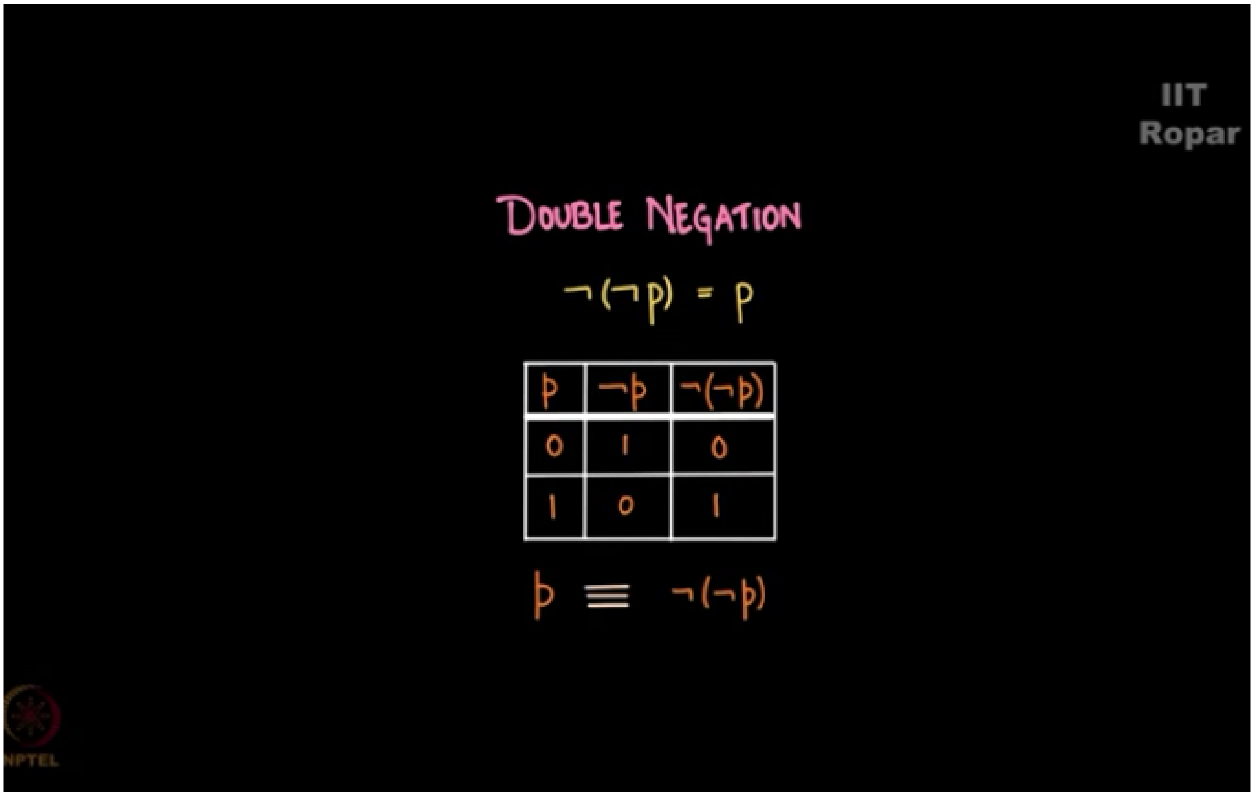

We also learnt what is called the double negation. Negation is quite simple, if a statement is given negate that statement and you get negation P. Double negation is you negated twice, what happens is you get back the original statement. Here is the truth table of double negation, P naught P, naught naught P, 0 1 is a 0, 1 0 is a 0, and you get back P here.

We also learnt rules of inference, we solved several problems on it and through them we learnt all the rules of inference.

I am now going to summarize the relations chapter.
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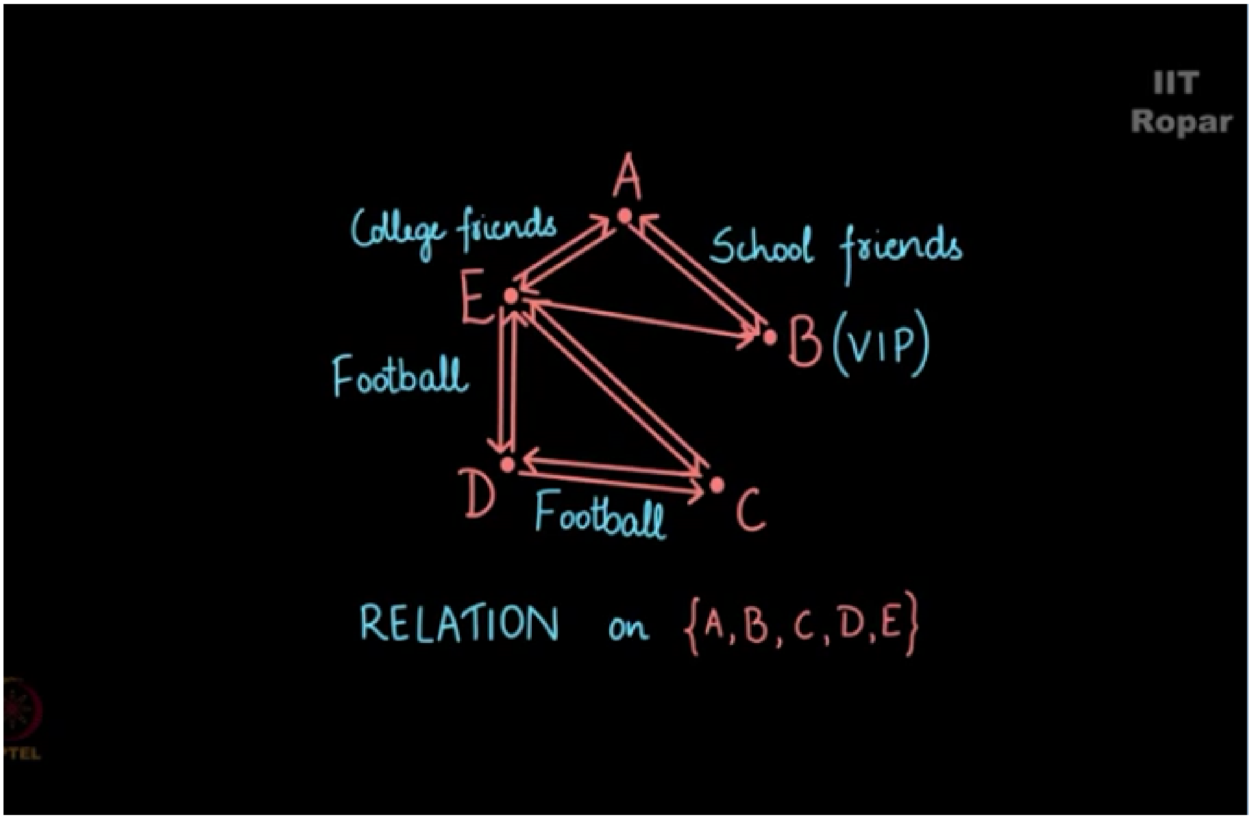

We started of like this, we considered these 5 people A, B, C, D, E, A and B were school friends, A and E were college friends, ED and EC, sorry DC use to play football together, and EC hence knew each other. What we are doing is we are formulating a relation on this set of people A, B, C, D, E, right.

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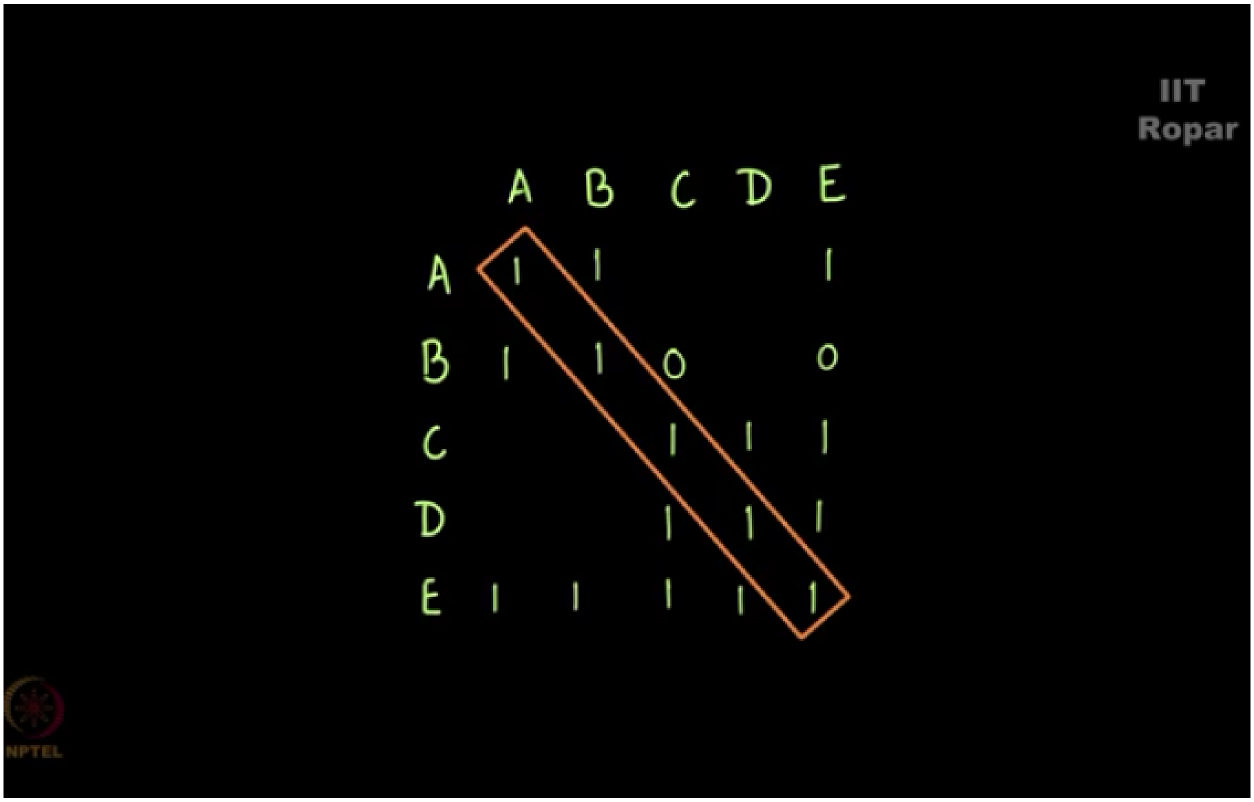

Now what we are going to do next is we have seen a matrix representation of the relation between these people A, B, C, D, E, B knew C, B does not know C and hence there is a 0 there, B does not know E either so there is a 0 there. D knows C and hence it's a 1, E knows all of them and hence it's a 1 and so on, right.

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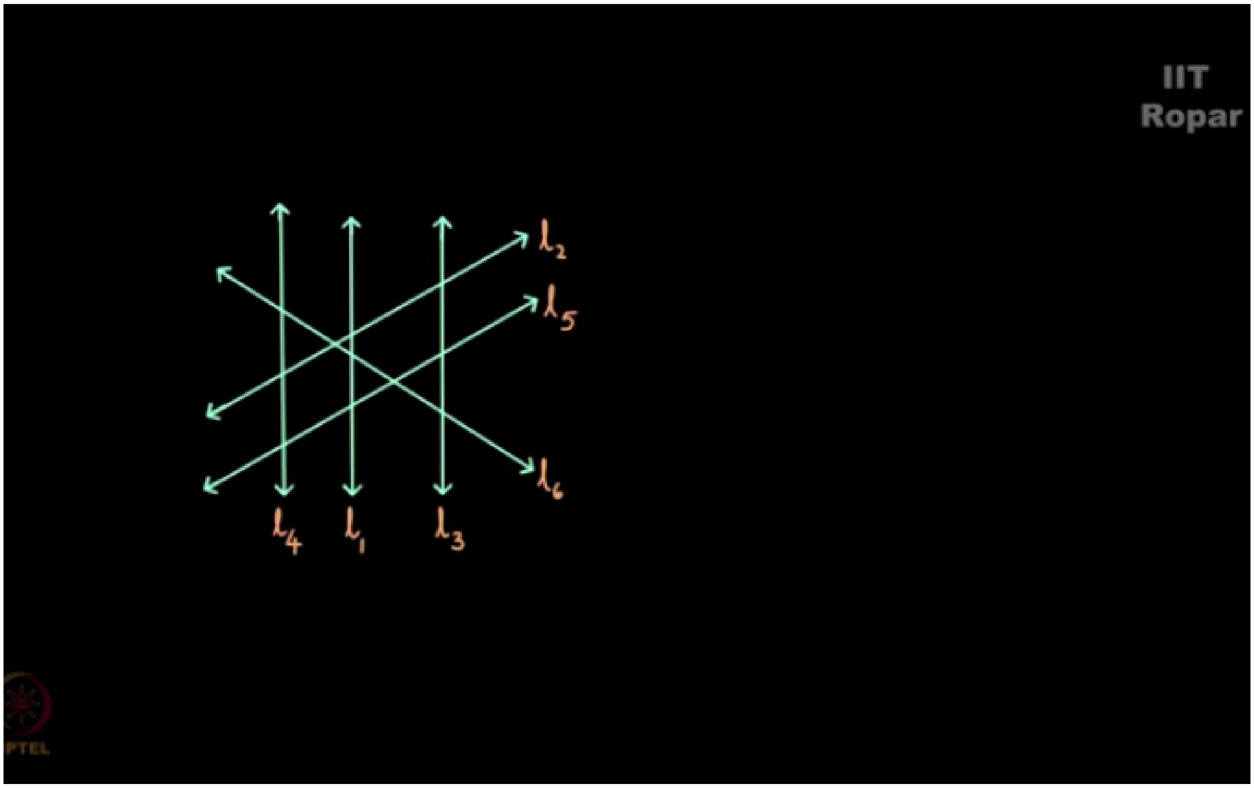

Now we considered these lines parallel lines, and one is not parallel so on, we considered these 6 lines and we took the parallelism as the relation like one line is parallel to other line, implies it is related to the other line and then we considered the matrix of these lines,

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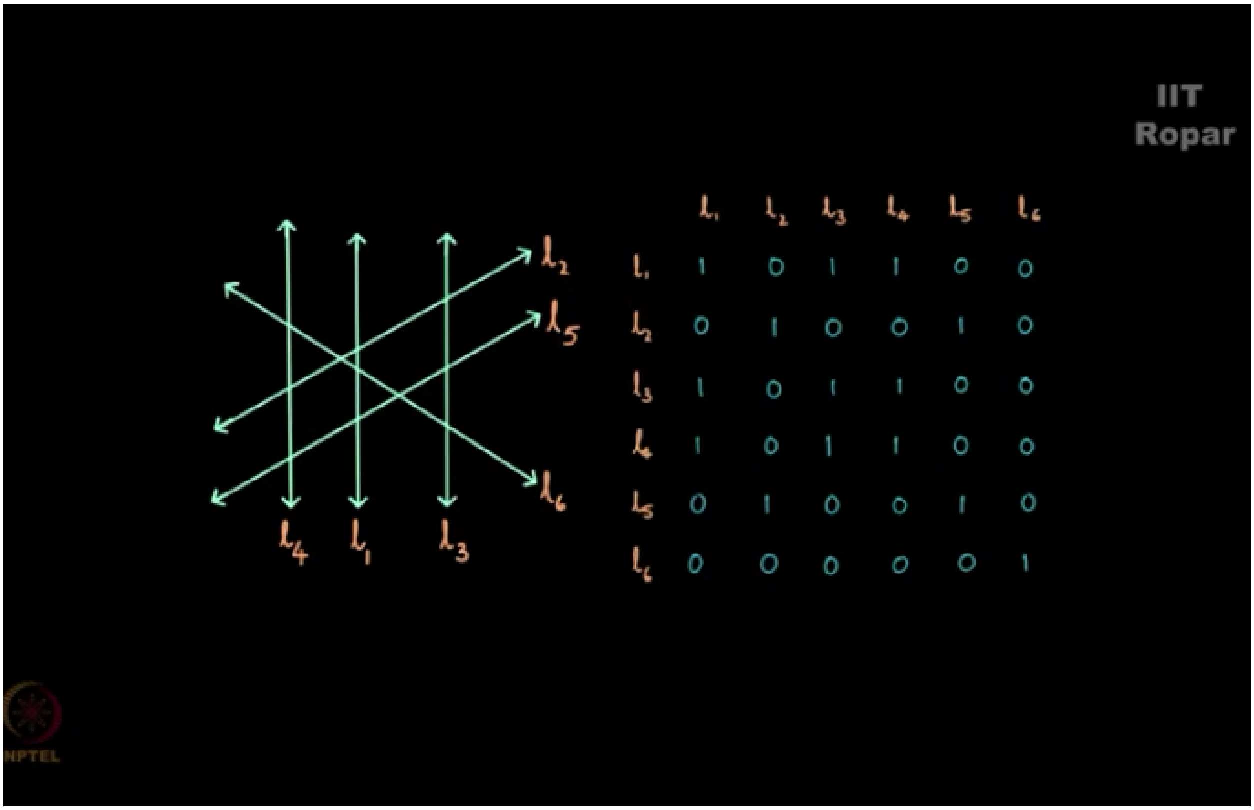

right, so if L1 is parallel to L2 then it gets a 1, right, though in the matrix L1, L2 is 0 the entry there is 0, I just gave an example if LI is parallel to LJ then the corresponding entry in the matrix LI, LJ will be 1, if not it is a 0.

So you see the diagonal here all are ones, how? Every line is considered to be parallel to itself and one more observation you see if L2 is parallel to L5, L5 is also parallel to L2, right. Now this is a symmetric or it is a symmetric matrix, what do I mean by that? A transpose if you take it is equal to A, therefore it is a symmetric matrix.

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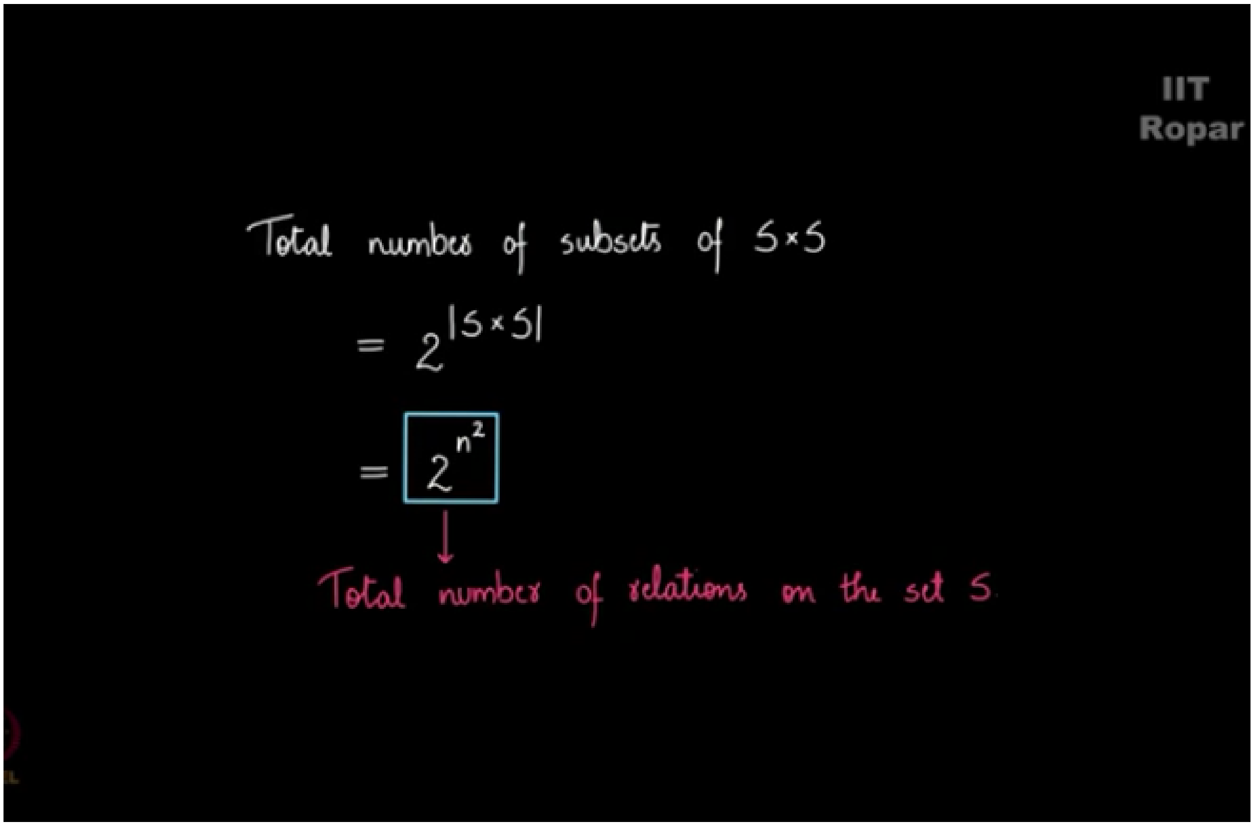

Now we studied that relations are a subset of S x S, and the total number of subsets of S x S are 2 to the power of S x S we know this from set theory chapter, right. And how many elements are there in S x S? It is N square, and therefore 2 to the N square is the total number of relations on the set S.

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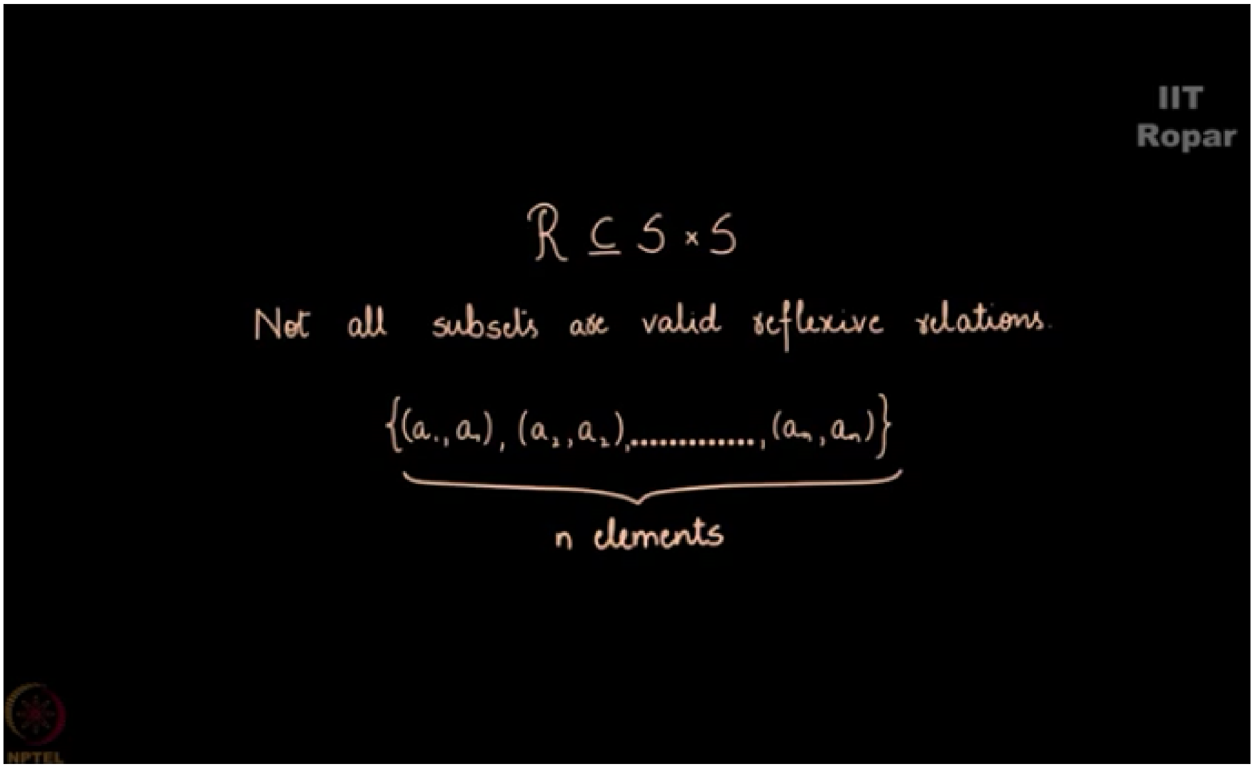

Now we are now considering how many reflexive relations are there, we have seen this. A1,A1, A2,A2 so on AN,AN, these are the elements if it is a reflexive relation. How many such elements are possible? N elements are possible and hence you have these elements in, if a relation is reflexive, right, but note not all subsets are valid reflexive relations.

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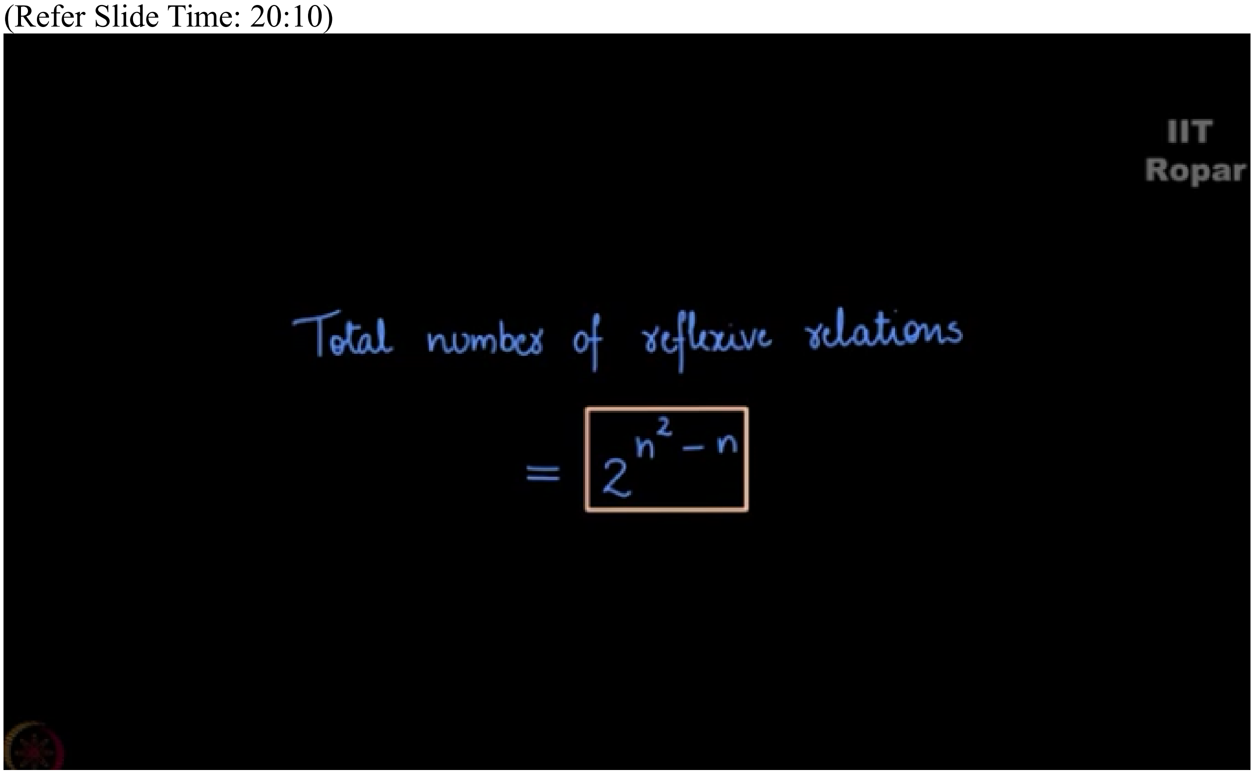

So the total number of reflexive relations possible is 2 to the N square – N, we have seen this how it is 2 to the N square – N in the relations chapter.

