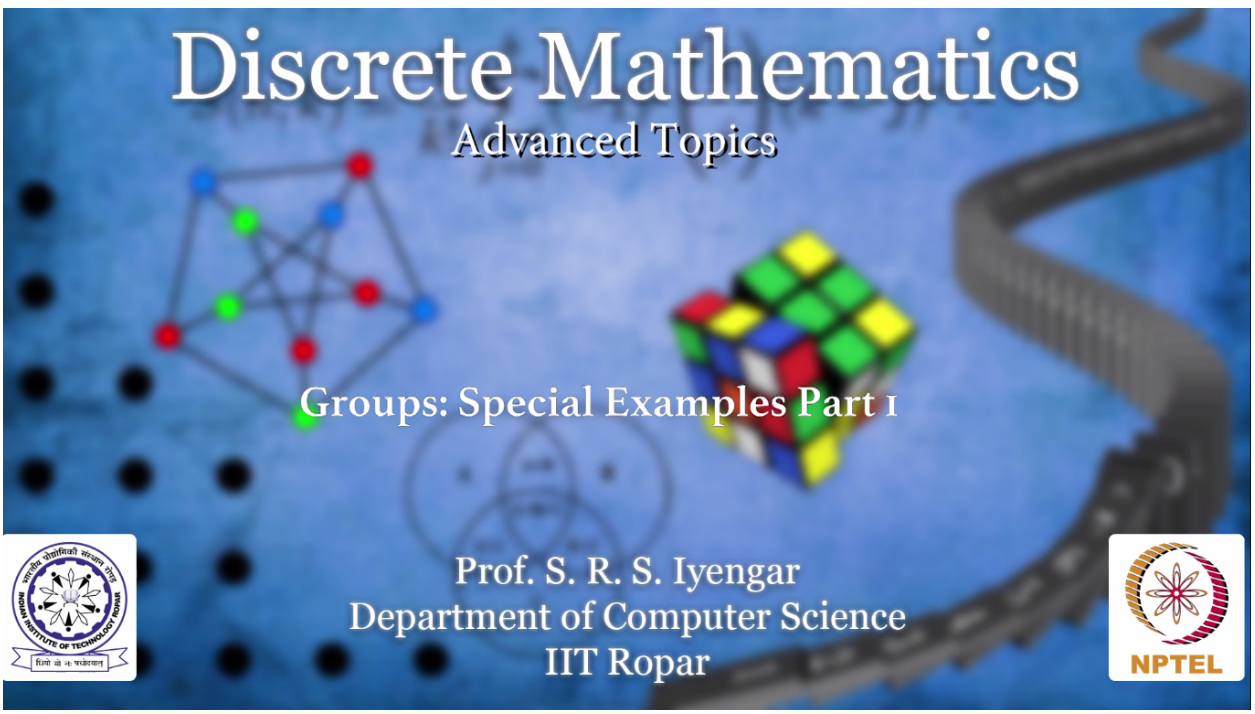
Discrete Mathematics

Functions

Advanced Topics



Groups Special Examples Part 1

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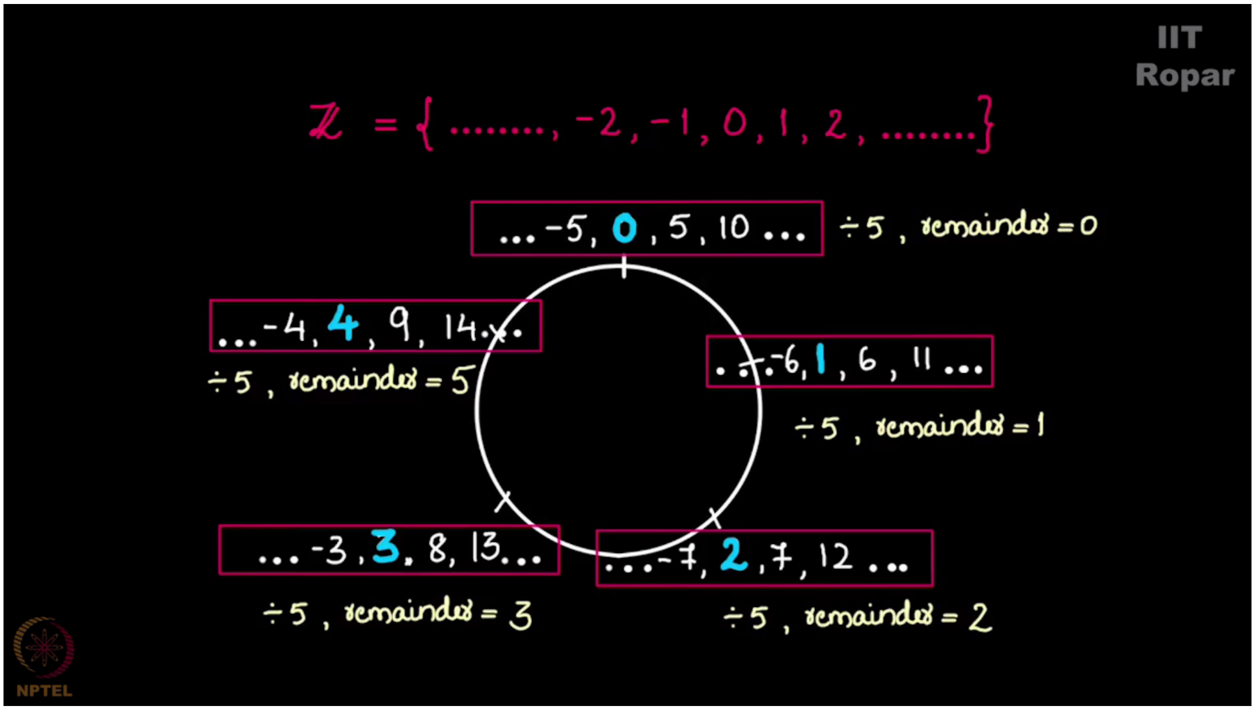
We have seen that integers have these elements; 0, 1, 2 so on in the positive side and at the negative side minus 1, minus 2 so on. This is quite familiar to all of us. Now let me show you something. Consider the circle just go on observing what I am going to write now. You might have several questions in your mind all of them will be clear once you watch the entire video.

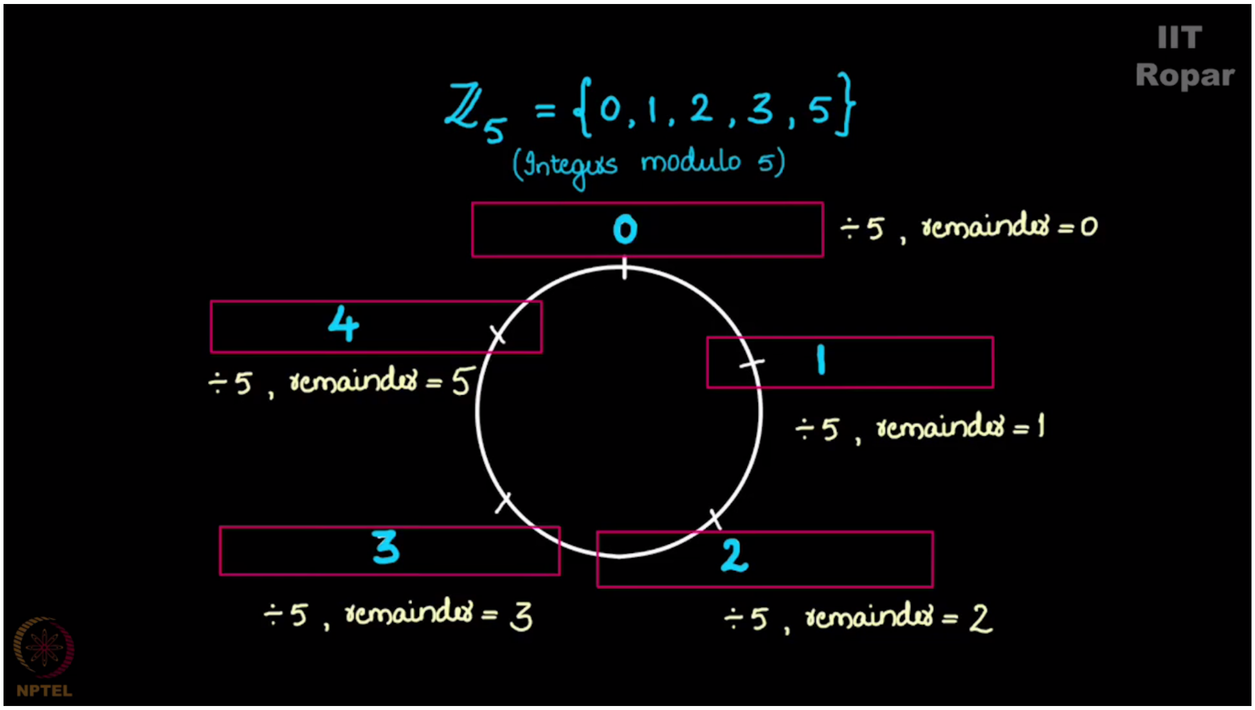
Watch what happens now. I am going to write these numbers like this 0, 1, 2, 3, 4 you must be seeing that 3 and 4 are quite away there is a reason why it is written like that. You will be observing what happens. Now I am going to write the next number like this 5, 6, 7, 8, and 9 and the next round will be 10, 11, 12, 13, and 14 like this. Now not only the positive integers I'm going to write some negative ones also like this minus 5, minus 6, minus 7, minus 3, minus 4. And now I am going to continue like this. I can put dots which means that this will go on extending in the positive side and the negative side as well. Now observe these integers here at the circle at every point. Take a minute's pause and observe carefully. What is common here, here, here, here and here? Please take a minute's pause to observe. You must be able to find out the answer yourself.