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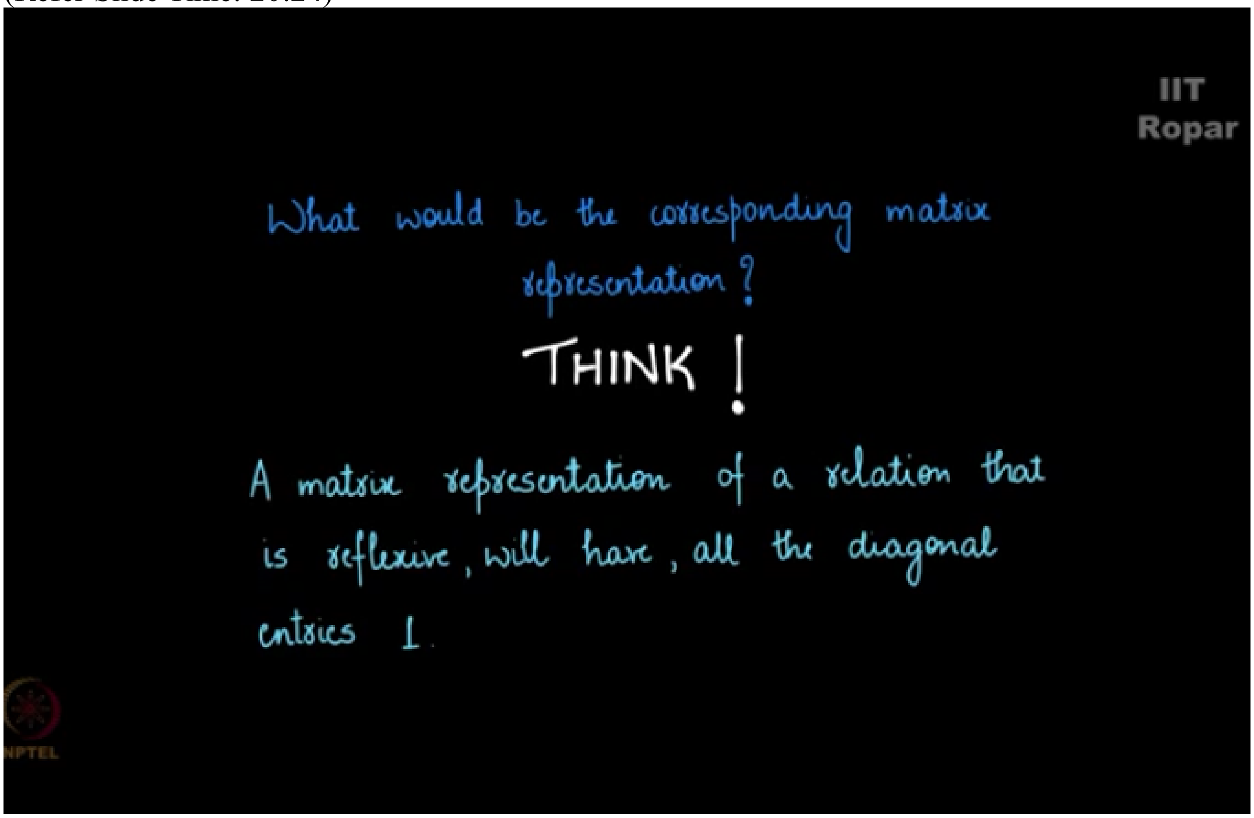

Now what would be the corresponding matrix representation of a reflexive relation? Think about it, a matrix representation is actually having all the diagonal entries 1, that's it, right, we cannot say anything about the other entries, but if all diagonal entries are 1 then definitely it is a reflexive relation, we can conclude at least this much.

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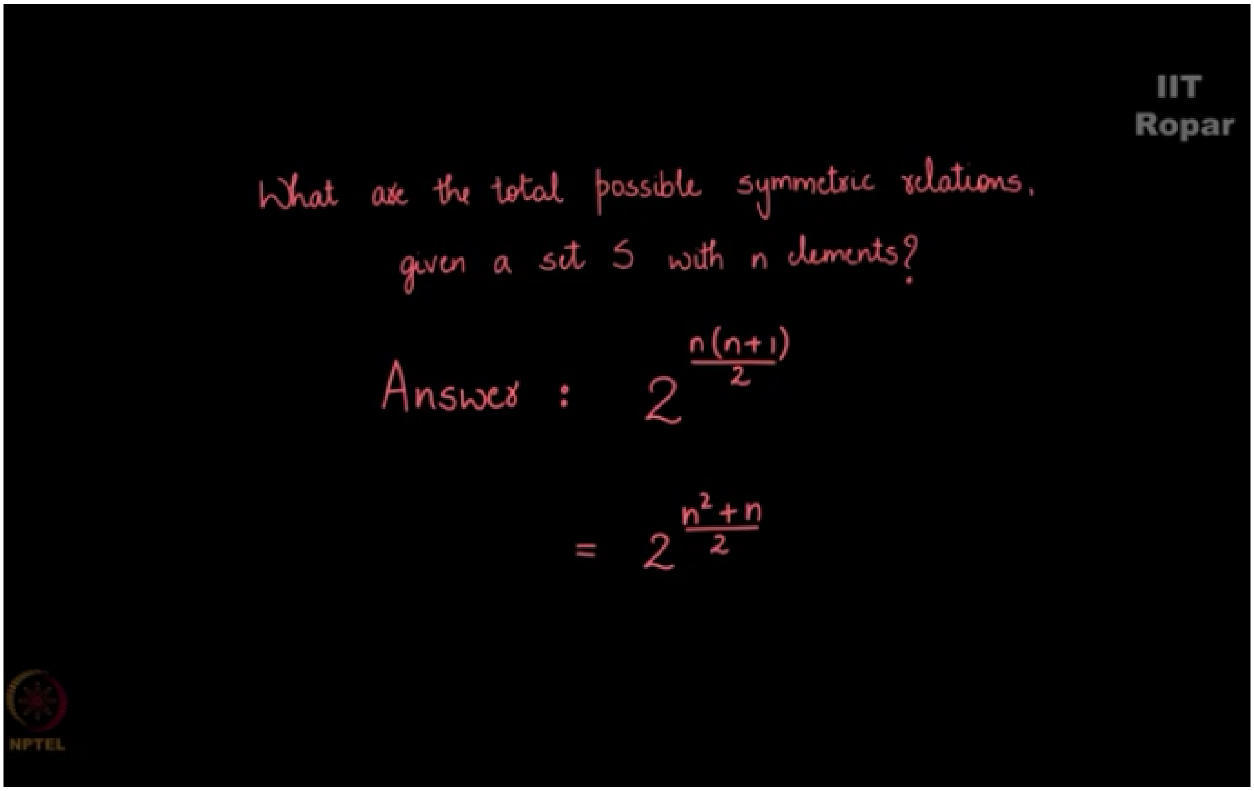

We then studied the total number of symmetric relations possible, what is a symmetric relation? If A, B is present then B, A is also present in the relation that is called a symmetric relation, right. Now if a set S has N elements, how many symmetric relations are possible? The answer is 2 to the N square + N/2, we have seen this.

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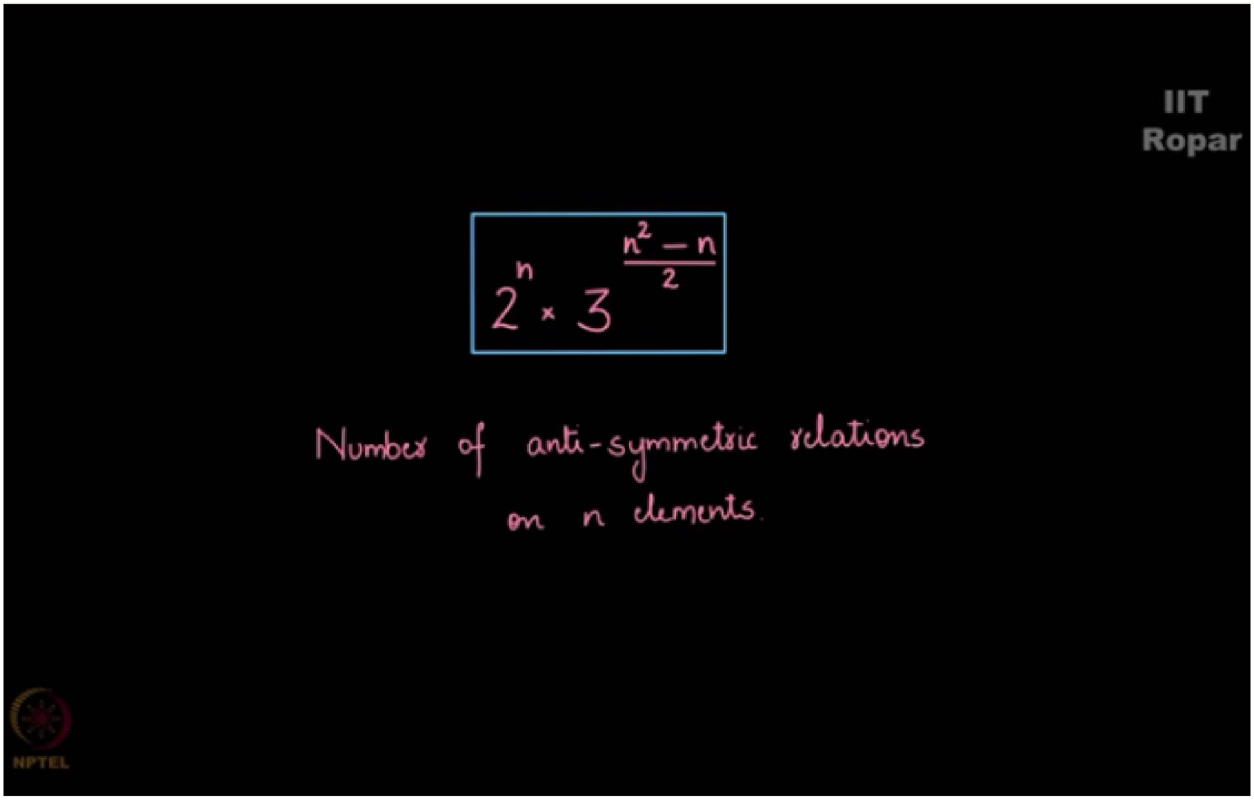

Now here comes anti-symmetric relations, what is anti-symmetric relations? If A, B is present, B, A cannot be present, right, and if A, B and B, A both are present then A = B, this is anti-symmetric relation, and the number of anti-symmetric relations on N elements are 2 to the N x 3 to the N square – N/2,

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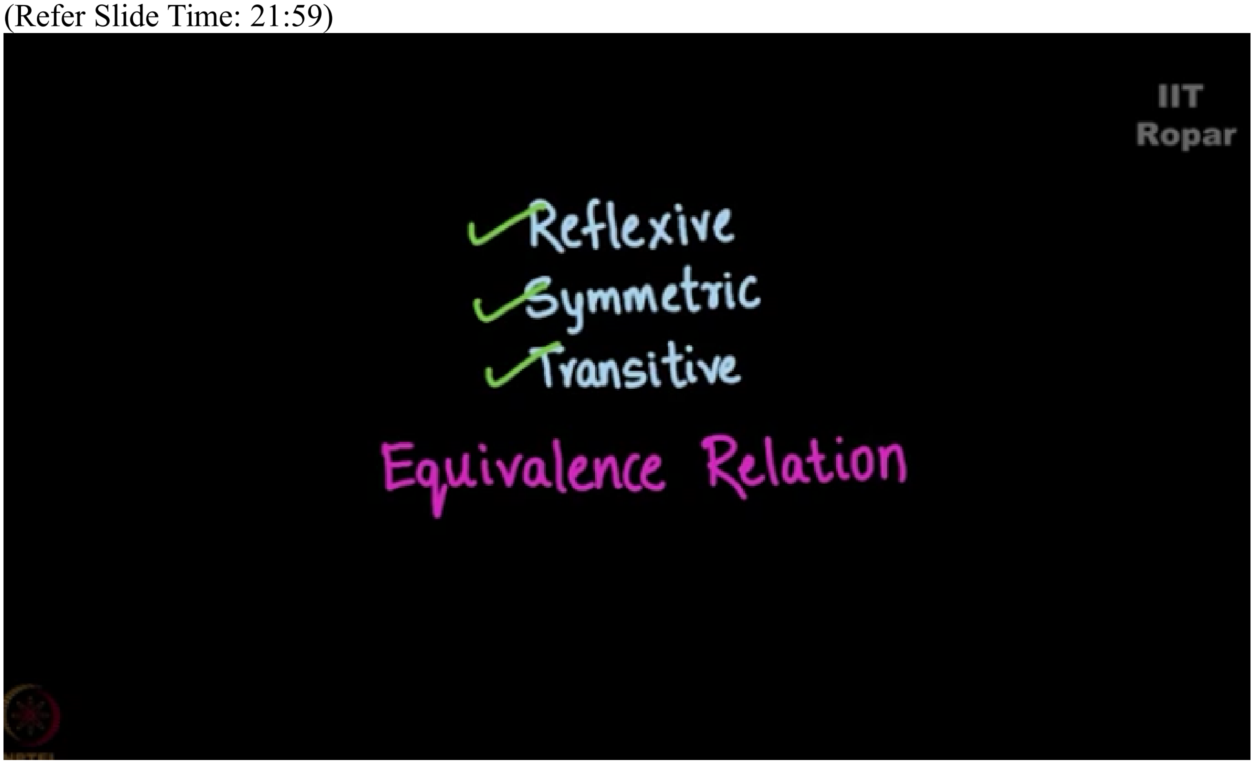

so if a relation satisfies all this properties reflexive, symmetric and transitive it is called an equivalence relation.

We have seen several examples of all this particular examples of all this relations and we've also solved several problems on equivalence relation as such, so if it satisfies all these three, let us called an equivalence relation. What do we mean by equivalence relation?

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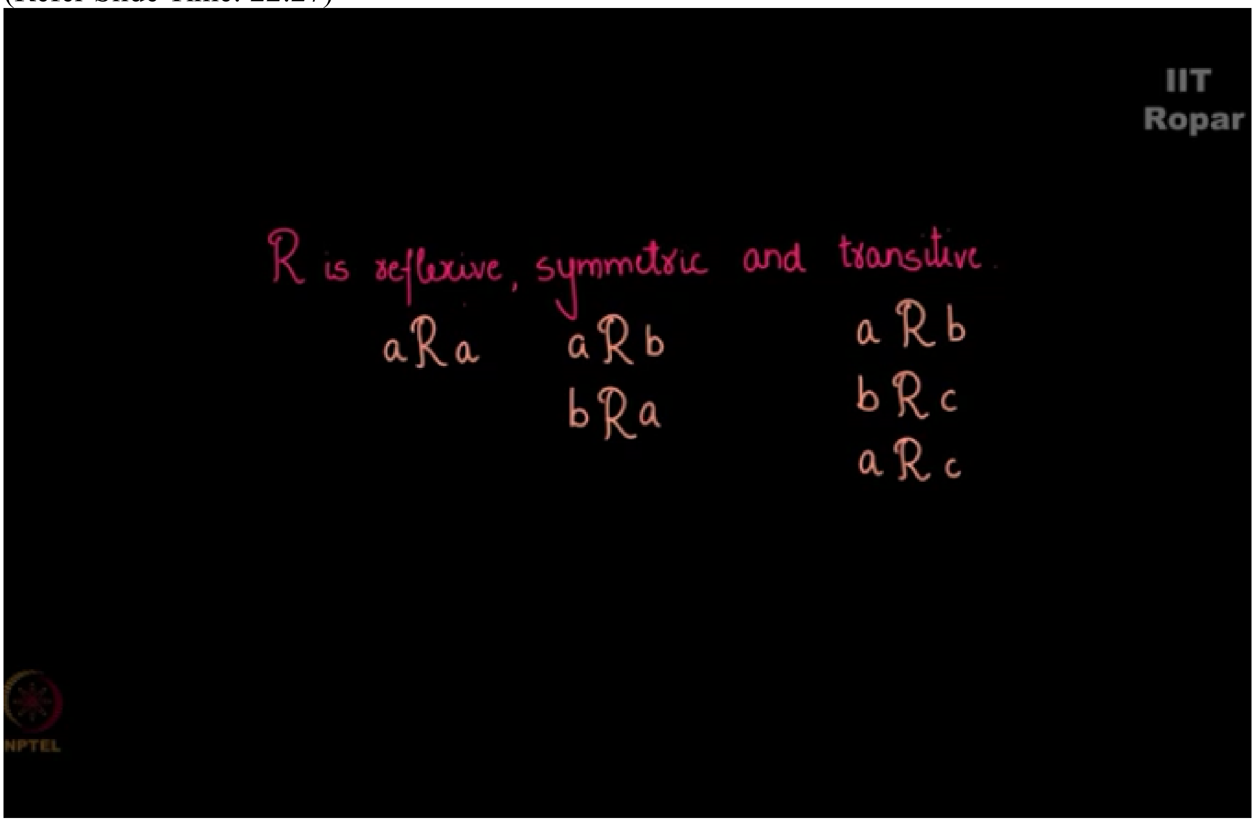

If R is reflexive that is A must be related to A for every A, right, and if it is symmetric it means that if A is related to B then B must also be related to A and by transitive we mean if A is related to B, and B is related to C, then A must be related to C, if these conditions are satisfied it is an equivalence relation.

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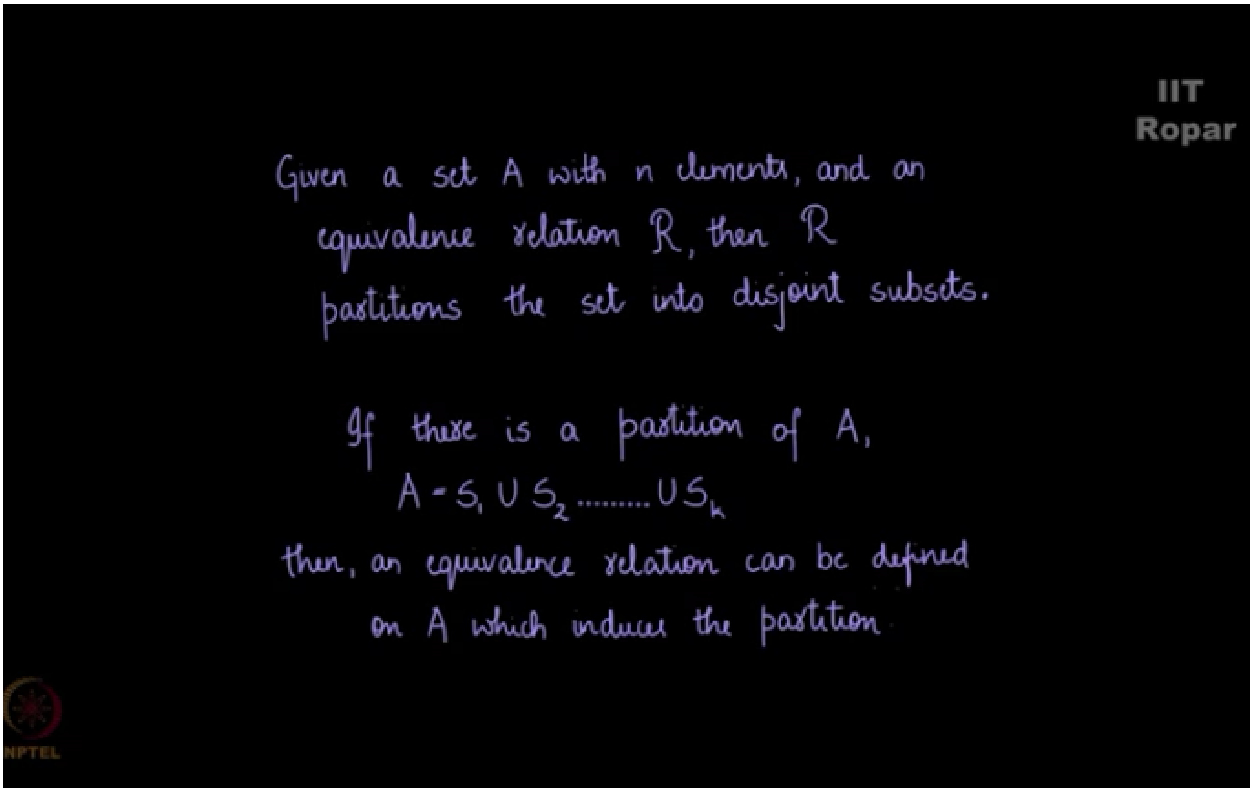

With equivalence relation comes a very nice concept called partitions, right, given a set A with N elements, an equivalence relation partitions the set into disjoints subsets and if there is a partition such that its union will give the entire set, then the equivalence relation can be defined, you can define an equivalence relation on A which induces this particular partition, so both ways it is possible, this theorem states that, so if you have an equivalence relation then your, the set is getting partitioned into disjoint subsets, and if you are able to partition the set into disjoints subsets you can induce, you can have an equivalence relation which can induce a partition.

So we have seen all these concepts in the relations chapter,
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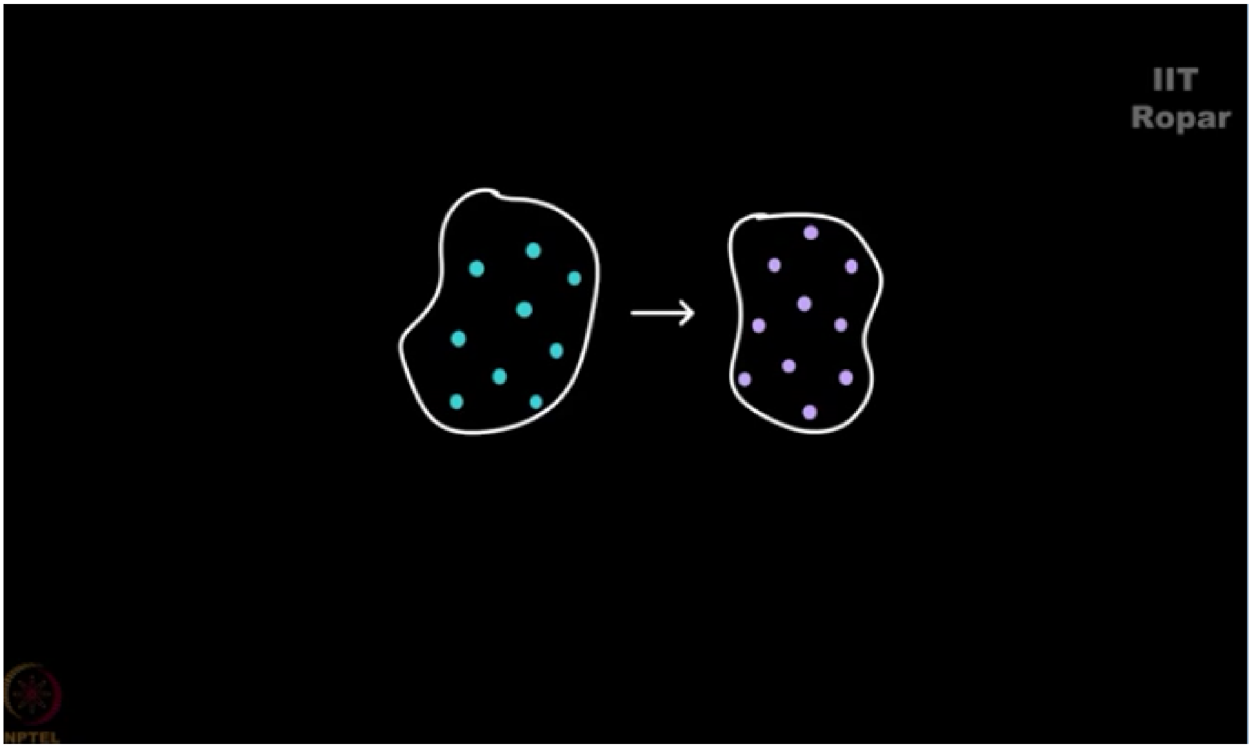

the moment you see this diagram it is for sure that I am talking about functions, yes I'm going to summarize this chapter functions now.

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We had seen what is the function, and we had seen that there is a difference between relations and functions, it is a mapping from one set to another set or it is a relation from one set to another set where one element is not related to two elements simultaneously, right. We had seen several examples and non-examples of functions.

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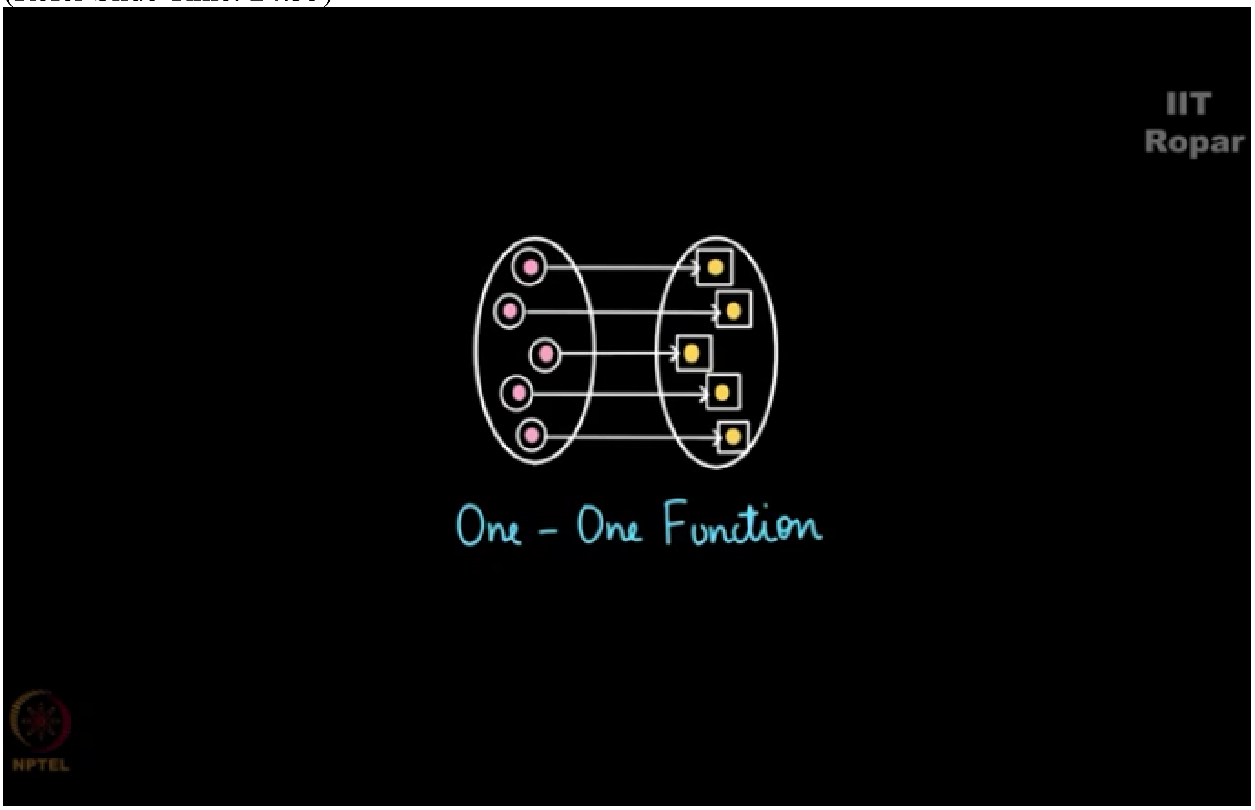

What comes next is one-one function, what it is, right. Where you have one element going exactly to one elements, it is called as a one-one function, please note when we talk of functions all the elements in the domain should be mapped to some element in the codomain, right,

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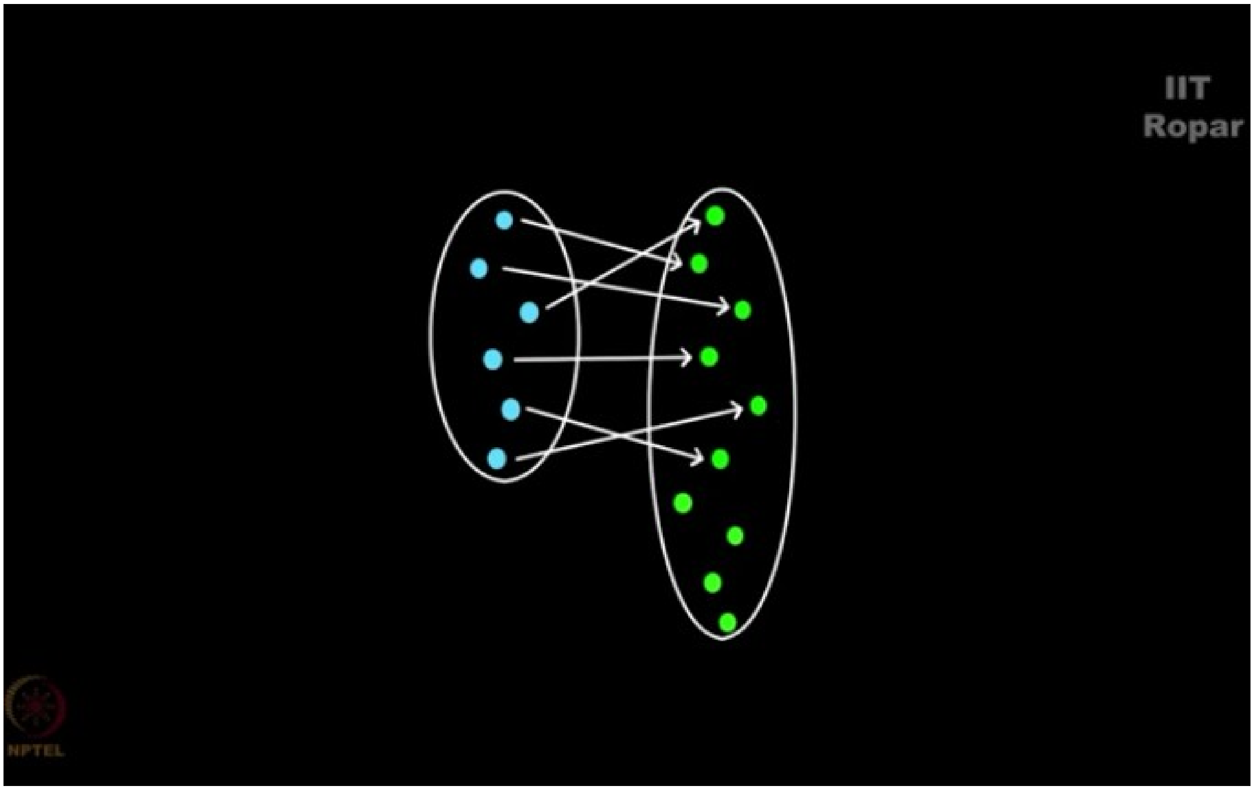

now in such a case what happens, what do you see, what are you seeing here? You're seeing that domain has less number of elements than the co-domain, in such a case a one-one function is possible.

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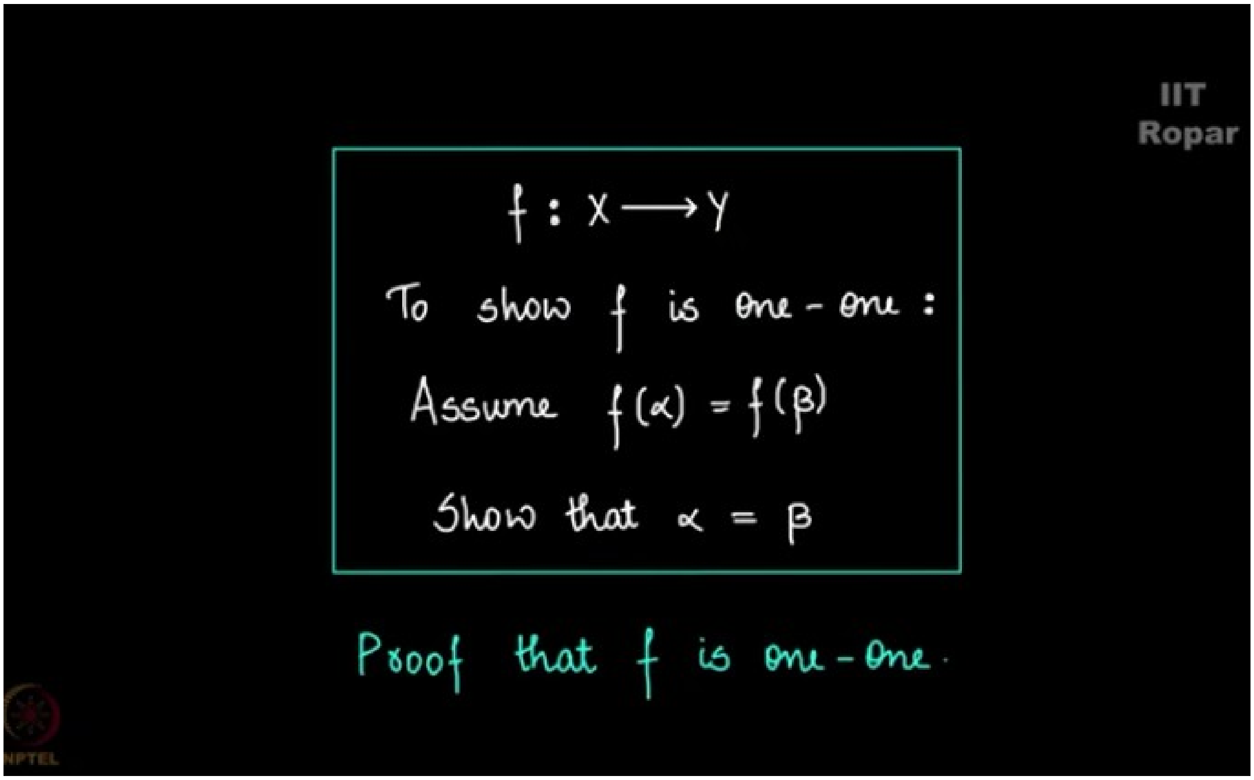

Now how do you show that a function is one-one? You have a function F from X to Y, you have to show that it is 1 1, what do you assume? You assume that F(alpha) = F(beta), what are alpha and beta? They are elements of X, F(alpha) and F(beta) are elements of Y, they are forming the range, right, you have to show that alpha is equal to beta, what happens next? You're sure that alpha is not equal to beta that is only when F(alpha) = F(beta), does alpha = beta com, right, and hence you are showing that F is 1 1.

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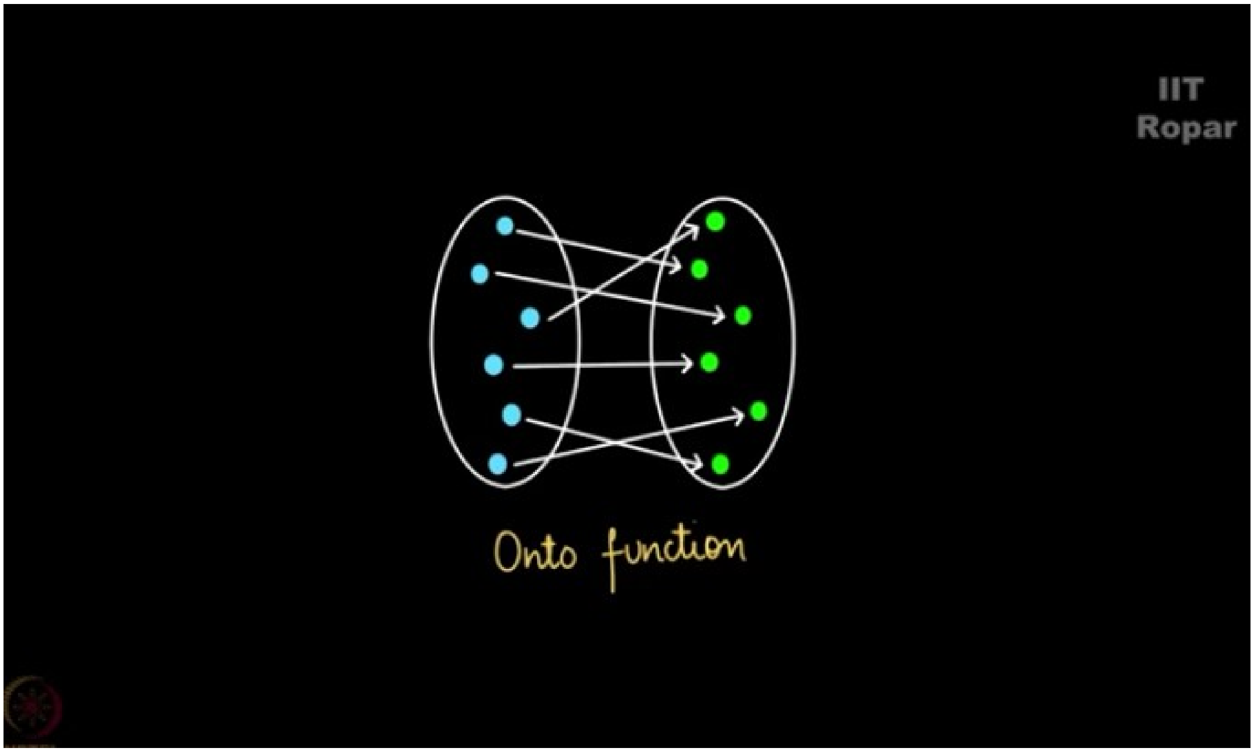

Now this is called the onto function where all your elements in the core domain are mapped to some elements or they are actually images of some elements of the domain, right, every element here has a pre-image in the domain such a function is called an onto function.

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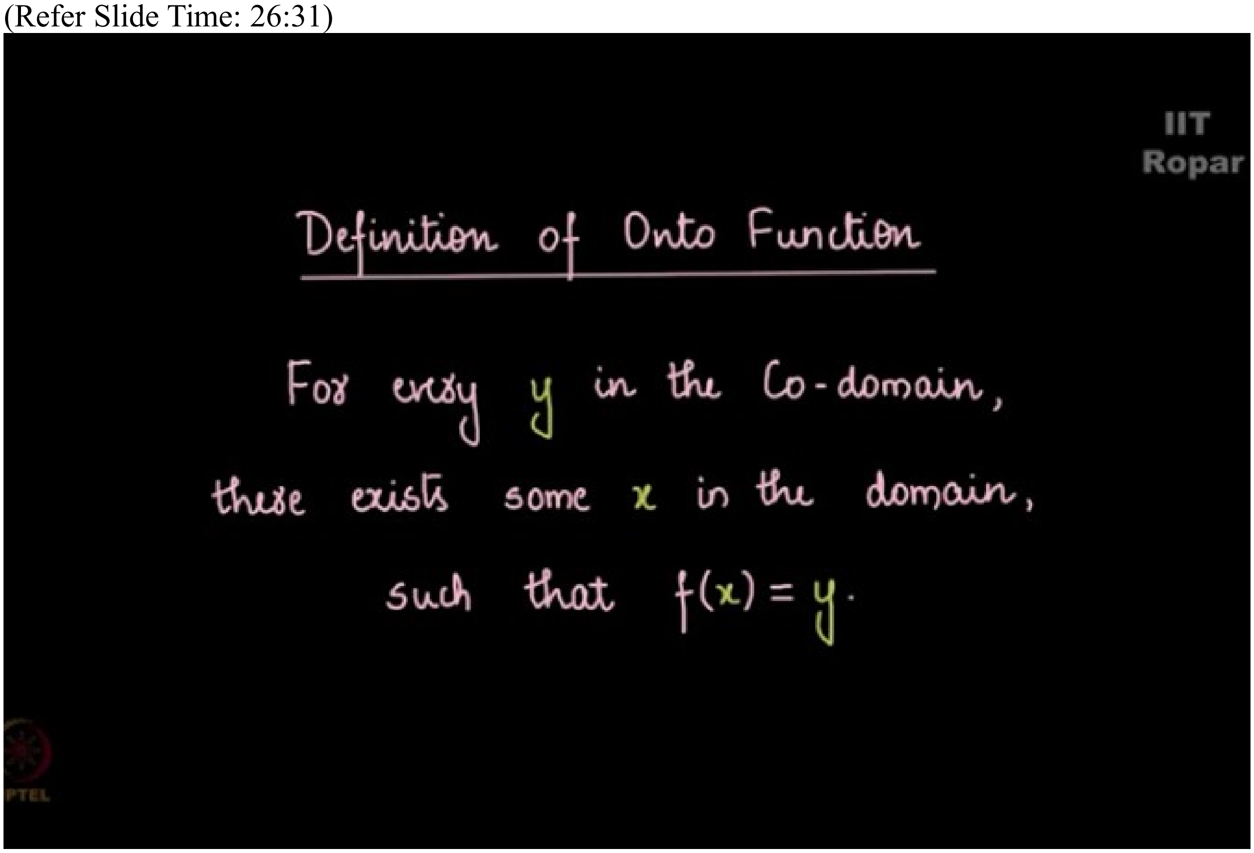

Again here we have seen several examples. For every Y in the co domain there exists some X in the domain such that F(x) = Y, this is the definition of an onto function.

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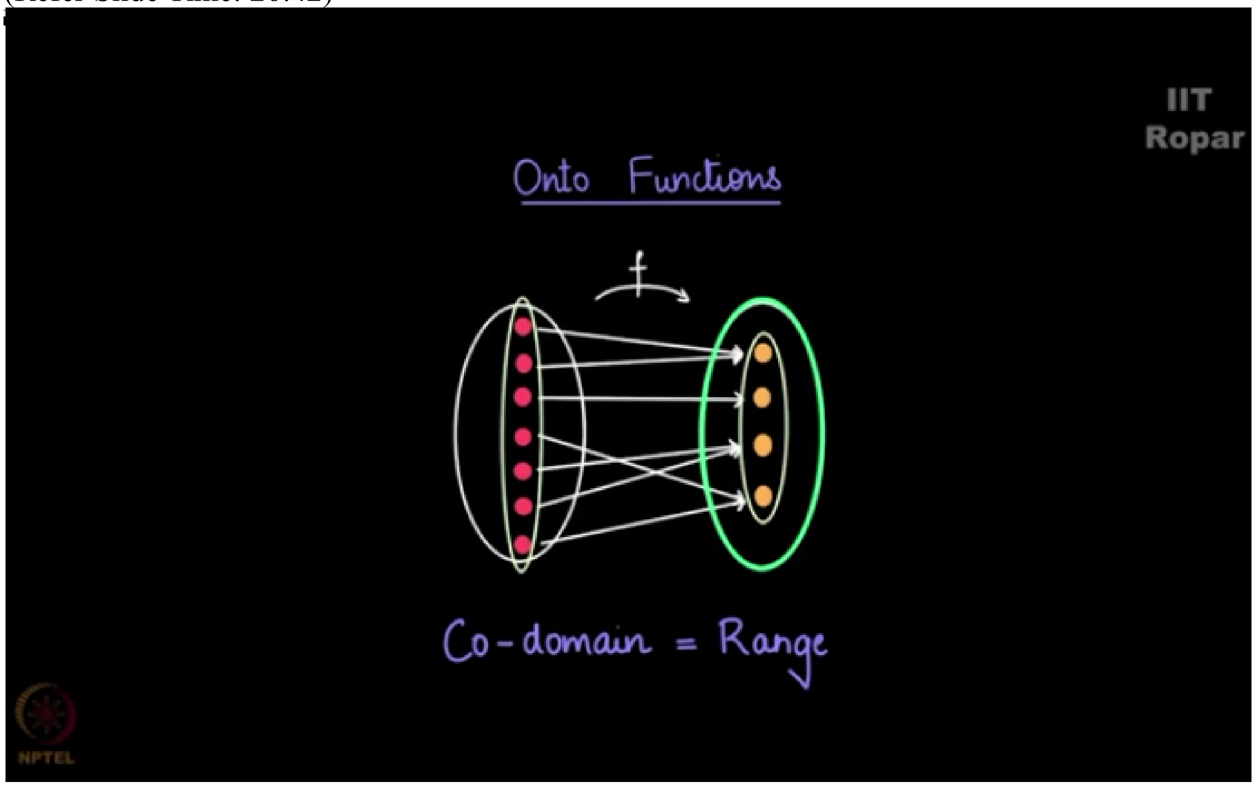

Now if a function is onto you have seen that co domain is equal to range, how? You are sure that every element in the co-domain has a pre-image in the domain and hence every element in the co-domain must be your range. So when you say that a function is onto, you can close your eyes and tell that co-domain equals range in this case.

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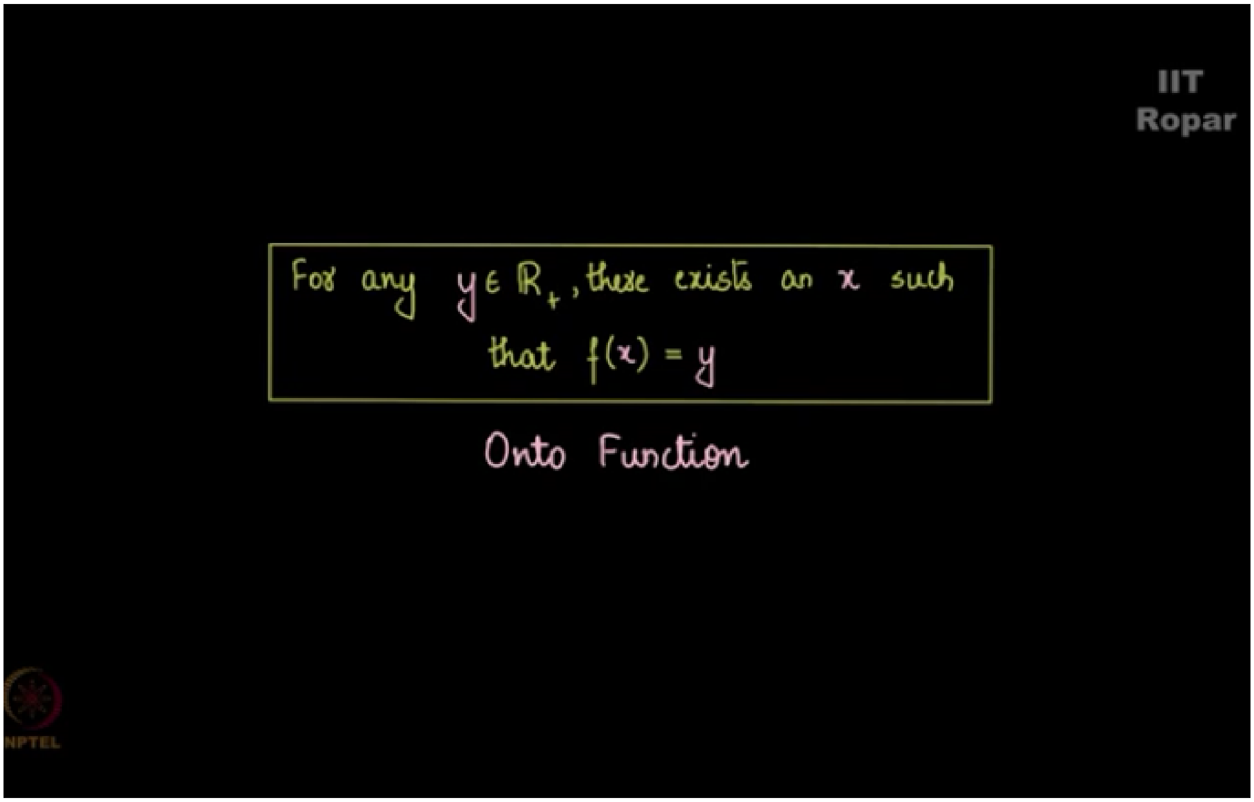

Now how do you prove it? For any Y belonging to the co-domain there exists an X, you must show that there exists such a X belonging to the domain such that F(x) = Y. If you show this then you have proved that a function is onto.

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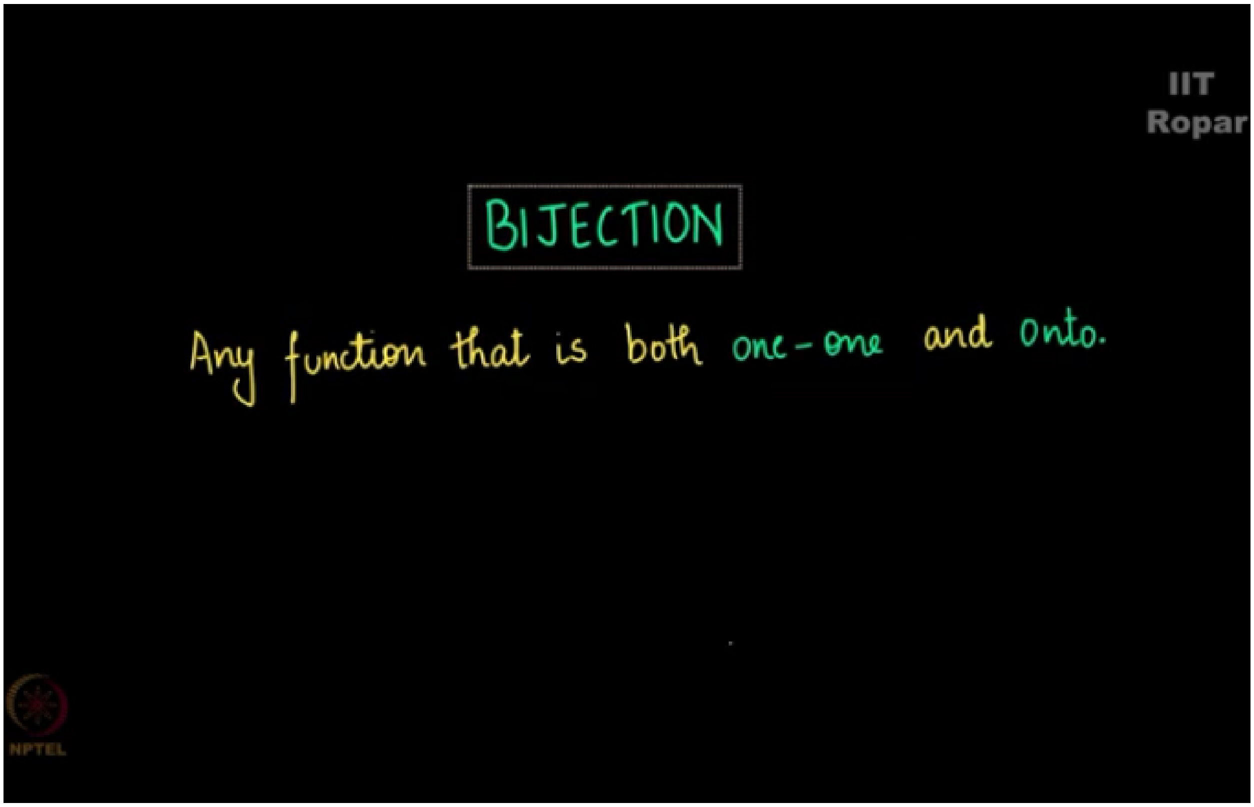

Now once you see a onto function and the one-one function, the immediate next step is defining a bijection.

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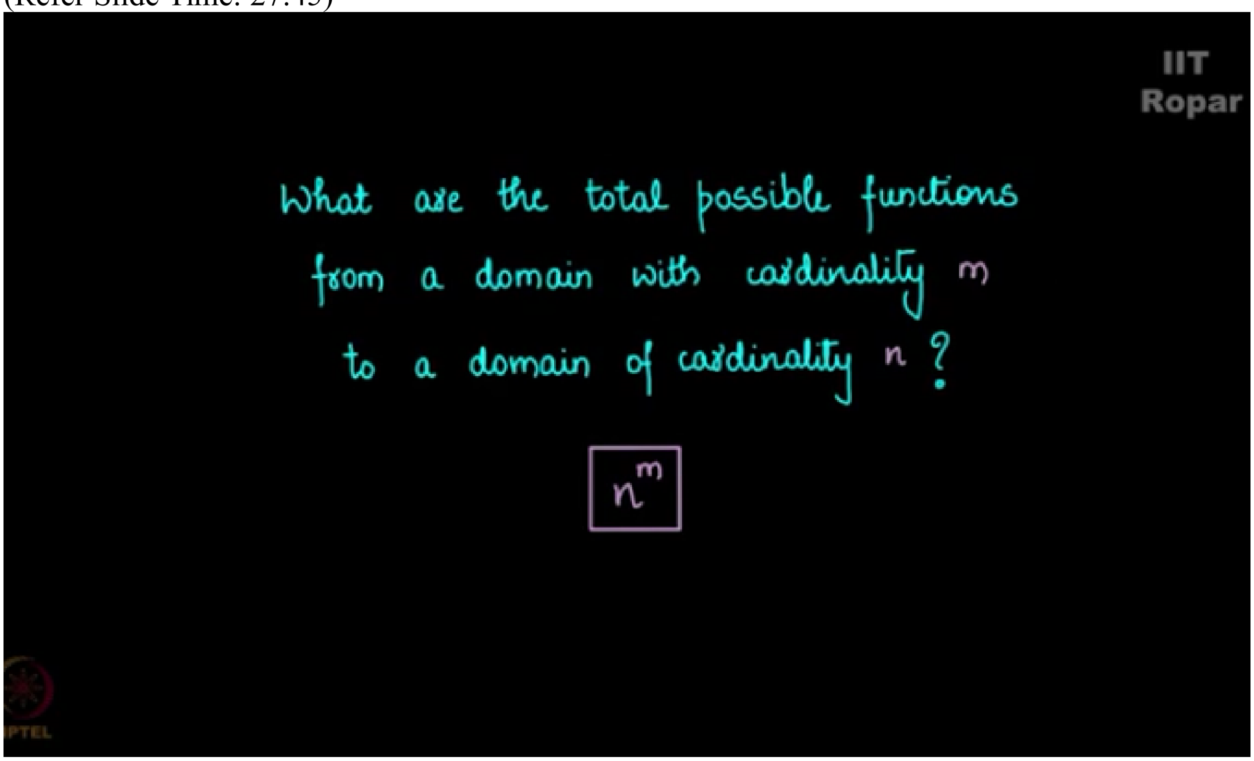

Any function that is both one-one and onto is called a bijection, we have seen several examples of bijections, right.

Now a counting question comes here,
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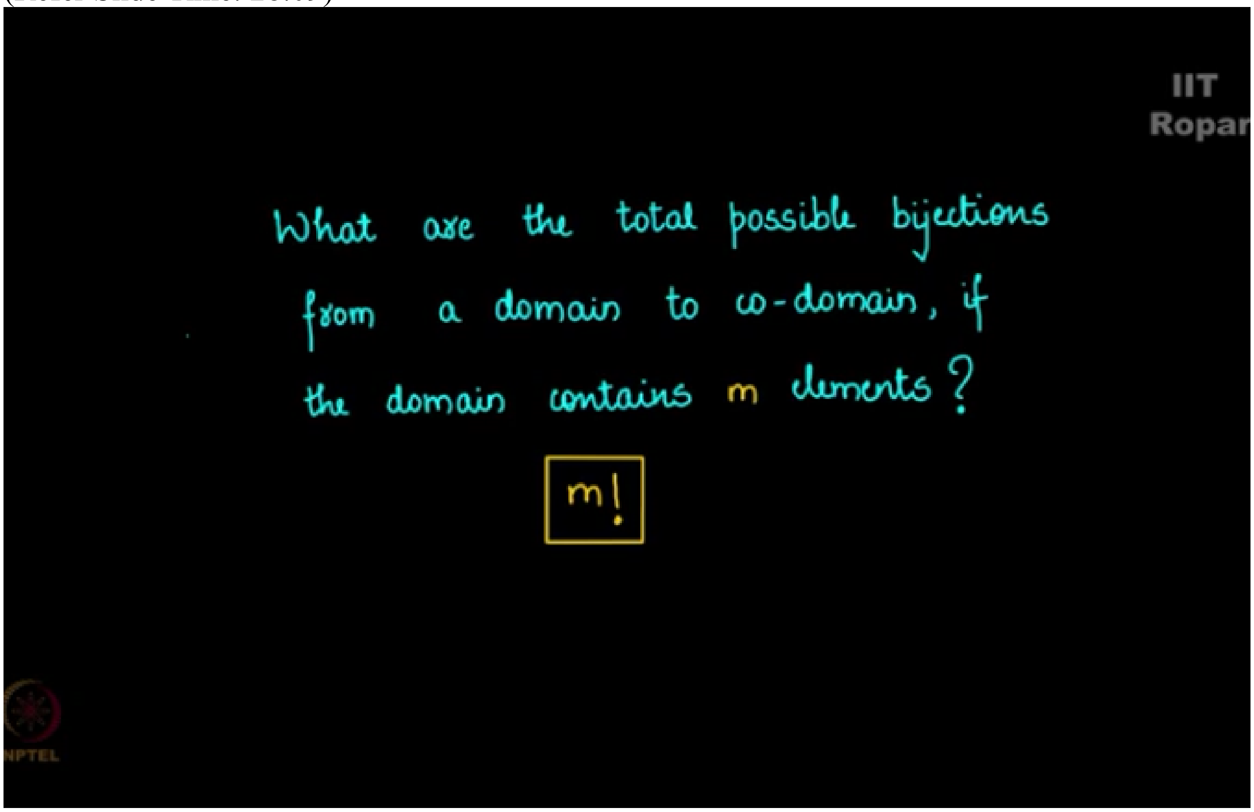

what are the total possible functions from a domain with cardinality M to a domain of cardinality N, right? What are the total possible functions? We see that it is N to the M. Now what are the total possible bijections? If the domain contains M elements it is M factorial in number, the total possible bijections you have seen how it is true, you've also seen the total possible one-one functions, right, and one case it is NPR follows permutation and in the other case it is a factorial itself, right. For the total possible onto functions we have seen that in the chapter principle of inclusion and exclusion.