Now observe something. If you have already figured it out well and good. If not you will get the answer now. What happens in the first box 0, 5, 10, minus 5 so on. What happens here? When I divide this by 5 what is the remainder? It is 0. Now if I divide 0 by 5 remainder is 0. If I divide 5 by 5 remainder is 0; all these integers if I divide by 5 remainder is 0. What I am going to do is I'm going to highlight 0 here. Next step if I divide 1, 6, 11, minus 6 this one by 5 the remainder happens to be 1. right 6/5 remainder is 1. 11 also it is 1. Now the hero here will be this number 1. now when I divide – 7, 2, 7, 12 this sequence by 5 the remainder is 2. So I'm going to highlight 2 here. Now I'm going to do the same thing for 3, 8, 13 I'm going to divide it by 5 and I see that remainder is 3. 13 divided by 3 gives me a remainder 3 and so on.

Now if I do – so I am going to highlight 3 here. Now when I divide – 4, 4, 9, 14, by 5 what happens? The remainder is going to be 5. 9 divided by I'm sorry the remainder happens to be 4 there. 9/5 happens the remainder is 4 and 14 divided by 5 the remainder is 4 so on. Now do you observe that in the first box remainder is 0. Second box remainder is 1. third one 2. Fourth one 3 and the fifth one is 4. 0, 1, 2, 3, 4. Well let me highlight 4 here. So all the highlighted ones represent the remainders of their respective boxes.