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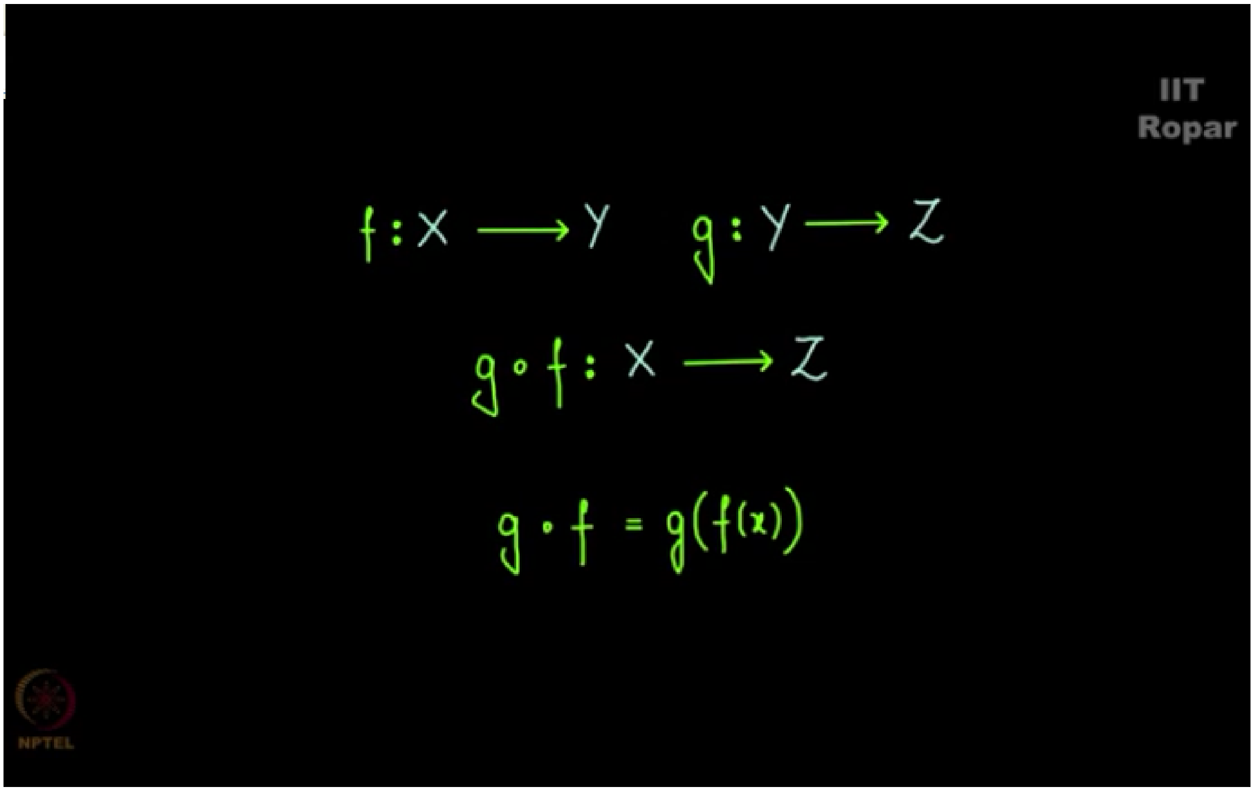

Now the next one is composition of functions, we have seen that if a function is given to you from X to Y, G is another function from Y to Z then G composition F is a function from X to Z, G composition F is actually G of F(x). The input value to G(f) is actually, G composition F is actually your values from the range of F, right, and from the range of F you take the values you apply G on it and you get the values of G composition F, that is elements in Z, so this is called G composition F.

If I remember we had seen the example of students name, birth date and birthday, right, you give a function, you give a student's name you finally get the birthday as your final output, that was the example which we had seen.

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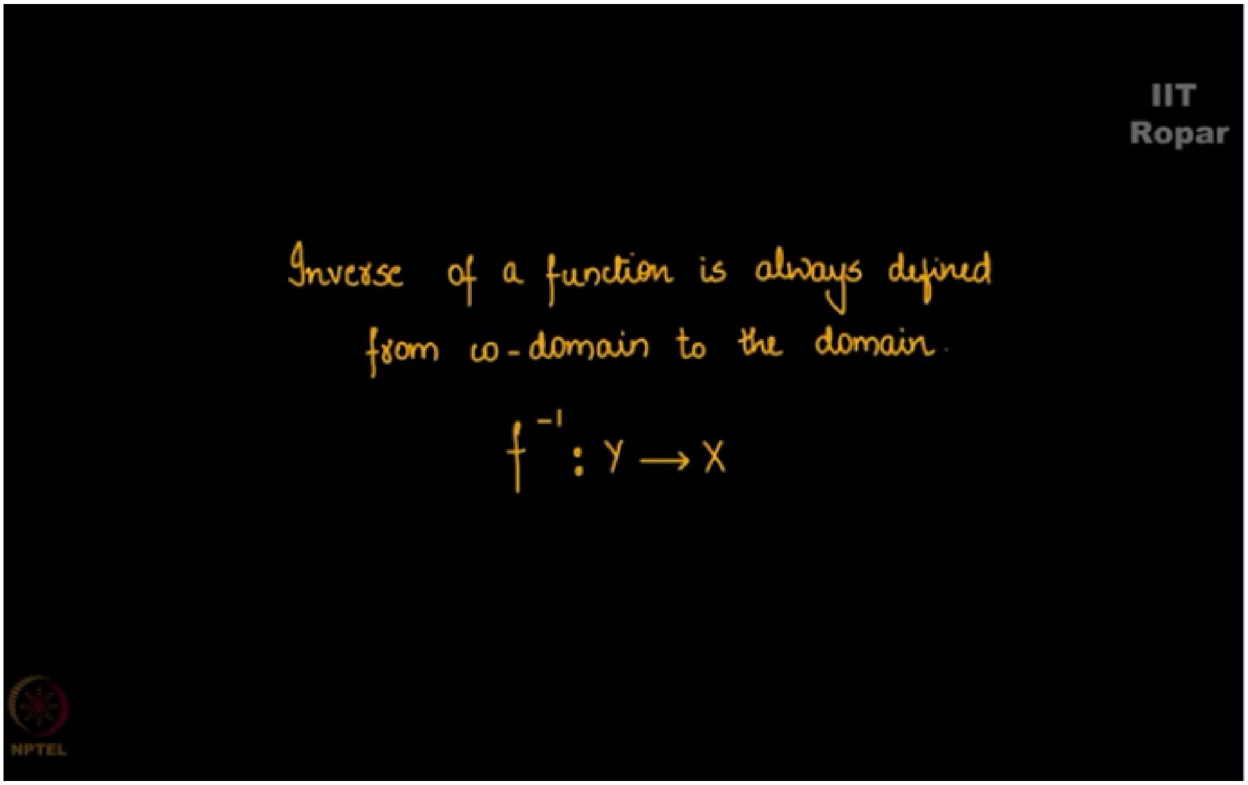

Now the last topic in functions is the invertibility of a function. Inverse of a function is always defined from co-domain to the domain, right, all this while we were talking of a function from domain to co-domain, now inverse is from codomain to domain. F inverse is from Y to X, and when is this possible? Inverse is possible only if the function is bijective, otherwise it is not.

If it is unclear to you, you must probably watch the videos once again. A great tool in mathematics which comes handy in solving problems is what is called the mathematical induction.
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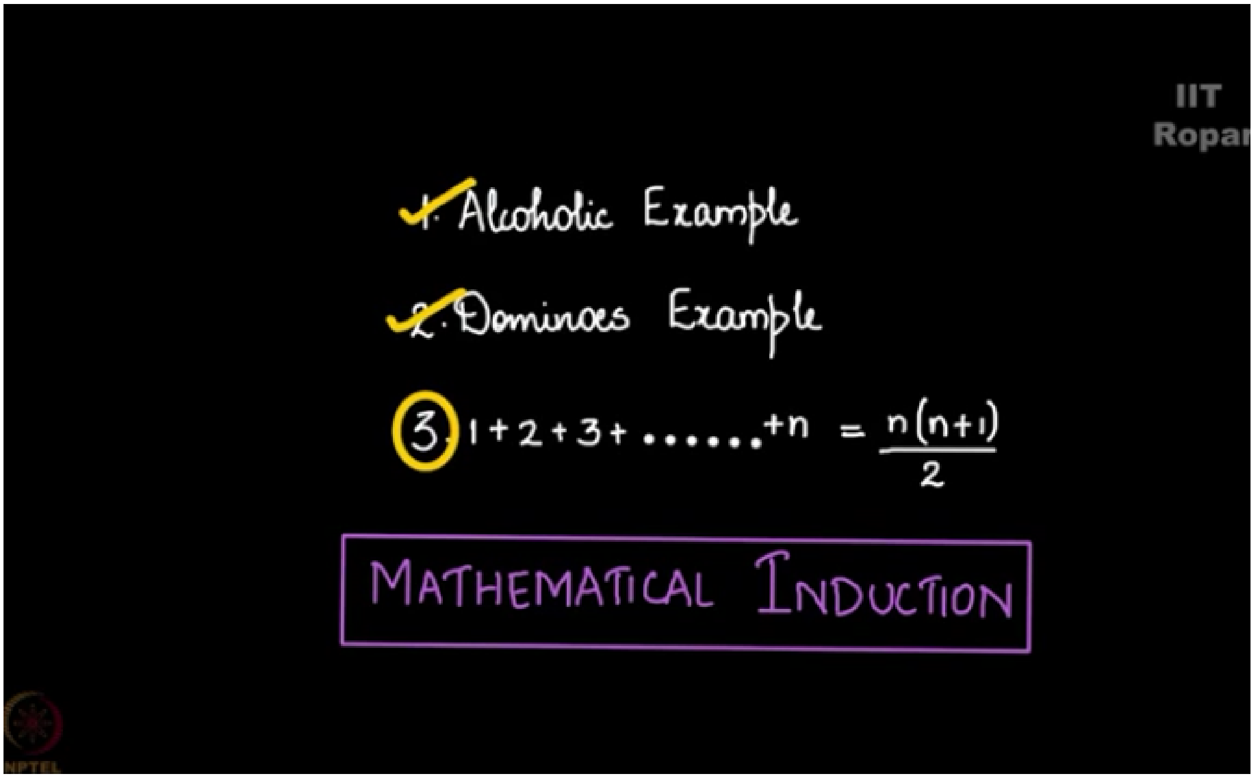

We started the chapter with seeing this example of alcoholics, we see that if a person de-alcohol, if a person stops drinking or stops becoming alcoholic then he can continue doing so for others and finally in the city you will not have any alcoholics, we saw this by induction.

We then saw the dominos falling, if you disturb one domino it falls under second one, second one falls under third one and so on it continues, right up to the last one. And we saw how 1 + 2 + 3 up to so on + N is N x N+1/2, well we didn't stopped with this, we saw several other examples using mathematical induction we proved them, right.

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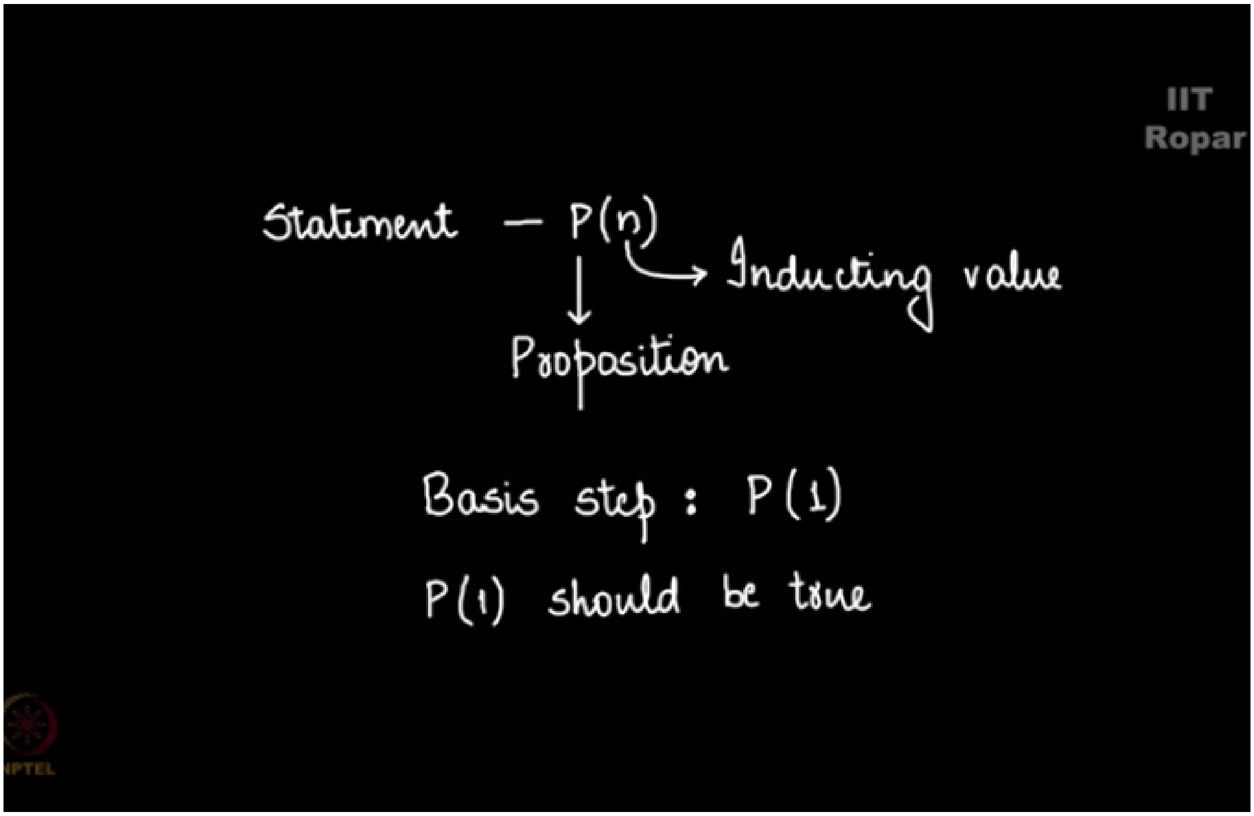

Well, what is induction? What constitutes induction is the following, you will be given a statement the P(n) which is a proposition, right, N is what you are inducting on, it is called the inducting value. Then you prove the proposition for N =1, it goes by the name basis step and you must ensure that P(1) is true. Once P(1) is true you can move to the induction hypothesis, what does it say? It says that the proposition is true for some K, right, you can assume this.

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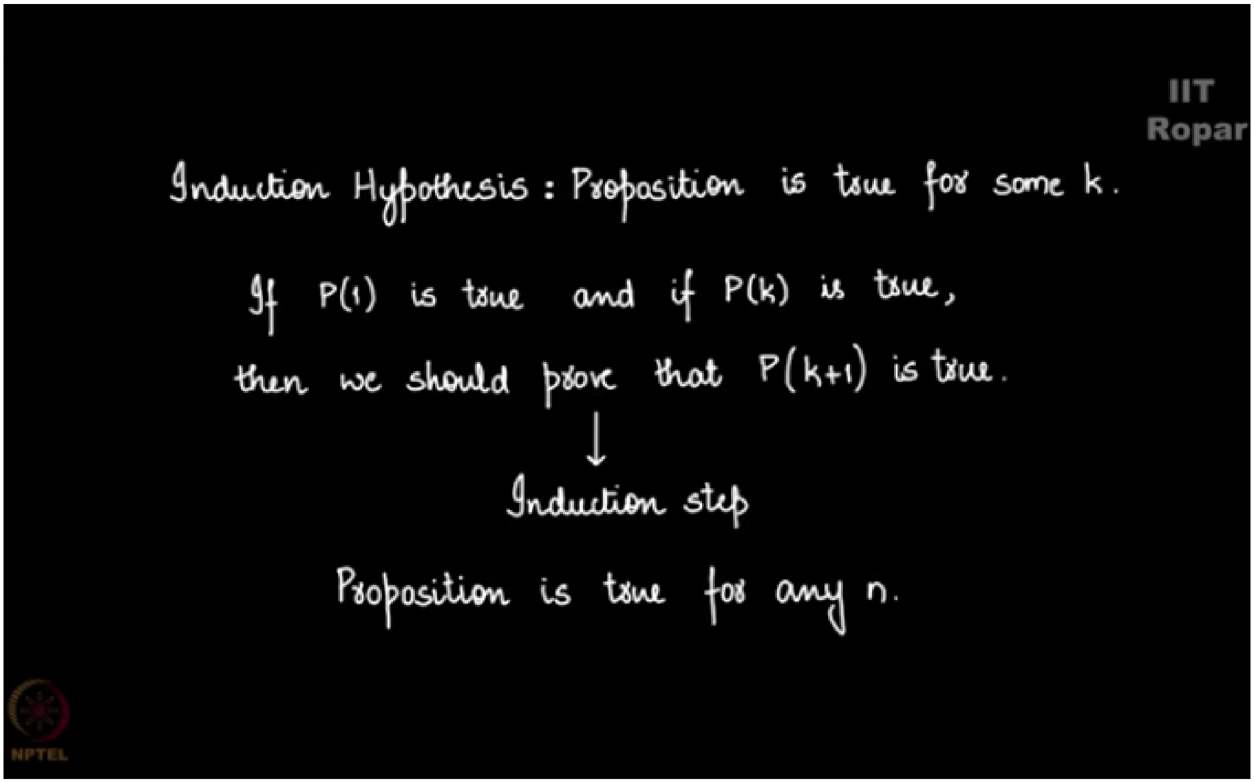

Now once it is true for P(1), once the proposition is true for P(1) for N = 1 and N = K, for both of these if it is true then we should prove that P(K+1) is true, this is called the induction step. Here is the major part where you must prove that the actual hypothesis is true, before this step whatever you have done is called simple, you just showed that P(1) is true then you assume that it is true for some K and then comes the portion where you have to show that P(K+1) is true, this goes by the name induction step.

Once you prove that induction step is true then you can tell that the proposition is true for any N, once this is done the proof is complete by mathematical induction, we had seen several examples on this.

The next topic in the week was the pigeonhole principle,
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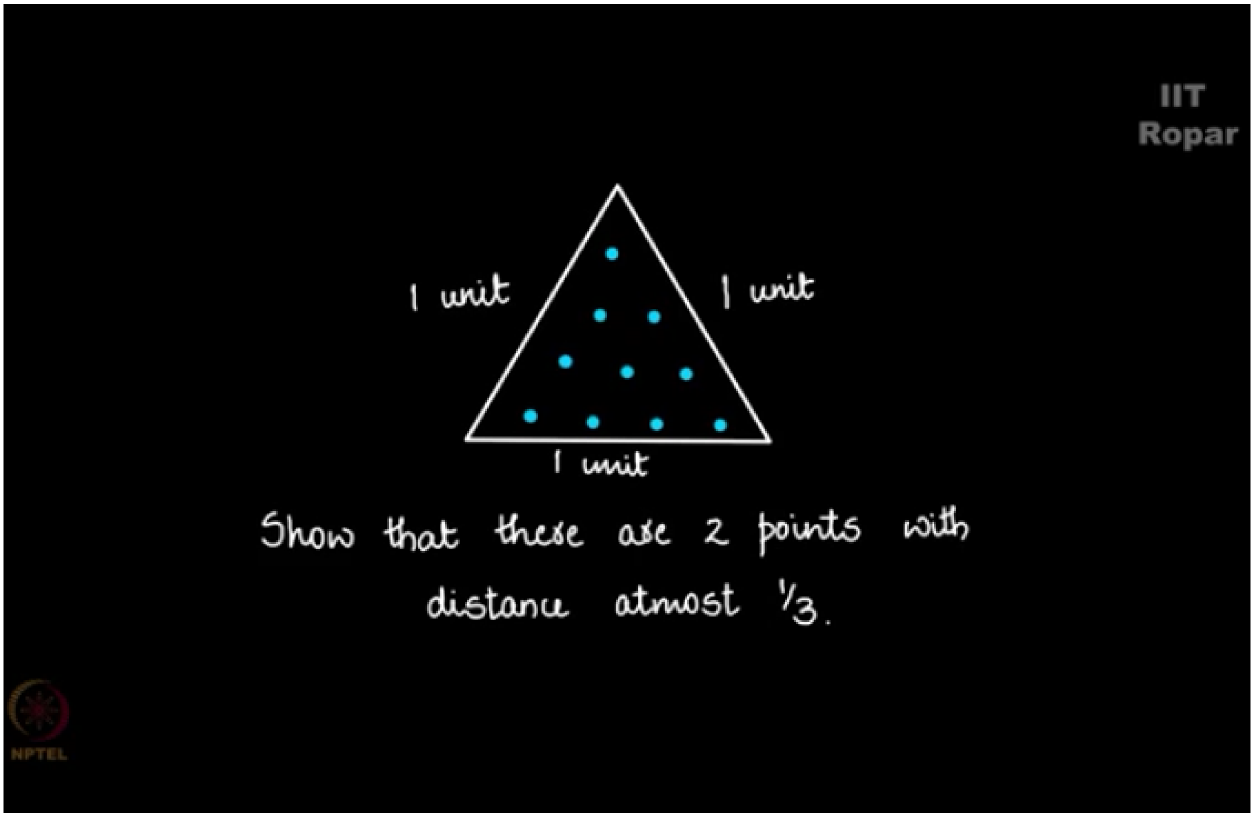

before moving to this example let me just paraphrase what is pigeonhole principle? If you are having N pigeons and M pigeonholes where M is less than N then there exists at least 1 pigeonhole which has 2 pigeons, right, it is very obvious looks very easy too.

Now we can use this in solving several problems, if you remember this equilateral triangle each of one unit site you must show that there are two points with distance at most 1/3, we didn't as using pigeonhole principle, what we did was we divided the triangle into 9 equivalent triangles, right, smaller triangles and then we observed that there exist at least one triangle where there are two points of distance less than 1/3 between them, so we used pigeonhole principle here, we ended the week with a very cute question,

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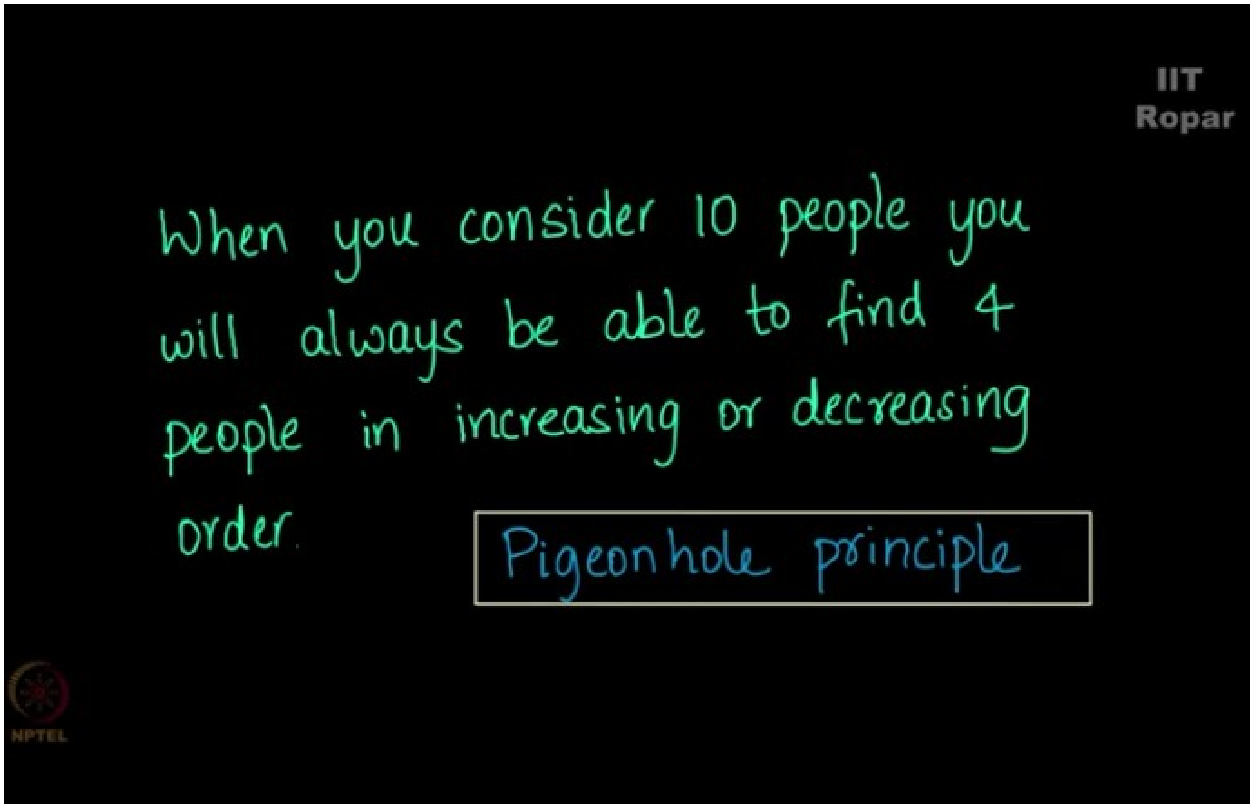

consider 10 people standing you will at least or rather you will always be able to find 4 people in increasing or decreasing order of their height, we had used pigeonhole principle out of the blue to solve this question, I mean when you read this question can you even guess that you can use PHP to rather pigeonhole principle to solve this.

If you remember how we had done, we considered their heights, right, it was not an increasing or decreasing order then we saw that their height, if you consider the proof again you will be able to recollect it we had done it using PHP, that was with the induction and pigeonhole principle.

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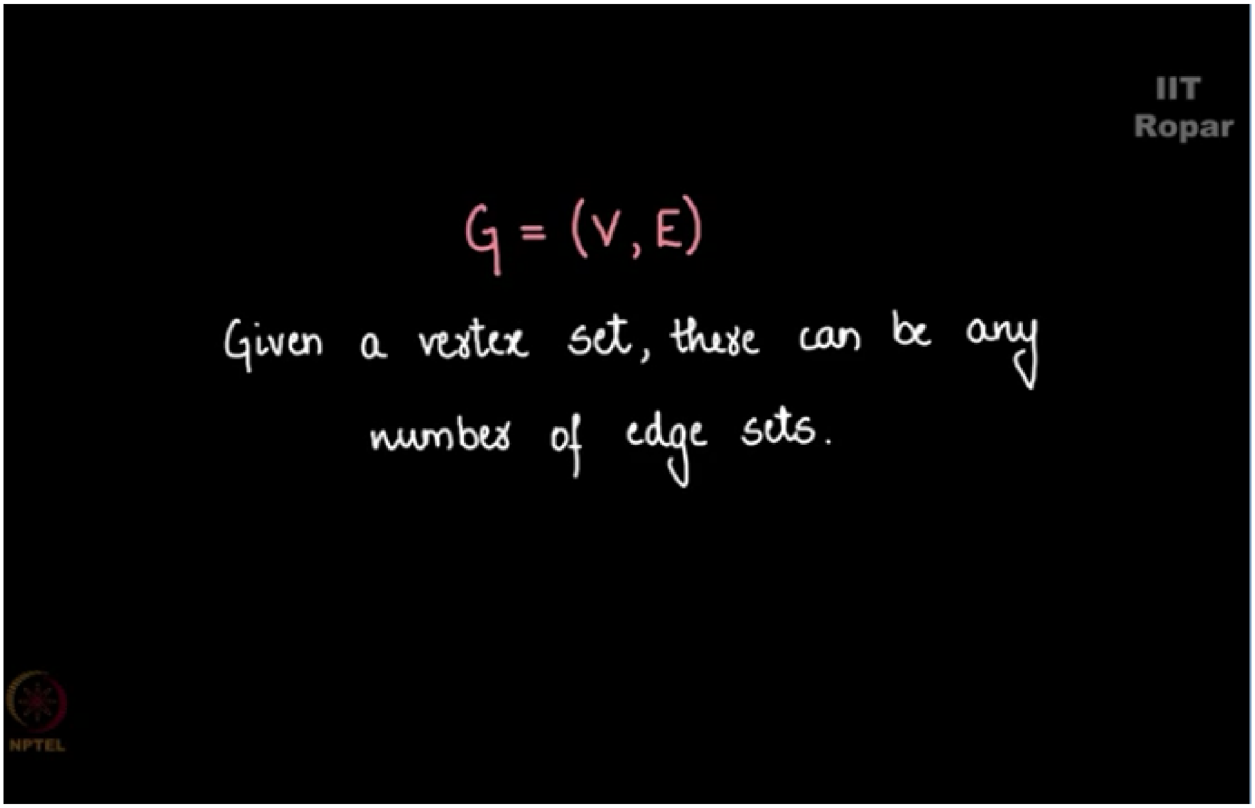

After mathematical induction and pigeonhole principle we moved on to the topic called graph theory, we studied this tubal V, E it is a graph where there is a vertex set and there are edges between the vertices. We take some vertices and we draw lines between them and they are called as edges.

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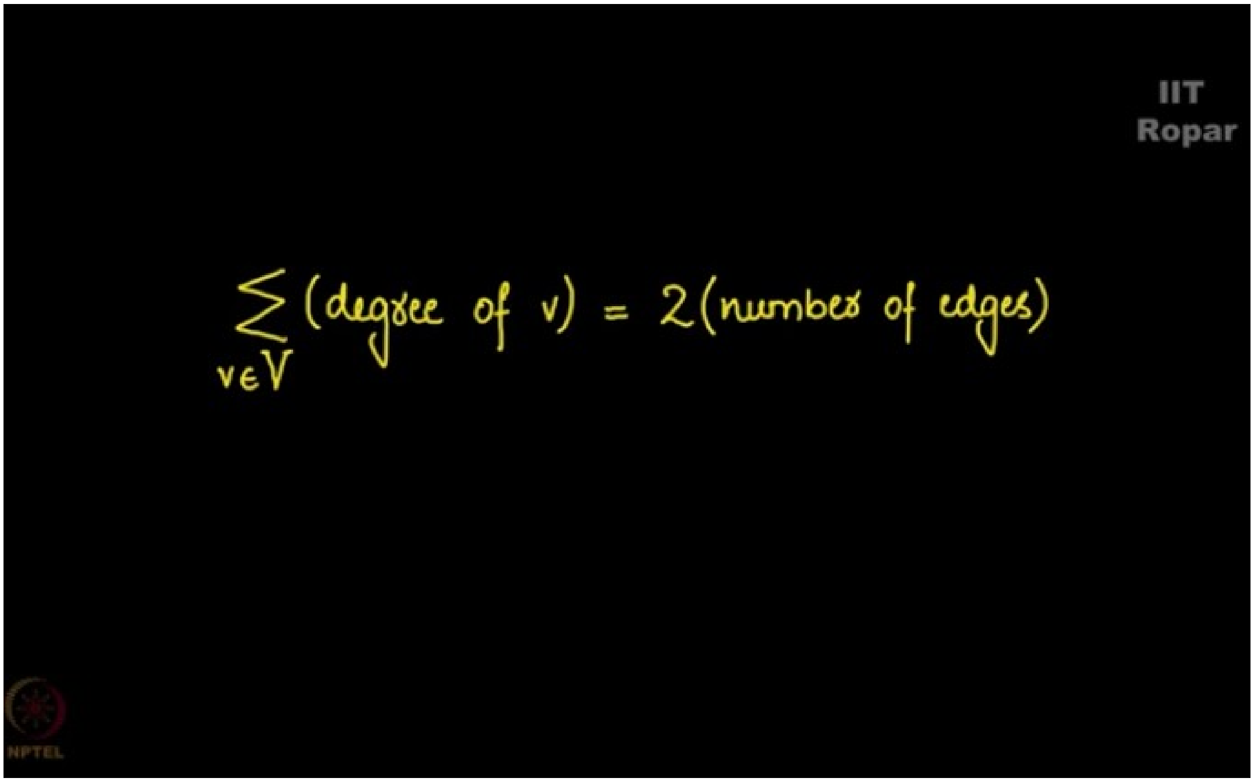

Now we studied what is called the handshaking lemma, what was that? Summation of all the degrees of the vertices, you take all the vertices given in the graph, sum up their degrees that will be equal to twice the number of edges in the graph. We have seen this, we have even proved it, so this is the handshaking lemma.

Before I move ahead we have seen several examples of graphs, a simple graph is the one which we majorly concentrated on, what is a simple graph? There are no two edges between two vertices, there is only one edge that is a simple graph.

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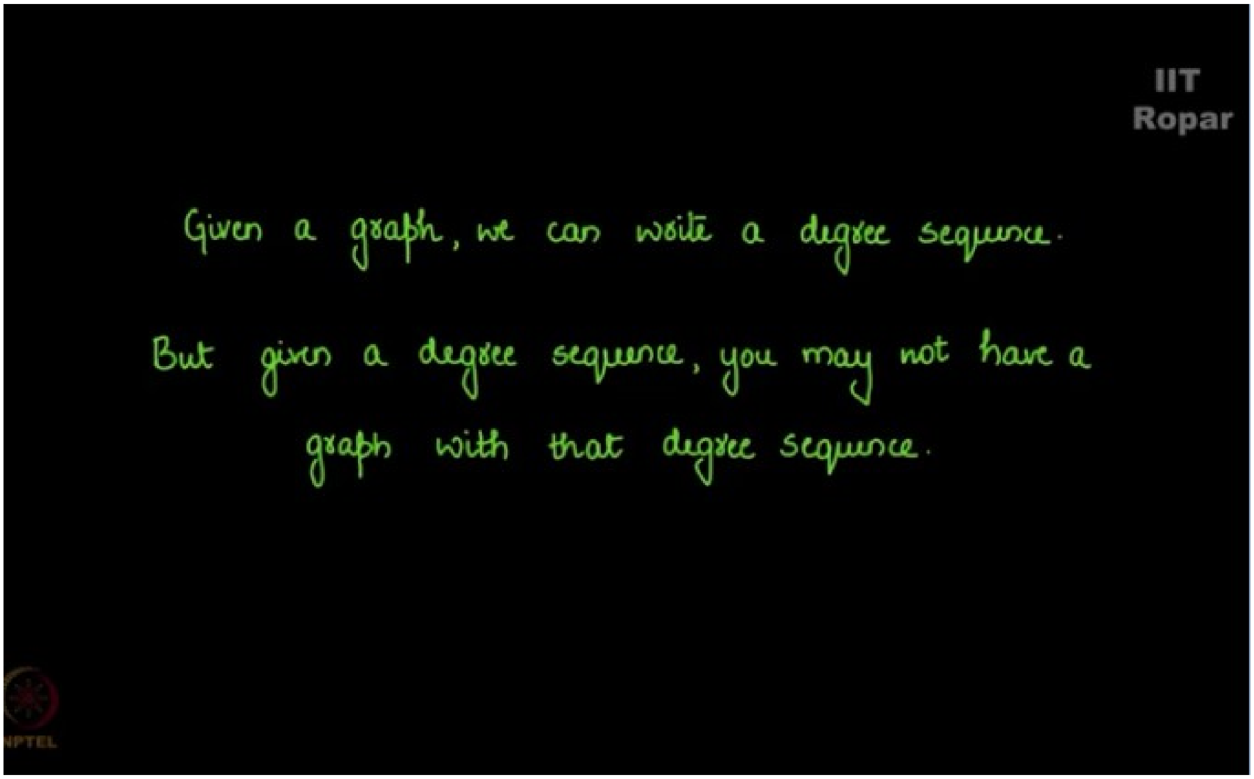

Now given a graph we saw that we can write a degree sequence, but given a degree sequence you may not have a graph with that degree sequence, we learn this. Now what is a degree sequence? You take all the degrees in the graph and you write them in ascending order, in connection with this we saw what is called the Havel Hakimi Theorem and we took a graph like this,

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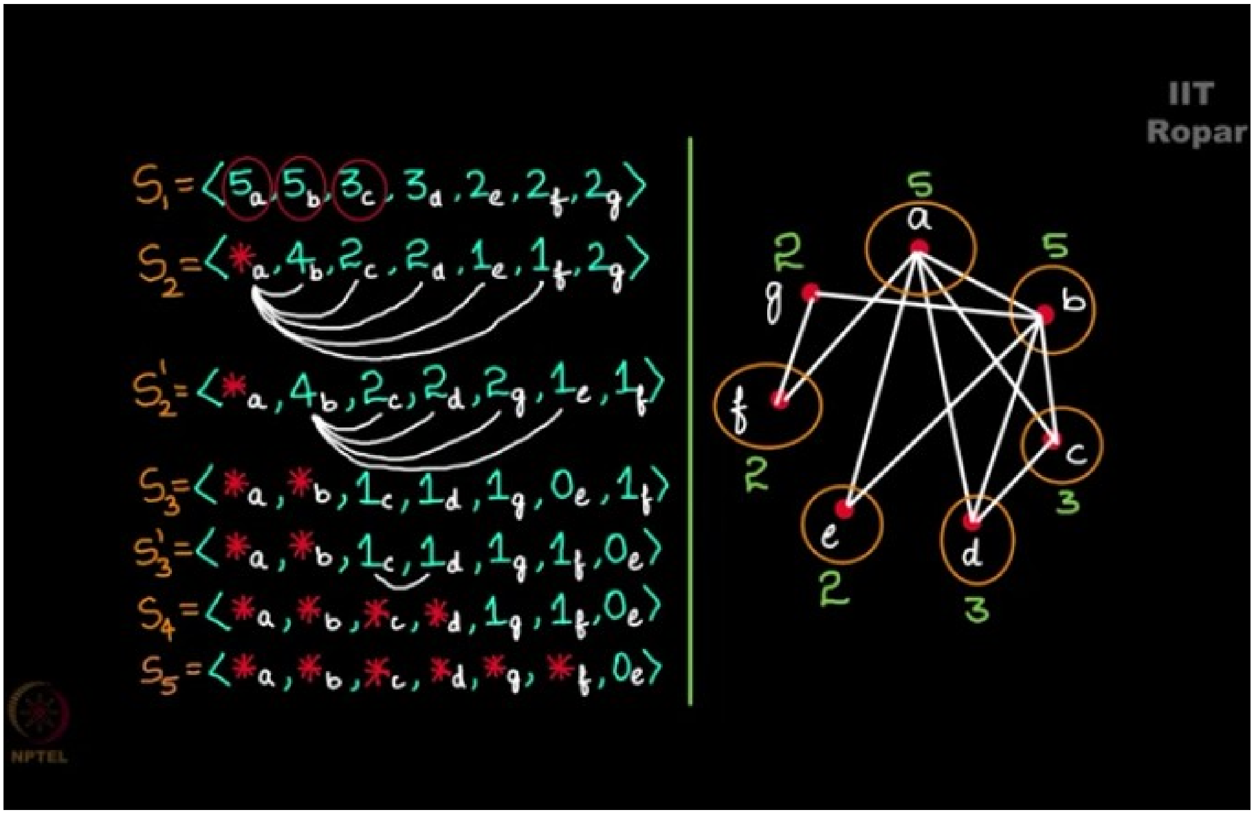

and we saw if we can, we actually started with the sequence S1, then we saw if we can actually construct this graph here, right, then it was a huge process, you start with this vertex A exhaust the degree of it, go to the next vertex B, go to the next vertex C, you like this exhaust all the degrees of A, B, C, D, E, F, G here and then you finally obtain the graph with the given degree sequence, so this was it with the Havel Hakimi theorem.

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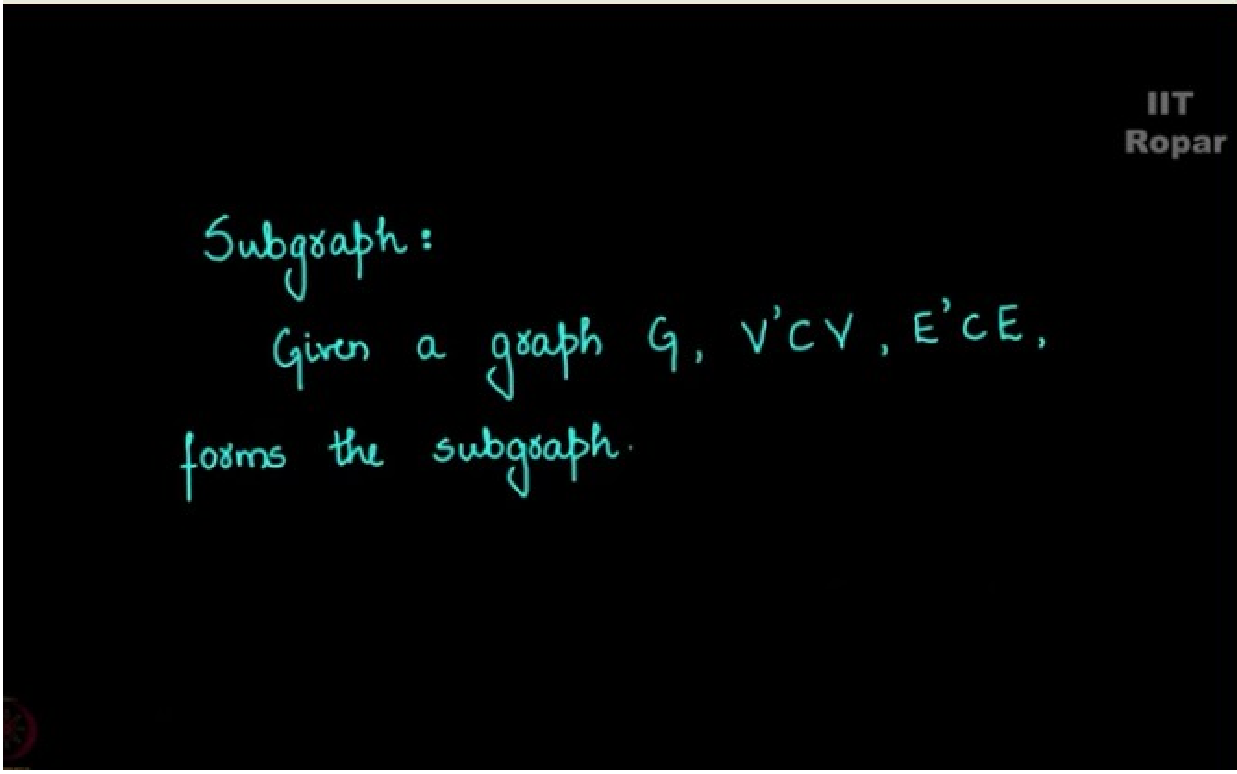

We then learnt what is called a subgraph it is a smaller graph, in simple words or you have the vertex set you take fewer number of vertices or even the same number of vertices and construct a graph based on that, what you get is a subset of your edge set also, it forms the sub graph. Here we had two categories, the induced subgraph and this spanning subgraph, what is this spanning subgraph? A subgraph which covers all the vertices, right, but it has fewer number of edges is the spanning subgraph, what is the induced subgraph? It is a subgraph where between 2 vertices if you have any two vertices in your subgraph you have all the edges between them in the subgraph, you are not going to leave out any edges between those two vertices this is called the induced subgraph, so for subgraph we studied two of them induced and spanning.

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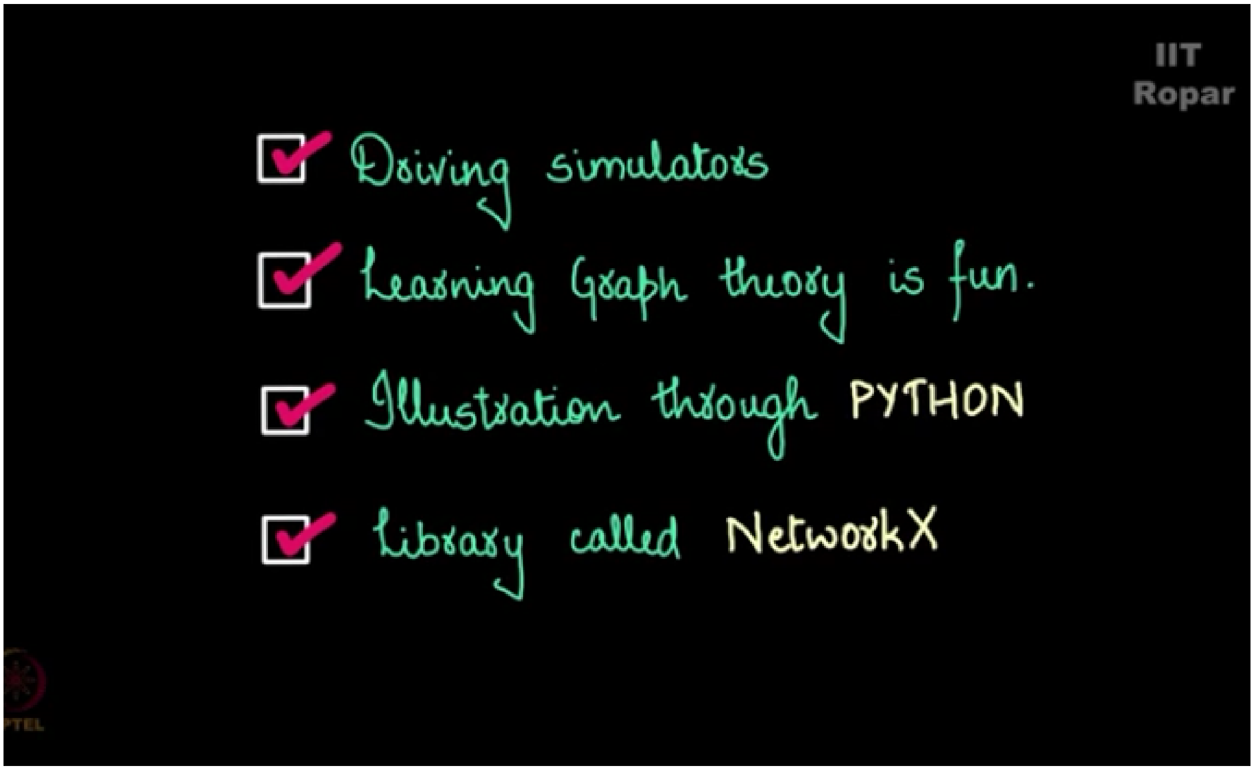

We then learnt that, we had seen what is the motivation behind learning graph theory? We actually saw that graph theory is fun. We had seen python, right, there were a few programming sessions, we had lengthy videos on some commands of graph theory there, and we also learnt this library called the NetworkX, basically we learnt a lot of things on python, to summarize we learnt how to draw a graph? How to add vertices? How to add edges? How can we check if a graph is connected? How can we check if it is Eulerian and so on? We saw everything there, we could even see if we can actually draw the graph or not given a degree sequence, right, we had long sessions with python.

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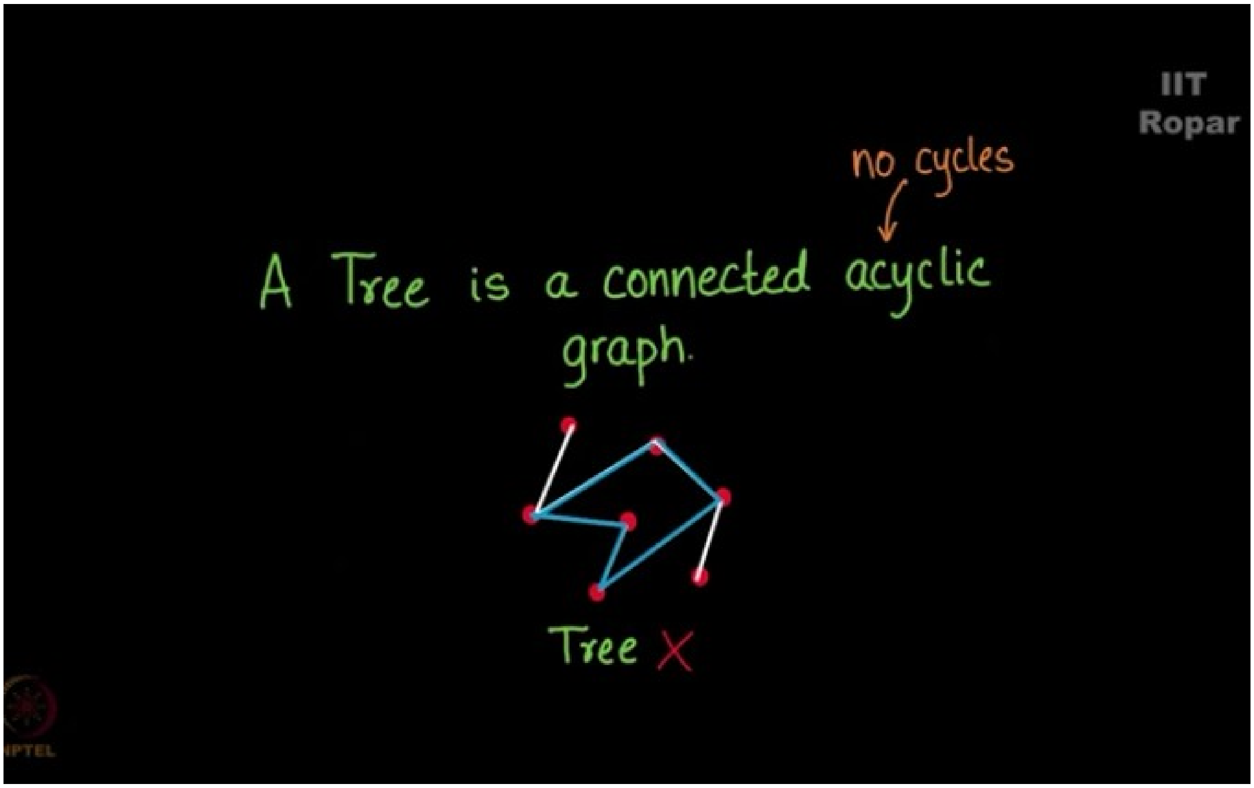

We learnt what is called a tree, it is a connected acyclic graph, there are no cycles in it and it is a connected graph.

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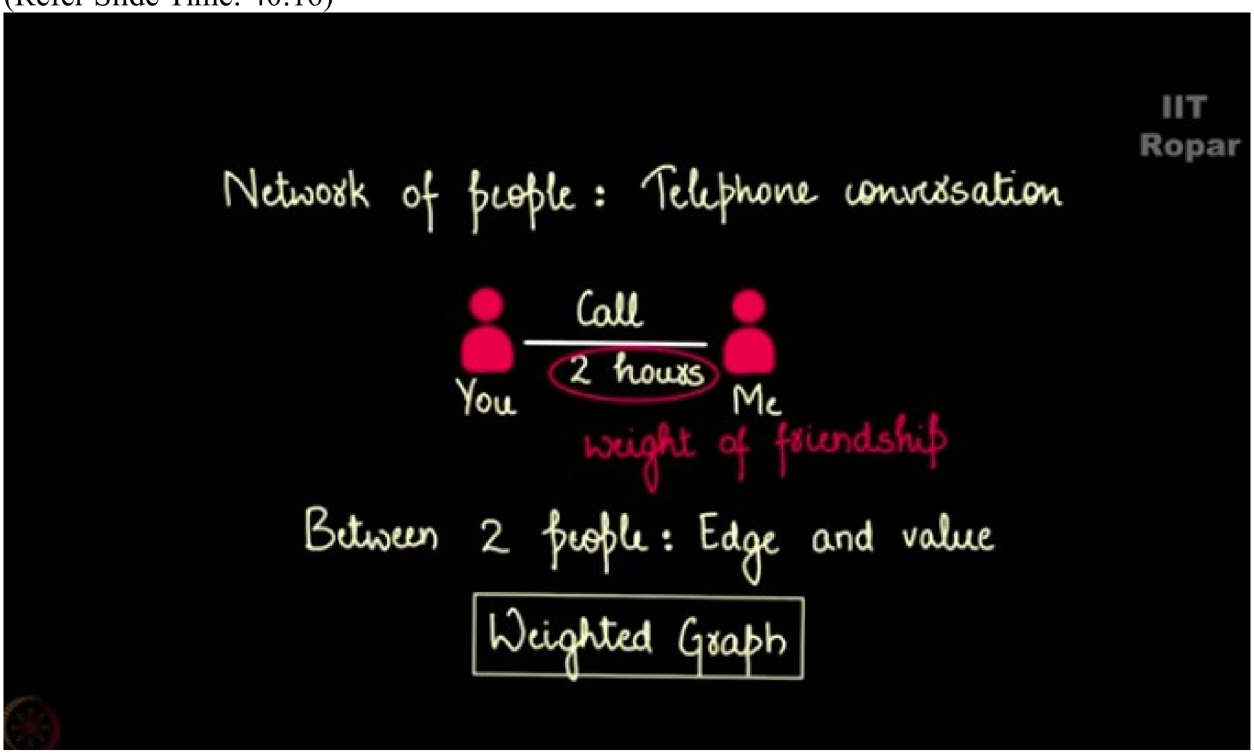

Now not just simple graph, we also learnt what is called the weighted graph. Basically between any two vertices on an edge there is something called the weight, we started with this motivational example of telephonic conversation between people, if you call me and the number of minutes or hours we speak to each other becomes the weight of the edge or the weight of the friendship graph here, right, so between 2 people you always have an edge and a value to the edge, so this is called the weighted graph, you assign weights to the edges.
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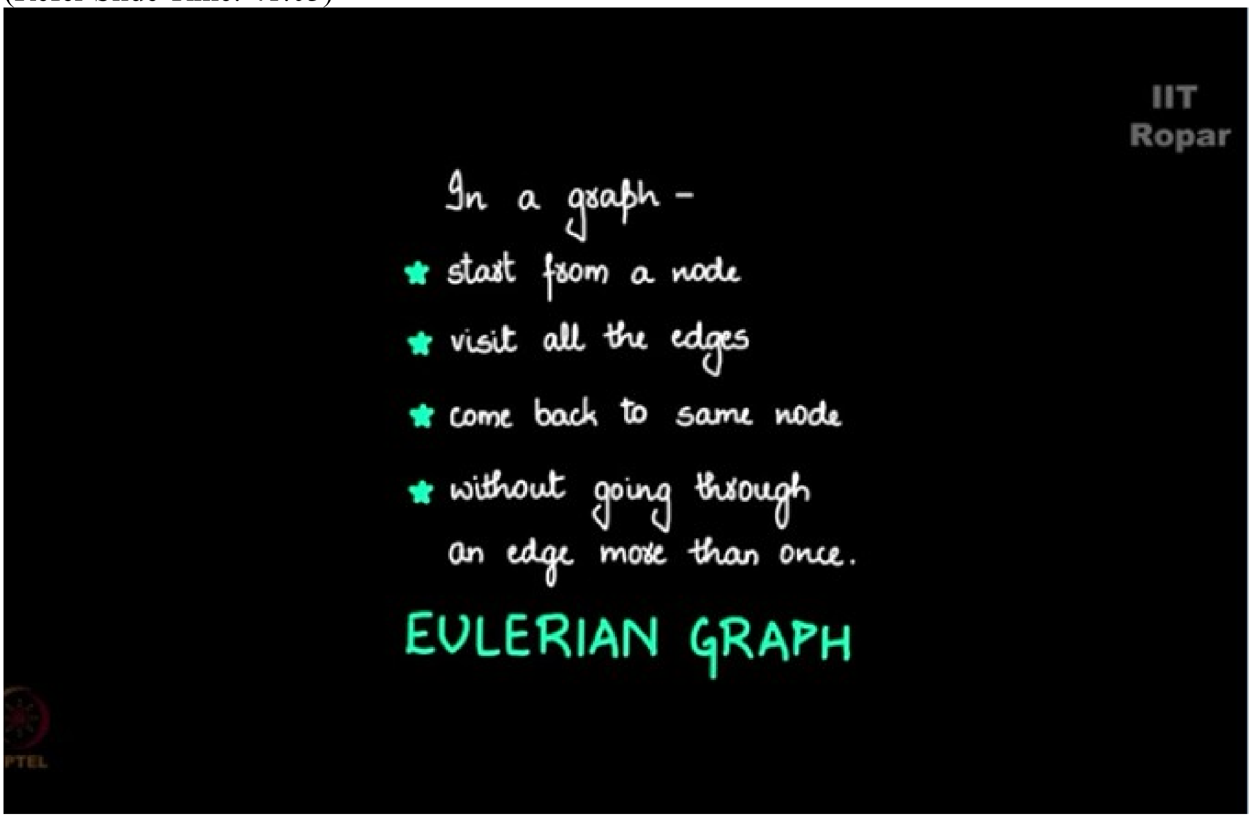

Then we learnt the Eulerian graph, what happens in an Eulerian graph? You start from a node, you exhaust all the edges that is you visit all the edges and you come back to the same node without going through an edge more than once, the rule is you must not repeat an edge more than once, but you must visit all the edges, and you must come back to the same point where you initially started, so this was it with the Eulerian graph, and we have seen several examples where we actually starts from a node go through all the vertices and comeback to the same node, we checked even on python whether a graph is Eulerian or not, if I remember the command was is Eulerian.