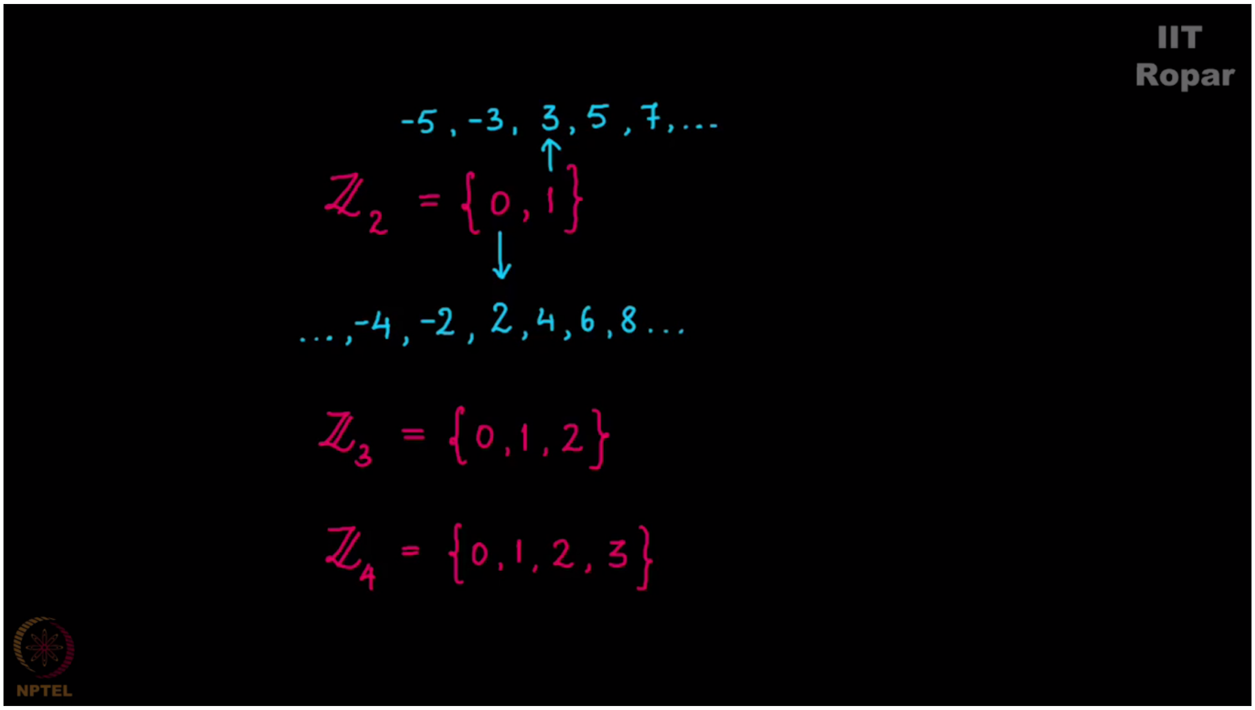


Let me take only them here. So 0, 1, 2, 3, 4 we have them when divided by 5 gives me the remainders as you can see the highlighted ones themselves. This represents a very very important group in group theory. It is called as Z5 and the elements of Z5 are 0, 1, 2, 3 and 4. There is 5 over here but it is actually 4. Now this is called as integers modulo 5. Modulo? you must be knowing that we have studied in our lower classes modulo represents division and what are these elements 0, 1, 2, 3, 4 they represent your remainders when divided by 5. So this is a special group. It is an example of a group. It forms a group. Let me say in the next video probably you will be seeing how it is a group. Now for now you must learn what is integers modulo 5.



For simplicity let me show you what is Z2. It is 0, 1. Why? Here 0 constitutes 2, 4, 6, 8 also minus 2, minus 4, so on. When divided by 2 it will give you the remainder 0. Now this one constitutes 3, 5, 7 so on. Minus 3, minus 5, so on. The 0 and 1 what you can see here they represent the remainders and Z3 is 0, 1, 2. So whatever number you take in your integers let's say 23 when divided by 3 gives the remainder 2 and hence 2 is there in Z3. So your entire set of integers when divided by 3 gives you these numbers 0, 1, 2 as the remainders.

What is it with Z4? It is 0, 1, 2, 3 because any integer when divided by 4 will not give you any other remainder other than these four integers. It is 0, 1, 2, 3. One of these numbers will be the remainder when you divide the entire integers by 4. If you divide 10 by 4 let me say the remainder is 2. If you divide let's see 47 by 4 remainder is 3. So on. So Zn in general is called the integers modulo n. You are dividing by n and the elements in this set will be all those elements which are the remainders. So you might want to pause the video, try out a few more examples and it will be clear to you.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-5 , -3 , 3 , 5 , 7 , ...
ℤ2 = {0 , 1}
... , -4 , -2 , 2 , 4 , 6 , 8 ...
ℤ3 = {0 , 1 , 2}
ℤ4 = {0 , 1 , 2 , 3}
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