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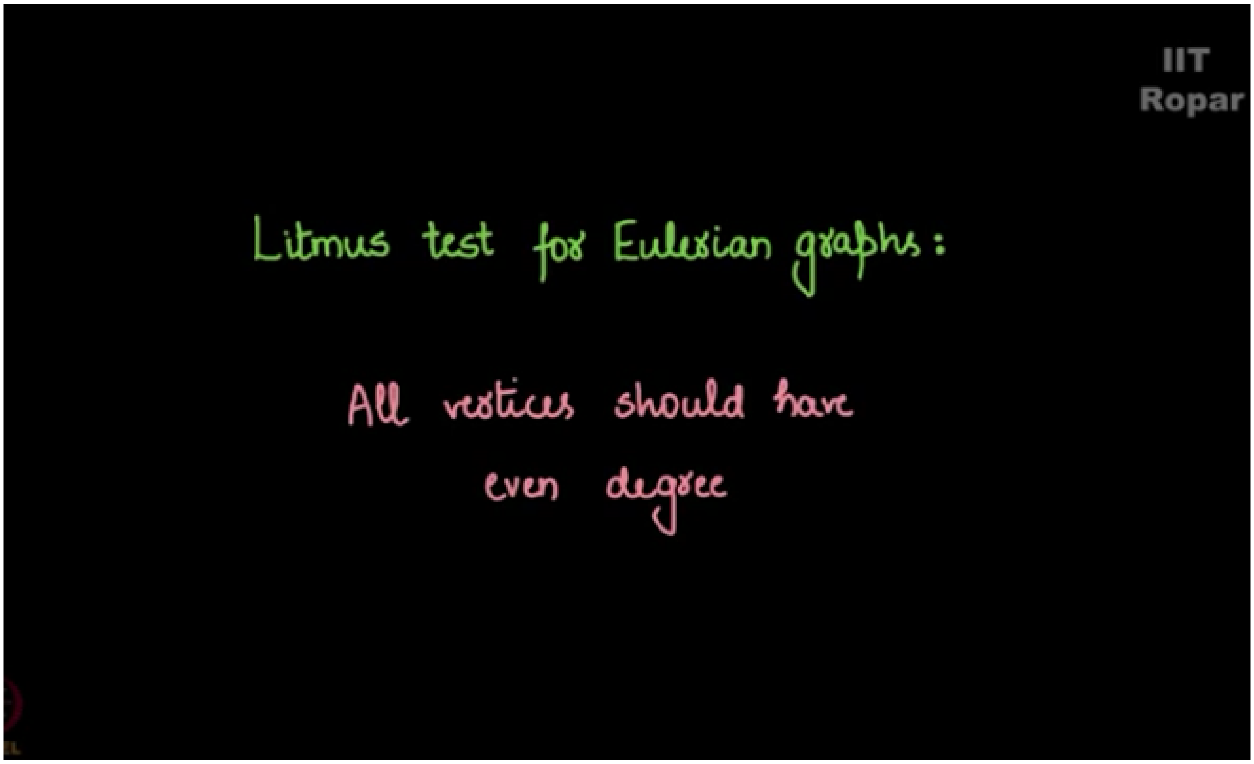

And then moving on we saw that the litmus test for Eulerian graphs is that, all the vertices should have even degree also if all the vertices have even degree the graph is Eulerian, it is an if and only if condition here, so this is a litmus test.

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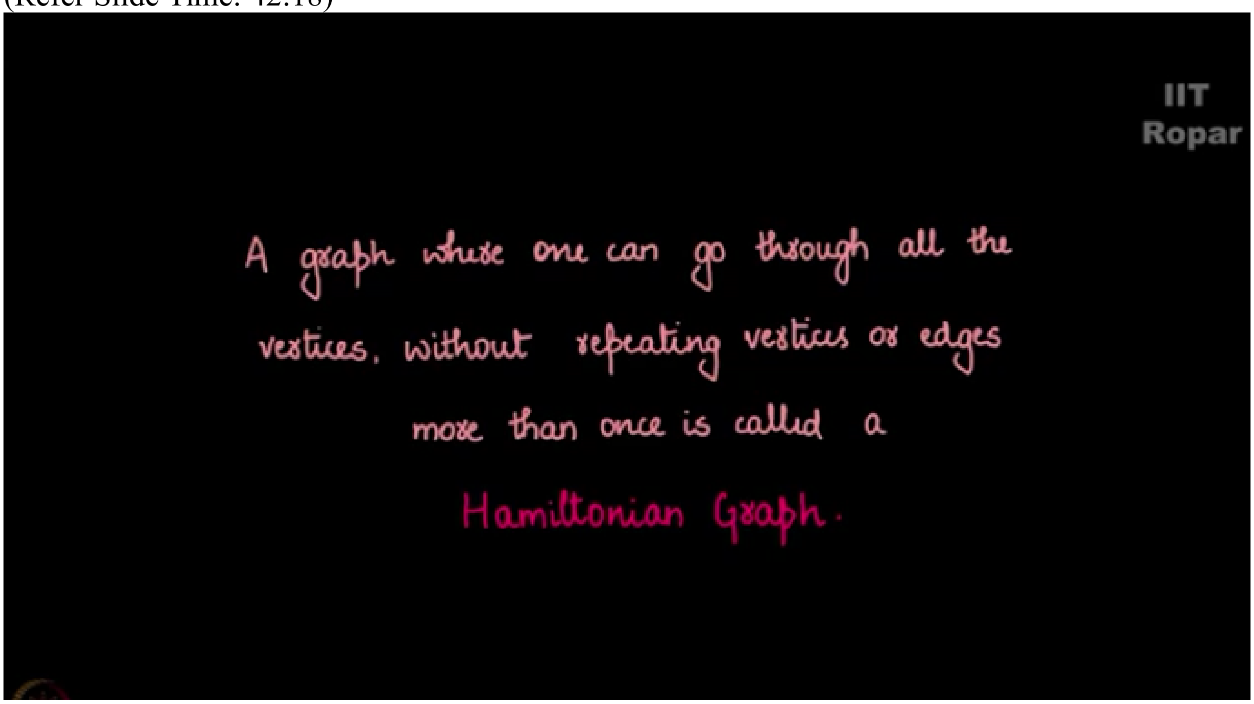

A graph where one can go through all the vertices without repeating vertices, obviously you will not repeat edges more than once is called a Hamiltonian graph, so we studied several examples where you start from a node, you go through all the nodes and comeback to it, right, basically you are searching for a Hamiltonian cycle in the graph. A Hamiltonian cycle is what? You start from a node, take a cycle where you do not repeat the vertices and you come back to the initial point, right, after exhausting all the vertices.

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We then learnt what is called as the isomorphism. Iso means same, morphism means different, it’s actually an oxymoron you see, isomorphism, we saw a graphs which are isomorphic as well which are not, we even played a game if you remember right and the website was Eric Nicholson. We can check if two graphs are isomorphic or not, even on python the command goes like this is isomorphic, you get a true or a false value there.

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When a graph is disconnected its complement is always connected, you saw complement of several graphs, you see that if a graph is disconnected its complement is always connected. For example you take K5 rather any KN, you see that the graph is connected and its complement is disconnected. Whereas when you take a disconnected graph let's say on 4 vertices its complement is a complete graph on 4 vertices, but it's not always necessary that to complement of a connected graph is always connected, you must be really careful about the settle points here.

We also saw C5 where I had to rather tell this when I was talking of isomorphism, C5 is a cycle on 5 vertices, a star is also a C5. By star I mean not the graph theoretic star where there is a center vertex and all the other edges connected to it, I mean a little star.

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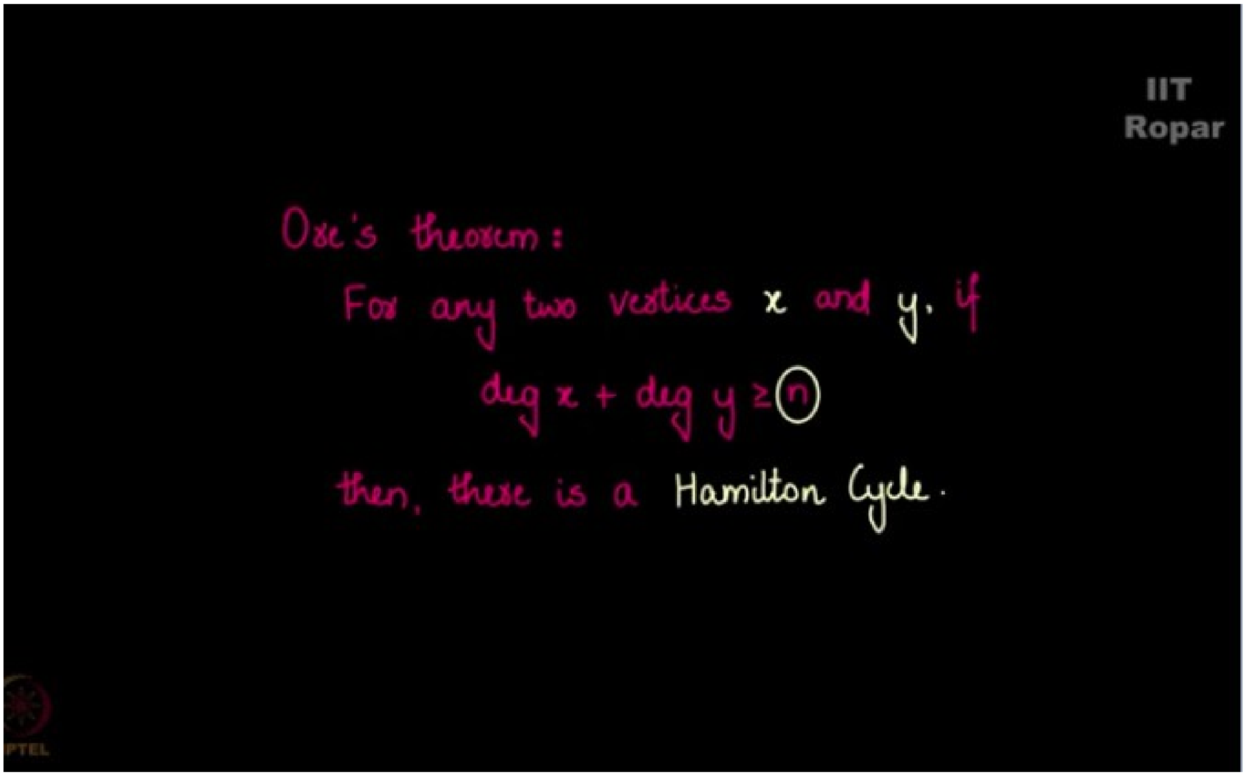

And then we saw what is called the Dirac's theorem and the Ore’s theorem. The Ore’s theorem is a stronger condition of the Dirac’s theorem, for any two vertices X and Y if degree of X + degree of Y is greater than or equal to N then there is a Hamiltonian cycle, so in Dirac’s what happens is, they say it for one vertex if degree of X is greater than or equal to N/2 that is, then there is a Hamiltonian cycle, so we had also seen the proof of Dirac’s theorem.

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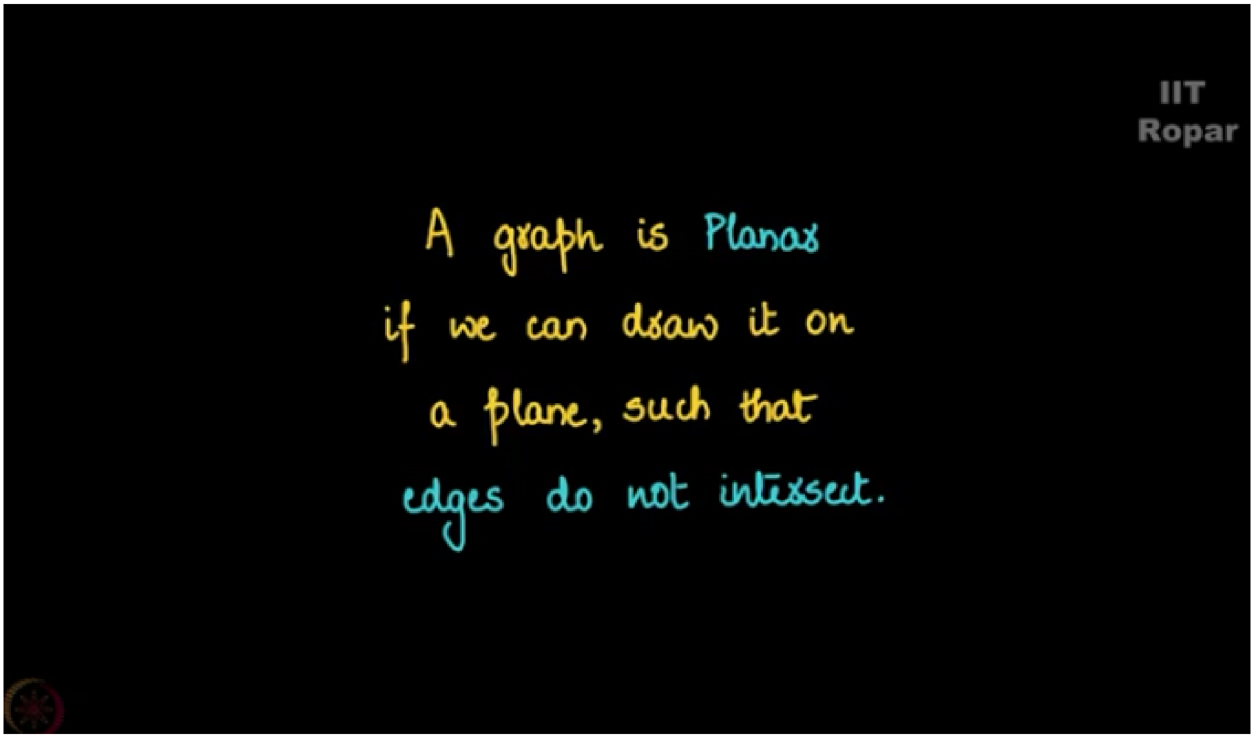

Moving on to planarity, we saw that a graph is planar if we can draw it on a plane such that no two edges intersect, basically we do not want crossovers, right, now when we speak of planarity regions emerge, right,

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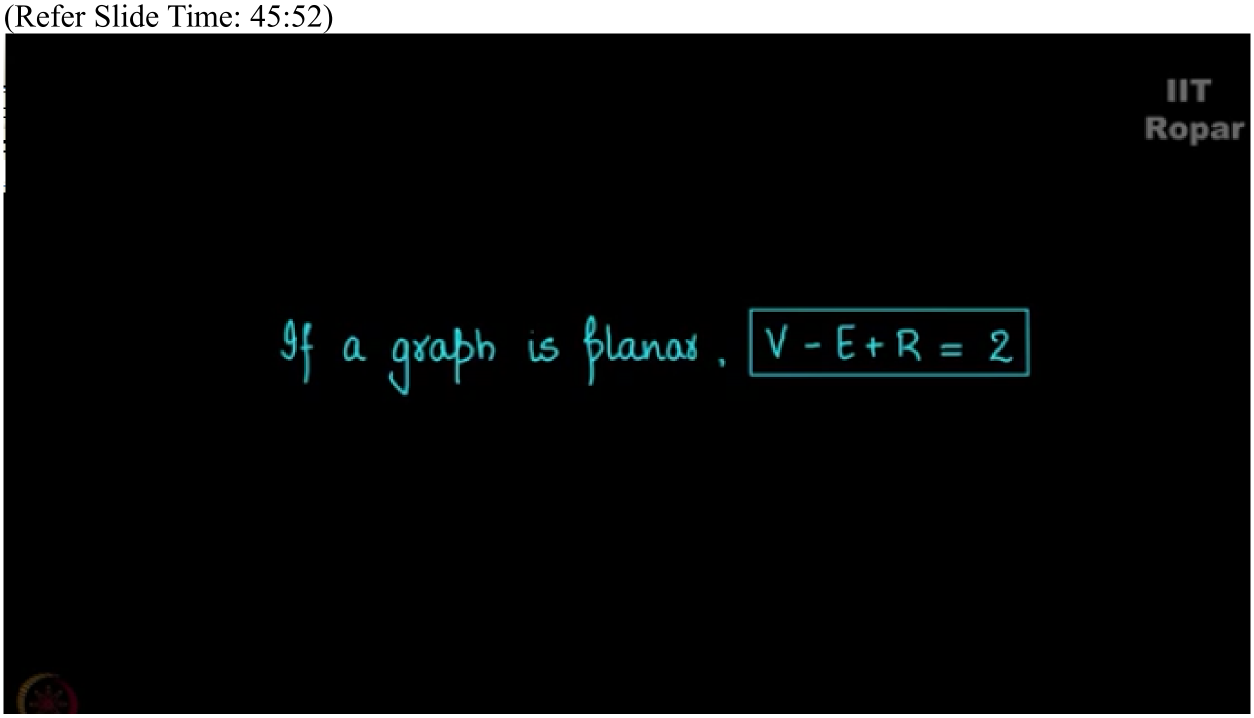

and what we can tell next is if a graph is planar V-E+R = 2, the famous Euler's formula, we also proved this using induction.

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The last topic that we covered was that of coloring, we studied what is proper coloring where we initially started with the question that if you take India's map, you take all the states, you put an edge, you put a vertex in every state, right, you connect all the neighboring states and you see if you can properly color it. What was the minimum number of colors that is required to color India's map? Properly that is, we did that and we saw that 4 colors are sufficient to color India's map.

So all the topics that we saw were Eulerian, Hamiltonian, coloring, planarity, disconnected graphs, connected graphs, we also saw star graph and we saw coloring or chromatic numbers for a few graphs, with this week we have also seen a chromatic polynomials, right, it is an extended version of coloring topic and so on.

After graph theory comes the advanced counting techniques, in this we have studied principle of inclusion and exclusion, generating functions and recurrence relations. I am now going to start with generating functions,
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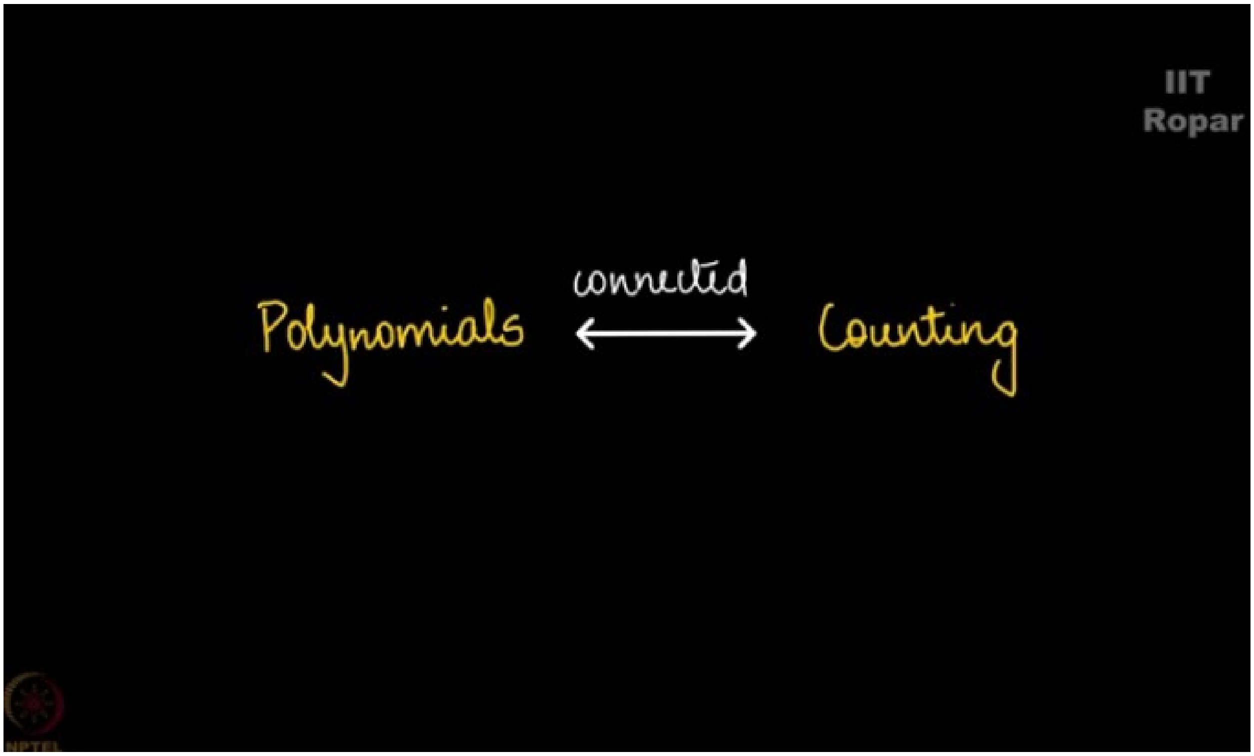

so what we saw was the connectedness between polynomials and a counting question, you are asked to count certain things and you frame a polynomial related to that, and then solve the problem.

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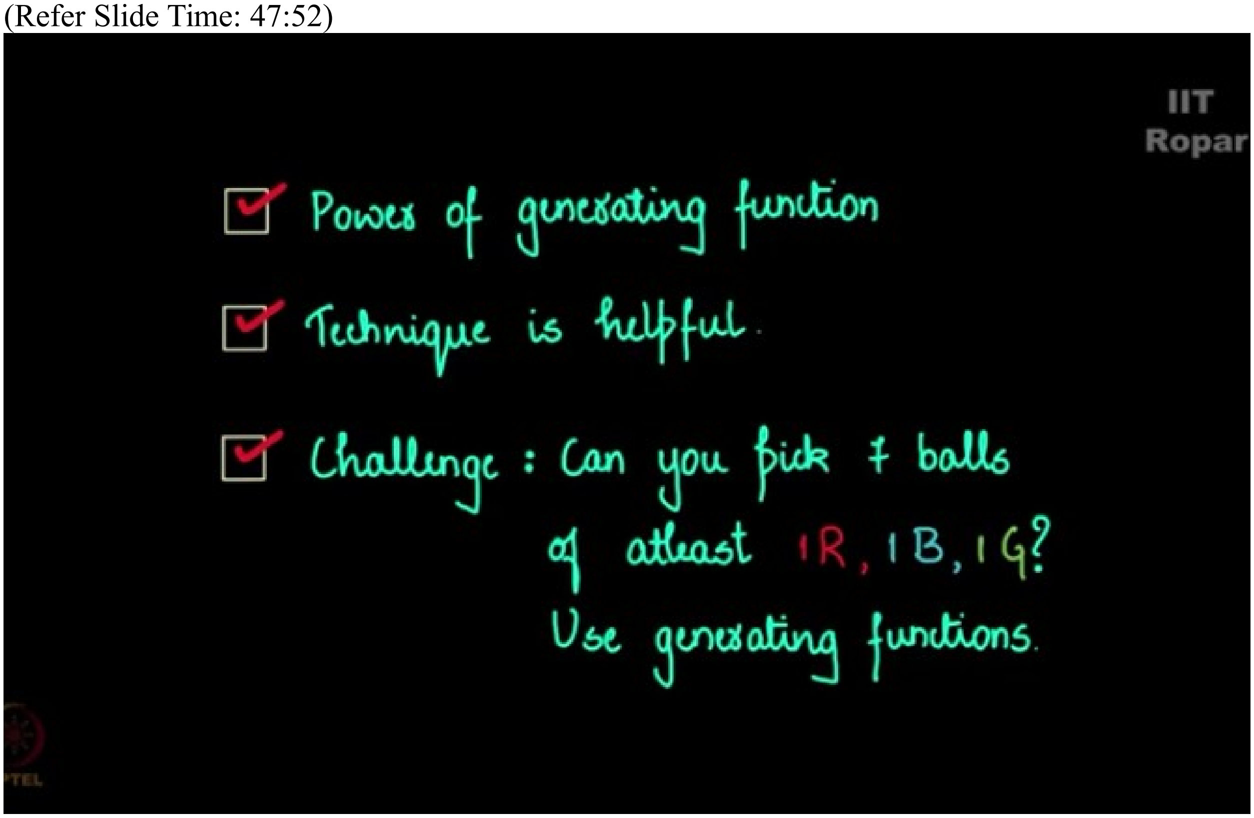

We have seen the power of generating functions and solving problems, we see that the technique is extremely useful. Now if I give you the question that can you pick seven balls of at least one red, one blue, one green color among the huge heap of balls, you can close your eyes, use generating functions and solve this problem. We have seen several problems of this zoner.

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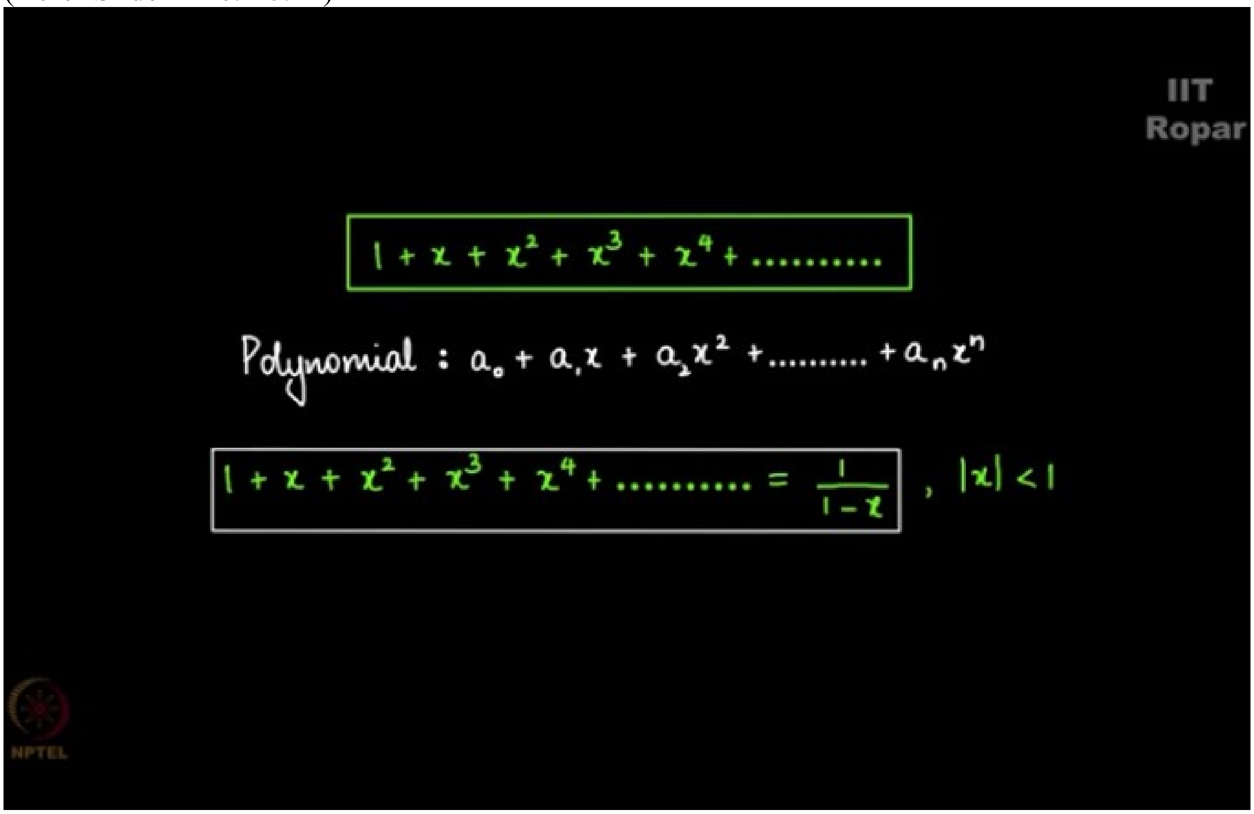

Now we started with framing this polynomial 1 + X + X square + X cube + so on, right, we initially started with the word example if you remember, they were certain conditions given that you can start framing the words with this letter, you have to follow with these vowels and end with these letters, and you were asked how many such words can you frame, right and there comes polynomial to use, you saw the creative link between this words creation question to polynomials, from there on started generating functions, so the polynomial here is A naught + A1X + A2X square so on + ANXN for the sequence A naught, A1, A2, so on up to AN, right.

Now 1+X+X square + X cube + so on it has the closed form 1/1-X, how? You can apply the formula of geometric series, right, this is the closed form of this generating function, we always asked for closed forms, why? Because it is very useful, it's very convenient for us.
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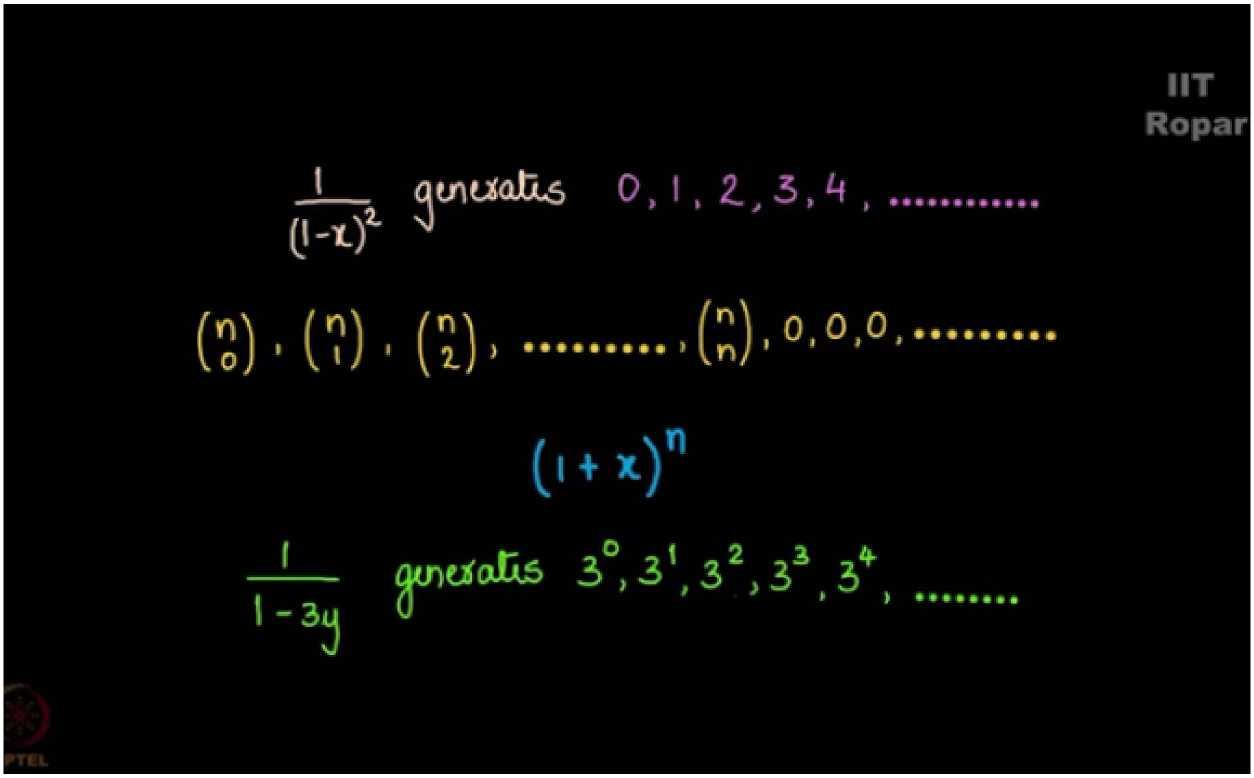

And we saw by differentiating the previous one that is 1 + X + X square so on if you differentiate that you will get this power generating function which has the closed form 1/1-X the whole square, it generates 0, 1, 2, 3, 4 so on this sequence.

Now the most interesting part is seeing binomial theorem here 1 + X to the N this is the polynomial, this is the generating function for the sequence N choose 0, N choose 1, N choose 2 so on, N choose N, 0, 0, 0 so on, this is the sequence. We also saw 1 – 1/1-AY this in general generates A to the 0, A, A square, A cube, A to the 4 so on, in particular 1/1-3Y generates 3 to the 0, 3 to the 1, 3 square, 3 cube, 3 to the 4 so on. We saw several other generating functions and the corresponding sequence which they generate.
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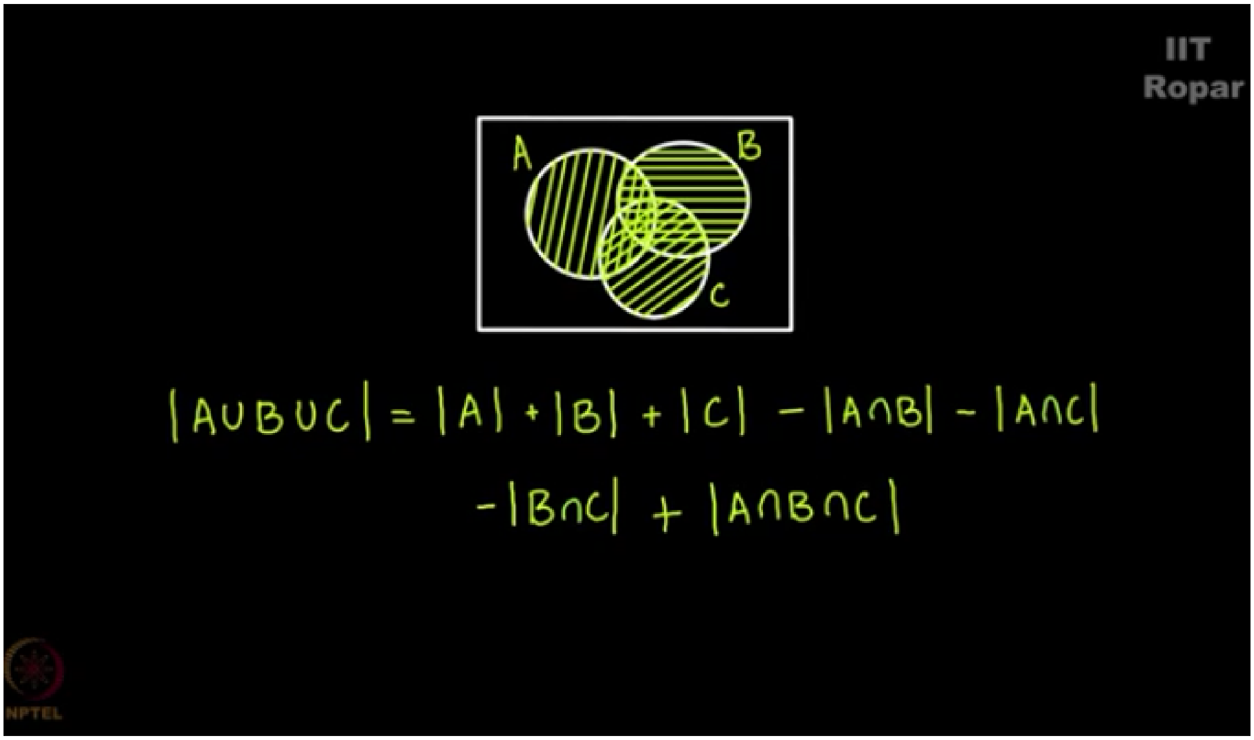

Now before I move ahead to principle of inclusion and exclusion, let me tell you what we have studied in generating functions this week in the advanced topics, we have seen partitions, right, we have seen how we can find out P(n) using generating functions, we have seen what is PD(n), what is PU(n), how they are equal, we have given a combinatorial proof for it, right, and we have seen what is the generating function for PD(n) and PU(n), we have also seen the exponential generating function and a problem on it.

Now moving ahead to principle of inclusion and exclusion, this is another advanced counting technique. You see this Venn diagram here consisting of three sets A, B, C, cardinality of A union B union C is cardinality of A + cardinality of B + cardinality of C – cardinality of A intersection B – cardinality of A intersection C – cardinality of B intersection C + cardinality of A intersection B intersection C, we know this, right.

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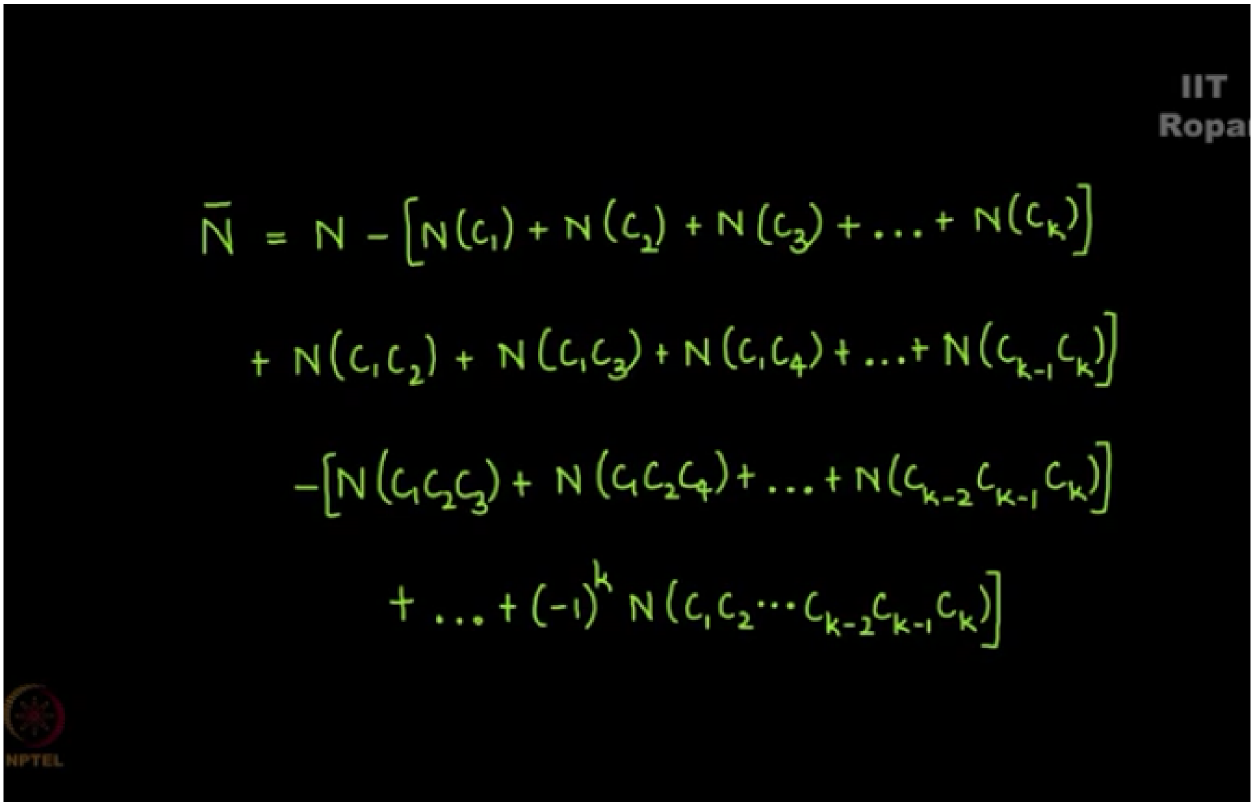

Now what we do is we are more interested in counting what we do not want, right, then what we want and that is when this formula creeps in, N bar is N – summation N(CI), right, I from 1 to K + summation N(CI, CJ) – summation of N(CI,CJ,CK) so on up to – 1 to the K summation N(CI,CJ,CK,CL) so on, right, you see this formula here you can use this to find out N bar.

N is the total number of elements in the set, right, N(C1) is those elements which satisfy condition 1 so on up to N(CK). N(C1,C2) it means those elements, the number of elements which satisfy both conditions C1 and C2, and it holds for other elements. Other elements I mean N(C3) the same thing what I told, and N(C1,C2,C3) what do we mean here? It means those elements which satisfy condition 1, 2, and 3 so on, the formula proceeds.

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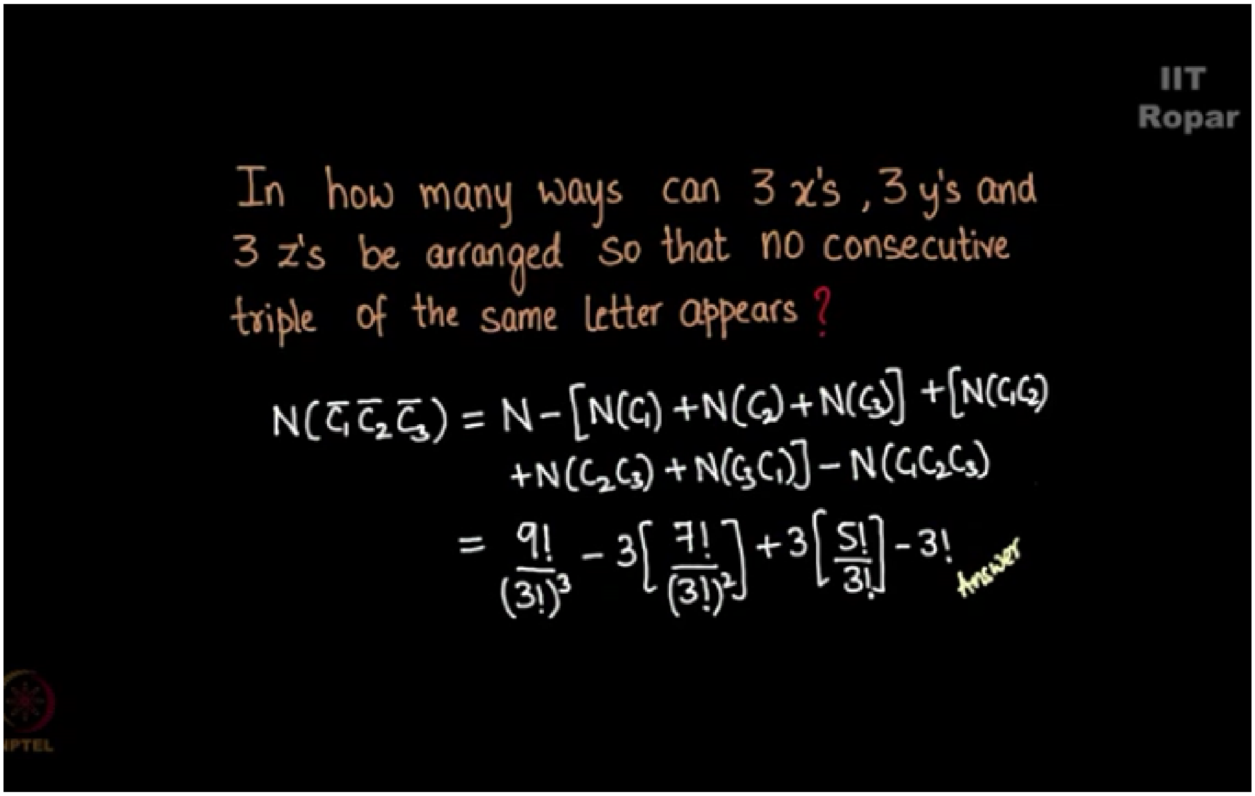

We've seen several such problems where I have asked, where we have seen that in how many ways can 3X's, 3Y's and 3Z's can be arranged so that no consecutive triple of the same letter appears, so we have to find out N(C1 bar, C2 bar, C3 bar) here, we have seen several such questions in the chapter P I E.

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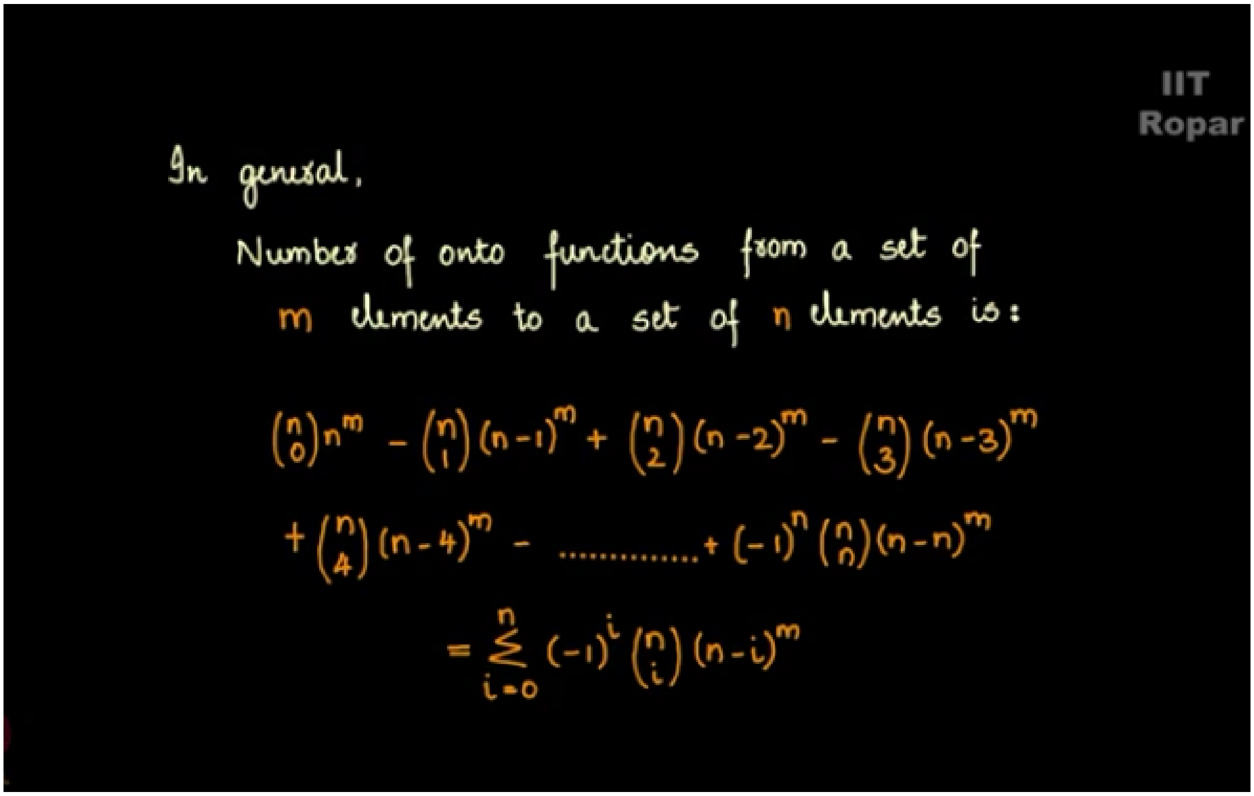

Now we also found out a general formula for the number of onto functions from a set of M elements to a set of N elements, right, it is summation I from 0 to N – 1 to the I, N choose I x N – I whole to the M, we use this formula to find the number of onto functions from a set of M elements to a set of N elements.

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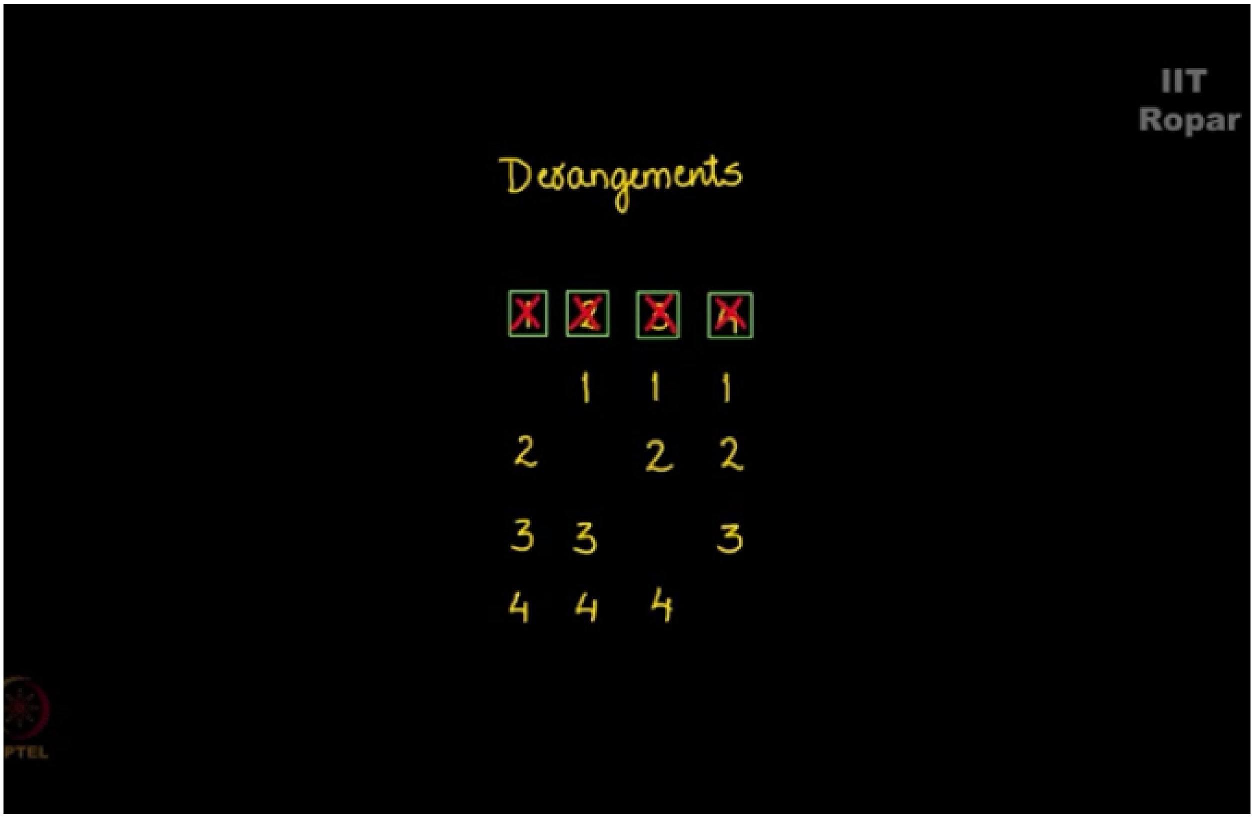

Next we saw something called derangements, what was that? I'll give you 4 numbers 1, 2, 3, 4, and I tell you that none of them should be in their original place, 1 should not be in one's place, 2 should not be in second place, 3 should not be in third place, and 4 should not be in fourth place, in how many ways can you do these arrangements then? This is called as derangements.

You see for four numbers one can occupy 2 3 4 this place, 2 can occupy 1 3 4 this place, 3 can occupy 1 2 4, and 4 can occupy 1 2 3. All these possibilities are there, so this goes by the name derangements,

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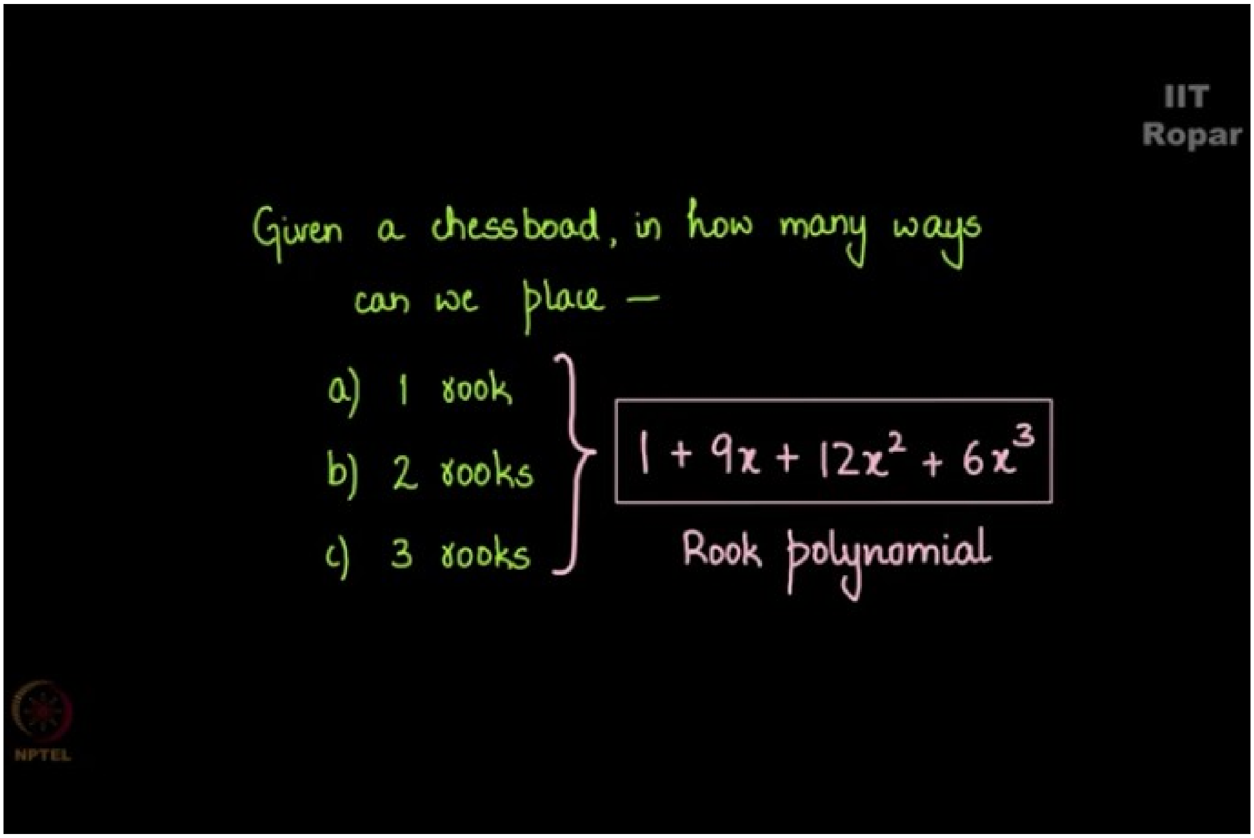

and we ended this chapter by studying something called the Rook polynomials, on the chessboard basically you have a pond called the rook.

In how many ways can you place a rook, one rook in 3 x 3 chessboard? It was 9 ways, and then we star a position in the chessboard and you ask that is you block it and you ask in how many ways can you place 2 rooks, right, in how many ways can you place 3 rooks? In a 3 x 3 chessboard you can place 3 rooks in 6 ways, 2 rooks in 12 ways, right, and hence the corresponding rook polynomial is 1 + 9X + 12X square + 6X cube, so this was with the rook polynomials.

So in inclusion and exclusion chapter we have seen that we do over counting sometimes, we do undercounting sometimes and hence we have to balance between the two, how do we do that? We have to add sometimes, we have to subtract sometimes, right, that depends on what we have to add and what we have to subtract, right, you can write down the Venn diagram and check, if you remember we had written a Venn diagram for probably around 5 sets, right, and we have seen several problems where you can use the formula.

Let me give you a tip here, if you solve many problems the topic you will really gain an insight yourself on the topic. The last chapter of the course is recurrence relation, it is a part of adverse counting techniques.

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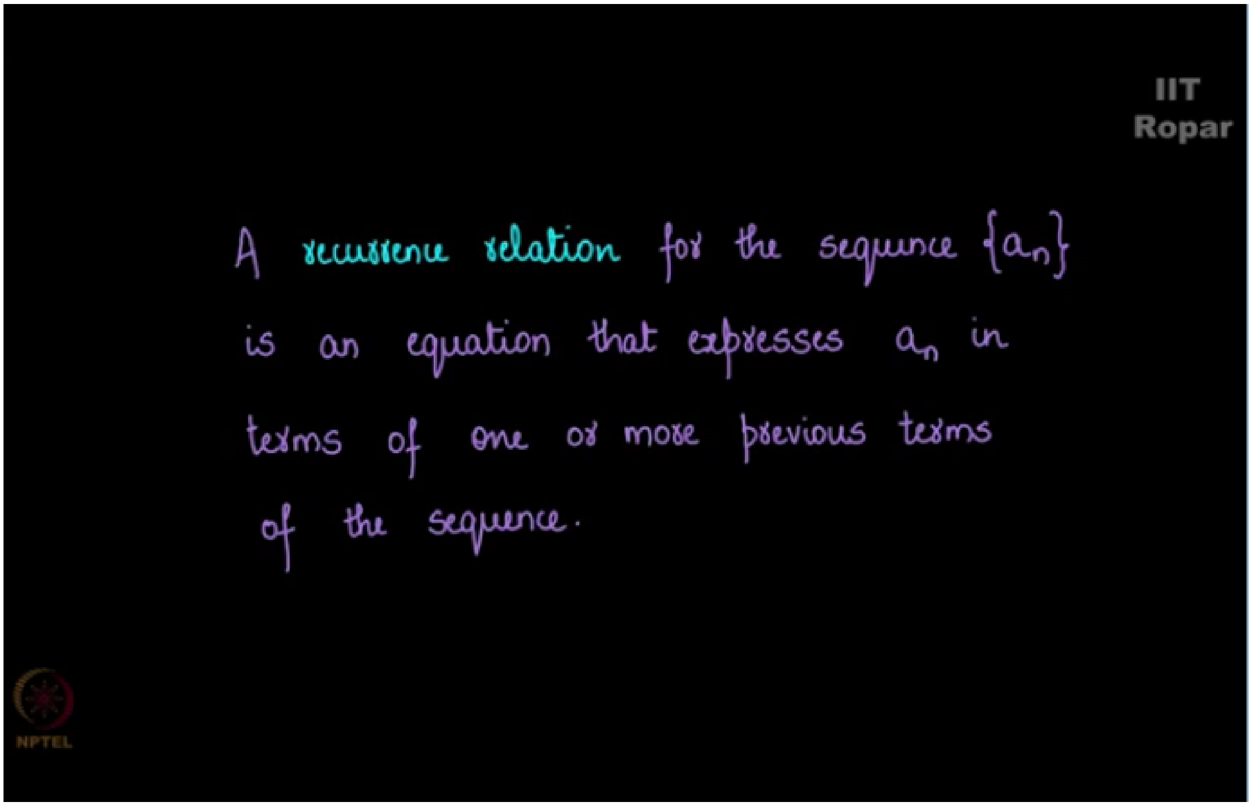

Let me start the summary with or rather the revisiting with the definition of recurrence relation, a recurrence relation for the sequence AN is an equation that expresses AN in terms of one or more previous terms of the sequence. So if there is a sequence, right, any term AN can be represented as in terms of the previous terms, then such a sequence is called as a relation for such a sequence is called a recurrence relation.

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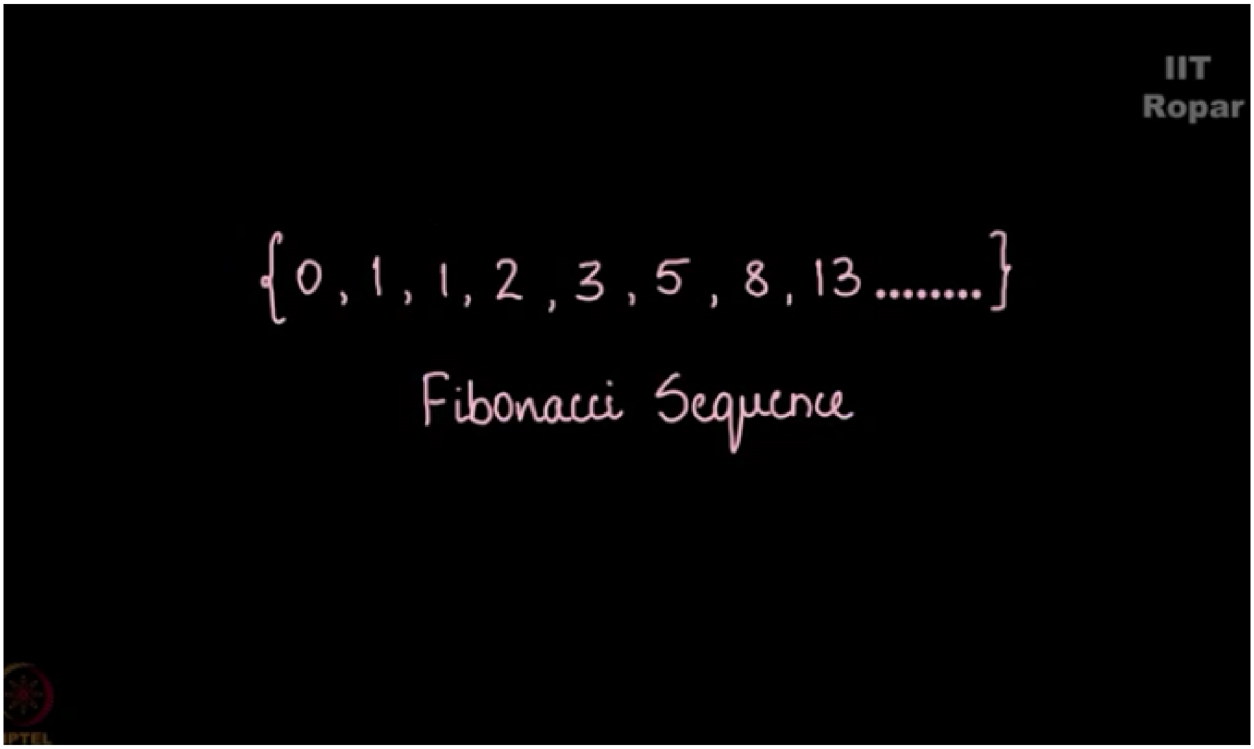

We saw that Fibonacci sequence is a recurrence relation, how? If I keep adding the previous two terms then I get the successive terms.

I start with 0 and 1 and I add them I get 1, I add 1 and 1 I get 2, 1 and 2 I get 3 and so on, it is an example of recurrence relation. Well, we have seen several other examples also.
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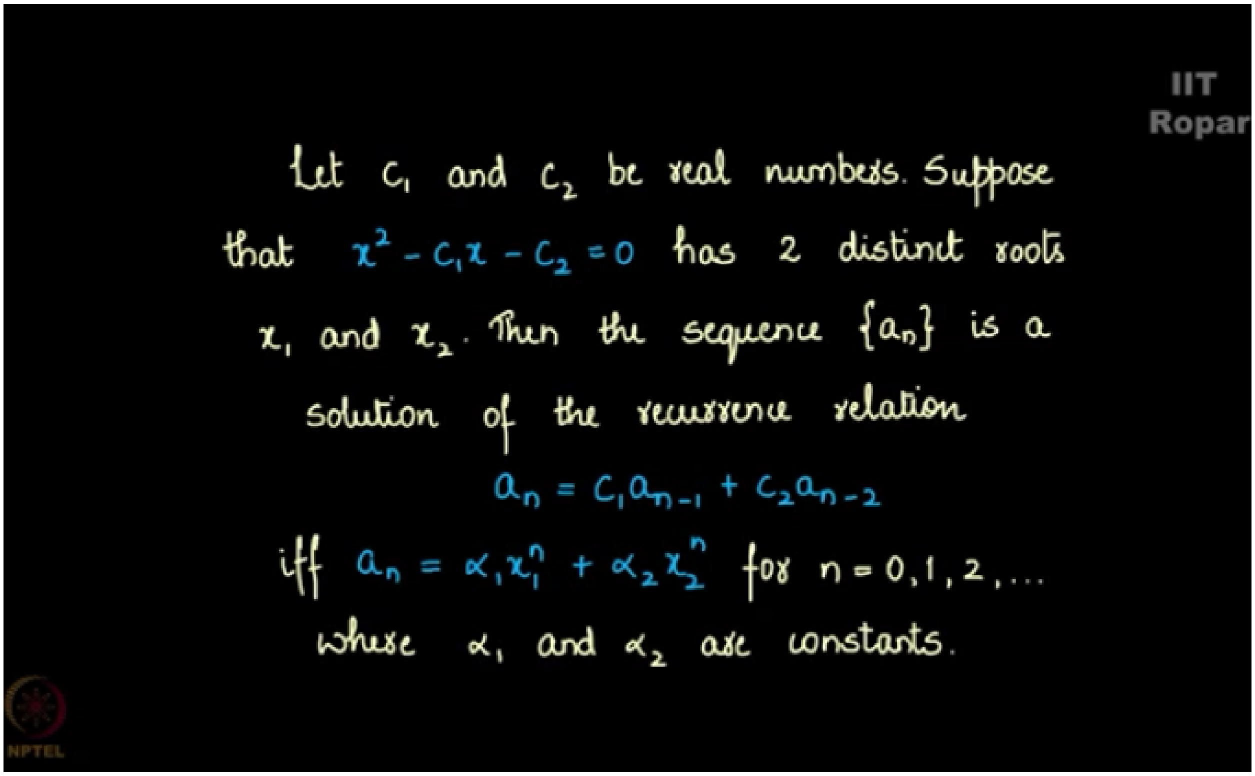

We saw a major theorem which is very important in this chapter, if C1 and C2 are real numbers then this quadratic equation X square – C1X –C2 has two distinct roots X1 and X2, right, the sequence AN is a solution of the recurrence relation, AN = C1, A1 – 1 + C2 AN-2 if and only if AN is written as alpha 1, X1 to the N + alpha 2 X2 to the N for N = 0, 1, 2, so on, what are these alpha ones and alpha twos? They are constants.

So if this equation quadratic equation has two distinct roots then the sequence is a solution of the recurrence relation given like this, C1 and C2 are what? They are these terms here as you can see in the quadratic equation. Well, we actually did not prove the theorem rigorously, but we have also seen how to prove it, right, some tips we have seen. We have also solved problems based on this theorem, so how did the proof go like?
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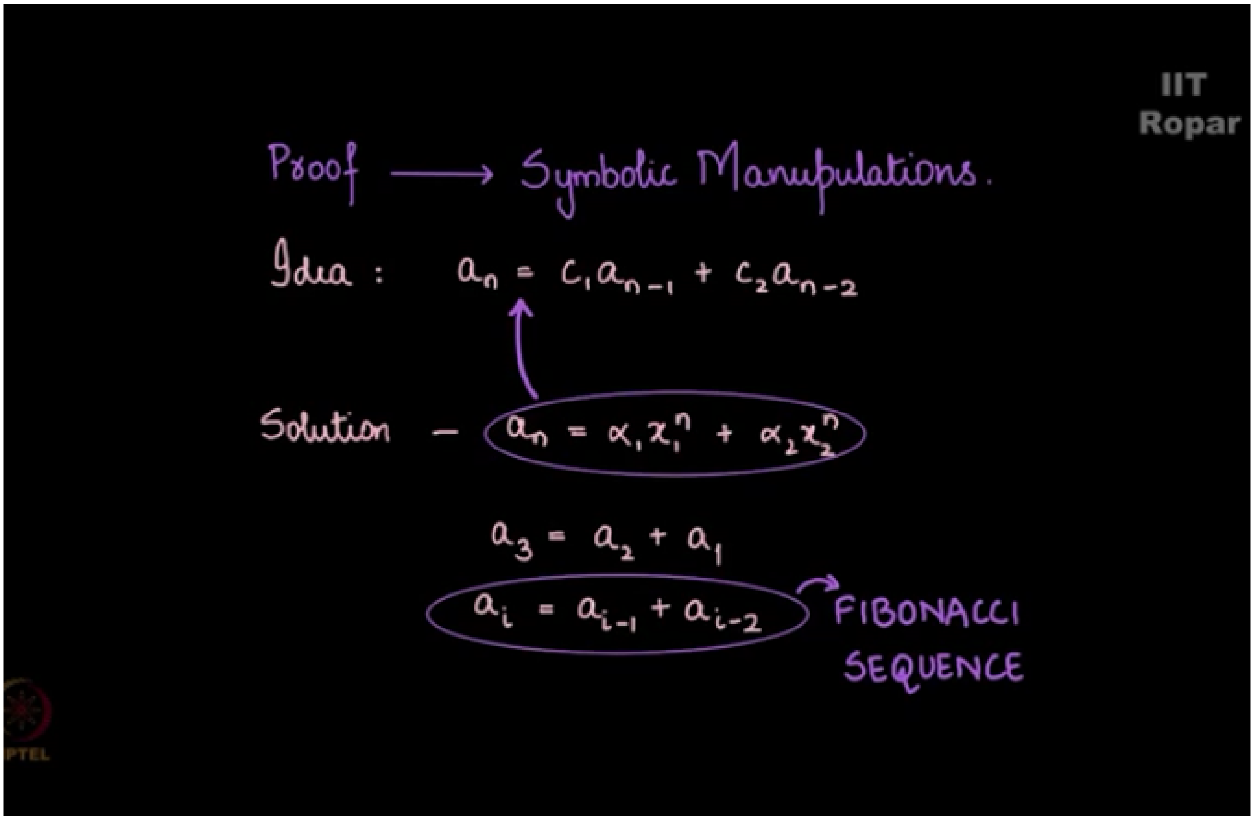

We just have to do a few symbolic manipulations here, we will be given a recurrence relation AN = C1, AN-1 + C2 AN-2, if this is not given the corresponding quadratic equation will definitely be given, you can frame this equation and then find out the solution for this, how? AN = alpha 1 X1 to the N + alpha 2 X2 to the N, X1 and X2 are the roots of the quadratic equation.

How do you find the roots? Using this formula X = -B, + or – root of B square – 4AC/2A you will get the roots X1 and X2, and hence you can substitute here and find out alpha 1 and alpha 2, right you will get simultaneous equations, we have seen a lot of this.

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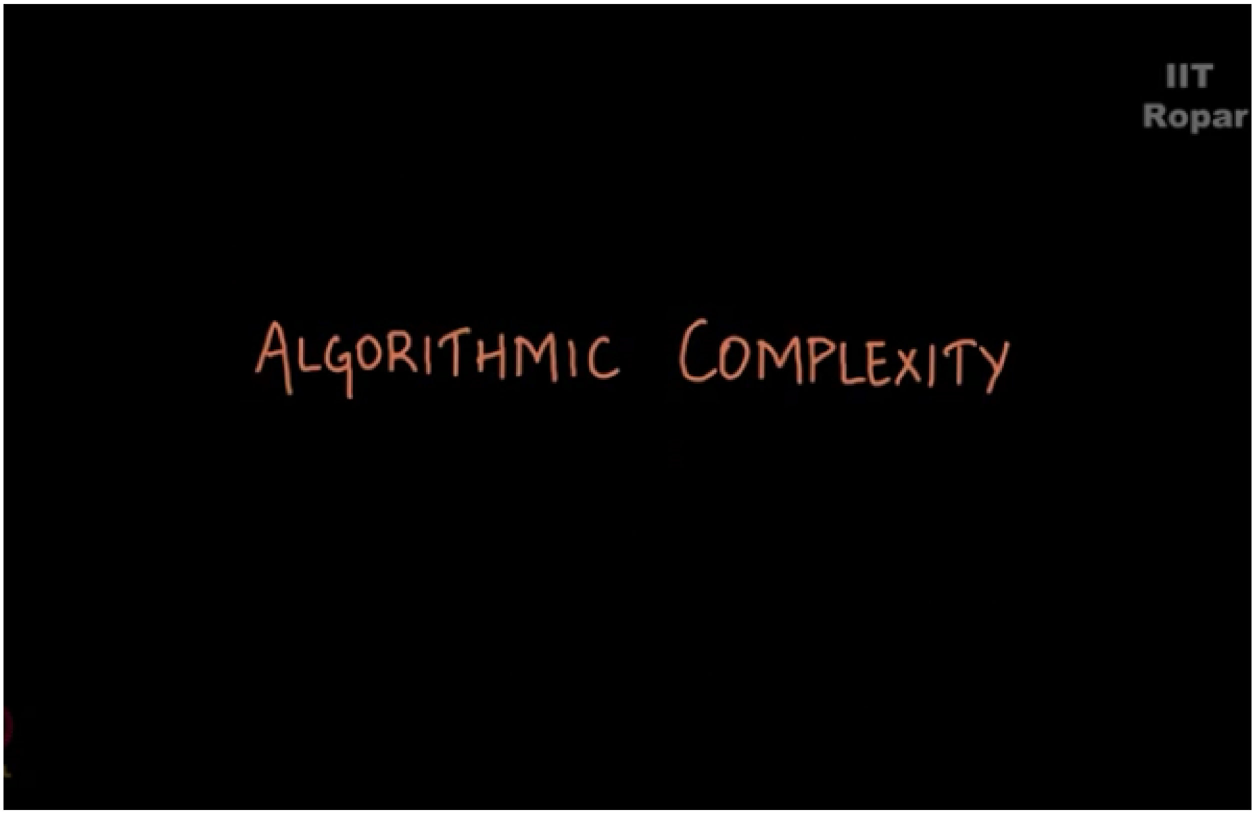

Now we have also studied something called the algorithmic complexity, right, the order or it is also called as the order, what does it mean? You can find out how complex your algorithm is or how long it will take for the process to finish, we have seen this.

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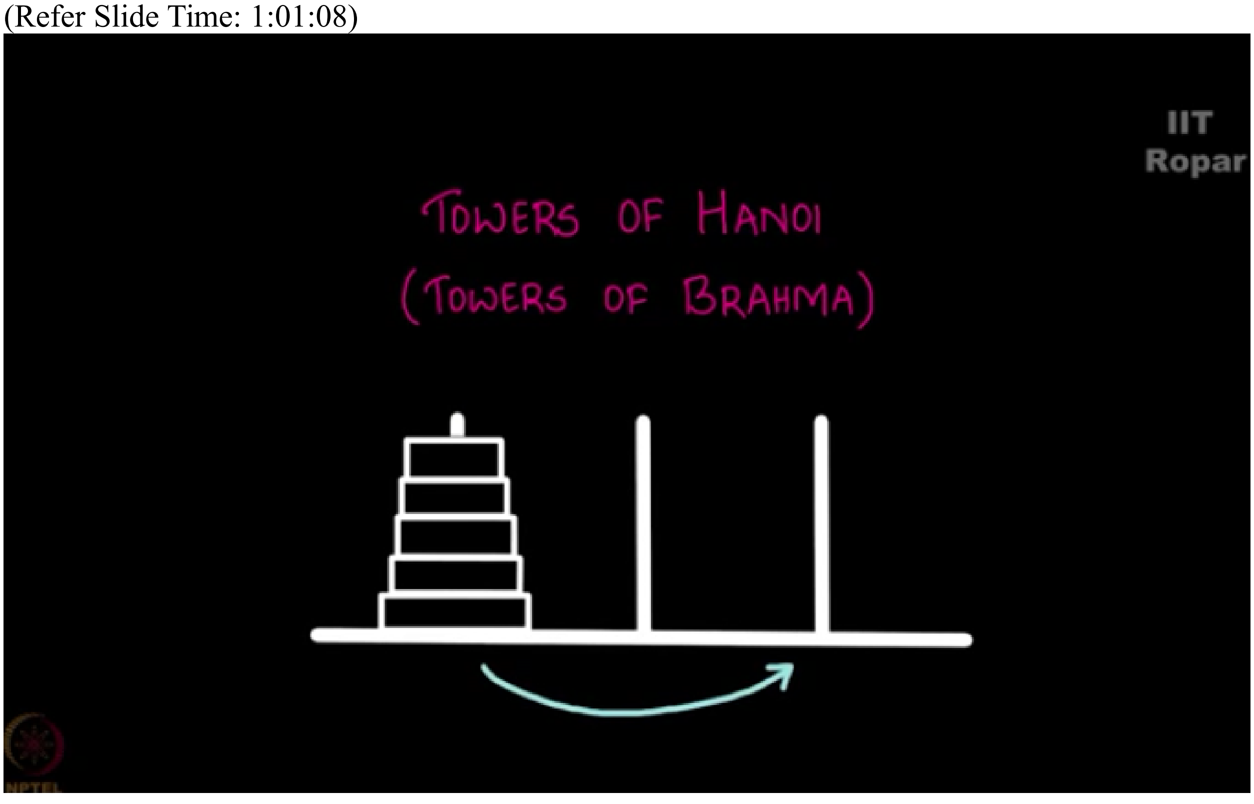

We saw something called the Towers of Hanoi also called the Towers of Brahma, you see these sticks here and these discs, you must basically move them to the last stick, and there are some rules to be followed, what are those rules? You cannot place a smaller disc and on top of it a bigger disc, this is not allowed, whereas you can place a bigger disc and on top of it you can place a smaller disc, so you have to move all of these discs to the last one, right, that is the challenge in this question of Towers of Hanoi.

Basically you will see a recurrence relation hidden here, we have seen that, we have solved it also.
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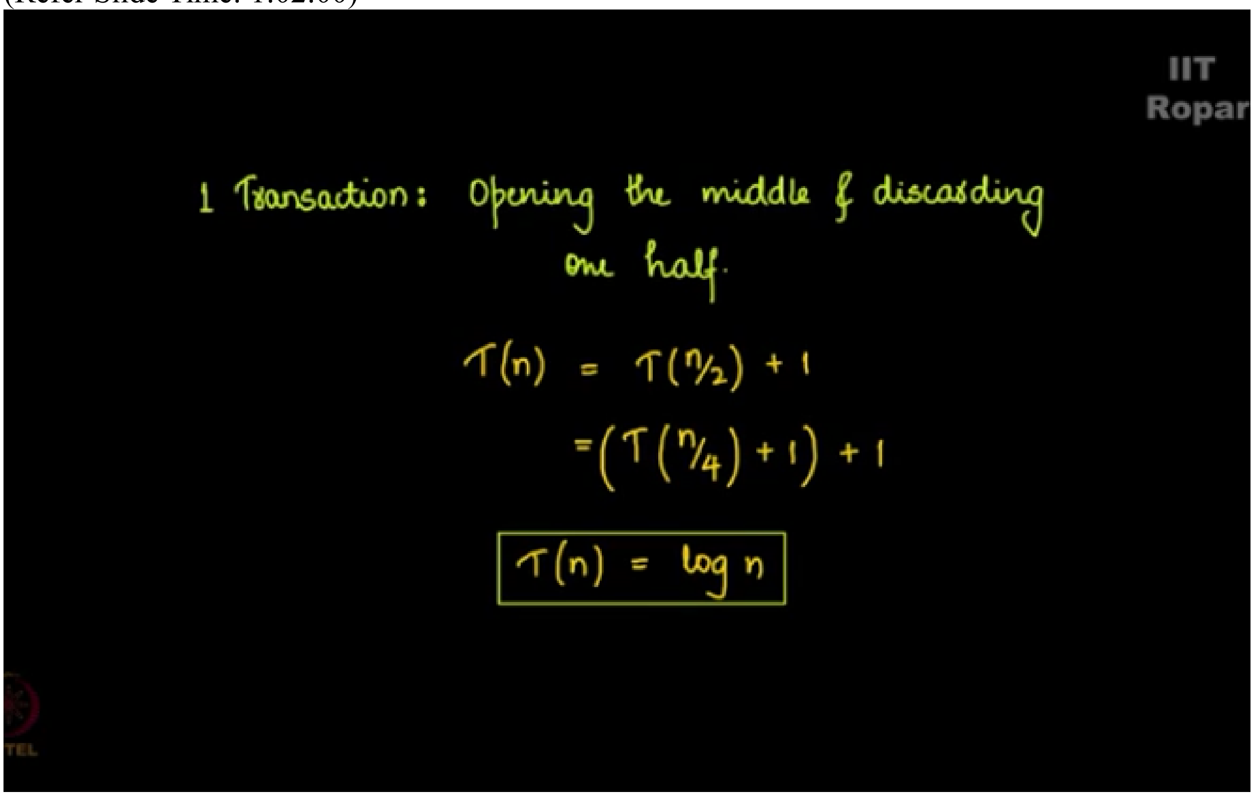

And then we moved on to something called the binary search, how did we do that? There was, you saw the example where there was a girl and she was finding out a word in the dictionary, she randomly opens, she randomly chooses a word and she wants to find that in the dictionary, she opens the middle of the dictionary and she sees where the word lies, she discards one half, right, and then she discards that half based on what actually or letter is, the starting letter of the word, whether you have to choose the right one or the left one.

Now based on what you have retained you will again go to the center, discard one half of it and keep one half of it again depending on what letter is the word containing in the beginning, right. Now you will continue this process and yes you will end up finding the letter in the end. Now this algorithm has the recurrence relation T(n) = log N, it is called the binary search.
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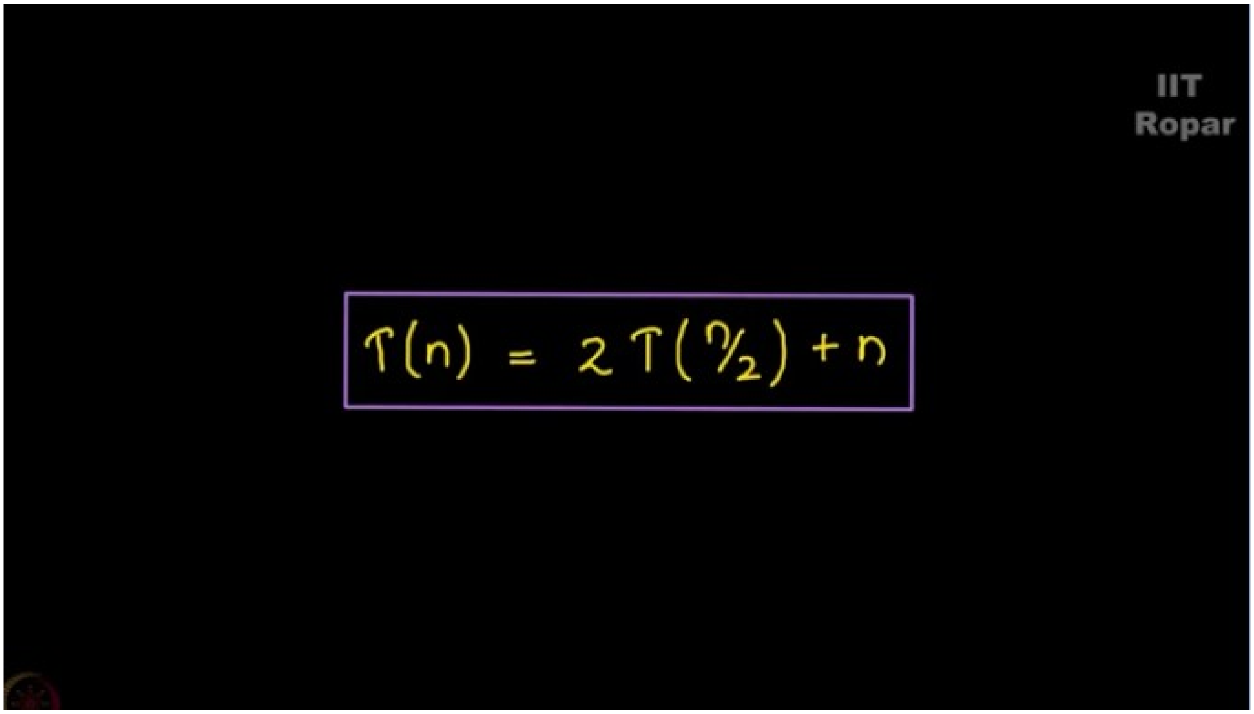

And we also saw something called the merge sort, a teacher had to sort her answer scripts, right, she had some answer scripts and she had to sort it based on their marks. Now she divides it into two half gives it to her two children and ask them to sort it, well a child A has sorted it and student B is also sorted it, now they gave it back to her, we see that the entire sheet of, entire pile of answer scripts is not actually sorted, we have to sort it, right, and here comes the merge sort, the relation for it is T(n) = 2 x T(n)/2 + N, so it's two times T(n/2) + N.

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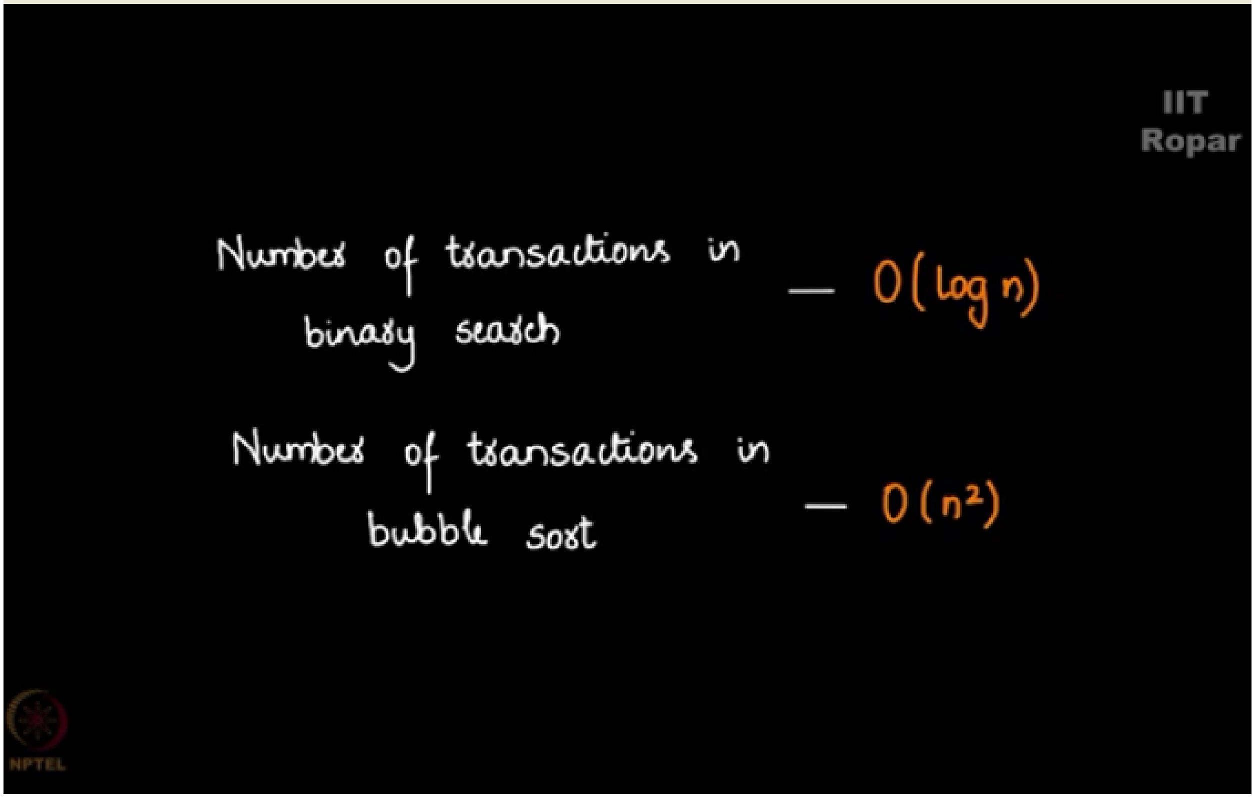

Now we saw that the number of transactions in binary search is actually order of log N and the number of transactions in bubble sort is order N square. So this was with the recurrence relations, we have seen several problems here and we have seen some settle concepts you might face difficulty initially, but do not worry things will be clear if you watch the videos once or twice more. So this was the entire course in a small video, in a small chunk, about one hour of this video gives you the entire glimpse of this course, but please note we might not have covered all the topics in this entire chunk of one video, but we have seen to it that we cover the most important ones. This course has been very colorful in terms of the content, the animations, and examples and analogies that we have given.

Now in front of you are several colors, choose the right colors and you have a canvas, choose the right canvas and the right colors and move ahead to paint a beautiful portrait.

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